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MEMORANDUM

HEAT TRANSFER IN A LIQUID METAL FLOWING TURBULENTLY THROUGH A CHANNEL WITH A STEP FUNCTION BOUNDARY TEMPERATURE

By H. F. Poppendiek

CONVAIR, A Division of General Dynamics Corporation
San Diego, Calif.

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ABSTRACT

An analytical heat transfer solution is derived and evaluated for the general case of a turbulently flowing liquid metal which suddenly encounters a step-function boundary temperature in a channel system. Local Nusselt moduli, dimensionless mixed-mean fluid temperatures, and arithmetic-mean Nusselt moduli are given as functions of Reynolds and Prandtl moduli and a dimensionless axial-distance modulus. These solutions are compared with known solutions of more specific systems as well as with a set of experimental liquid-metal heat transfer data for a thermal entrance region.

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NOMENCLATURE

SYMBOLS

a,	thermal molecular diffusivity, $k/\gamma c_p$, ft^2/hr
A_n,	series coefficients in equation (14) defined by equation (15), dimensionless
b,	half the channel breadth (see Figure 1), ft
b_n,	series coefficients in equations (11) and (13), dimensionless
c,	constant in equation (37)
C_1,	constant in equation (30), dimensionless
c_1, c_2,	constants in the general solution, equation (9), dimensionless
c_p,	heat capacity, Btu/lb°F
$f(r)$,	a function of the variable r, dimensionless
h_{am} ,	arithmetic mean unit thermal conductance or heat transfer coefficient, $\frac{q}{A}/\Delta t_{am}$, Btu/hr ft^2°F
h_x,	local unit thermal conductance or heat transfer coefficient, $\left(\frac{q}{A}\right)_x/(t_w - t_m)$, Btu/hr ft^2°F
J_o, Y_o,	zero order Bessel functions of the first and second kind, respectively
J'_o, Y'_o,	derivatives of the zero order Bessel functions of the first and second kind, respectively, with respect to the argument
k,	thermal conductivity, Btu/hr ft^2 (°F/ft)
p,	constant in equation (37)
$\left(\frac{q}{A}\right)_x$,	local heat flux, Btu/hr ft^2
S,	a function of x and y given by equation (37), dimensionless
t,	liquid-metal temperature at some point x, y, °F
t_m,	mixed-mean fluid temperature, °F

CONTENTS

SYMBOLS (Continued)

t_w,	uniform wall temperature (step function), °F
t_o,	initial liquid-metal temperature (see Figure 1), °F
u,	liquid-metal velocity profile (function of y), ft/hr
U,	mean or uniform liquid-metal velocity, ft/hr
U_o,	sum of J_o and Y_o functions in equation (11)
w,	independent variable defined in equation (39), dimensionless
x,y,	Cartesian coordinates, ft
Δt_{am},	arithmetic-mean temperature difference, $\frac{(t_w - t_o) + (t_w - t_m)}{2}$, °F
ϵ,	eddy diffusivity, ft^2/hr
ν,	kinematic viscosity, ft^2/hr
β,	parameter arising in the separation-of-variables technique, dimensionless
β_n,	eigenvalues of equation 12, dimensionless
γ,	weight density, lb/ft^3
Γ,	gamma function

DIMENSIONLESS MODULI

$$F_o = \frac{4}{Pe}$$

$$F_1 = \frac{4C_1}{Re^{0.1}}$$

$$Nu_X = \frac{h_X 4b}{k}$$, local Nusselt modulus

$$Nu_{am} = \frac{h_{am} 4b}{k}$$, arithmetic-mean Nusselt modulus

$Pe = Re \cdot Pr$, Peclet modulus

$$Pr = \frac{\gamma c_p \nu}{k}$$, Prandtl modulus

DIMENSIONLESS MODULI (Continued)

$$Re = \frac{U\,4b}{\nu}$$, channel Reynolds modulus

$$r = \sqrt{F_o + F_1 Y}$$

$$T = \frac{t - t_w}{t_o - t_w}$$

$$T_m = \int_0^1 T dY$$

$$X = \frac{x}{b}$$

$$Y = \frac{y}{b}$$

INTRODUCTION

It is well known that the convective heat transfer in the entrance regions of duct systems where thermal and hydrodynamic boundary layers are not yet established can be far superior to heat transfer in the established flow regions. A quantitative understanding of this type of heat transfer, sometimes called entrance region heat transfer, is essential when designing high heat-flux cooling systems for rocket motors, nuclear reactors, exhaust nozzles, and missile nose cones. Because the liquid metals are the most effective high-temperature coolants known, they are considered exclusively in this report. Further, in many practical flow systems the hydrodynamic boundary layers have been completely or almost completely established before the thermal entrance region is encountered. Therefore, the work presented deals with heat transfer in the thermal entrance region with an established velocity field.

A thermal entrance region results when a thermally established fluid, flowing in a duct of uniform cross section, suddenly encounters duct surface with some new boundary temperature distribution. Under these circumstances the temperature field in the fluid is no longer established and thus greatly influences the local convective heat transfer. As an example, for a step function boundary temperature entrance region the heat transfer can be very high because the thin thermal boundary layers have low thermal resistances.

A limited number of turbulent flow entrance region solutions for ducts are available in the literature. Sanders (1) has obtained a turbulent flow solution for a step function entrance region in a pipe by transforming the turbulent core to a laminar core of equivalent thermal resistance. Seban and Shimazaki (2) have obtained some specific numerical solutions for the uniform wall-heat-flux entrance region in a pipe. Elser (3) obtained a simplified solution for only the initial portion of the thermal entrance region of a pipe containing a turbulent fluid and a step function wall temperature distribution. Several mathematical analyses for forced convection heat transfer in thermal entrance regions for low Prandtl modulus (liquid metal) systems were presented previously (4); three dealt with low, turbulent Reynolds moduli (radial heat flow by conduction only) and two others dealt with all turbulent Reynolds moduli (radial heat flow by eddy transfer as well as conduction). The solutions for the latter two problems had been completed but not evaluated. One solution has since been evaluated and the results are presented. The solution describes the general case of turbulently flowing liquid metal which suddenly encounters a step function boundary temperature in a channel system. The derivation of the mathematical solution is presented. Local Nusselt moduli, dimensionless mixed-mean fluid temperatures, and arithmetic-mean Nusselt moduli are given as functions of Reynolds and Prandtl moduli and a dimensionless axial-distance modulus. The solution is shown to reduce correctly to known specific solutions of the general case. Also the solution is compared with a set of experimental liquid-metal heat transfer data previously obtained in a thermal entrance region.

ANALYSIS

The idealized system which defines heat transfer in a liquid metal flowing turbulently through a channel (between two parallel plates of infinite extent) with a step function boundary temperature is based on the following postulates:

1) The wall temperature distribution is a simple step function, $t = t_o$ for $x < 0$ and $t = t_w$ for $x > 0$. The fluid approaching the entrance region has an established temperature, t_o (see Figure 1).

2) Longitudinal heat conduction is small compared to convection and is neglected (see Appendix 1).

3) The established turbulent velocity profile is represented by a uniform distribution $u = U$, (see Appendix 2).

4) The eddy diffusivity distribution varies linearly with distance from the wall and as the nine tenths power of the Reynolds modulus, $\frac{\epsilon}{\nu} = C_1 \, Re^{0.9} \, \frac{y}{b}$, (see Appendix 3).

5) The fluid properties are invariant with temperature.

6) Steady state exists.

The differential equation describing the convective heat transfer in dimensionless form is (see Appendix 3)

$$\frac{\partial T}{\partial X} = \frac{\partial}{\partial Y}\left[\left(\frac{4}{Pe} + \frac{4C_1}{Re^{0.1}} \, Y\right) \frac{\partial T}{\partial Y}\right] \tag{1}$$

This equation can be expressed in a simpler form by making the change of variable,

$$r = \sqrt{\frac{4}{Pe} + \frac{4C_1}{Re^{0.1}} \, Y} = \sqrt{F_o + F_1 Y} \tag{2}$$

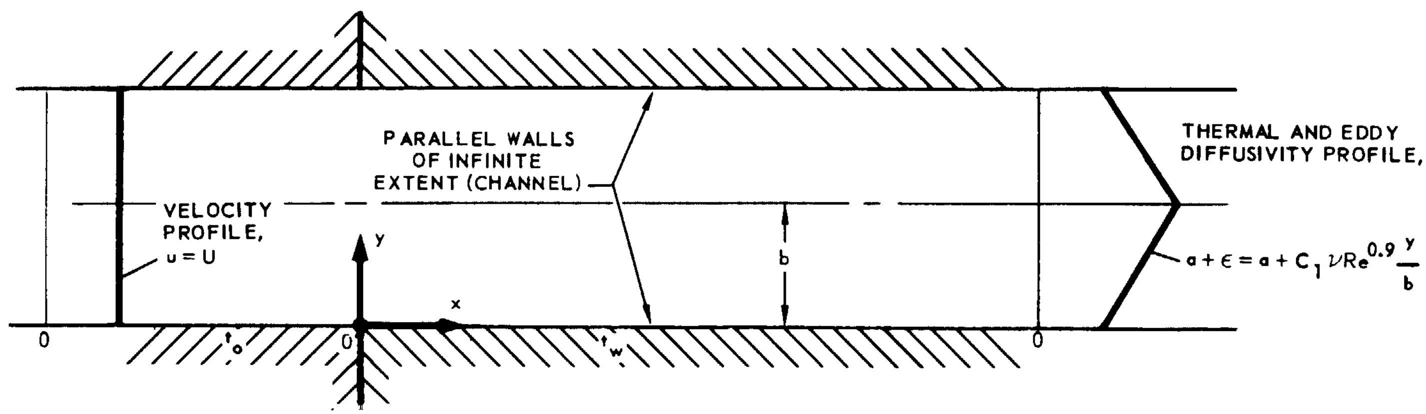

Figure 1.- Idealized heat transfer system.

The resulting boundary value problem to be solved is,

$$\frac{\partial T}{\partial X} = \frac{F_1^2}{4}\frac{\partial^2 T}{\partial r^2} + \frac{F_1^2}{4}\frac{1}{r}\frac{\partial T}{\partial r} \tag{3}$$

$$T(o, r) = 1 \tag{4}$$

$$T(X, \sqrt{F_o}) = 0 \tag{5}$$

$$\frac{\partial T}{\partial r}(X, \sqrt{F_o + F_1}) = 0 \tag{6}$$

$$\lim_{X \to \infty} T(X, r) = 0 \tag{7}$$

This problem can be solved by the separation-of-variables technique. Let

$$T = \phi_1(X), \ \phi_2(r) \tag{8}$$

where $\phi_1(X)$ and $\phi_2(r)$ are functions of X and r, respectively. Upon substituting equation (8) into equation (3), two total differential equations result. One involves $\phi_1(X)$ and the other $\phi_2(r)$. Their solution yields,

$$T = e^{-\beta^2 X}\left(c_1 J_o\left(\frac{2\beta}{F_1} r\right) + c_2 Y_o\left(\frac{2\beta}{F_1} r\right)\right) \tag{9}$$

where β is the parameter arising in the separation-of-variables technique and c_1 and c_2 are constants in the general solution.

From equation (5), the constant, c_1, is found to be

$$c_1 = -c_2 \frac{Y_o\left(\frac{2\beta}{F_1}\sqrt{F_o}\right)}{J_o\left(\frac{2\beta}{F_1}\sqrt{F_o}\right)} \tag{10}$$

Thus

$$T = \frac{c_2}{J_o\left(\frac{2\beta}{F_1}\sqrt{F_o}\right)} e^{-\beta^2 X}\left(-Y_o\left(\frac{2\beta}{F_1}\sqrt{F_o}\right) J_o\left(\frac{2\beta}{F_1} r\right) + J_o\left(\frac{2\beta}{F_1}\sqrt{F_o}\right) Y_o\left(\frac{2\beta}{F_1} r\right)\right)$$

$$= \frac{c_2}{J_o\left(\frac{2\beta}{F_1}\sqrt{F_o}\right)} e^{-\beta^2 X} U_o\left(\frac{2\beta}{F_1} r\right) \tag{11}$$

The constant β can be evaluated by substituting the temperature function, T, into the boundary condition given by equation (6). The resulting expression is,

$$-Y_o\left(\frac{2\beta_n}{F_1}\sqrt{F_o}\right) J'_o\left(\frac{2\beta_n}{F_1}\sqrt{F_o + F_1}\right) + J_o\left(\frac{2\beta_n}{F_1}\sqrt{F_o}\right) Y'_o\left(\frac{2\beta_n}{F_1}\sqrt{F_o + F_1}\right) = 0 \tag{12}$$

which is the eigenfunction. The terms β_n are the eigenvalues (n = 1, 2, 3 . . .) .

The constant c_2 in equation (11) is now replaced by the constants b_n (n = 1, 2, 3, . . .) corresponding to the values, β_n. The constants, b_n, can be evaluated from the boundary condition given by equation (4),

$$1 = \sum_{n=1}^{\infty} \frac{b_n}{J_o\left(\frac{2\beta_n\sqrt{F_o}}{F_1}\right)} U_o\left(\frac{2\beta_n}{F_1} r\right) \tag{13}$$

It can be shown (see Appendix 4) that a function f(r) can be expanded into a $U_o\left(\frac{2\beta_n}{F_1} r\right)$ series over the interval r_1 to r_2,

$$f(r) = \sum_{n=1}^{\infty} A_n U_o\left(\frac{2\beta_n}{F_1} r\right) \tag{14}$$

where

$$A_n = \frac{\int_{r_1}^{r_2} r\, f(r) U_o\left(\frac{2\beta_n}{F_1} r\right) dr}{\int_{r_1}^{r_2} r\, U_o^2\left(\frac{2\beta_n}{F_1} r\right) dr} \tag{15}$$

For the boundary value problem being considered $f(r) = 1$. From equations (13) and (14),

$$b_n = J_o\left(\frac{2\beta_n}{F_1}\sqrt{F_o}\right) A_n \tag{16}$$

Thus, the temperature solution is,

$$T = \sum_{n=1}^{\infty} A_n e^{-\beta_n^2 X}\left(-Y_o\left(\frac{2\beta_n}{F_1}\sqrt{F_o}\right) J_o\left(\frac{2\beta_n}{F_1} r\right) + J_o\left(\frac{2\beta_n}{F_1}\sqrt{F_o}\right) Y_o\left(\frac{2\beta_n}{F_1} r\right)\right) \tag{17}$$

where the coefficients A_n are given by equation (15) and the eigenvalues are defined by equation (12).

As X approaches zero, many terms are required to obtain convergence in the series solution given by equation (17). Thus, it is convenient to use an asymptotic solution in that region (see Appendix 5).

The mixed-mean fluid temperature[1], t_m, can be expressed in dimensionless form as

$$T_m = \int_0^1 T\,dY = \frac{2}{F_1}\int_{\sqrt{F_o}}^{\sqrt{F_o + F_1}} T\,r\,dr \qquad (18)$$

The local Nusselt modulus, Nu_x, can be expressed as,

$$Nu_x = \frac{h_x 4b}{k} = \frac{4\left(\frac{\partial T}{\partial Y}\right)_{Y=0}}{T_m} = \frac{\frac{2F_1}{\sqrt{F_o}}\left(\frac{\partial T}{\partial r}\right)_{r=\sqrt{F_o}}}{T_m} \qquad (19)$$

where

$$\left(\frac{\partial T}{\partial r}\right)_{r=\sqrt{F_o}} = \sum_{n=1}^{\infty} A_n e^{-\beta_n^2 X} \frac{2\beta_n}{F_1}\left(-Y_o\left(\frac{2\beta_n}{F_1}\sqrt{F_o}\right) J'_o\left(\frac{2\beta_n}{F_1}\sqrt{F_o}\right) + J_o\left(\frac{2\beta_n}{F_1}\sqrt{F_o}\right) Y'_o\left(\frac{2\beta_n}{F_1}\sqrt{F_o}\right)\right) = \sum_{n=1}^{\infty} A_n e^{-\beta_n^2 X} \frac{2\beta_n}{F_1}\left(\frac{2}{\pi}\,\frac{F_1}{2\beta_n\sqrt{F_o}}\right) \qquad (20)$$

Upon substituting equations (20) and (18) into equation (19), there results

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1. The mixed-mean fluid temperature for a channel system is defined as

$$t_m = \frac{\int_0^b u\,t\,dy}{\int_0^b u\,dy}$$

$$Nu_X = \frac{\dfrac{2F_1}{\sqrt{F_o}} \sum_{n=1}^{\infty} A_n e^{-\beta_n^2 X} \dfrac{2}{\pi\sqrt{F_o}}}{\dfrac{2}{F_1}\displaystyle\int_{\sqrt{F_o}}^{\sqrt{F_o + F_1}} \sum_{n=1}^{\infty} A_n e^{-\beta_n^2 X} r \left[-Y_o\left(\frac{2\beta_n}{F_1}\sqrt{F_o}\right) J_o\left(\frac{2\beta_n}{F_1} r\right) + J_o\left(\frac{2\beta_n}{F_1}\sqrt{F_o}\right) Y_o\left(\frac{2\beta_n}{F_1} r\right)\right] dr} \tag{21}$$

The arithmetic mean Nusselt modulus, Nu_{am}, which is based on an arithmetic mean wall-fluid temperature difference, is expressed as

$$Nu_{am} = \frac{h_{am}\, 4b}{k} \tag{22}$$

where

$$h_{am} = \frac{q}{A} \frac{1}{\Delta t_{am}} \tag{23}$$

$$\Delta t_{am} = \frac{(t_w - t_o) + (t_w - t_m)}{2} \tag{24}$$

$$\frac{q}{A} = \frac{1}{x} \int_0^x -k\left(\frac{\partial t}{\partial y}\right)_{y=o} dx \tag{25}$$

Upon substituting equations (23), (24), and (25) into equation (22), there results

$$Nu_{am} = \frac{\dfrac{4}{X}\displaystyle\int_0^X \left(\frac{\partial T}{\partial Y}\right)_{Y=0} dX}{\dfrac{1 + T_m}{2}} \tag{26}$$

RESULTS

The eigenvalues and series coefficients for this boundary value problem were evaluated for a wide range of Reynolds and Prandtl moduli. The quantities T_m, Nu_X, and Nu_{am} were calculated as functions of the parameter X, and Reynolds and Prandtl moduli. The results are presented in Figures 2 to 20 and in Appendix 6 (Tables I to VI). From the Nu_X graphs it may be observed that the entrance length increases as Reynolds modulus increases. The established Nusselt moduli, Nu_∞, are shown as a function of Reynolds and Prandtl moduli, in Figure 8. The T_m graphs show that the fluid temperature approaches the wall temperature in a shorter distance from the entrance as the Reynolds modulus decreases.

