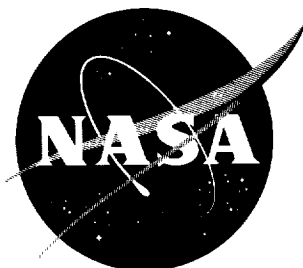


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# TECHNICAL NOTE

D-470

ANALYSIS OF TRAJECTORY PARAMETERS FOR PROBE AND  
ROUND-TRIP MISSIONS TO VENUS

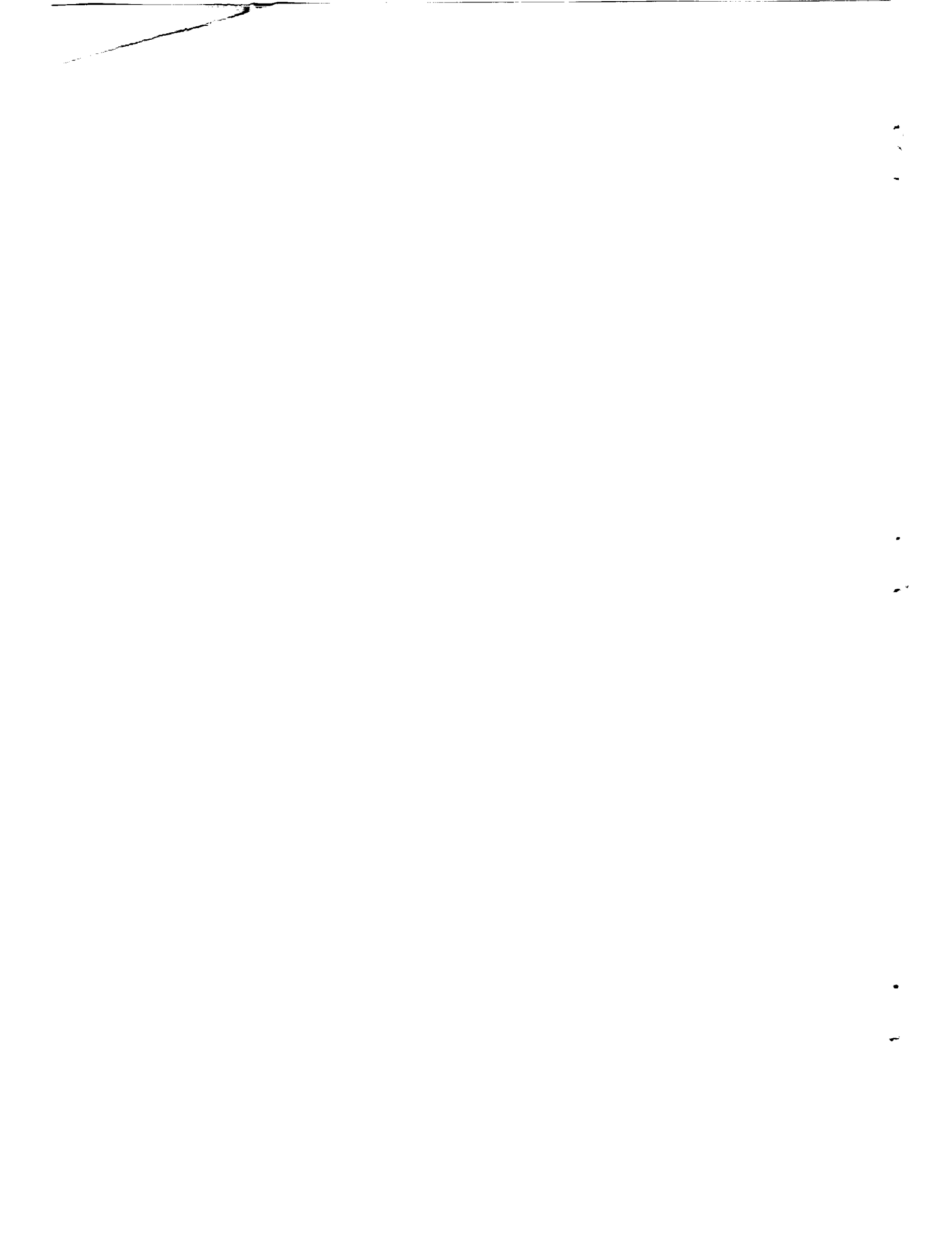
By James F. Dugan, Jr., and Carl R. Simsic

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Cleveland, Ohio

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION  
WASHINGTON

November 1960



NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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TECHNICAL NOTE D-470

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ANALYSIS OF TRAJECTORY PARAMETERS FOR PROBE AND  
ROUND-TRIP MISSIONS TO VENUS

By James F. Dugan, Jr., and Carl R. Simsic

SUMMARY

For one-way transfers between Earth and Venus, charts are obtained that show velocity, time, and angle parameters as functions of the eccentricity and semilatus rectum of the Sun-focused vehicle conic. From these curves, others are obtained that are useful in planning one-way and round-trip missions to Venus. The analysis is characterized by circular coplanar planetary orbits, successive two-body approximations, impulsive velocity changes, and circular parking orbits at 1.1 planet radii. For round trips the mission time considered ranges from 65 to 788 days, while wait time spent in the parking orbit at Venus ranges from 0 to 467 days. Individual velocity increments, one-way travel times, and departure dates are presented for round trips requiring the minimum total velocity increment.

For both single-pass and orbiting Venusian probes, the time span available for launch becomes appreciable with only a small increase in velocity-increment capability above the minimum requirement. Velocity-increment increases are much more effective in reducing travel time for single-pass probes than they are for orbiting probes. Round trips composed of a direct route along an ellipse tangent to Earth's orbit and an aphelion route result in the minimum total velocity increment for wait times less than 100 days and mission times ranging from 145 to 612 days. Minimum-total-velocity-increment trips may be taken along perihelion-perihelion routes for wait times ranging from 300 to 467 days. These wait times occur during missions lasting from 640 to 759 days.