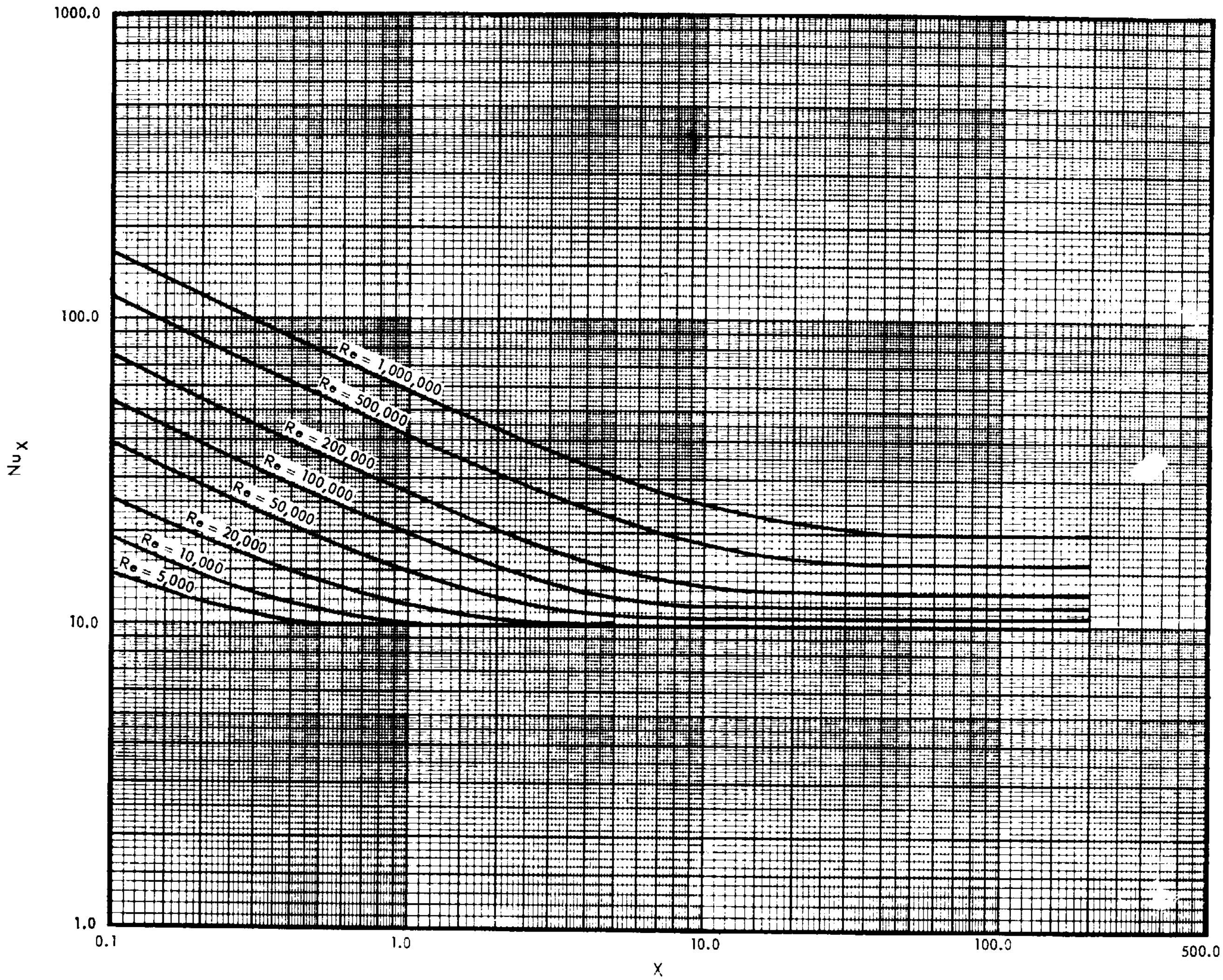

Figure 2.- Nu_X vs X and Re for Pr = 0.002.

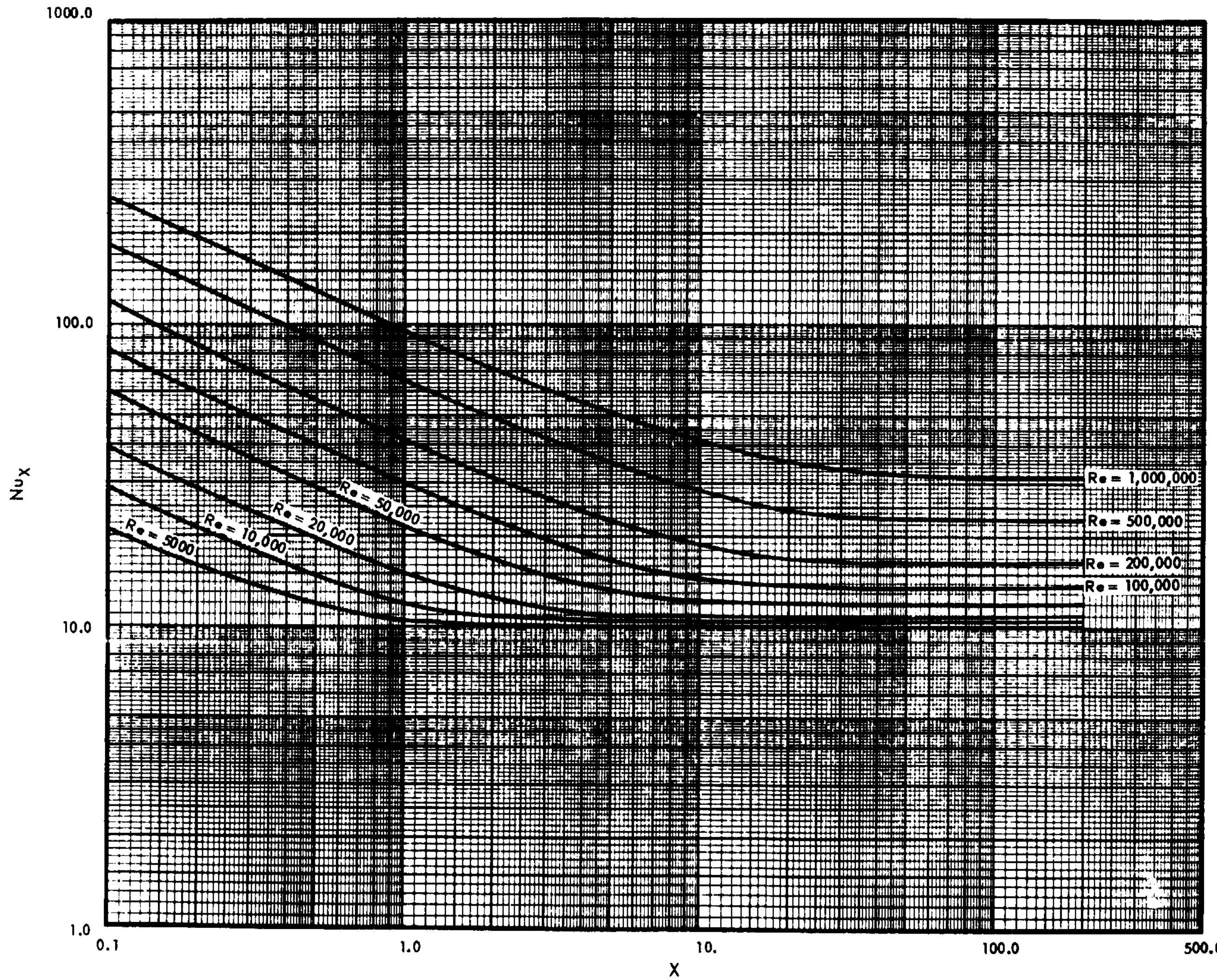

Figure 3.- Nu_X vs X and Re for Pr = 0.005.

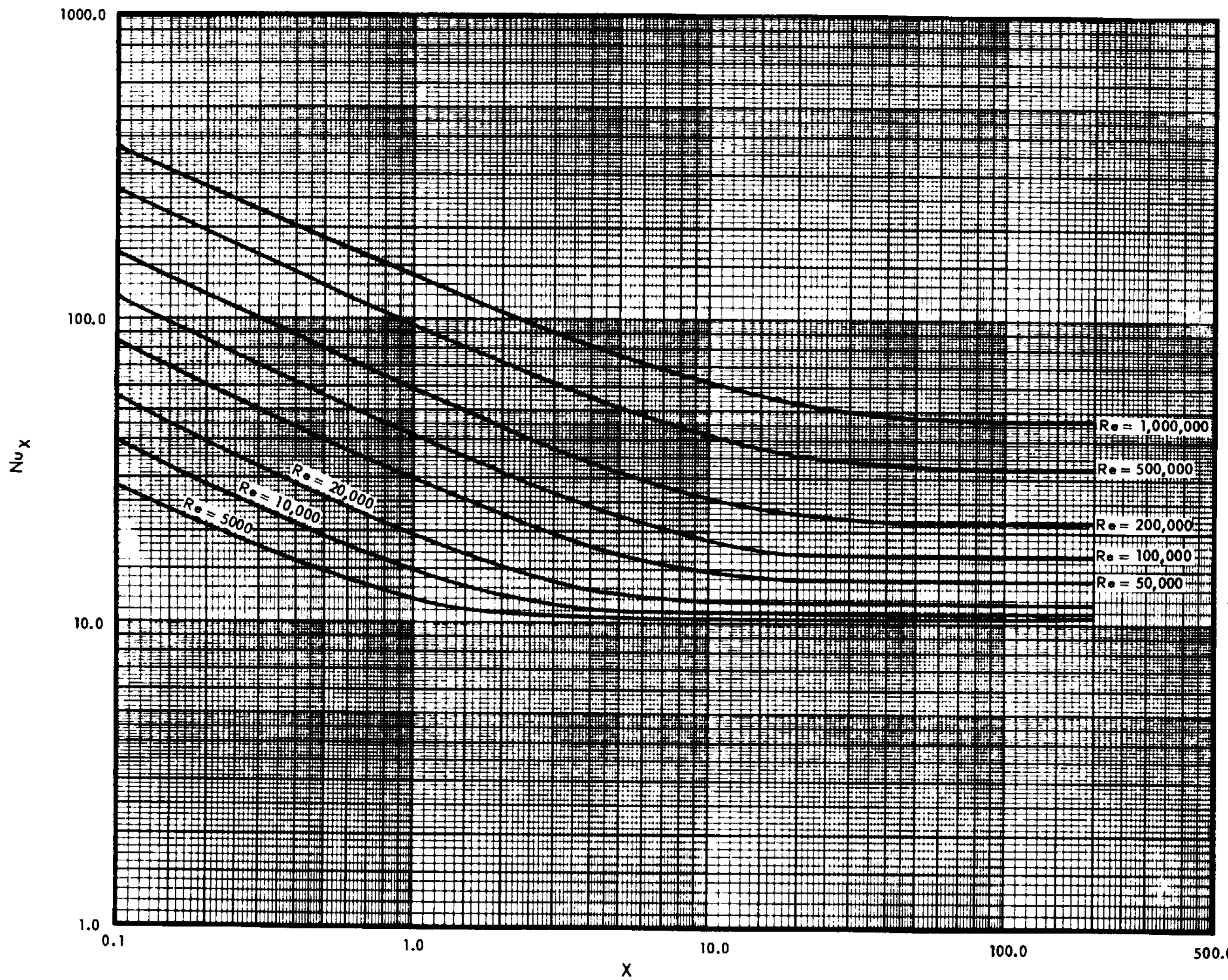

Figure 4.- Nu_X vs X and Re for Pr = 0.01.

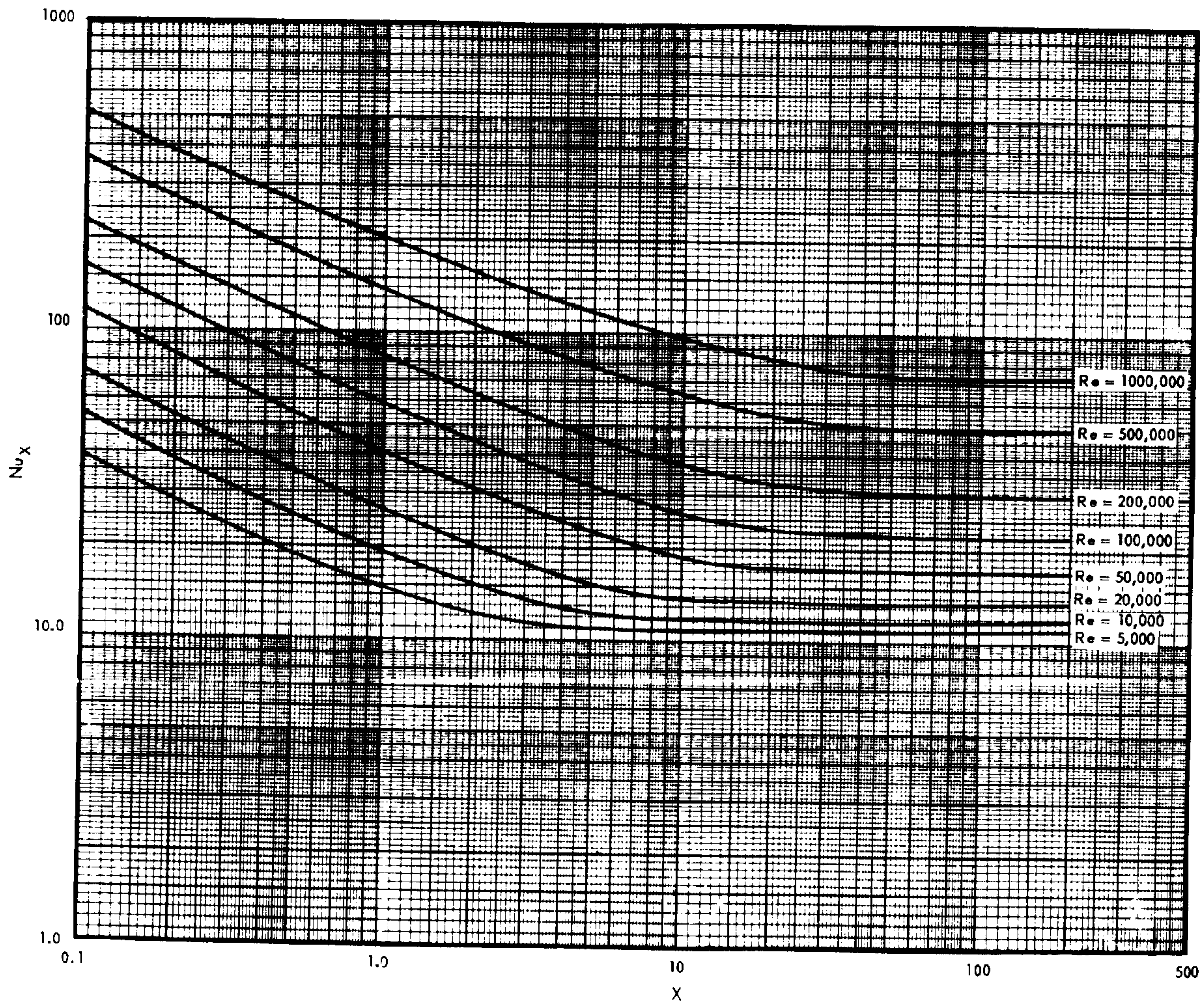

Figure 5.- Nu_X vs X and Re for Pr = 0.02.

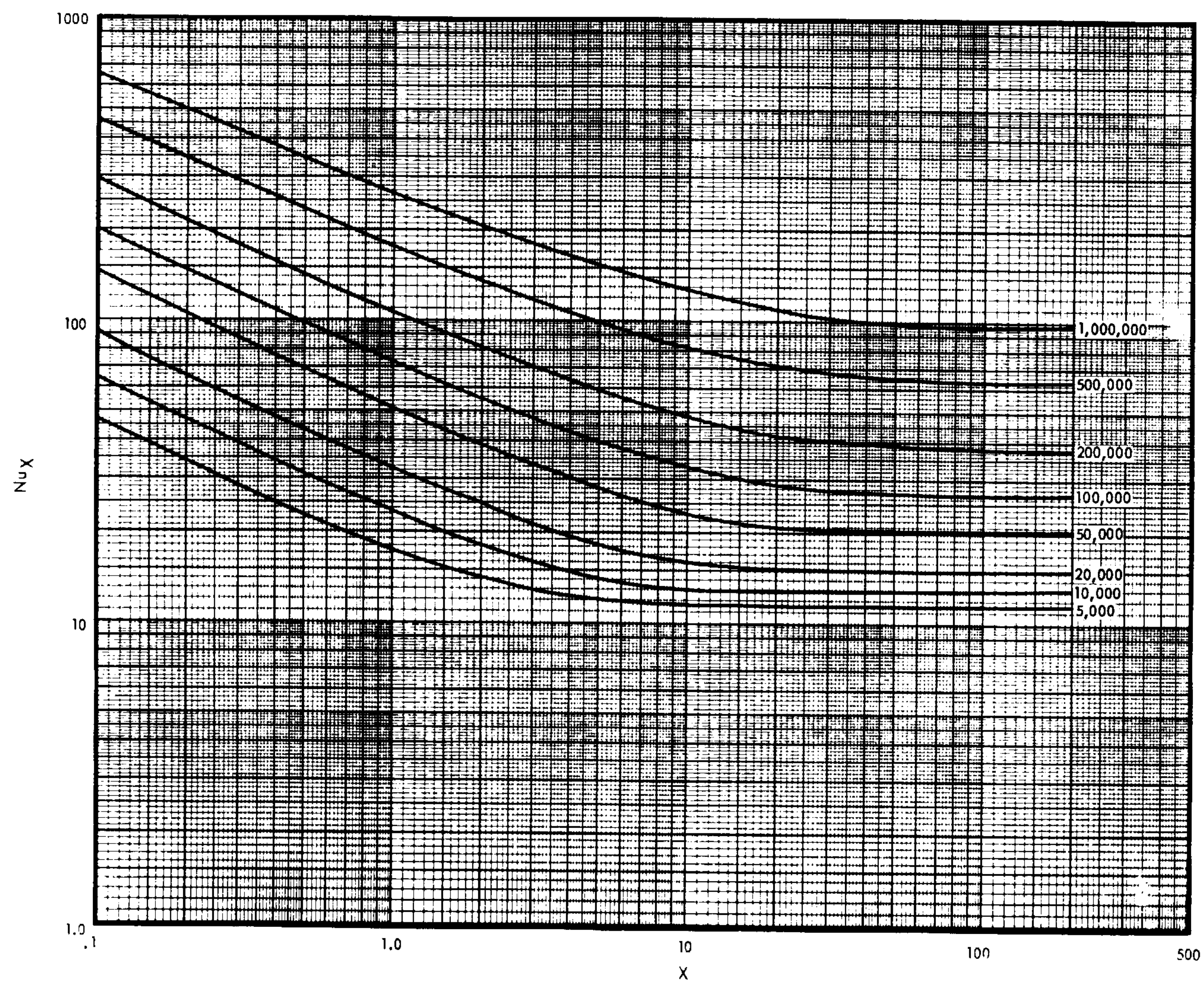

Figure 6.- Nu_X vs X and Re for Pr = 0.03.

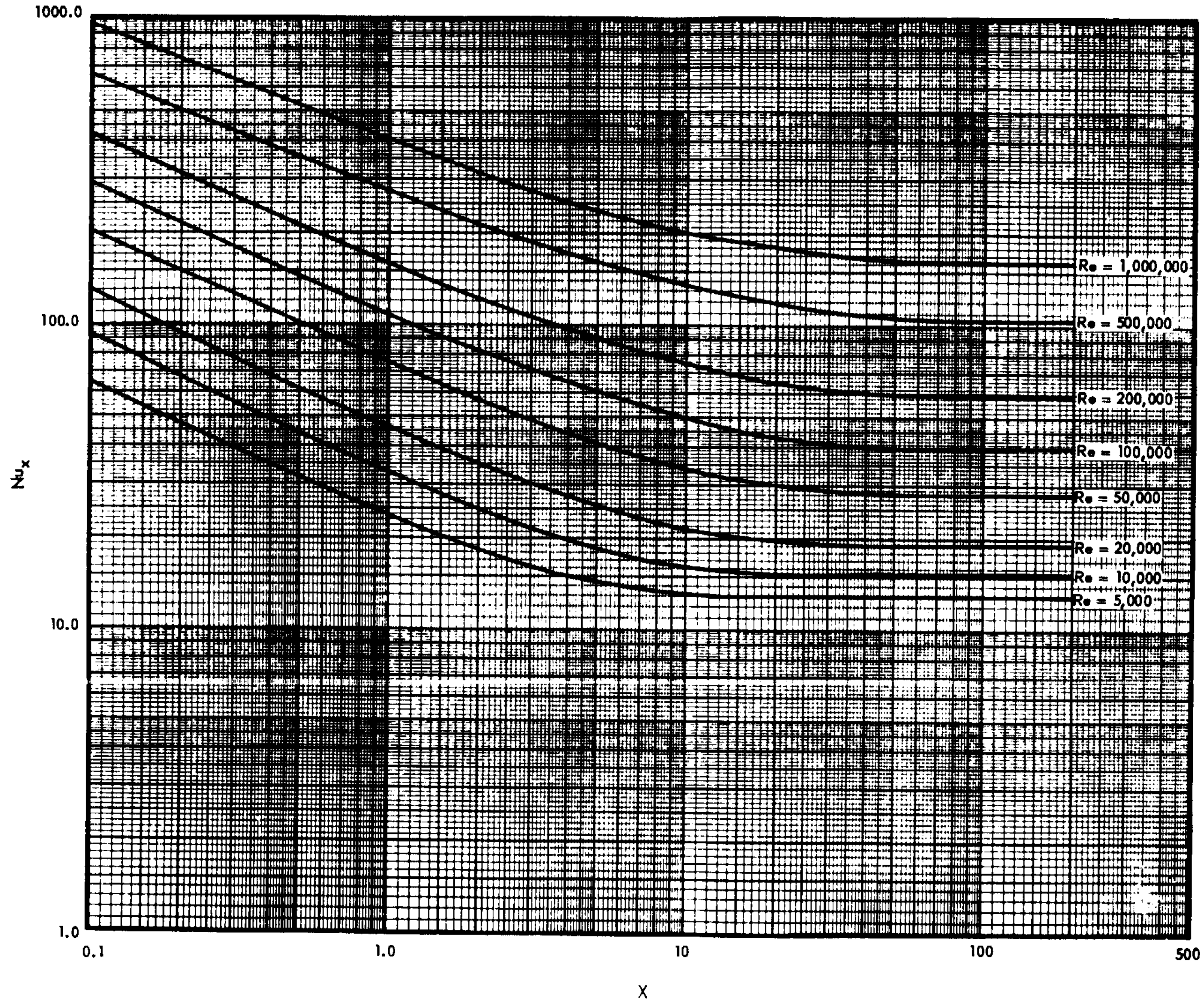

Figure 7.- Nu_X vs X and Re for Pr = 0.06.

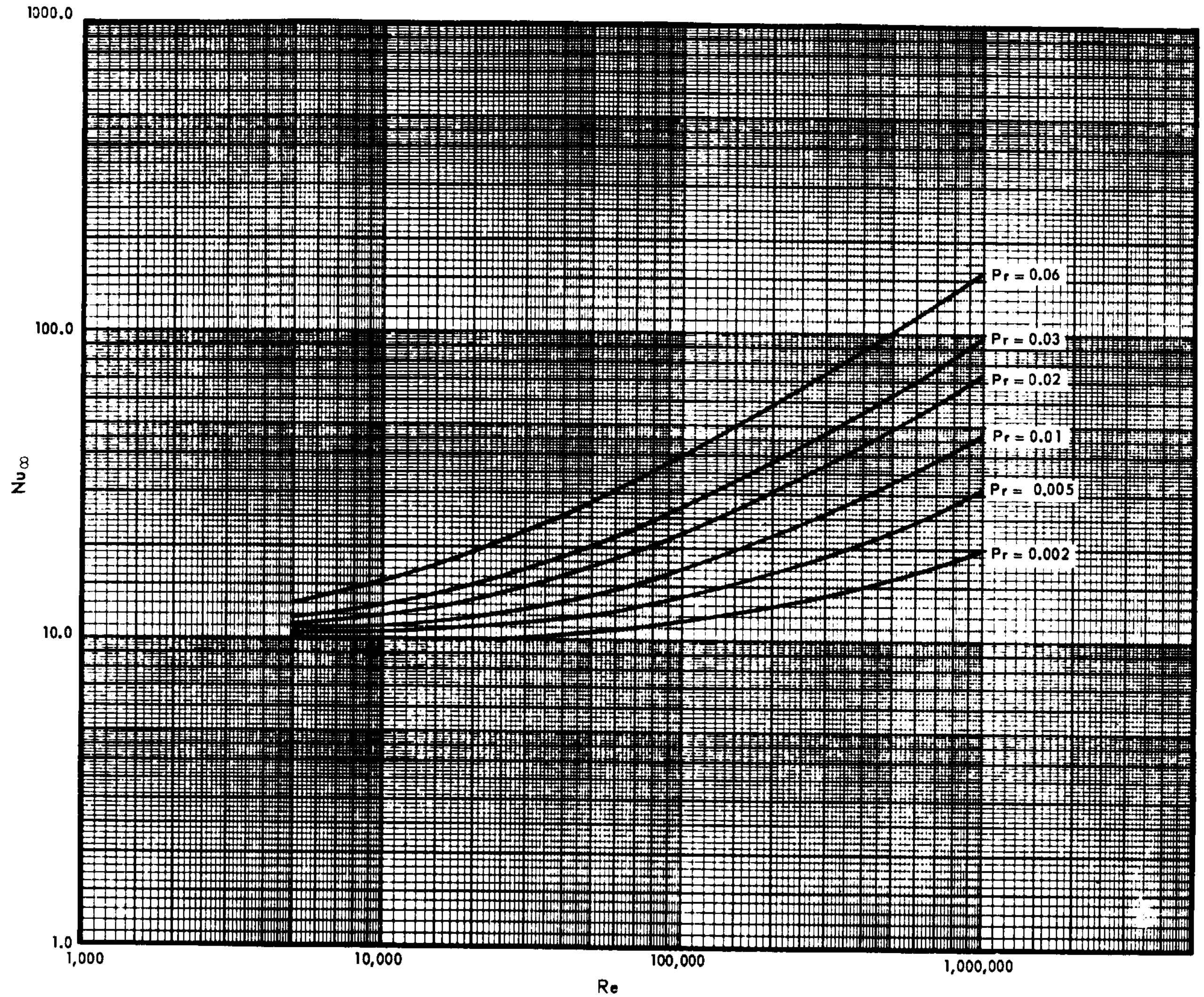

Figure 8.- Nu_{∞} vs Re and Pr.

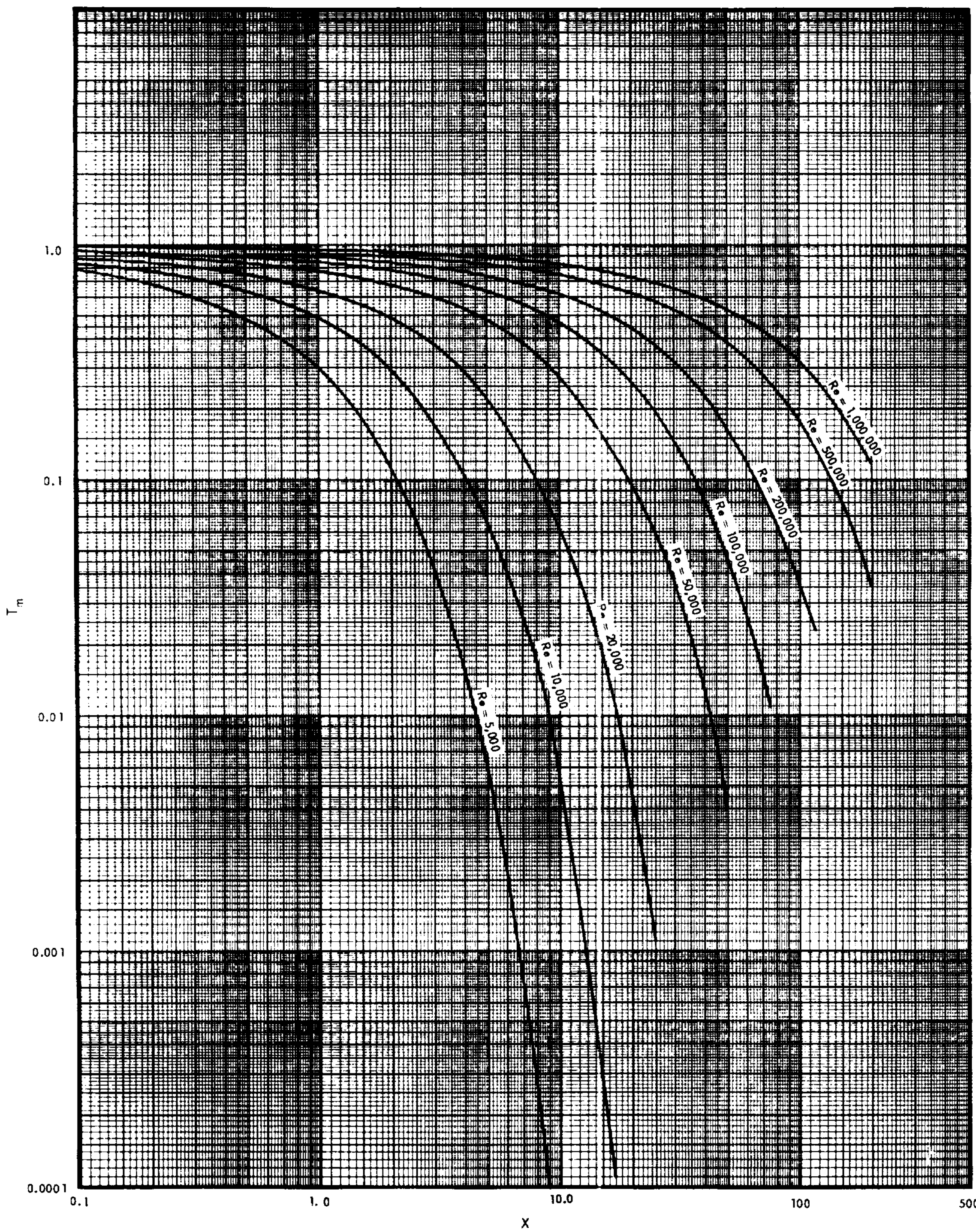

Figure 9.- T_m vs X and Re for Pr = 0.002.

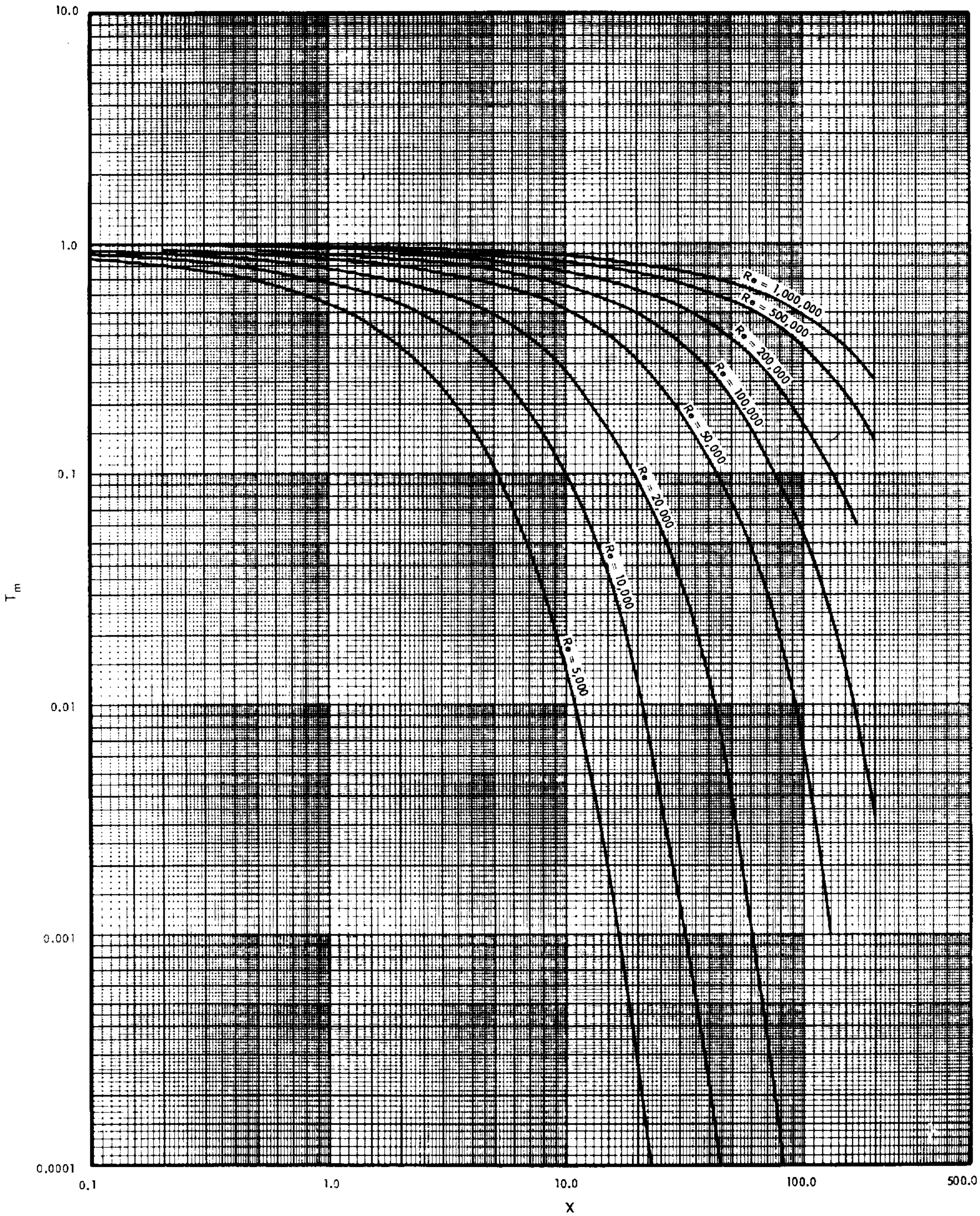

Figure 10.- T_m vs X and Re for Pr = 0.005.