INTRODUCTION

In planning any space mission, a trajectory analysis is a prerequisite for deciding upon specific features of the mission. The analysis presents interrelations among such parameters as departure date, elapsed time for separate phases of the mission, total mission time, individual

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and total velocity increments, and Earth-vehicle separation distance at different times during the mission. The velocity parameters influence the choice of propulsion system and permit preliminary estimates of propellant requirements. Time and separation-distance parameters have a bearing on reliability and communication considerations.

Some of the trajectory information needed to plan various missions to Venus is found in references 1 to 6. In this report, charts of the kind suggested by Vertregt in reference 7 are presented for one-way trajectories between Earth and Venus. These charts facilitate the preliminary planning of one-way missions and give approximate values of launch date, duration of voyage, and required velocity increments. Modifications of these charts were used to find the interrelations among trajectory parameters of interest for many one-way single-pass, one-way orbiting, and round-trip missions between Earth and Venus.

The equations of reference 7 and several additional equations needed to obtain parameters useful in round-trip calculations were programmed for a digital computer. Tables of trajectory-parameter data were then obtained for trips between Earth and Venus. The trajectory charts suggested by Vertregt were constructed from these tables. The velocity, time, and angle parameters were plotted as functions of the semilatus rectum and eccentricity of the Sun-focused vehicle conic.

Working curves, in which the parameter of interest appears as the ordinate, were then used to obtain information for a wide range of one-way and round-trip missions between Earth and Venus. For single-pass and orbiting Venusian probes, the relations among departure date, travel time, total velocity increment, and Earth-Venus separation distance when the probe arrives at Venus are presented. Probe and round-trip missions requiring minimum velocity increment are discussed. Variations of travel time, departure date, and individual velocity increment with mission time are presented for round trips with 0-, 50-, and 100-day wait times at Venus.

Because of the simplifying assumptions, these results should be used only during the preliminary planning of a particular Venus mission. An analysis based on the n-body equations of motion and a three-dimensional model of the solar system must be used for more exact results. In spite of their approximate nature, the results of this report do show the interrelations among the important trajectory parameters, and they should serve as a useful guide in making more precise calculations.

A similar analysis and set of working curves for probe and round-trip missions to Mars are presented in reference 8.

## ANALYSIS

### Assumptions

The following simplifying assumptions were made:

(1) The planets describe circular orbits around the Sun. The orbits of Earth, Venus, and the space vehicle are coplanar.

(2) The vehicle is attracted by only one inverse-square central force field at a time. When "near" a planet, the vehicle is attracted by the planet; when "far" from the planets, it is attracted only by the Sun.

(3) The impulsive-velocity-increment concept is used; that is, the time during which thrust is applied to the vehicle is insignificant compared with the total duration of the voyage.

(4) The travel time spent under the influence of the planets is negligible compared with the travel time spent under the influence of the Sun.

### Possible Heliocentric Trajectories

In traveling between Earth and Venus (or any two planets), an infinite number of Sun-focused conic sections may be traversed. For a given ellipse, which will in general intersect the orbits of Earth and Venus at four points, four alternative routes can be considered (fig. 1(a)). (A list of symbols appears in appendix A; fig. 1(b) aids in defining some of the trajectory angles.) The direct route D from Earth to Venus is along 1-2; the perihelion route P along 1-2-3; the aphelion route A along 4-1-2; and the indirect route I along 4-1-2-3. If the ellipse is tangent to Earth's orbit (fig. 1(a)), routes D and A become identical, and also routes P and I. Similarly, if the ellipse is tangent to Venus' orbit, routes D and P become identical, and also routes A and I. For a given parabolic or hyperbolic path, only direct and perihelion routes are possible.

### Charts of Eccentricity Against Semilatus Rectum

In reference 7, Vertregt shows how interplanetary trajectory parameters along all possible conics may be expressed as functions of the eccentricity and the semimajor axis of a Sun-focused conic, and of the radius of the destination planet's orbit. An addendum in reference 7 indicates that a considerable simplification results from constructing

diagrams for constant values of trajectory parameters as functions of eccentricity  $\epsilon$  and semilatus rectum  $p$  (instead of semimajor axis). However, only one of these diagrams is included by Vertregt to illustrate the advantages.

The equations from reference 7 and several additional equations, needed to obtain parameters useful in round-trip calculations, were programmed for a digital computer. Tables of trajectory-parameter data were then obtained for trips between Earth and Venus. The equations and the procedure for constructing these tables are found in appendix B. From the tables were constructed  $\epsilon$ - $p$  trajectory charts such as those shown in figures 2(a) to (c). A wide range of useful trajectories is shown within the arbitrary boundaries of  $p_{\max} = 2.0$  A.U. and  $\epsilon_{\max} = 2.0$ . A set of 22  $\epsilon$ - $p$  charts for Earth-Venus trajectories may be obtained upon request from NASA, Washington 25, D. C. The trajectory parameters included in these charts are:  $\Delta v_E$ ,  $\Delta v_V$ ,  $(\Delta v_E + \Delta v_V)$ ,  $V_E$ ,  $V_V$ ,  $(V_E + V_V)$ ,  $T_D$ ,  $T_P$ ,  $T_A$ ,  $T_I$ ,  $\psi_D$ ,  $\psi_P$ ,  $\psi_A$ ,  $\psi_I$ ,  $\lambda_D$ ,  $\lambda_P$ ,  $\lambda_A$ ,  $\lambda_I$ ,  $\phi_1$ ,  $\phi_2$ ,  $\alpha_1$ , and  $\alpha_2$ .

#### Probe Missions

To find the interrelations among trajectory parameters of interest for specific Earth-Venus missions, it was found convenient to use working curves in which the various parameters were plotted against eccentricity for constant values of semilatus rectum. These curves are not included in the report, since they depict the same information as contained in the  $\epsilon$ - $p$  charts. Because the procedure for analyzing the one-way single-pass missions is very similar to that for the one-way orbiting missions, only the former is discussed. Four trajectory parameters of special interest are: (1) velocity increment required near Earth when starting from a circular parking orbit of 1.1 Earth radii, (2) travel time, (3) configuration angle at departure, and (4) Earth-Venus separation distance at arrival. The configuration angle at departure defines the launch date (fig. 2(d)). The Hohmann date is defined as that date on which Venus' heliocentric longitude is less than Earth's by  $54.1^\circ$ , this being the configuration angle required for a Hohmann transfer from Earth to Venus.

For a given  $\Delta v_E$ , values of  $\epsilon$  and  $p$  are read from the working curve of  $\Delta v_E$  against  $\epsilon$  for constant  $p$ . These values of  $\epsilon$  and  $p$  permit values of  $\psi_{1,i}$  and  $T_i$  to be read from their respective working curves. The configuration angle when the probe arrives at Venus may be calculated from the following equation:

$$\psi_{2,i} = \psi_{1,i} + (\dot{\theta}_E - \dot{\theta}_V) T_i \quad (1)$$

where

$$\dot{\theta}_E = 0.9856 \text{ deg/day}$$

$$\dot{\theta}_V = 1.6021 \text{ deg/day}$$

Earth-Venus separation distance at arrival may then be found from figure 2(e), which is a plot of

$$d = (R_E^2 + R_V^2 - 2R_ER_V \cos \psi)^{1/2} \quad (2)$$

where

$$R_E = 92,900,000 \text{ miles (80,730,000 Int. naut. miles)}$$

and

$$R_V = 67,200,000 \text{ miles (58,395,000 Int. naut. miles)}$$

These data are then plotted as  $T$  against  $\psi_1$  for constant  $\Delta v_E$  (fig. 3(a)) and  $d_2$  against  $\psi_1$  for constant  $\Delta v_E$  (fig. 3(b)).

#### Round-Trip Missions

For round-trip missions to Venus, it is convenient to introduce an angle parameter  $\lambda$  defined by

$$\lambda_1 = \Delta\phi_1 - \dot{\theta}_E T_1 \quad (3)$$

Therefore,  $\lambda$  is the angular difference between the vehicle's travel and Earth's travel during a one-way trip between planet orbits. The relation that must be satisfied during a round-trip mission to Venus with a wait time  $T_w$  spent in a parking orbit around Venus is

$$\lambda_{out} + (\dot{\theta}_V - \dot{\theta}_E)T_w + \lambda_{back} = (N)360 \quad (4)$$

where  $N$  may be 0 or  $\pm$  an integer.

To illustrate the technique employed for round-trip missions, consider the combinations involving direct and aphelion routes. For a specific value of  $\lambda_D$ , values of  $\epsilon_D$  and  $p_D$  may be read from the working curve;  $T_D$  and  $(\Delta v_E + \Delta v_V)_D$  may then be read for one pair of  $\epsilon_D$  and  $\lambda_D$  values. For a specific  $T_w$ , the value of  $\lambda_A$  may be calculated from equation (4). The  $\lambda_A$  working curve then yields pairs of values for  $\epsilon_A$  and  $p_A$ . For each pair, values of  $(\Delta v_E + \Delta v_V)_A$  and  $T_A$  may be read

from their respective working curves. The total velocity increment and total mission time are calculated and then plotted as  $\Delta v_T$  against  $T_T$ . Curves may then be drawn for each pair of  $\epsilon_D$  and  $p_D$  values. The envelope of these curves forms a single curve for a specific  $\lambda_D$  value. This procedure is repeated for numerous  $\lambda_D$  values. An envelope of all the individual envelope curves may then be drawn for the complete direct-aphelion round trip.

By repeating the entire procedure for other combinations of D, P, A, and I routes, the minimum total velocity increment required to accomplish a round trip of given duration and wait time at Venus can be found.

## RESULTS AND DISCUSSION

For all three types of mission considered, emphasis is placed upon the interrelations among velocity parameters, travel times, vehicle-planet separation distance, and planetary configuration angle at departure, which is equivalent to departure date (fig. 2(d)).

### Single-Pass Probes

The interrelations among trajectory parameters for a single-pass Venusian probe launched from an orbit of 1.1 Earth radii are shown in figure 3. The contours are lines of constant velocity increment. The dashed lines, which converge at the point representing a Hohmann path, separate the plot into four regions representing direct, aphelion, perihelion, and indirect routes. Along each dashed line, the Sun-focused conic section is tangent to either Earth's or Venus' orbit, and the route is either direct or indirect (fig. 1(a)). For example, the dashed line between the direct routes and the aphelion routes represents direct routes along Sun-focused ellipses that are tangent to Earth's orbit.

Figure 3(a) shows that, for a given velocity increment, the minimum travel time occurs along a direct route. Since the Hohmann path is indicated by a point, departure along this route can occur only at one instant of time during the synodic period of 584 days. A small increase in probe velocity increment above the Hohmann value results in an appreciable time span during which launch is possible. For example, a velocity-increment increase of only 810 feet per second (0.26 km/sec) above the Hohmann value gives a launch span of approximately 76 days. This same velocity-increment increase can also diminish the Earth-Venus separation distance at arrival from 55 to 31 million miles (48 to 27 million International nautical miles) (fig. 3(b)).

The routes requiring minimum velocity increments for travel times less than the Hohmann value of 146 days are of special interest (fig. 4).



Increasing the probe velocity-increment capability 3200 feet per second (0.975 km/sec) above the Hohmann value will decrease the travel time from 146 to 70 days and decrease Earth-Venus separation distance at arrival from 55.0 to 27.8 million miles (47.8 to 24.2 million Int. naut. miles).