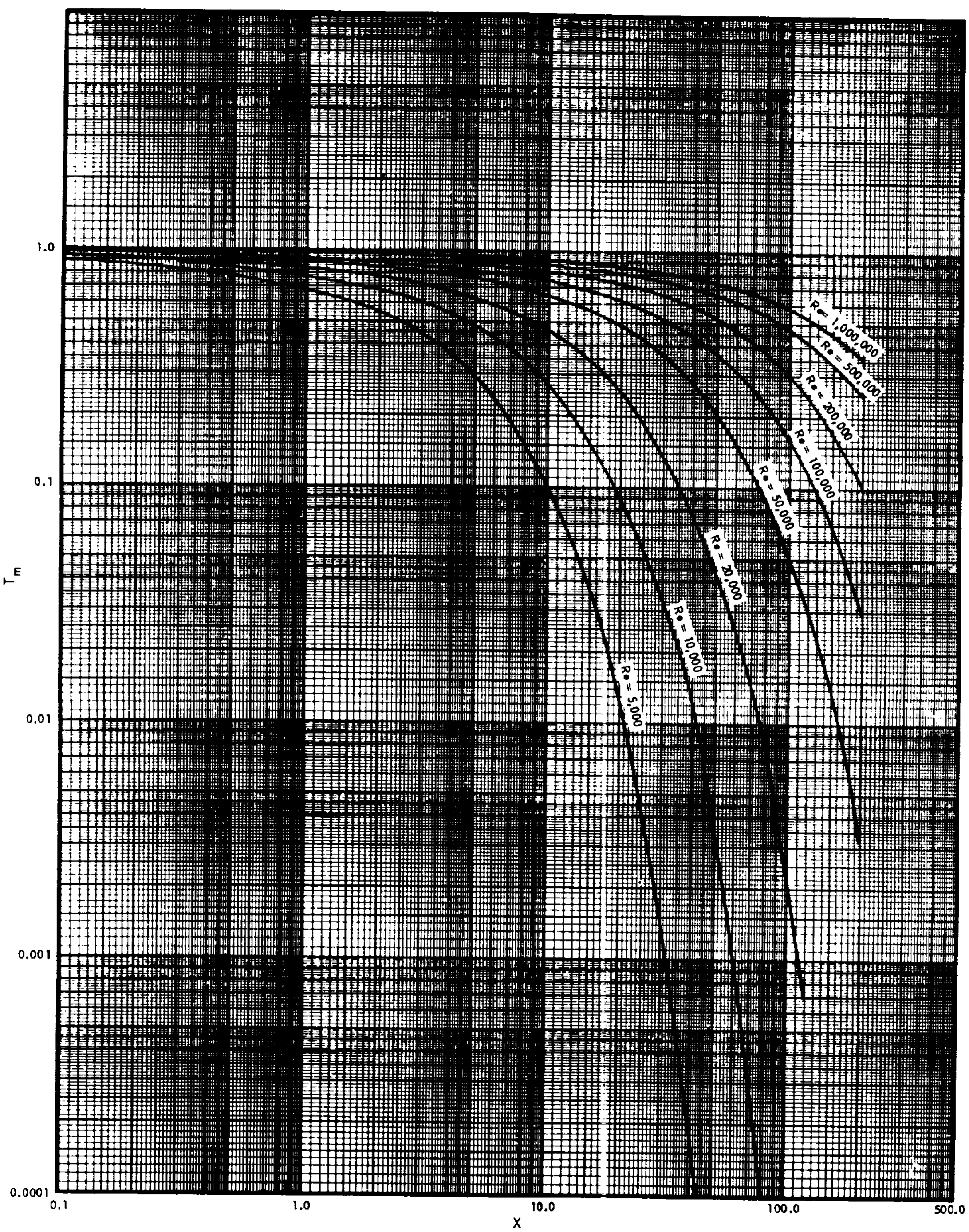

Figure 11.- T_m vs X and Re for Pr = 0.01.

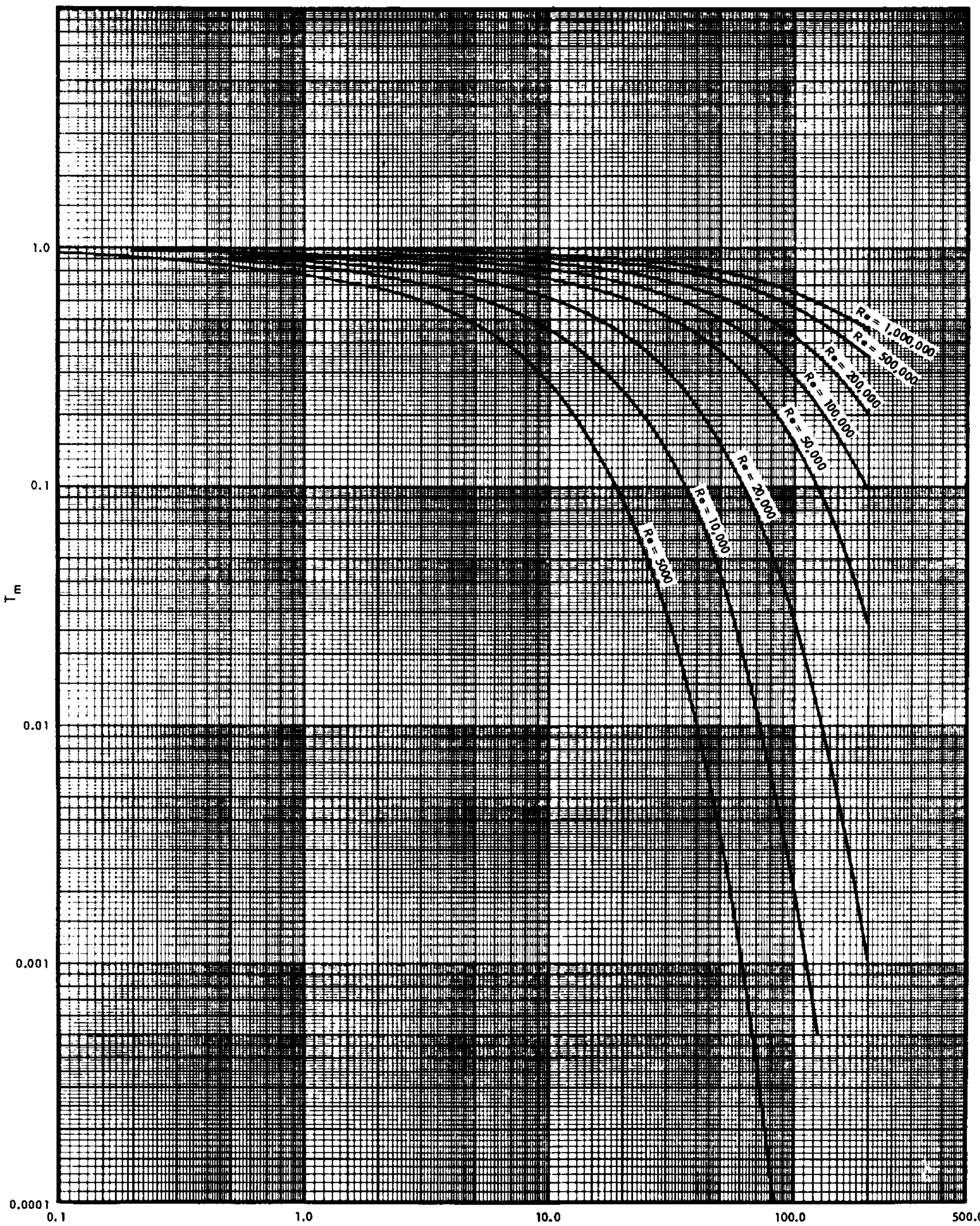

Figure 12.- T_m vs X and Re for Pr = 0.02.

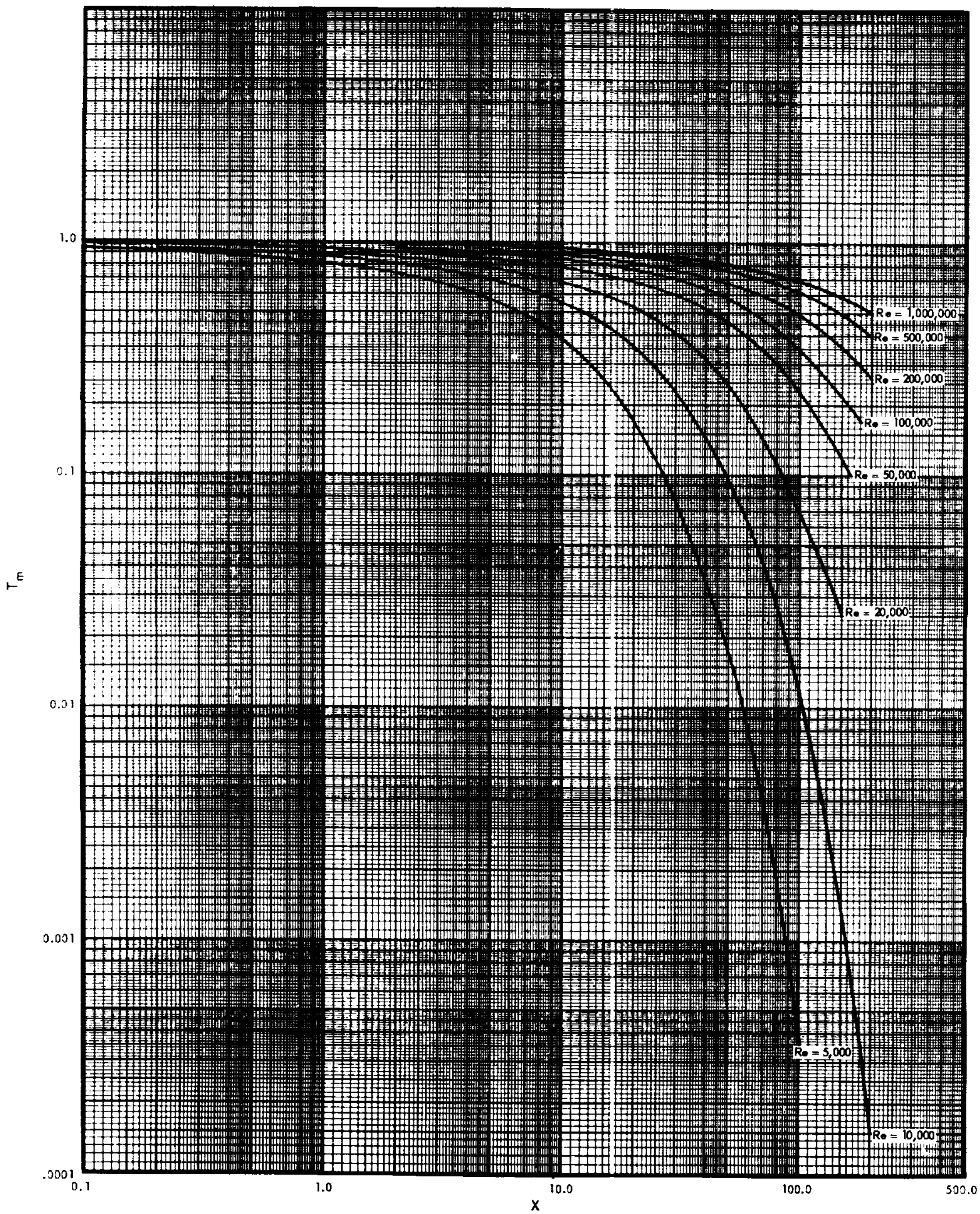

Figure 13.- T_m vs X and Re for Pr = 0.03.

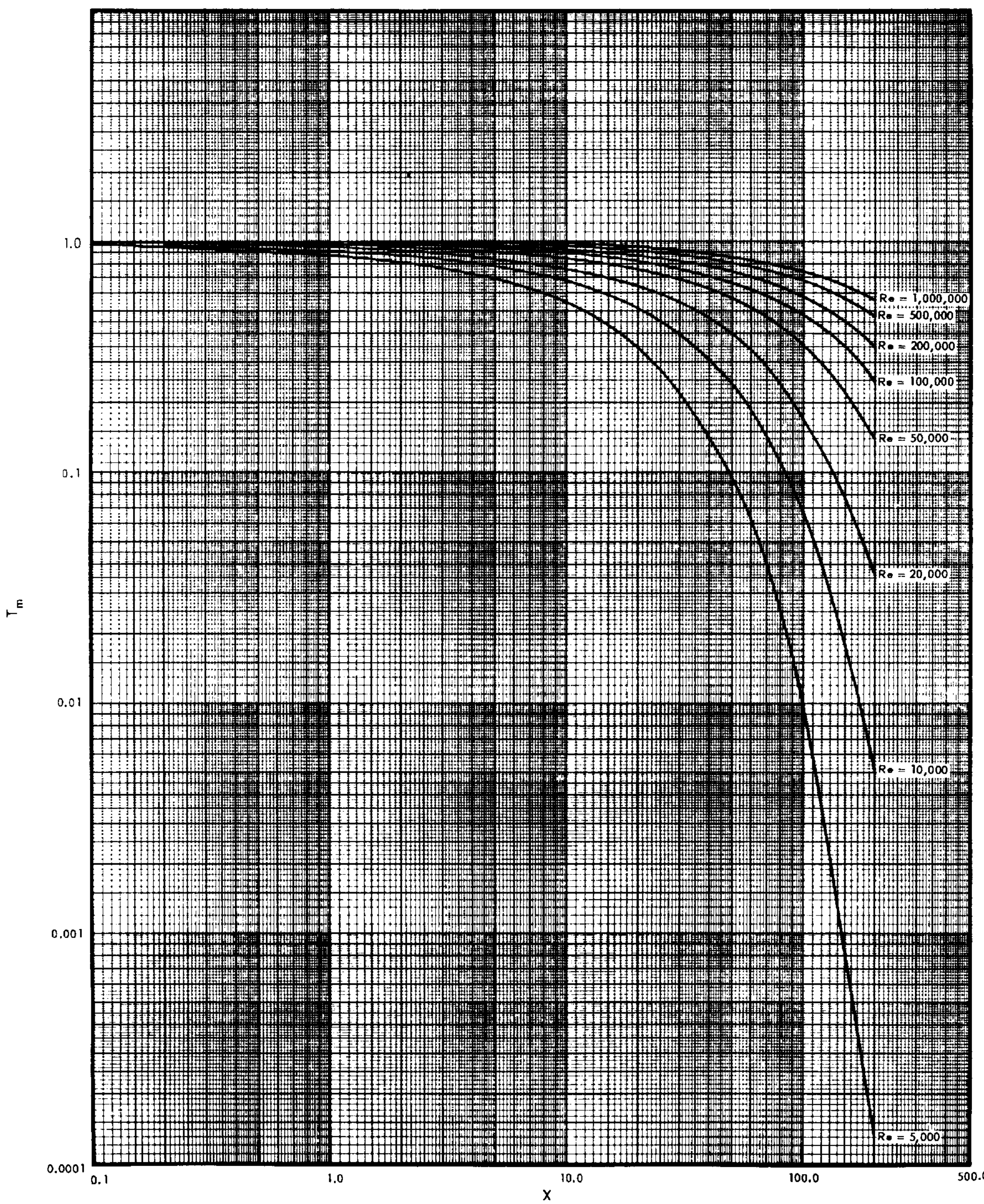

Figure 14.- T_m vs X and Re for Pr = 0.06.

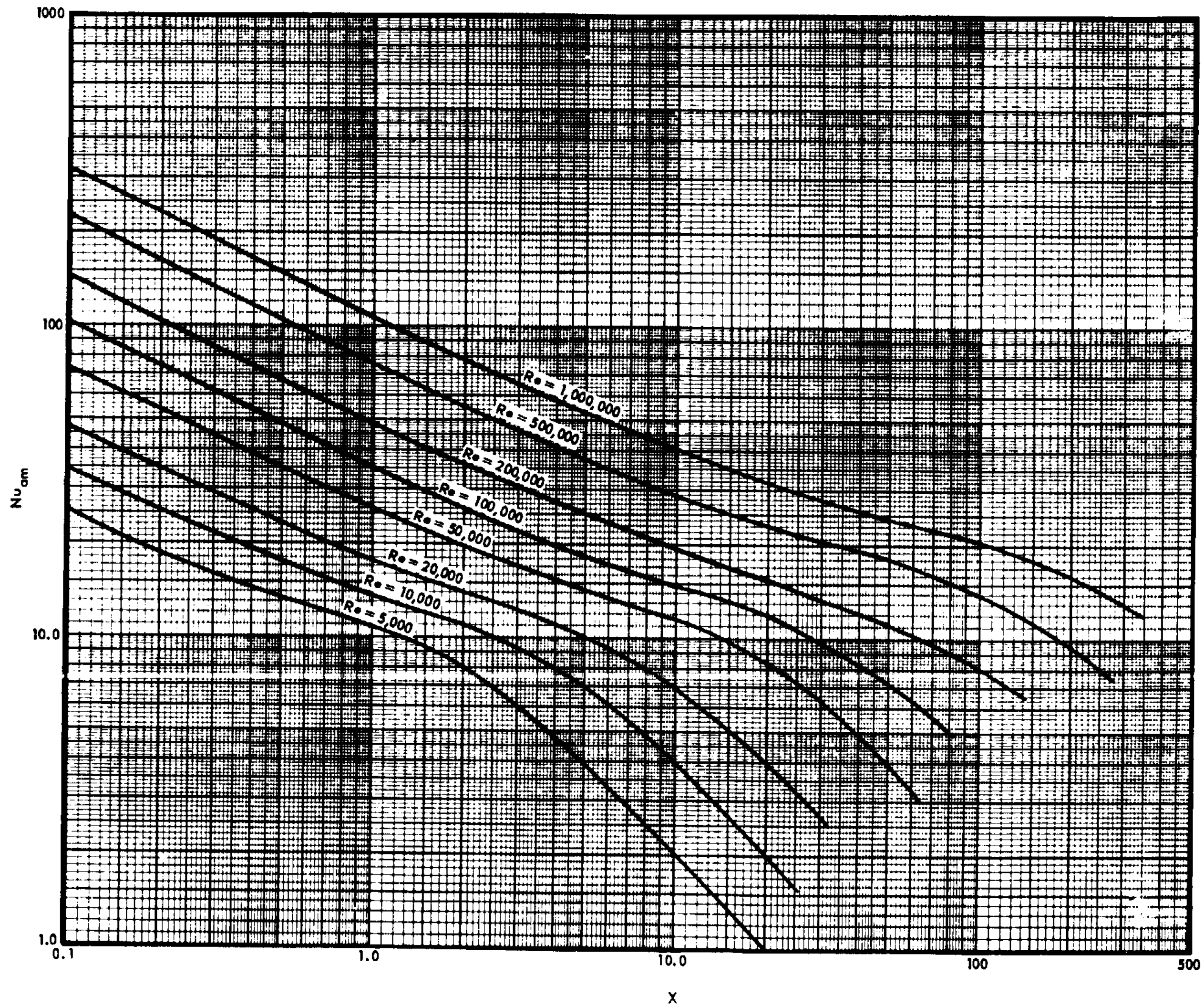

Figure 15.- Nu_{am} vs X and Re for Pr = 0.002.