### Orbiting Probes

The relations among trajectory parameters for an orbiting Venusian probe are shown in figure 5. The probe is assumed to be launched from a parking orbit at 1.1 Earth radii with the terminal parking orbit at 1.1 Venus radii. Because two impulses are required for such a mission, the velocity parameter used is the sum of the velocity increments required to leave the orbit around Earth and enter the orbit around Venus. Features of the orbiting probe trajectories are similar to those for the single-pass probe. Direct routes result in minimum travel time for a given velocity-increment capability. Increases in the probe velocity-increment capability above the Hohmann value will lengthen the time span during which departure is possible and can decrease the Earth-Venus separation distance at probe arrival.

Minimum-total-velocity-increment data are shown in figure 6 for travel times less than 146 days. To decrease travel time from 146 to 70 days, the velocity-increment capability of the probe must be increased from 4.12 to 5.84 miles per second (6.63 to 9.40 km/sec) (fig. 6(a)). This increase can also result in a decrease from 55 to 37 million miles (48 to 32 million Int. naut. miles) in the Earth-Venus separation distance upon arrival (fig. 6(b)).

### Round-Trip Missions

The round-trip mission to Venus begins with the space vehicle in orbit around the Earth at 1.1 Earth radii. The first velocity increment sends the vehicle on the outward trajectory from Earth to Venus. A second velocity increment settles the vehicle into an orbit about Venus at 1.1 Venus radii. After a specific wait time ranging from 0 to 467 days, a third velocity increment sends the vehicle on its return trajectory from Venus to Earth. The final velocity increment causes the vehicle to enter an orbit around Earth at 1.1 Earth radii. The total velocity increment is the sum of the four impulsive velocity increments, while mission time is the sum of the individual travel times for the outward and return trajectories and the wait time spent in the parking orbit around Venus.

The specification of parking orbits at 1.1 planet radii is arbitrary. If atmospheric braking is used to attain these parking orbits, the propellant required for a round-trip mission can be greatly reduced. The features of optimum round-trip missions employing atmospheric braking will change markedly from those discussed in this report.

Variation of minimum total velocity increment with mission time. -

Outward and return trajectories may be combined in an infinite number of combinations involving direct, aphelion, perihelion, and indirect routes. Particular combinations of these routes lead to envelope curves such as those shown in figure 7 for a wait time at Venus of 0 days. For a given mission time, the total velocity increment shown is the minimum value required for the designated combination of outward and return trajectories. An envelope of the curves shown in figure 7 shows the minimum total velocity increment required for mission times ranging from 65 to 650 days. Direct-direct round trips possess minimum-total-velocity-increment requirements for mission times less than 145 days. The single line in the range of 145 to 439 days' mission time ( $D_tA$ ) is a combination of a trip along an ellipse tangent to Earth's orbit and an ellipse tangent to neither planet's orbit. The particular trip along the ellipse tangent to Earth's orbit is a direct route or an aphelion route. But, since these two routes are identical (fig. 1(a)), throughout the remainder of this report this special trip will be called a direct route along an ellipse tangent to Earth's orbit. In the abovementioned range of mission times, this direct route and an aphelion route must be taken for a minimum-total-velocity-increment trip. A direct-indirect combination from 439 to 463 days, an aphelion-aphelion combination from 463 to 482 days, and an indirect-indirect combination of routes for missions from 482 to 650 days are required for trips of minimum total velocity increment.

It is interesting to note that the total velocity increment does not decrease continuously as mission time increases. This indicates that, for a wait time of 0 days, missions longer than 439 days may be of little practical value, since they would require total velocity increments equal to or greater than that required for shorter missions. A region of special interest in figure 7 is around a mission time of 439 days. The total velocity increment at this point is a minimum value of 10.3 miles per second (16.6 km/sec).

The effect of wait time at Venus on total velocity increment and mission time is shown in figure 8. A portion of the curves from figure 7 is replotted along with selected envelope curves for PP and  $D_tA$  round trips. A combination of a direct route along an ellipse tangent to Earth's orbit and an aphelion route will require the minimum total velocity increment for wait times less than 100 days. Minimum-total-velocity-increment trips may be taken along perihelion-perihelion routes for wait times ranging from 300 to 467 days. For wait times between 100 and 300 days, the direct route along an ellipse tangent to Earth's orbit and an aphelion route will possess relatively low velocity increments and long mission times, while perihelion-perihelion trips yield higher velocity increments for shorter mission times.

As the wait time at Venus is increased from 0 days, the envelopes for the direct route along an ellipse tangent to Earth's orbit and an aphelion route are displaced toward longer mission time and toward higher velocity increments. However, the effect of wait time on perihelion-perihelion round trips is not the same. The envelopes for these routes shift toward longer mission times and toward lower velocity increments as the wait time at Venus increases. For example, a mission time of 660 days may be flown on a direct route along an ellipse tangent to Earth's orbit and an aphelion route at a total velocity increment of 11.80 miles per second (18.99 km/sec) and a wait time of 129 days. The same mission time may be expended on a perihelion-perihelion trip, but at a total velocity increment of only 9.92 miles per second (15.96 km/sec) and a longer wait time of 467 days.

For a given mission time and wait time at Venus, other trajectory parameters besides total velocity increment are of interest. Among these are the individual velocity increments comprising the total, the time required for outward and return trajectories, and the configuration angle required to begin the mission. The variation of these parameters with wait time at Venus will be shown for round trips yielding the minimum total velocity increment.

Round trips on direct routes along ellipse tangent to Earth's orbit and aphelion route. - A breakdown of the various parameters for direct routes along an ellipse tangent to Earth's orbit and an aphelion route for wait times ranging from 0 to 200 days is shown in figure 9. Two of the four velocity increments comprising the total are constant within the given range of wait times (fig. 9(b)). Mission time is seen to increase at a greater rate than wait time (fig. 9(c)). As wait time increases from 0 to 100 days, mission time increases 173 days. Since one path of the round trip is along a Hohmann ellipse, one of the travel times remains constant at 146 days (fig. 9(d)). The other ranges from 287 to 440 days. Because either trajectory may be selected for the outward path, every round trip of this type could begin on a Hohmann date. For a particular round trip of interest, the Earth-Venus separation distance during the wait time spent in orbit around Venus can be found from equation (1) and figure 2(e).