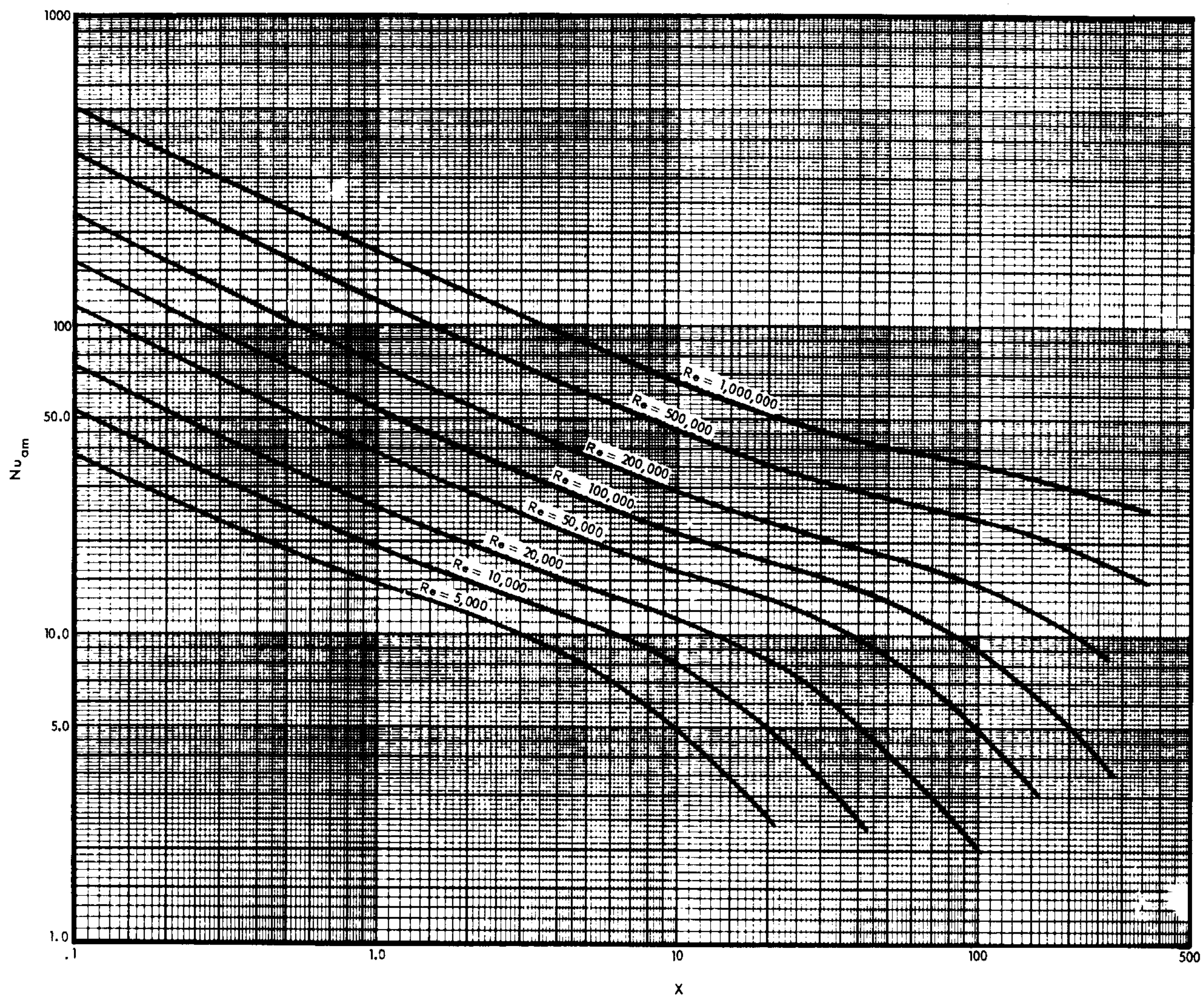

Figure 16.- Nu_{am} vs X and Re for Pr = 0.005.

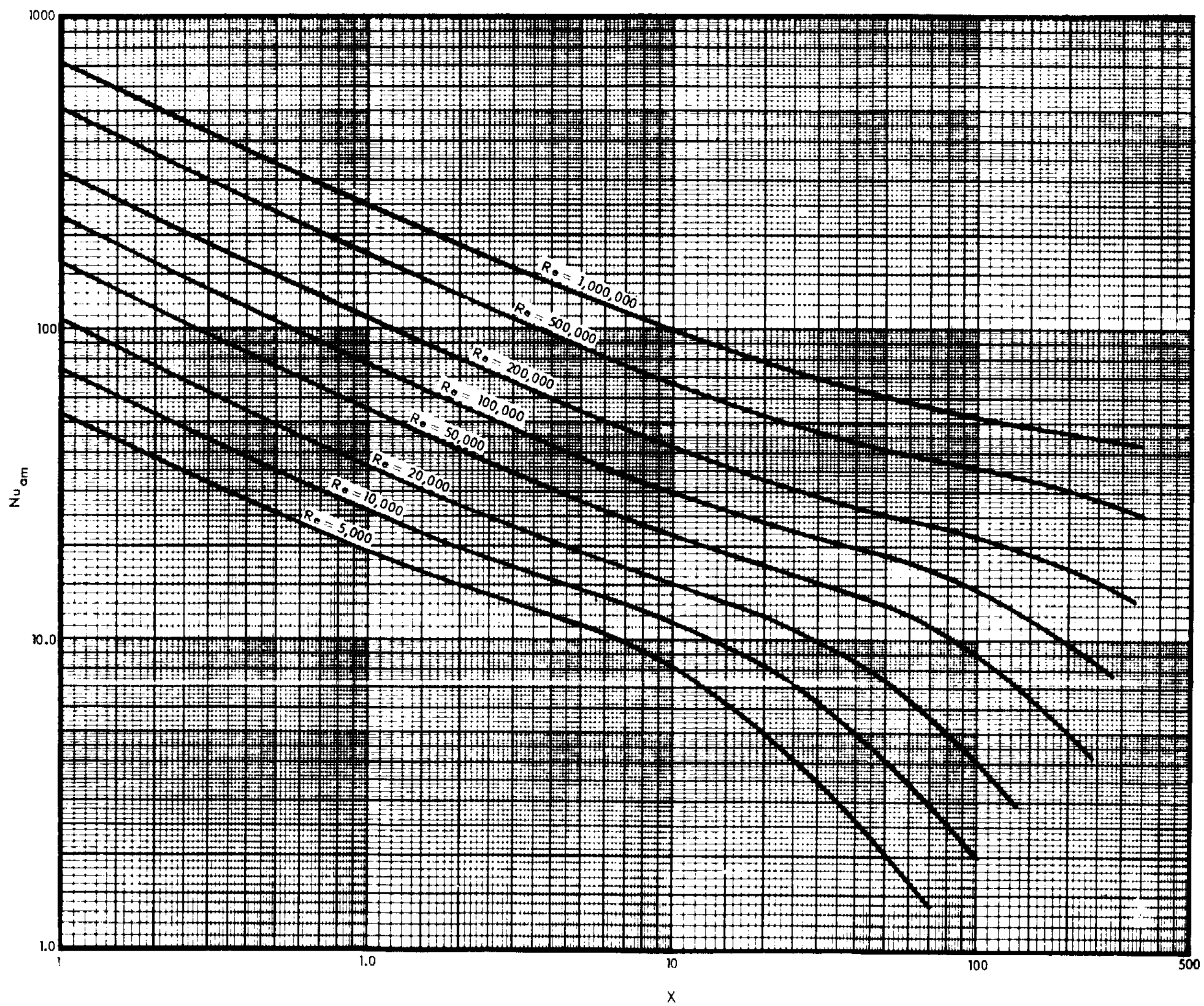

Figure 17.- Nu_{am} vs X and Re for Pr = 0.01.

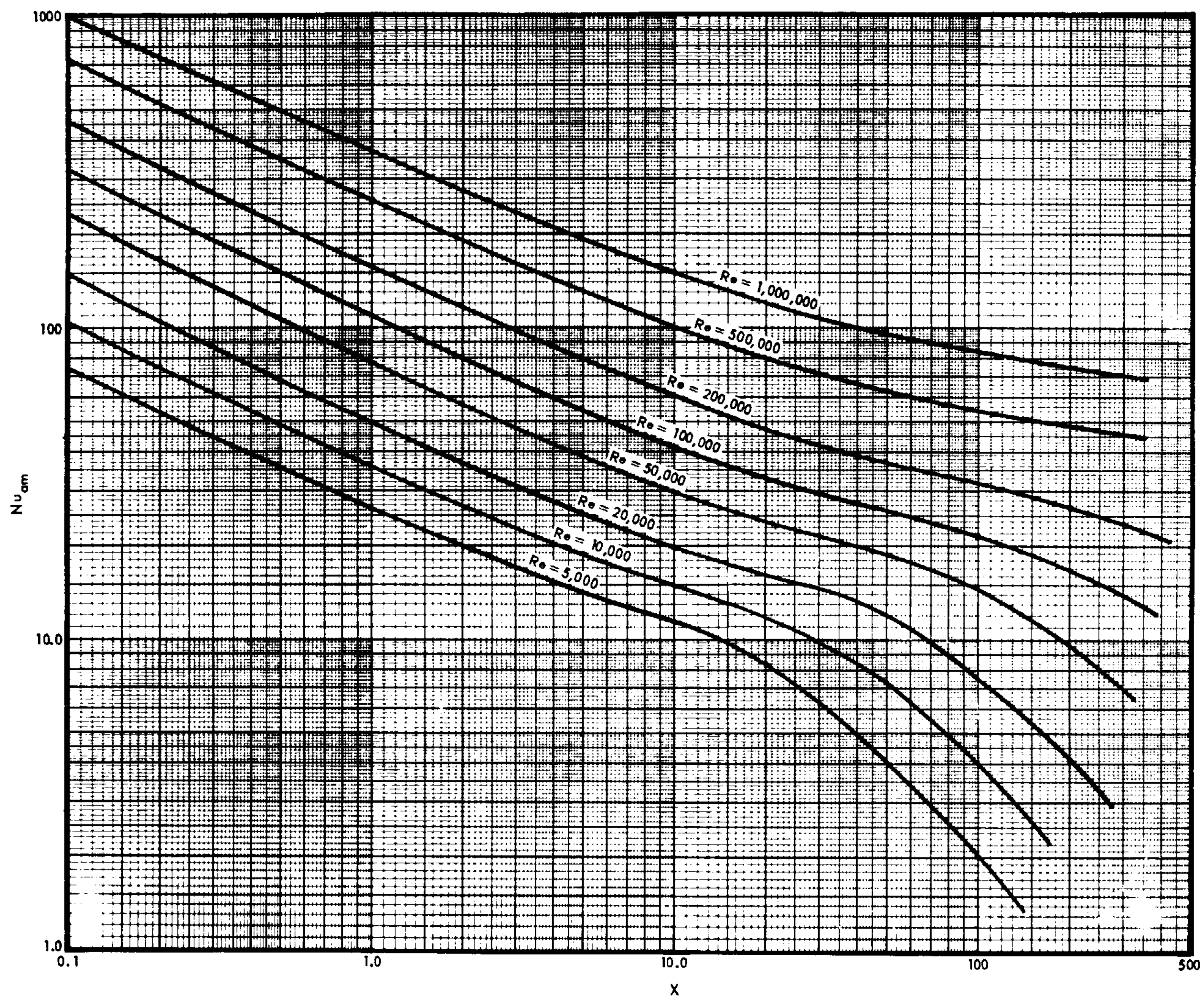

Figure 18.- Nu_{am} vs X and Re for Pr = 0.02.

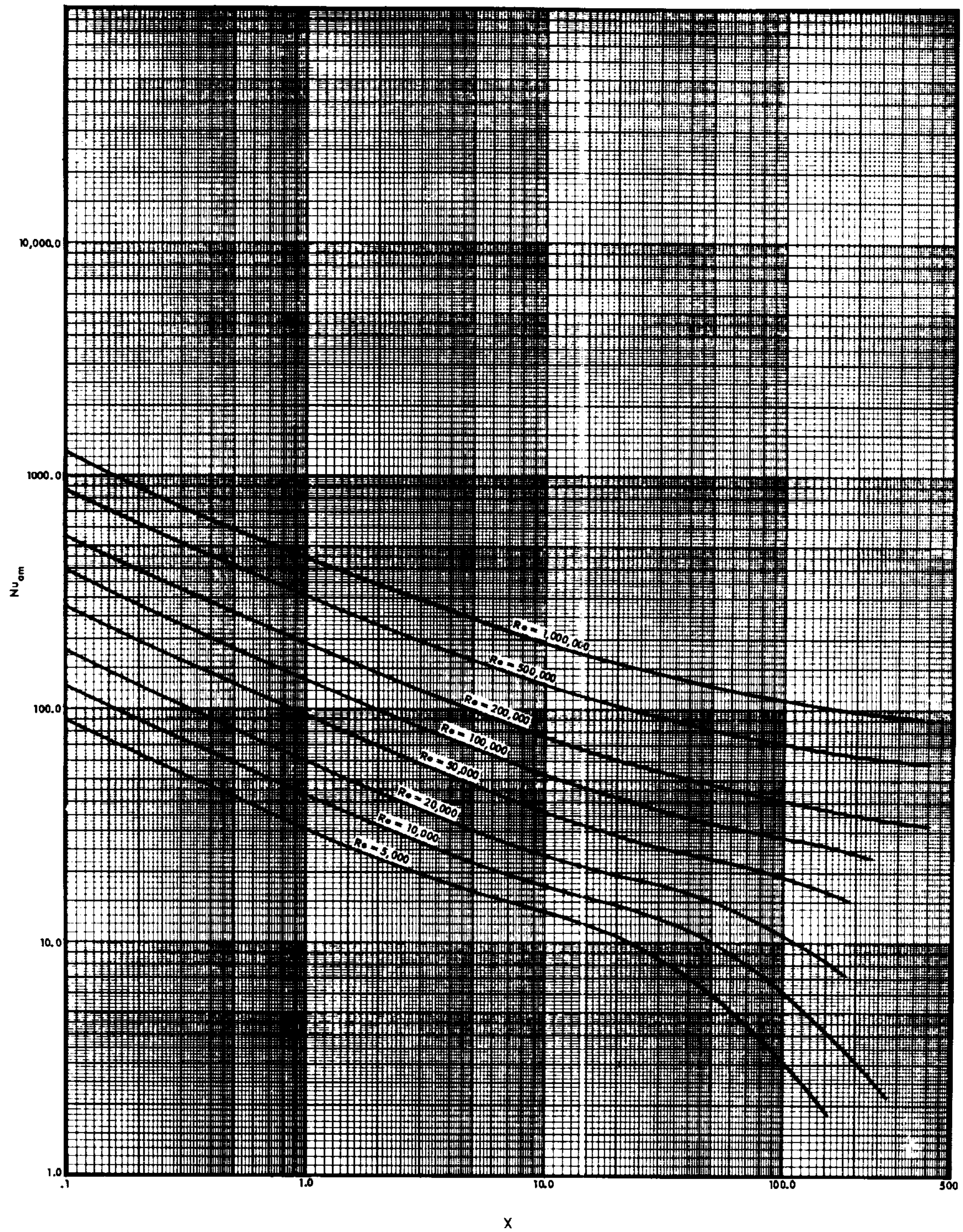

Figure 19.- Nu_{am} vs X and Re for Pr = 0.03.

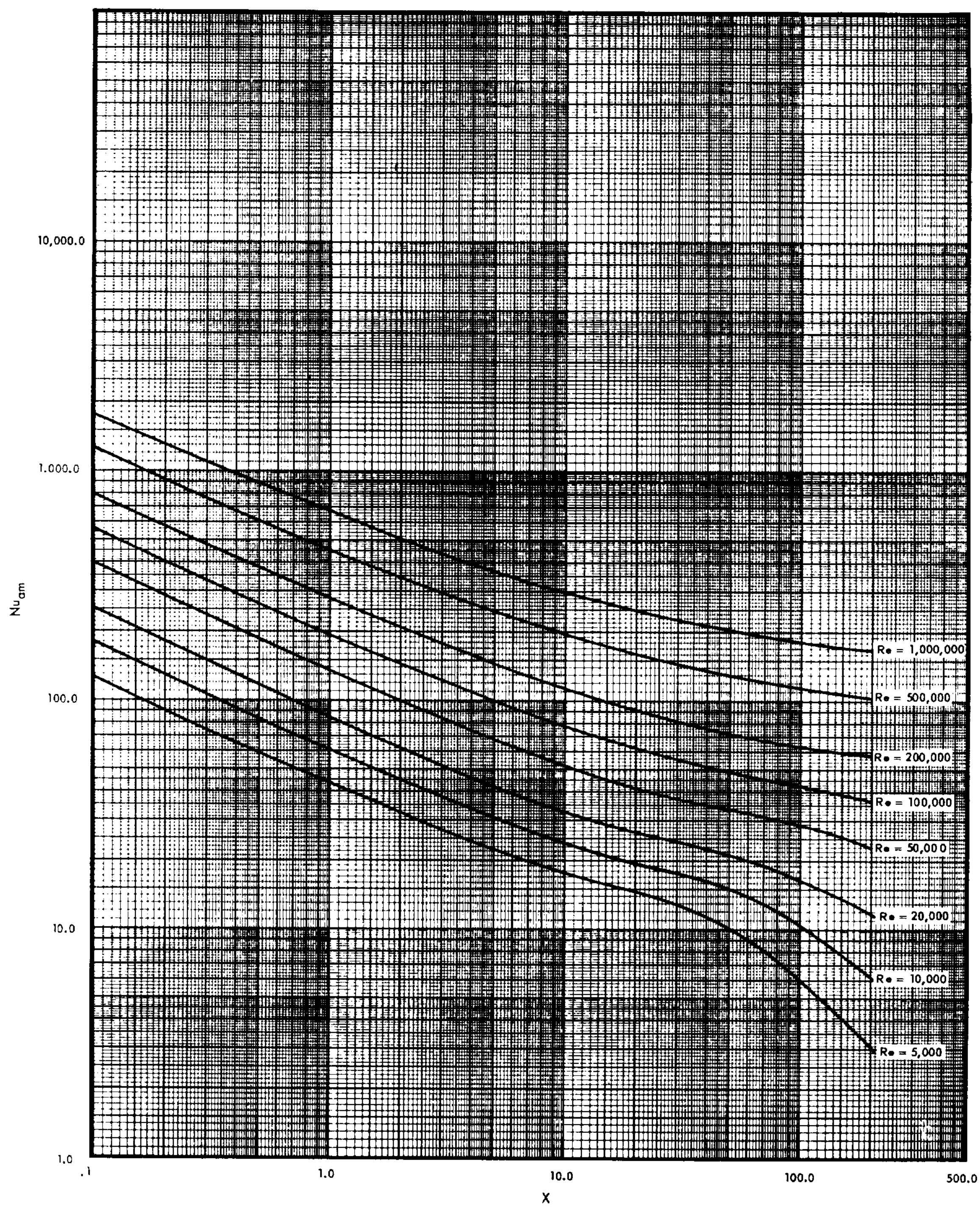

Figure 20.- Nu_{am} vs X and Re for Pr = 0.06.

DISCUSSION

Comparison of Solution with Specific Analytical and Numerical Work

The well known solution for the special case of flow in a channel under low Reynolds and Prandtl moduli conditions (radial heat flow by conduction only) with a uniform velocity profile was compared with the general convection solution derived above for a condition where eddy transfer is small compared to conduction, namely, Re = 5000 and Pr = 0.002; the respective temperature and Nusselt modulus solutions for the two cases were in complete agreement for all values of X.

The established Nusselt modulus solution for long channels, Nu_{∞} for uniform wall-heat-flux conditions by Martinelli (5) was also compared to the established Nusselt modulus results (Figure 8) obtained from the general solution. Figure 21 illustrates that the two sets of calculations are in good agreement, and the uniform wall temperature Nusselt moduli fall about 15 percent below the uniform wall-heat-flux Nusselt moduli.[2]

A discussion of the linear approximation of the complicated eddy diffusivity profile was presented in Appendix 3. It was concluded that the agreement between the linear and actual eddy diffusivity functions in the important heat transfer layers was good. To substantiate this conclusion further, a numerical analysis of the convective heat transfer problem was made with the actual eddy diffusivity function for the specific case of Re = 200,000 and Pr = 0.01. The agreement between the analytical solutions (using the linear eddy diffusivity function) and the numerical solutions (using the actual eddy diffusivity function) was good. For example, the local Nusselt moduli for the two cases shown in Figure 22 fall within about 10 percent of each other.

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2. It has been found (6) that for the case of established flow $X \rightarrow \infty$ the Nusselt moduli for uniform wall-heat-flux boundary conditions are a little greater than those quantities for uniform wall-temperature boundary conditions; this result was based on analytical conduction solutions as well as on numerical turbulence solutions.

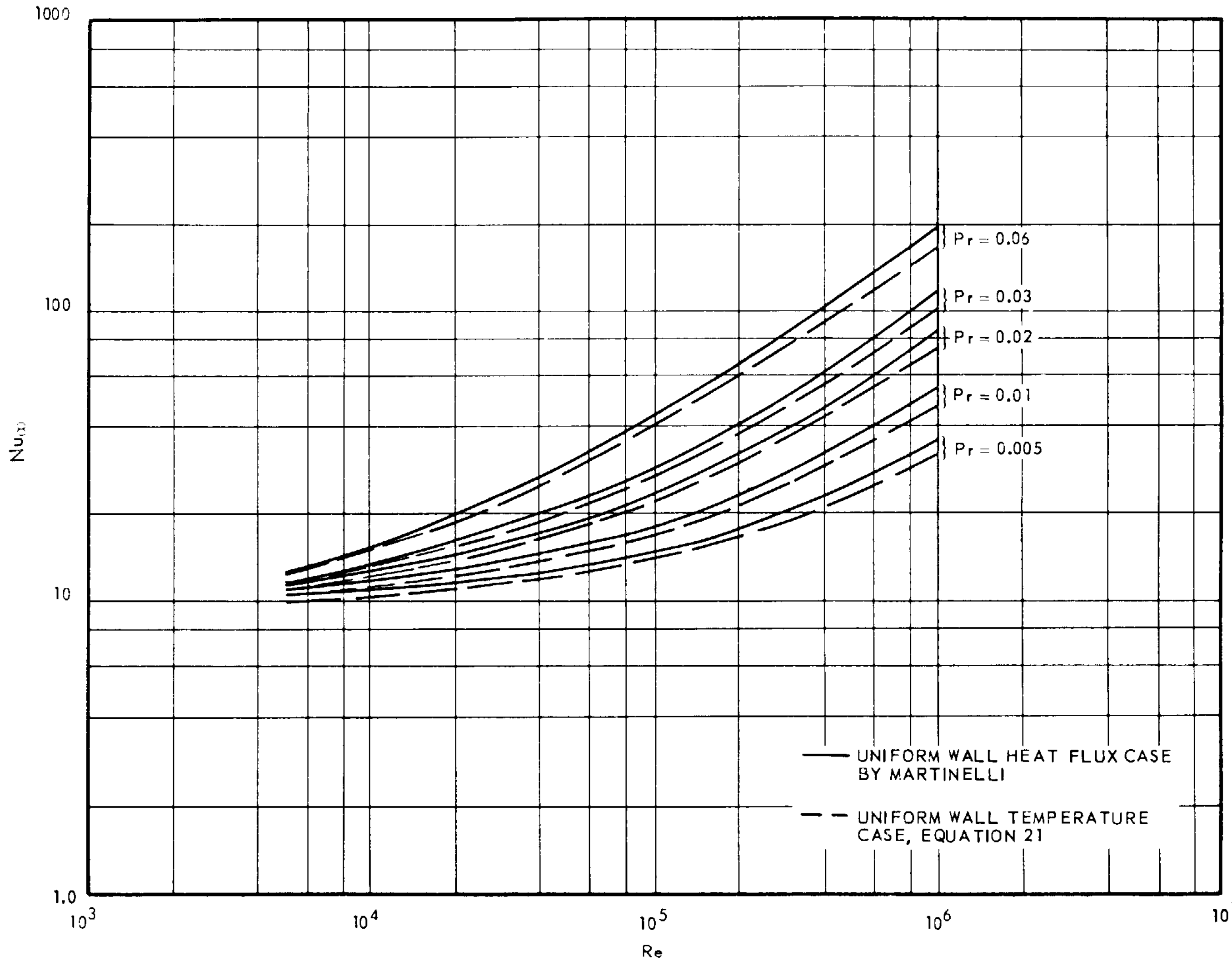

Figure 21.- Nu_∞ vs Re and Pr for uniform wall heat flux and uniform wall temperature cases.

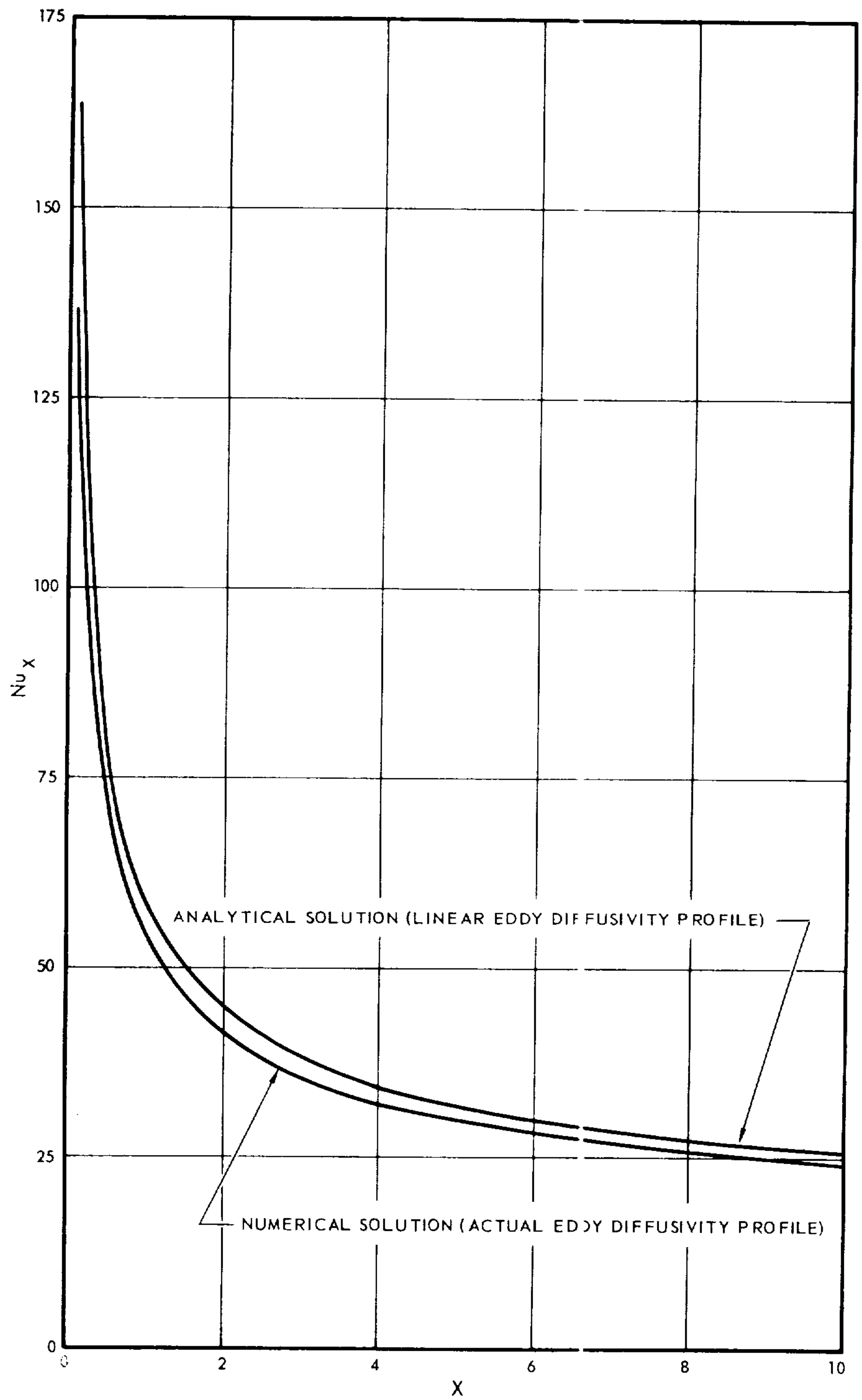

Figure 22.- Nu_X vs X for analytical and numerical solutions ($Re = 200{,}000$ and $Pr = 0.01$).

Comparison of Solution with Experimental Data

The only experimental heat transfer data that could be found for a uniform wall-temperature system were those of Harrison (8) for turbulently flowing mercury in small diameter pipes with short heating sections. These experimental data occurred in the initial part of the entrance region[3] where the asymptotic solutions (Appendix 5) hold. These solutions compared favorably with Harrison's experimental results (4) as shown in Figure 23.

Application

There are a number of practical high-heat-flux cooling systems to which the convection solution presented may be applied. One good example is a nuclear reactor core whose fuel elements are plates which are cooled by a liquid metal. It is possible to calculate ideal fission heat source distributributions in the axial direction in fuel plates of nuclear reactors which yield maximum uniform fuel plate temperature distributions. Maximum reactor powers would result from such axial temperature profiles.

Another example is a liquid-metal cooled target of an accelerator. For high-conductivity targets, the power that can be absorbed without exceeding limiting target temperatures can also be determined.

The solution is also applicable to special cooling problems encountered with missile nose cones and exhaust nozzles of propulsion systems; the excellent heat transfer that can be obtained in the thermal entrance regions of liquid-metal convection systems can be used to advantage. In such cases, the liquid metal would flow between plates (fins) which would be attached to the heat transfer surface to be cooled.

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3. Mean Nusselt moduli, Nu_{am}, in Harrison's system were from about two to four times higher than those for established flow values, Nu_{∞}.

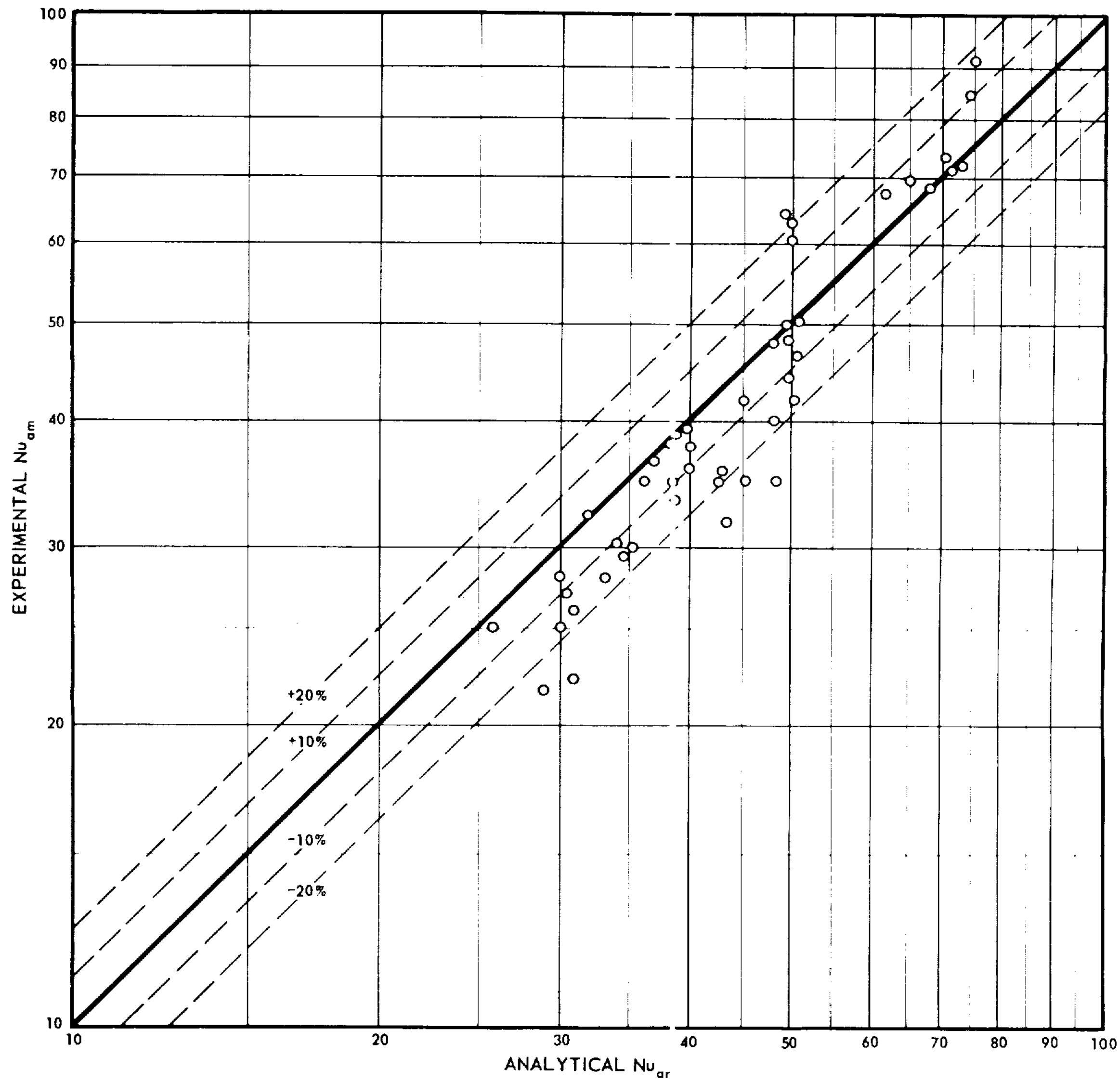

Figure 23.- A comparison of Harrison's experimental mercury heat transfer data with the asymptotic solution for a pipe with a uniform velocity profile.

ACKNOWLEDGMENT

The writer wishes to thank Miss Andrea B. Hart of General Atomic for her fine work in calculating the eigenvalues and series coefficients that arise in the solution and also for making most of the Nusselt modulus and temperature calculations that appear in the report.

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APPENDIX 1

LONGITUDINAL HEAT CONDUCTION

Mathematical convection solutions are usually based on the postulate that the longitudinal heat conduction term is negligible compared with the convection term. The validity of this premise was investigated for channel and pipe systems for low Prandtl modulus conditions and uniform velocity profiles (reference 4). Temperature solutions were derived for the case where the axial conduction term was not neglected and the results were compared to the solution for the case where axial conduction was neglected. The comparison revealed that axial conduction was relatively unimportant for turbulent flow conditions. For example, it was found that for a Prandtl modulus of 0.005 (which represents a practical minimum value for liquid metals) and the low turbulent Reynolds modulus of 8,000, the two Nusselt moduli for a pipe system differed by 2.5 percent at 0.4 of a diameter from the entrance, 1.3 percent at one diameter from the entrance, and a still smaller percentage at greater distances from the entrance.

APPENDIX 2

FLUID VELOCITY DISTRIBUTION

The blunt-nosed turbulent velocity distribution in a channel can be represented satisfactorily by a uniform distribution. In reference 4, entrance region heat transfer solutions for a channel were presented for low turbulent Reynolds modulus conditions for both uniform and blunt-nosed (one seventh power law) velocity distributions. The results are graphed in Figure 24. Note that the Nusselt moduli for these two profiles[4] differ from each other by percentages varying from about 6 to 20 over the wide range of Re Pr/X values shown. The higher difference-percentages correspond to the initial portion of the entrance region which normally represents only a small fraction of the total heat transfer surface in the entrance region. However, if one desires to calculate convective heat transfer in the very initial portion of the entrance region, where a non-uniform velocity profile should be used, the asymptotic solutions given in reference 4 can be used. At the higher turbulent Reynolds moduli, the actual turbulent velocity profiles become very flat (reference 7); in this region the idealized uniform velocity profile very closely represents the actual ones.

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4. The results for a third velocity profile, the parabola, are also graphed in Figure 24 for purposes of comparison.