The effects of mission time on velocity increment, travel time, and configuration angle at departure for wait times at Venus of 0, 50, and 100 days are shown in figures 10, 11, and 12, respectively.

Round trips along perihelion-perihelion routes. - The effect of wait time on velocity increment, mission time, travel time, and configuration angle parameters is shown in figure 13 for perihelion-perihelion round trips to Venus. The routes shown are for round trips requiring the minimum total velocity increment. Both the outward and the return trips on this route are made along the same perihelion path. As the wait time spent in orbit around Venus becomes longer, the total velocity

increment required for the complete round trip steadily decreases until the Hohmann value of 8.24 miles per second (13.26 km/sec) is reached (fig. 13(a)). The one-way travel time (fig. 13(d)) is seen to increase as wait time at Venus increases until a wait of 325 days is reached. At this point, the travel time begins to decrease until it reaches a value of 146 days for a wait time of 467 days. A total velocity increment of 8.24 miles per second (13.26 km/sec) and a wait time of 467 days can only be attained by following a Hohmann ellipse along both the outward and the return portions of the round trip.

#### CONCLUDING REMARKS

The analysis of trajectory parameters for probe and round-trip missions to Venus has revealed some interesting features. For probes that pass near Venus only once, the time span for launching is appreciable even for small increases in velocity-increment capability above the minimum requirement of 11,190 feet per second (5.41 km/sec). A velocity-increment increase of only 3200 feet per second (0.975 km/sec) may be used to decrease the travel time from 146 to 70 days and the Earth-Venus separation distance at arrival from 55 to 27.8 million miles (47.8 to 24.2 million Int. naut. miles).

For orbiting probes, a small increase in the velocity-increment capability will provide a long time span for launching. However, the effectiveness of velocity-increment increases in reducing travel time and Earth-Venus separation distance is less pronounced. An increase of 38 percent above the minimum velocity increment will cause a 50-percent decrease in travel time and a corresponding 31-percent decrease in Earth-Venus separation distance at arrival.

In the range of wait time from 0 to 100 days, round trips using a direct route along an ellipse tangent to Earth's orbit and an aphelion route must be taken to keep the total velocity increment at its minimum value. A mission time of 365 days with no wait time at Venus may be flown for a total velocity increment of 10.50 miles per second (16.9 km/sec). If a total mission time of 465 days with 100 days' wait at Venus were considered, the minimum velocity increment required would be 14.96 miles per second (24.08 km/sec). Mission times ranging from 640 to 759 days should be flown along a perihelion-perihelion route to obtain missions of minimum total velocity increment.

Lewis Research Center

National Aeronautics and Space Administration  
Cleveland, Ohio, June 22, 1960

## APPENDIX A

## SYMBOLS

A	aphelion route (fig. 1(a))
D	direct route (fig. 1(a))
d	Earth-Venus separation distance, miles (or Int. naut. miles)
I	indirect route (fig. 1(a))
N	an integer (eq. (4))
n	ratio of mean radius of destination planet's orbit around the Sun to that of Earth
P	perihelion route (fig. 1(a))
p	semilatus rectum of heliocentric conic section, A.U.
q	semimajor axis of heliocentric ellipse or hyperbola, A.U.
R	mean distance of planet from Sun, miles (or Int. naut. miles)
T	heliocentric travel time, days
$T_w$	waiting time in parking orbit at Venus, days
V	ratio of hyperbolic velocity to mean orbital velocity of Earth, (miles/sec)/(18.5 miles/sec) (or (km/sec)/(29.77 km/sec))
v	velocity, miles/sec (or km/sec)
$\Delta v$	velocity increment, miles/sec (or km/sec)
$\alpha$	heliocentric trajectory angle relative to circumferential direction (fig. 1(b)), deg
$\epsilon$	eccentricity of heliocentric conic section
$\dot{\theta}$	angular velocity of planet, deg/day
$\lambda$	angular difference between vehicle's travel and Earth's travel during one-way trip between planet orbits (eq. (3)), deg
$\tau$	angle (eq. (B3c)), radians

$\varphi$  angular position of vehicle measured counterclockwise from perihelion of heliocentric conic section (fig. 1(b)), deg

$\Delta\varphi$  change in angular position of vehicle, deg

$\psi$  configuration angle (fig. 1(b)), deg

$\oplus$  Earth's position

$\ominus$  Venus' position

Subscripts:

A aphelion route

c circular

D direct route

E Earth

e escape

I indirect route

i index signifying A, D, I, or P

j index signifying 1 or 2

max maximum

min minimum

P perihelion route

T total

t tangent to Earth's orbit

V Venus

1 instant of space vehicle's departure from Earth's orbit

2 instant of space vehicle's arrival at Venus' orbit

## APPENDIX B

## CALCULATION PROCEDURE

The trajectory-parameter calculation procedure is based mainly on the equations in reference 7. These and several additional equations were programmed for a digital computer to yield trajectory-parameter tables with  $p$  and  $\epsilon$  as arguments for values of  $n \leq 1.0$ . The value of  $n$  for Earth-Venus trajectories is 0.7233. The initial value of  $p$  is set equal to 0.1. The minimum value of  $\epsilon$  is calculated from equation (B1a) or (B1b), whichever yields the larger value:

$$\left. \begin{aligned} \epsilon_{\min} &= \pm(p - 1) & \text{(B1a)} \\ \text{or} & & \\ \epsilon_{\min} &= \pm\left(1 - \frac{p}{n}\right) & \text{(B1b)} \end{aligned} \right\} \begin{array}{l} + \text{ for } n > 1.0 \\ - \text{ for } n < 1.0 \end{array}$$