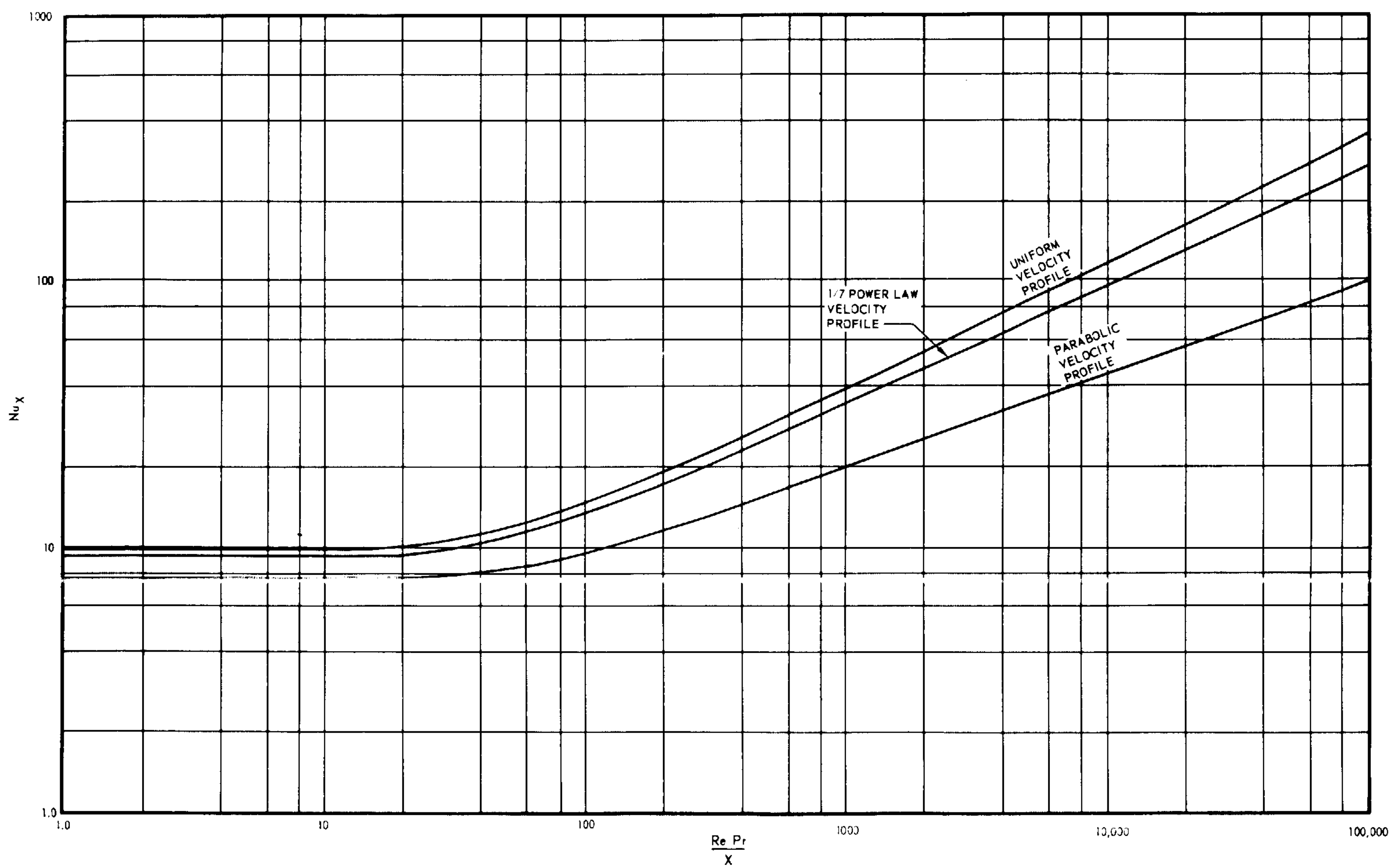

Figure 24.- Comparison of low Reynolds number (conduction) heat transfer solutions for uniform, one seventh power law, and parabolic velocity profiles.

APPENDIX 3

EDDY DIFFUSIVITY DISTRIBUTION

The general differential equation describing convective heat transfer for the idealized system described previously is

$$U \frac{\partial t}{\partial x} = \frac{\partial}{\partial y} \left[(a + \epsilon) \frac{\partial t}{\partial y} \right] \tag{27}$$

This equation can be expressed in a dimensionless form,

$$\frac{\partial T}{\partial X} = \frac{\partial}{\partial Y} \left[\frac{4}{Re} \left(\frac{1}{Pr} + \frac{\epsilon}{\nu} \right) \frac{\partial T}{\partial Y} \right] \tag{28}$$

The analogy between heat and momentum transfer has been firmly established in many experimental turbulent-flow systems. Thus it is postulated in this analysis that heat transfer and momentum transfer eddy diffusivities are identical. Momentum transfer eddy diffusivities (in dimensionless form) for a channel system can be represented as follows:[5]

$$\left.\begin{array}{lll}
\underline{\text{Laminar Sublayer}} & & \\
(0 < Y < 131.5/Re^{0.9}) & \frac{\epsilon}{\nu} = 0 & \\
\underline{\text{Buffer Layer}} & & \\
(131.5/Re^{0.9} < Y < 789/Re^{0.9}) & \frac{\epsilon}{\nu} = 0.0076\, Re^{0.9} \frac{y}{b} - 1 & \\
\underline{\text{Outer Turbulent Layer}} & & \\
(789/Re^{0.9} < Y < 0.5) & \frac{\epsilon}{\nu} = 0.0152\, Re^{0.9} \left(1 - \frac{y}{b}\right) \frac{y}{b} & \\
\underline{\text{Inner Turbulent Layer}} & & \\
(0.5 < Y < 1.0) & \frac{\epsilon}{\nu} = 0.0038\, Re^{0.9} &
\end{array}\right\} \tag{29}$$

5. These relations are obtained from the shear stress expression, the generalized velocity profile for turbulent flow, and the shear equation. The results pertain to smooth channels over a Reynolds modulus range of 5×10^{3} to 10^{6}.

Because the important heat transfer layers are those nearest the wall,[6] an idealized eddy diffusivity function is postulated which approximates the actual one in that region; the idealized eddy diffusivity relation is,

$$\frac{\epsilon}{\nu} = C_1 \, Re^{.9} \frac{y}{b} = 0.01 \, Re^{0.9} \frac{y}{b} \tag{30}$$

Note, from equation (28), the dimensionless eddy diffusivity always appears together with the reciprocal of the Prandtl modulus in the form of a sum, $\frac{1}{Pr} + \frac{\epsilon}{\nu}$. A comparison of this sum for the actual and idealized eddy diffusivity functions is shown in Figure 25 for a typical liquid-metal system. Note, that in the important outer half of the flow channel, the idealized sum, $\frac{1}{Pr} + \frac{\epsilon}{\nu}$, is a good approximation of the actual quantity. Upon substituting equation (30) into equation (28), equation (1) can be obtained.

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6. The heat transfer layers between the wall and about half the distance to the duct center are the important ones because a large fraction of the total radial temperature drop is found there. This is true because 1) the radial heat flows and 2) the thermal resistances are large in this region in comparison to the central duct core.

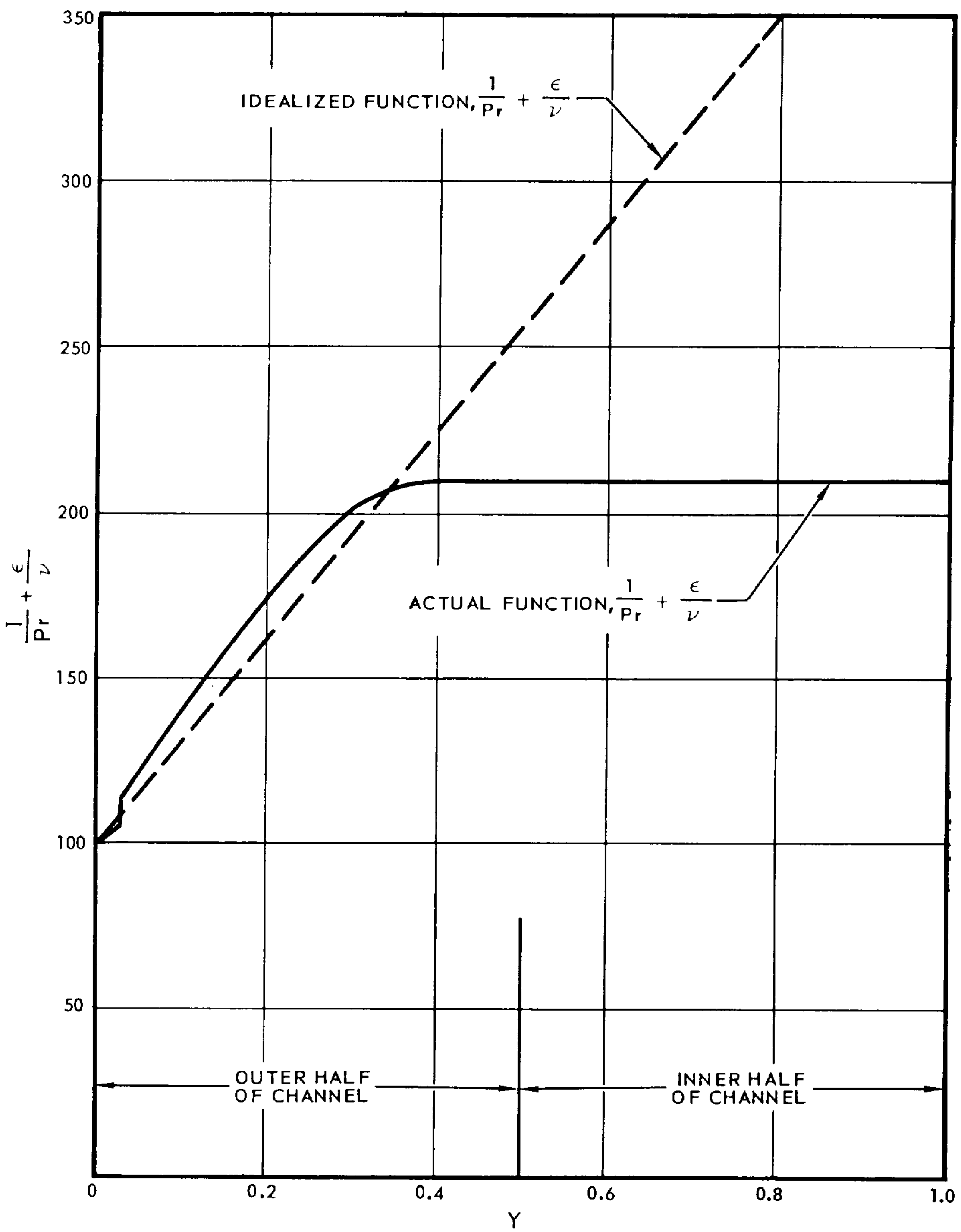

Figure 25.- Comparison of actual and idealized functions, $\frac{1}{Pr} + \frac{\epsilon}{\nu}$ for a channel with Re = 100,000 and Pr = 0.01.

APPENDIX 4

SERIES EXPANSION OF f(r)

In equation (13) it was found necessary to expand unity into series of $U_o\left(\frac{2\beta_n}{F_1} r\right)$ functions. The known procedure for doing this (see reference 9 for example) is discussed here for the case of a general function, f (r). It is desired to expand f (r) into a $U_o\left(\frac{2\beta_n}{F_1} r\right)$ series in the interval r_1 to r_2,

$$f(r) = \sum_{n=1}^{\infty} A_n U_o\left(\frac{2\beta_n}{F_1} r\right) \tag{31}$$

If both sides of equation (31) are multiplied by the term, $r U_o\left(\frac{2\beta_n}{F_1} r\right)$, and the equation is integrated from r_1 to r_2, all integrals involving $U_o\left(\frac{2\beta_n}{F_1} r\right)$ terms with two different values of β_n are zero. Only the integral having $U_o\left(\frac{2\beta_n}{F_1} r\right)$ terms with the same values of β_n are not zero. Thus the equation for the series coefficient is found to be

$$A_n = \frac{\int_{r_1}^{r_2} r f(r) U_o\left(\frac{2\beta_n}{F_1} r\right) dr}{\int_{r_1}^{r_2} r U_o^2\left(\frac{2\beta_n}{F_1} r\right) dr} \tag{32}$$

The proof that the integral $\int_{r_1}^{r_2} r U_o\left(\frac{2\beta_n}{F_1} r\right) U_o\left(\frac{2\beta'_n}{F_1} r\right) dr$ is equal to zero is outlined below. The quantities β_n and β'_n represent two different eigenvalues. The proof usually involves writing down two differential Bessel equations for two different solutions, one in terms of β_n and another in terms of β'_n. Upon multiplying each equation by the solution of the other, subtracting, and integrating over the range r_1 to r_2, there results

$$\int_{r_1}^{r_2} r\, U_o\left(\frac{2\beta_n}{F_1} r\right) U_o\left(\frac{2\beta'_n}{F_1} r\right) dr$$

$$= \frac{\left[r \left(\frac{dU_o\left(\frac{2\beta_n}{F_1} r\right)}{dr} \cdot U_o\left(\frac{2\beta'_n}{F_1} r\right) - U_o\left(\frac{2\beta_n}{F_1} r\right) \frac{dU_o\left(\frac{2\beta'_n}{F_1} r\right)}{dr} \right) \right]_{r_1}^{r_2}}{\frac{4}{F_1^2}\left(\beta'^2_n - \beta^2_n\right)} \tag{33}$$

Upon substitution of equations (5) and (12) into the bracket of equation (33), the integral is found to be,

$$\int_{r_1}^{r_2} r\, U_o\left(\frac{2\beta_n}{F_1} r\right) U_o\left(\frac{2\beta'_n}{F_1} r\right) dr = 0 \tag{34}$$

The integral $\int_{r_1}^{r_2} r\, U_o^2\left(\frac{2\beta_n}{F_1} r\right) dr$ is evaluated as follows. If the original differential Bessel equation is multiplied by $2r^2 \frac{dU_o\left(\frac{2\beta_n}{F_1} r\right)}{dr}$, and the resulting equation is integrated by parts over the range r_1 to r_2, there results,

$$\int_{r_1}^{r_2} r\, U_o^2\left(\frac{2\beta_n}{F_1} r\right) dr$$

$$= \frac{F_1^2}{8\beta_n^2} \left[r^2 \left(\frac{dU_o\left(\frac{2\beta_n}{F_1} r\right)}{dr} \right)^2 + \frac{4\beta_n^2}{F_1^2} r^2 U_o^2 \left(\frac{2\beta_n}{F_1} r\right) \right]_{r_1}^{r_2} \tag{35}$$

where the limits for the specific problem being studied are

$$r_2 = \sqrt{F_o + F_1} \quad \text{and} \quad r_1 = \sqrt{F_o}$$

APPENDIX 5

ASYMPTOTIC CONVECTION SOLUTION

Equation (17) converges very slowly for small values of X. A general asymptotic solution has been derived (reference 4) which can be used to evaluate the temperature and heat transfer in this region. For small values of X, the turbulent flow system being studied here reduces to the case of convection over a single flat plate with radial heat flow being achieved entirely by conduction. This is true because for small values of X 1) the influence of heat flow from the other wall would not exist and 2) the thermal boundary layer would not have diffused into the turbulent flow region. The boundary conditions for the asymptotic case are

$$\left.\begin{aligned} T(X, 0) &= 0 \\ \lim_{X \to \infty} T(X, Y) &= 0 \\ T(0, Y) &= 1 \\ \lim_{Y \to \infty} T(X, Y) &= 1 \end{aligned}\right\} \tag{36}$$

The solution of equation (1) together with the boundary equations (36) can be accomplished by making a change in variable,

$$S = c \frac{y}{x^p} \tag{37}$$

where p and c are constants to be determined. Upon substituting equation (37) into equation (1), the solution for the boundary equations (36) is found to be (reference 10)

$$T = \frac{\int_0^w e^{-w^2} dw}{\Gamma\left(\frac{3}{2}\right)} \tag{38}$$

where

$$w = \sqrt{\frac{Ub}{4\,a\,X}} \quad Y \tag{39}$$

Further it can be shown that for small values of X,

$$Nu_X = \frac{Re^{1/2}\ Pr^{1/2}}{\Gamma\left(\frac{3}{2}\right)\ X^{1/2}} \tag{40}$$

and

$$Nu_{am} = \frac{2\ Re^{1/2}\ Pr^{1/2}}{\Gamma\left(\frac{3}{2}\right)\ X^{1/2}} \tag{41}$$

CONVAIR,
A Division of General Dynamics Corporation,
San Diego, Calif., May 1958.