The  $p, \epsilon$  pair of values ( $0.1, \epsilon_{\min}$ ) defines a specific Sun-focused conic section, and all trajectory parameters can be expressed as functions of  $p$  and  $\epsilon$ . The nondimensional hyperbolic velocities are calculated from

$$V_1 = \left(3 - 2\sqrt{p} - \frac{1 - \epsilon^2}{p}\right)^{1/2} \quad \text{(B2a)}$$

$$V_2 = \left(\frac{3 - 2\sqrt{p/n}}{n} - \frac{1 - \epsilon^2}{p}\right)^{1/2} \quad \text{(B2b)}$$

The velocity increments required to leave the parking orbit around Earth and to enter the parking orbit around Venus are calculated from the following equations derived from the constant-energy property of two-body motion:

$$\Delta v_E = \left[ (18.50 V_1)^2 + (v_{e,E})^2 \right]^{1/2} - v_{c,E} \quad \text{(B2c)}$$

$$\Delta v_V = \left[ (18.50 V_2)^2 + (v_{e,V})^2 \right]^{1/2} - v_{c,V} \quad \text{(B2d)}$$

At both Earth and Venus, parking orbits of 1.1 planet radii were assigned. The values of  $v_{e,E}$ ,  $v_{c,E}$ ,  $v_{e,V}$ , and  $v_{c,V}$  used in the calculations are 6.624, 4.683, 6.043, and 4.273 miles per second, respectively (10.660, 7.537, 9.725, and 6.877 km/sec, respectively).

While the velocity parameters are independent of route, the travel times are different for direct, perihelion, aphelion, and indirect

routes. Moreover, the equation for travel time for a direct route changes form, depending on whether  $\epsilon$  is less than, equal to, or greater than 1.0.

For  $\epsilon < 1.0$ ,

$$T_D = \pm 58.13 q^{3/2} \left[ (\tau_1 - \tau_2) - \epsilon (\sin \tau_1 - \sin \tau_2) \right] \begin{cases} + \text{ for } n < 1 \\ - \text{ for } n > 1 \end{cases} \quad (\text{B3a})$$

where

$$q = \frac{p}{1 - \epsilon^2} \quad (\text{B3b})$$

$$\tau_1 = \cos^{-1} \left( \frac{q - 1}{q\epsilon} \right) \quad (\text{B3c})$$

$$\tau_2 = \cos^{-1} \left( \frac{q - n}{q\epsilon} \right) \quad (\text{B3d})$$

The equation for  $\epsilon = 1.0$  was not programmed. As a result of using  $p$  rather than  $q$  in the diagrams (fig. 2), the curves run continuously from the area of ellipses ( $\epsilon < 1.0$ ), through the line of parabolas ( $\epsilon = 1.0$ ), to the area of the hyperbolas ( $\epsilon > 1.0$ ).

For  $\epsilon > 1.0$ ,

$$T_D = \pm 58.13 q^{3/2} \left[ \epsilon (\sinh \tau_1 - \sinh \tau_2) - (\tau_1 - \tau_2) \right] \begin{cases} + \text{ for } n < 1 \\ - \text{ for } n > 1 \end{cases} \quad (\text{B3e})$$

where

$$q = \frac{p}{\epsilon^2 - 1} \quad (\text{B3f})$$

$$\tau_1 = \cosh^{-1} \left( \frac{q + 1}{q\epsilon} \right) \quad (\text{B3g})$$

$$\tau_2 = \cosh^{-1} \left( \frac{q + r}{q\epsilon} \right) \quad (\text{B3h})$$

The travel times for perihelion, aphelion, and indirect routes are calculated from

$$T_P = T_D + 116.26 q^{3/2} (\tau_j - \epsilon \sin \tau_j) \begin{cases} j = 1 & \text{for } n > 1 \\ j = 2 & \text{for } n < 1 \end{cases} \quad (\text{B3i})$$



$$T_A = T_D + 116.26 q^{3/2} (\pi - \tau_j + \epsilon \sin \tau_j) \begin{cases} j = 1 & \text{for } n > 1 \\ j = 2 & \text{for } n < 1 \end{cases} \quad (\text{B3j})$$

$$T_I = 365.26 (q^{3/2} - T_D) \quad (\text{B3k})$$

Because aphelion and indirect routes are not possible for hyperbolic orbits, travel times for these routes are calculated only for values of  $\epsilon < 1.0$ .

The change in heliocentric angle also depends upon the route:

$$\varphi_1 = \cos^{-1} \left[ \frac{q(1 - \epsilon^2) - 1}{\epsilon} \right] \quad (\text{B4a})$$

$$\varphi_2 = \cos^{-1} \left[ \frac{q(1 - \epsilon^2) - n}{n\epsilon} \right] \quad (\text{B4b})$$

$$\Delta\varphi_D = |\varphi_2 - \varphi_1| \quad (\text{B4c})$$

$$\Delta\varphi_P = \varphi_1 + \varphi_2 \quad (\text{B4d})$$

$$\Delta\varphi_A = 360 - \Delta\varphi_P \quad \text{for } \epsilon < 1.0 \quad (\text{B4e})$$

$$\Delta\varphi_I = 360 - \Delta\varphi_D \quad \text{for } \epsilon < 1.0 \quad (\text{B4f})$$

Since the configuration angle depends on both travel time and change in heliocentric angle, it depends on the route, also:

$$\psi_{1,i} = \frac{0.9856}{n^{3/2}} T_i - \Delta\varphi_i \quad (\text{B5})$$

Another trajectory parameter depending on both  $T$  and  $\varphi$  is  $\lambda$ , which is useful in round-trip calculations (eqs. (3) and (4)):

$$\lambda_i = \Delta\varphi_i - 0.9856 T_i \quad (\text{B6})$$

The trajectory parameter  $\alpha$  is calculated from

$$\alpha_1 = \tan^{-1} \frac{\pm\epsilon \sin \varphi_1}{1 + \epsilon \cos \varphi_1} \quad (\text{B7a})$$

$$\alpha_2 = \tan^{-1} \frac{\pm\epsilon \sin \varphi_2}{1 + \epsilon \cos \varphi_1} \quad (\text{B7b})$$

where plus is for  $n > 1.0$  and minus is for  $n < 1.0$ .

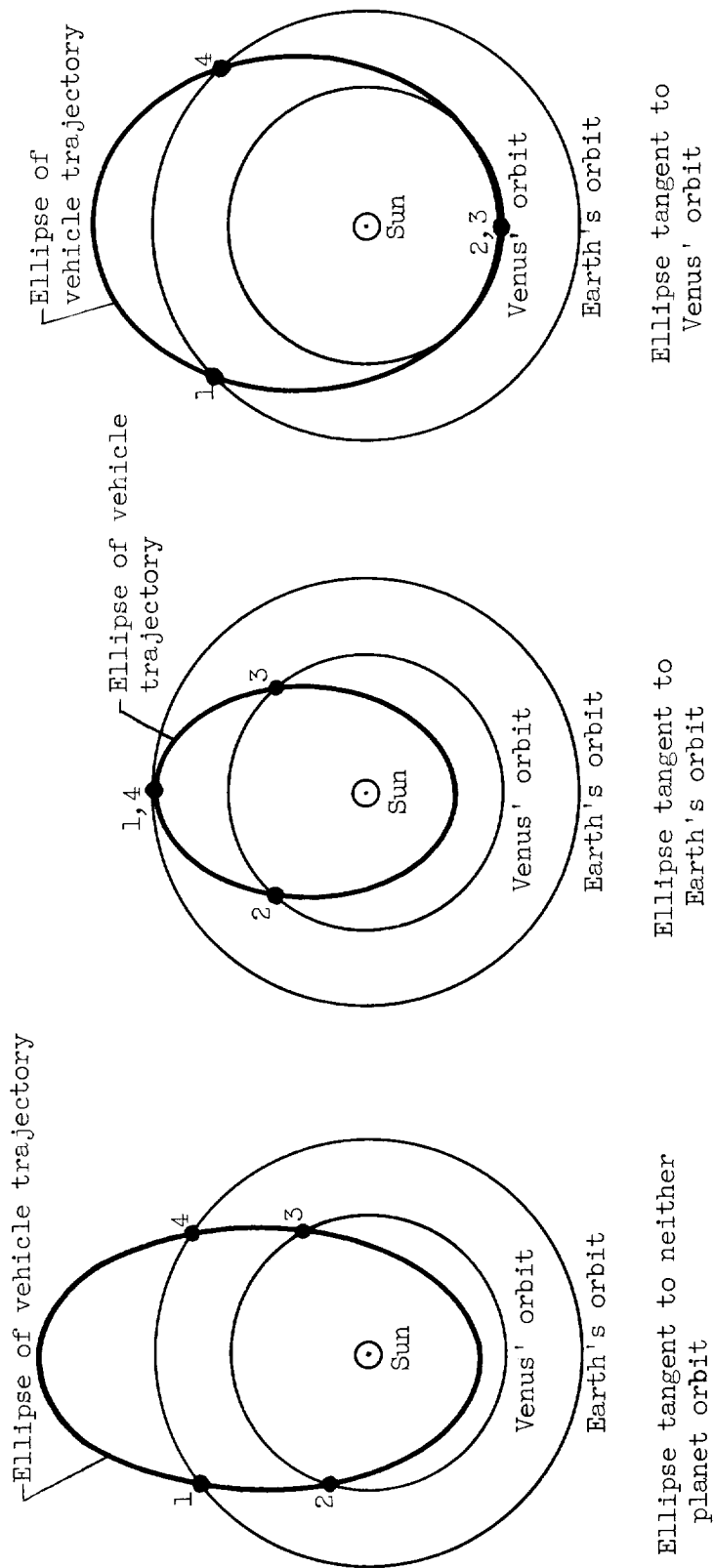
Having calculated all trajectory parameters of interest for  $p = 0.1$  and  $\epsilon_{\min}$ , the value of  $\epsilon$  is increased just enough to make it divisible by  $\Delta\epsilon$ . In the present calculations,  $\Delta\epsilon$  is set equal to 0.1. With  $p = 0.1$  and  $\epsilon$  increased, all trajectory parameters are calculated. This procedure is repeated, the last calculation being for  $p = 0.1$  and  $\epsilon = 2.0$ . At this point the value of  $p$  is increased by  $\Delta p$ . Again, the increment is set equal to 0.1. For  $p = 0.2$ ,  $\epsilon_{\min}$  is calculated and all trajectory parameters are computed for this pair of  $p, \epsilon$  values. The procedure is repeated until a stipulated range of the arguments  $p$  and  $\epsilon$  is covered. For the Earth-Venus calculations,  $p$  ranges from 0.1 to 2.0 in increments of 0.1, and  $\epsilon$  ranges from  $\epsilon_{\min}$  to 2.0 in increments of 0.1.