APPENDIX 6

CALCULATED RESULTS OBTAINED FOR FIGURES 2 through 20

TABLE I

Nu_X, T_m, Nu_{am} vs X and Re for Pr = 0.002

	Re = 5000			Re = 10,000			Re = 20,000		
X	Nu_X	T_m	Nu_{am}	Nu_X	T_m	Nu_{am}	Nu_X	T_m	Nu_{am}
0.1	14.635	0.774	25.445	19.091	0.840	34.691	25.616	0.887	47.844
0.2	11.796	0.680	19.192	14.690	0.773	25.720	19.181	0.840	35.086
0.5	10.151	0.494	13.648	11.252	0.641	17.610	13.730	0.746	23.446
1.0	9.997	0.299	10.850	10.248	0.492	13.687	11.389	0.639	17.680
2.0	9.994	0.110	8.190	10.102	0.296	10.976	10.427	0.489	13.892
5.0	9.994	0.005	4.067	10.100	0.065	7.118	10.296	0.225	10.212
10.0	9.994		2.056	10.100	0.005	4.015	10.296	0.062	7.127
20.0	9.994		1.028	10.100		2.044	10.296	0.005	4.111
50.0	9.994		0.411						

	Re = 50,000			Re = 100,000			Re = 200,000		
X	Nu_X	T_m	Nu_{am}	Nu_X	T_m	Nu_{am}	Nu_X	T_m	Nu_{am}
0.1	38.714	0.928	74.022	52.980	0.949	103.567	75.219	0.964	145.348
0.2	28.460	0.898	53.792	38.992	0.929	74.739	54.415	0.949	105.087
0.5	19.427	0.839	35.278	26.191	0.886	48.687	36.009	0.919	68.044
1.0	15.072	0.771	25.978	19.805	0.837	35.513	26.836	0.884	49.313
2.0	12.331	0.674	19.693	15.500	0.768	26.508	20.503	0.835	36.455
5.0	10.912	0.480	14.191	12.340	0.628	18.461	15.319	0.733	24.845
10.0	10.818	0.279	11.352	11.636	0.467	14.638	13.407	0.615	19.218
20.0	10.818	0.095	8.480	11.578	0.261	11.862	12.915	0.444	15.587
50.0	10.818	0.004	4.096	11.578	0.046	7.415	12.890	0.169	11.498

TABLE I (Continued)

	Re = 500,000			Re = 1,000,000		
0.1	117.160	0.977	228.291	162.481	0.984	321.782
0.2	85.209	0.968	164.746	120.353	0.977	231.606
0.5	56.014	0.948	106.521	79.211	0.963	150.089
1.0	41.376	0.926	76.978	58.427	0.947	102.545
2.0	31.149	0.894	56.610	43.851	0.924	76.810
5.0	22.390	0.827	38.088	31.192	0.875	52.386
10.0	18.447	0.749	28.928	25.214	0.817	39.915
20.0	16.502	0.630	22.939	21.703	0.728	31.515
50.0	16.074	0.388	17.779	20.333	0.534	24.363
100.0	16.070	0.174	14.173	20.292	0.321	20.575
200.0	16.070	0.035	9.658	20.292	0.116	16.152

TABLE II

Nu_X, T_m, Nu_{am} vs X and Re for Pr = 0.005

	Re = 5000			Re = 10,000			Re = 20,000		
X	Nu_X	T_m	Nu_{am}	Nu_X	T_m	Nu_{am}	Nu_X	T_m	Nu_{am}
0.1	20.946	0.857	38.434	28.297	0.899	53.157	38.840	0.928	74.022
0.2	15.957	0.797	28.386	21.062	0.856	38.890	28.499	0.898	53.834
0.5	11.912	0.679	19.266	14.866	0.772	25.850	19.470	0.839	35.319
1.0	10.500	0.545	14.818	12.079	0.677	19.360	15.120	0.771	26.021
2.0	10.189	0.361	11.860	10.722	0.541	15.069	12.390	0.673	19.740
5.0	10.178	0.106	8.169	10.438	0.288	11.155	10.994	0.478	14.246
10.0	10.178	0.014	4.920	10.437	0.101	8.225	10.905	0.277	11.403
20.0	10.178		2.566	10.437	0.013	5.062	10.905	0.093	8.507
50.0	10.178			10.437			10.905	0.004	4.099
100.0							10.904		2.064

	Re = 50,000			Re = 100,000			Re = 200,000		
X	Nu_X	T_m	Nu_{am}	Nu_X	T_m	Nu_{am}	Nu_X	T_m	Nu_{am}
0.1	59.993	0.954	115.467	81.601	0.967	162.221	117.587	0.977	228.294
0.2	43.546	0.935	83.618	60.207	0.954	116.560	85.479	0.968	164.920
0.5	29.050	0.897	54.329	40.166	0.927	75.567	56.285	0.948	106.752
1.0	21.871	0.854	39.542	29.893	0.896	54.835	41.655	0.926	77.232
2.0	16.989	0.791	29.425	22.773	0.851	40.552	31.437	0.893	56.877
5.0	13.245	0.664	20.359	16.869	0.759	27.607	22.707	0.826	38.375
10.0	12.232	0.517	16.042	14.553	0.650	21.294	18.811	0.746	29.237
20.0	12.108	0.318	13.088	13.838	0.491	17.238	16.948	0.626	23.280
50.0	12.106	0.074	8.734	13.785	0.215	13.046	16.573	0.380	18.128
100.0	12.106	0.007	5.004	13.785	0.054	9.050	16.571	0.166	14.406
200.0				13.785	0.003	5.148			

TABLE II (Continued)

	Re = 500,000			Re = 1,000,000		
0.1	181.784	0.985	359.461	261.084	0.990	507.275
0.2	135.270	0.979	258.100	193.337	0.985	364.570
0.5	89.326	0.967	168.283	128.054	0.976	238.232
1.0	66.078	0.952	122.016	95.069	0.966	173.521
2.0	49.748	0.931	89.966	71.831	0.950	128.492
5.0	35.548	0.886	60.667	51.458	0.917	87.125
10.0	28.809	0.832	46.037	41.571	0.876	66.328
20.0	24.796	0.749	36.276	35.281	0.812	52.284
50.0	23.141	0.564	28.057	31.852	0.667	40.252
100.0	23.083	0.356	23.894	31.582	0.486	34.753
200.0	23.082	0.141	19.215	31.578	0.258	29.857

TABLE III

Nu_X, T_m, Nu_{am} vs X and Re for Pr = 0.01

X	Re = 5000 Nu_X	Re = 5000 T_m	Re = 5000 Nu_{am}	Re = 10,000 Nu_X	Re = 10,000 T_m	Re = 10,000 Nu_{am}	Re = 20,000 Nu_X	Re = 20,000 T_m	Re = 20,000 Nu_{am}
0.1	28.314	0.899	53.158	38.869	0.928	74.023	53.900	0.943	103.867
0.2	21.080	0.856	38.899	28.530	0.898	53.849	39.251	0.928	77.254
0.5	14.887	0.772	25.865	19.504	0.838	35.344	26.322	0.885	49.782
1.0	12.105	0.677	19.379	15.159	0.770	26.051	19.946	0.837	36.141
2.0	10.756	0.540	15.091	12.438	0.673	19.776	15.661	0.767	26.905
5.0	10.478	0.287	11.178	11.061	0.477	14.290	12.557	0.625	18.735
10.0	10.476	0.101	8.237	10.975	0.275	11.443	11.901	0.462	14.880
20.0	10.476	0.012	5.064	10.975	0.092	8.529	11.852	0.255	12.060
50.0	10.476		2.084	10.975	0.003	4.100	11.852	0.043	7.479
100.0				10.975		2.065	11.852	0.002	4.060

X	Re = 50,000 Nu_X	Re = 50,000 T_m	Re = 50,000 Nu_{am}	Re = 100,000 Nu_X	Re = 100,000 T_m	Re = 100,000 Nu_{am}	Re = 200,000 Nu_X	Re = 200,000 T_m	Re = 200,000 Nu_{am}
0.1	84.121	0.968	162.197	117.917	0.977	228.295	164.504	0.984	321.786
0.2	60.889	0.954	117.356	85.697	0.967	165.058	121.320	0.977	232.355
0.5	40.296	0.927	76.052	56.506	0.948	106.937	80.098	0.963	150.927
1.0	30.022	0.896	55.144	41.882	0.926	77.436	59.332	0.947	109.445
2.0	22.911	0.851	40.775	31.673	0.893	57.095	44.766	0.923	80.746
5.0	17.031	0.758	27.786	22.967	0.825	38.608	32.154	0.873	54.551
10.0	14.754	0.648	21.470	19.108	0.744	29.488	26.256	0.812	41.511
20.0	14.077	0.487	17.419	17.312	0.622	23.559	22.914	0.720	32.872
50.0	14.031	0.210	13.182	16.975	0.373	18.410	21.774	0.517	25.633
100.0	14.031	0.052	9.098	16.973	0.160	14.589	21.750	0.300	21.639
200.0	14.031	0.003	5.152	16.973	0.029	7.777	21.750	0.101	16.742

TABLE III (Continued)

	Re = 500,000			Re = 1,000,000		
0.1	265.322	0.990	507.282	370.444	0.993	716.336
0.2	194.937	0.985	365.502	278.897	0.989	515.574
0.5	128.944	0.976	239.176	186.839	0.983	340.163
1.0	95.944	0.965	174.432	139.989	0.975	249.843
2.0	72.703	0.950	129.384	106.878	0.963	186.646
5.0	52.336	0.916	88.005	77.645	0.938	128.243
10.0	42.478	0.874	67.211	63.250	0.906	98.678
20.0	36.262	0.809	53.190	53.798	0.858	78.533
50.0	33.022	0.660	41.231	47.937	0.737	61.007
100.0	32.798	0.475	35.729	47.246	0.581	53.178
200.0	32.796	0.247	30.636	47.227	0.362	47.216

TABLE IV

Nu_X, T_m, Nu_{am} vs X and Re for Pr = 0.02

	Re = 5000			Re = 10,000			Re = 20,000		
X	Nu_X	T_m	Nu_{am}	Nu_X	T_m	Nu_{am}	Nu_X	T_m	Nu_{am}
0.1	38.901	0.928	74.024	54.003	0.949	103.562	75.552	0.964	145.354
0.2	28.563	0.898	53.866	39.310	0.928	75.106	54.740	0.949	105.253
0.5	19.541	0.838	35.371	26.385	0.885	48.940	36.347	0.918	68.307
1.0	15.200	0.770	26.084	20.015	0.836	35.740	27.192	0.884	49.617
2.0	12.490	0.672	19.814	15.739	0.766	26.732	20.889	0.833	36.790
5.0	11.132	0.476	14.337	12.661	0.623	18.708	15.785	0.730	25.226
10.0	11.050	0.274	11.486	12.028	0.460	14.909	13.995	0.608	19.651
20.0	11.050	0.091	8.552	11.983	0.252	12.102	13.592	0.431	16.054
50.00	11.050	0.003	4.102	11.982	0.042	7.484	13.577	0.156	11.805
100.00	11.050		2.065	11.982	0.002	4.054	13.577	0.029	7.632
200.00	11.050						13.577	0.001	4.121

	Re = 50,000			Re = 100,000			Re = 200,000		
X	Nu_X	T_m	Nu_{am}	Nu_X	T_m	Nu_{am}	Nu_X	T_m	Nu_{am}
0.1	118.254	0.977	228.298	165.391	0.984	321.788	230.632	0.988	454.009
0.2	85.932	0.967	165.202	121.782	0.977	232.693	173.398	0.983	328.014
0.5	56.744	0.948	107.136	80.527	0.963	151.323	115.419	0.973	214.477
1.0	42.125	0.925	77.655	59.763	0.946	109.858	85.944	0.961	156.348
2.0	31.925	0.892	57.328	45.204	0.923	81.169	65.203	0.944	115.965
5.0	23.244	0.824	38.859	32.615	0.872	54.989	47.088	0.906	78.940
10.0	19.427	0.742	29.757	26.756	0.811	41.966	38.403	0.860	60.384
20.0	17.700	0.618	23.856	23.497	0.717	33.360	33.089	0.788	47.943
50.0	17.399	0.366	18.706	22.454	0.510	26.162	30.654	0.623	37.441
100.00	17.398	0.153	14.777	22.436	0.291	22.089	30.540	0.425	32.425
200.00	17.398	0.269	9.826	22.436	0.095	16.976	30.539	0.198	27.225

TABLE IV (Continued)

	Re = 500,000			Re = 1,000,000		
0.1	374.095	0.993	716.339	530.791	0.995	1011.982
0.2	278.338	0.989	521.903	400.613	0.992	740.276
0.5	188.504	0.983	344.165	276.416	0.988	493.500
1.0	141.639	0.975	252.679	209.772	0.981	365.558
2.0	108.505	0.963	188.886	162.422	0.973	275.840
5.0	79.257	0.937	130.110	120.267	0.953	192.626
10.0	64.878	0.905	100.419	99.217	0.928	150.258
20.0	55.500	0.853	80.231	85.051	0.887	121.141
50.0	49.898	0.730	62.756	75.537	0.788	95.454
100.0	49.315	0.570	54.956	74.088	0.654	83.968
200.0	49.302	0.348	48.796	74.027	0.452	76.038

TABLE V

Nu_X, T_m, Nu_{am} vs X and Re for Pr = 0.03

X	Re = 5000 Nu_X	Re = 5000 T_m	Re = 5000 Nu_{am}	Re = 10,000 Nu_X	Re = 10,000 T_m	Re = 10,000 Nu_{am}	Re = 20,000 Nu_X	Re = 20,000 T_m	Re = 20,000 Nu_{am}
0.1	47.107	0.941	90.050	65.725	0.958	126.237	92.233	0.970	177.430
0.2	34.403	0.917	65.397	47.705	0.941	91.477	66.814	0.958	128.482
0.5	23.258	0.868	42.742	31.812	0.906	59.468	44.243	0.933	83.348
1.0	17.808	0.811	31.330	23.928	0.866	43.292	32.973	0.904	60.478
2.0	14.227	0.731	23.572	18.546	0.808	32.212	25.161	0.862	44.747
5.0	11.919	0.567	16.719	14.345	0.689	22.267	18.659	0.776	30.512
10.0	11.608	0.384	13.436	13.108	0.550	17.524	16.063	0.673	23.575
20.0	11.597	0.177	10.666	12.925	0.357	14.366	15.217	0.520	19.134
50.0	11.597	0.017	59.261	12.922	0.098	9.976	15.147	0.243	14.718
100.0	11.597		30.674	12.922	0.011	5.938	15.147	0.069	10.535
200.0				12.922		3.077			

X	Re = 50,000 Nu_X	Re = 50,000 T_m	Re = 50,000 Nu_{am}	Re = 100,000 Nu_X	Re = 100,000 T_m	Re = 100,000 Nu_{am}	Re = 200,000 Nu_X	Re = 200,000 T_m	Re = 200,000 Nu_{am}
0.1	144.303	0.981	279.024	201.771	0.987	393.530	291.063	0.990	555.455
0.2	105.508	0.973	201.971	150.090	0.981	284.734	215.475	0.986	401.810
0.5	69.756	0.957	131.237	99.657	0.969	185.850	143.784	0.978	264.131
1.0	51.808	0.938	95.246	74.171	0.956	135.313	107.593	0.968	193.403
2.0	39.248	0.911	70.381	56.265	0.936	100.268	82.075	0.953	144.125
5.0	28.457	0.854	47.734	40.693	0.893	68.203	59.683	0.921	98.797
10.0	23.545	0.784	36.513	33.324	0.841	52.188	48.833	0.881	75.988
20.0	21.024	0.677	29.165	28.994	0.759	41.523	42.011	0.818	60.603
50.0	20.399	0.449	22.974	27.308	0.575	32.596	38.522	0.671	47.506
100.0	20.394	0.228	18.984	27.257	0.365	28.043	38.295	0.488	41.499
200.0							38.293	0.258	35.891

TABLE V (Continued)

	Re = 500,000			Re = 1,000,000		
0.1	461.719	0.994	876.753	659.178	0.996	1238.846
0.2	349.953	0.991	635.124	509.478	0.994	900.769
0.5	237.019	0.986	423.032	350.289	0.990	607.884
1.0	179.394	0.979	313.233	268.095	0.985	455.143
2.0	138.547	0.969	236.111	209.570	0.977	346.995
5.0	102.303	0.947	164.519	157.162	0.960	244.708
10.0	84.325	0.918	128.102	130.744	0.937	193.682
20.0	72.422	0.872	103.174	112.764	0.901	157.674
50.0	64.919	0.762	81.379	100.256	0.811	125.578
100.0	63.994	0.615	71.680	98.114	0.688	111.121
200.0	63.967	0.402	64.513			

TABLE VI

Nu_X, T_m, Nu_{am} vs X and Re for Pr = 0.06

Re = 5000

X	Nu_X	T_m	Nu_{am}
0.1	65.819	0.958	126.239
0.2	47.798	0.941	91.524
0.5	31.909	0.906	59.544
1.0	24.031	0.865	43.379
2.0	18.660	0.807	32.309
5.0	14.488	0.688	22.379
10.0	13.289	0.548	17.652
20.0	13.120	0.353	14.495
50.0	13.117	0.095	10.036
100.0	13.117	0.011	5.947
200.0	13.117		3.076

Re = 10,000

X	Nu_X	T_m	Nu_{am}
0.1	92.419	0.970	177.433
0.2	66.981	0.958	128.571
0.5	44.444	0.933	83.484
1.0	33.152	0.904	60.623
2.0	25.350	0.862	44.916
5.0	18.876	0.775	30.699
10.0	16.327	0.671	23.782
20.0	15.530	0.516	19.362
50.0	15.470	0.238	14.903
100.0	15.470	0.066	10.609
200.0	15.470	0.005	6.167

Re = 20,000

X	Nu_X	T_m	Nu_{am}
0.1	129.936	0.979	249.843
0.2	94.619	0.970	181.066
0.5	62.632	0.952	117.688
1.0	46.606	0.931	85.467
2.0	35.417	0.901	63.229
5.0	25.874	0.837	43.008
10.0	21.648	0.759	33.034
20.0	19.686	0.641	26.562
50.0	19.320	0.395	20.997
100.0	19.318	0.176	16.905
200.0	19.318	0.035	11.580

Re = 50,000

X	Nu_X	T_m	Nu_{am}
0.1	203.189	0.987	393.532
0.2	150.829	0.981	285.274
0.5	100.329	0.969	186.476
1.0	74.843	0.956	135.962
2.0	56.940	0.935	100.928
5.0	41.388	0.892	68.876
10.0	34.060	0.839	52.878
20.0	29.826	0.755	42.249
50.0	28.279	0.567	33.382
100.0	28.240	0.354	28.756
200.0	28.240	0.138	23.213

Re = 100,000

X	Nu_X	T_m	Nu_{am}
0.1	284.858	0.990	555.472
0.2	215.934	0.986	402.932
0.5	144.996	0.978	265.348
1.0	108.793	0.968	194.614
2.0	83.264	0.953	145.328
5.0	60.874	0.920	99.989
10.0	50.055	0.879	77.183
20.0	43.327	0.815	61.826
50.0	40.060	0.664	48.815
100.0	39.878	0.476	42.788
200.0	39.877	0.245	36.908

Re = 200,000

X	Nu_X	T_m	Nu_{am}
0.1	416.802	0.993	784.469
0.2	313.177	0.990	570.102
0.5	211.722	0.984	379.234
1.0	160.224	0.977	280.470
2.0	123.764	0.965	211.240
5.0	91.500	0.941	147.123
10.0	75.608	0.909	114.600
20.0	65.275	0.858	92.441
50.0	59.272	0.736	73.280
100.0	58.704	0.576	64.743
200.0	58.693	0.353	57.862

TABLE VI (Continued)

	Re = 500,000			Re = 1,000,000		
0.1	655.601	0.996	1238.891	964.916	0.997	1750.927
0.2	507.384	0.994	901.786	758.037	0.995	1289.515
0.5	354.279	0.990	609.881	532.192	0.992	887.962
1.0	272.720	0.984	458.391	414.287	0.988	675.753
2.0	214.054	0.977	350.894	305.733	0.982	504.283
5.0	161.454	0.959	249.886	234.323	0.968	365.521
10.0	134.948	0.936	197.884	213.736	0.950	304.974
20.0	116.997	0.898	161.876	186.447	0.920	252.498
50.0	104.881	0.805	129.859	166.552	0.843	204.918
100.0	103.042	0.678	115.533	162.677	0.736	183.219
200.0	102.966	0.481	105.743	162.428	0.561	169.362