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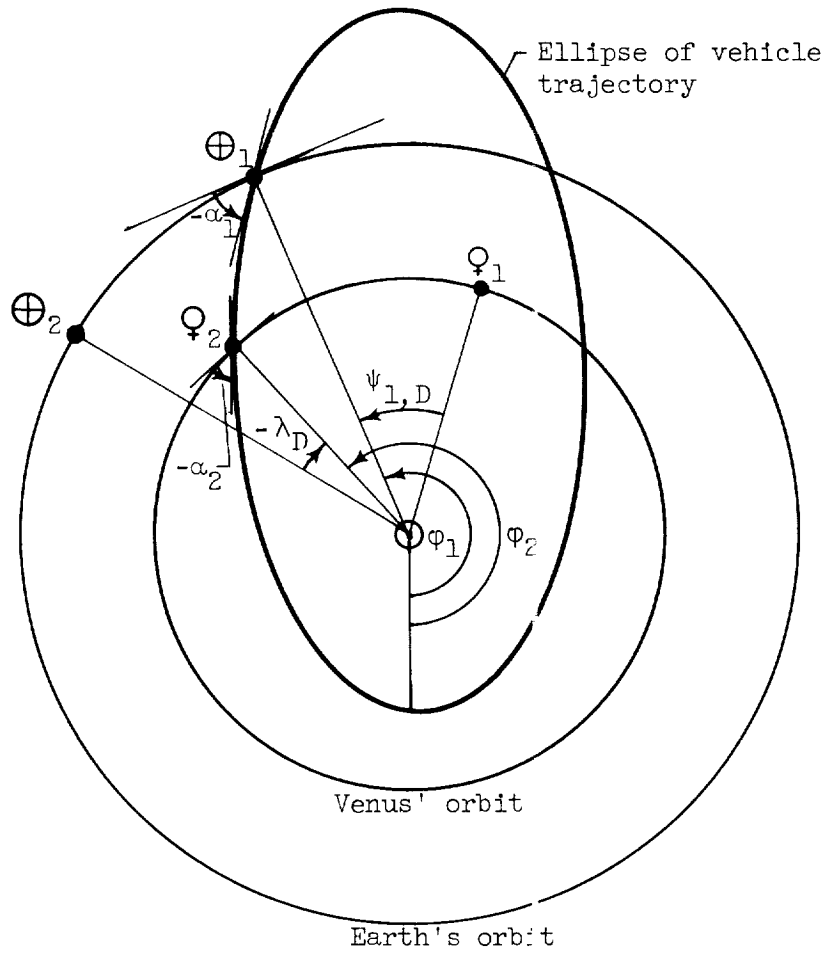
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Route Earth to Venus Venus to Earth

- Direct 1-2 3-4
- Perihelion 1-2-3 2-3-4
- Aphelion 4-1-2 3-4-1
- Indirect 4-1-2-3 2-3-4-1

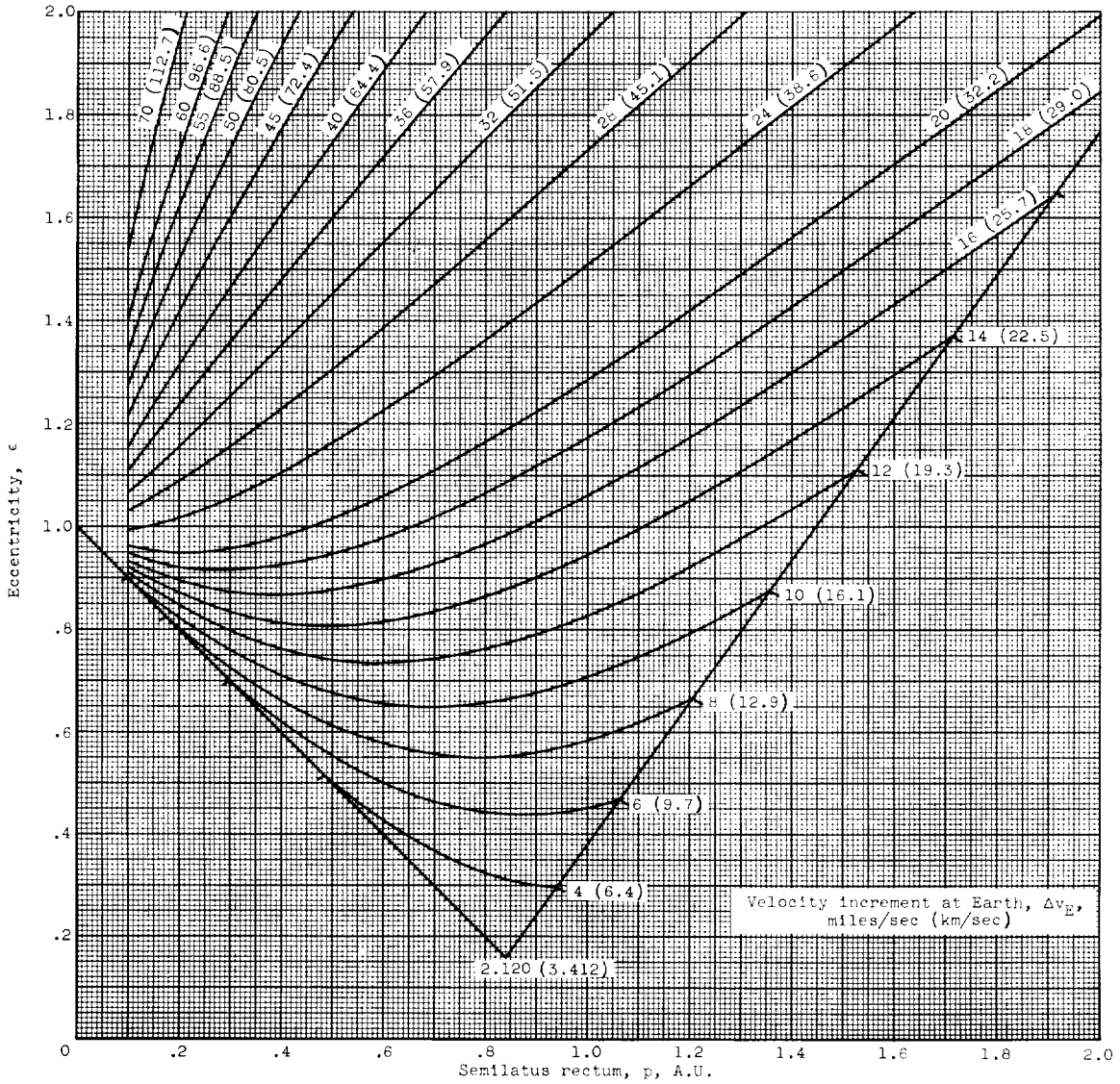


(a) Interplanetary routes.  
Figure 1. - Nomenclature.



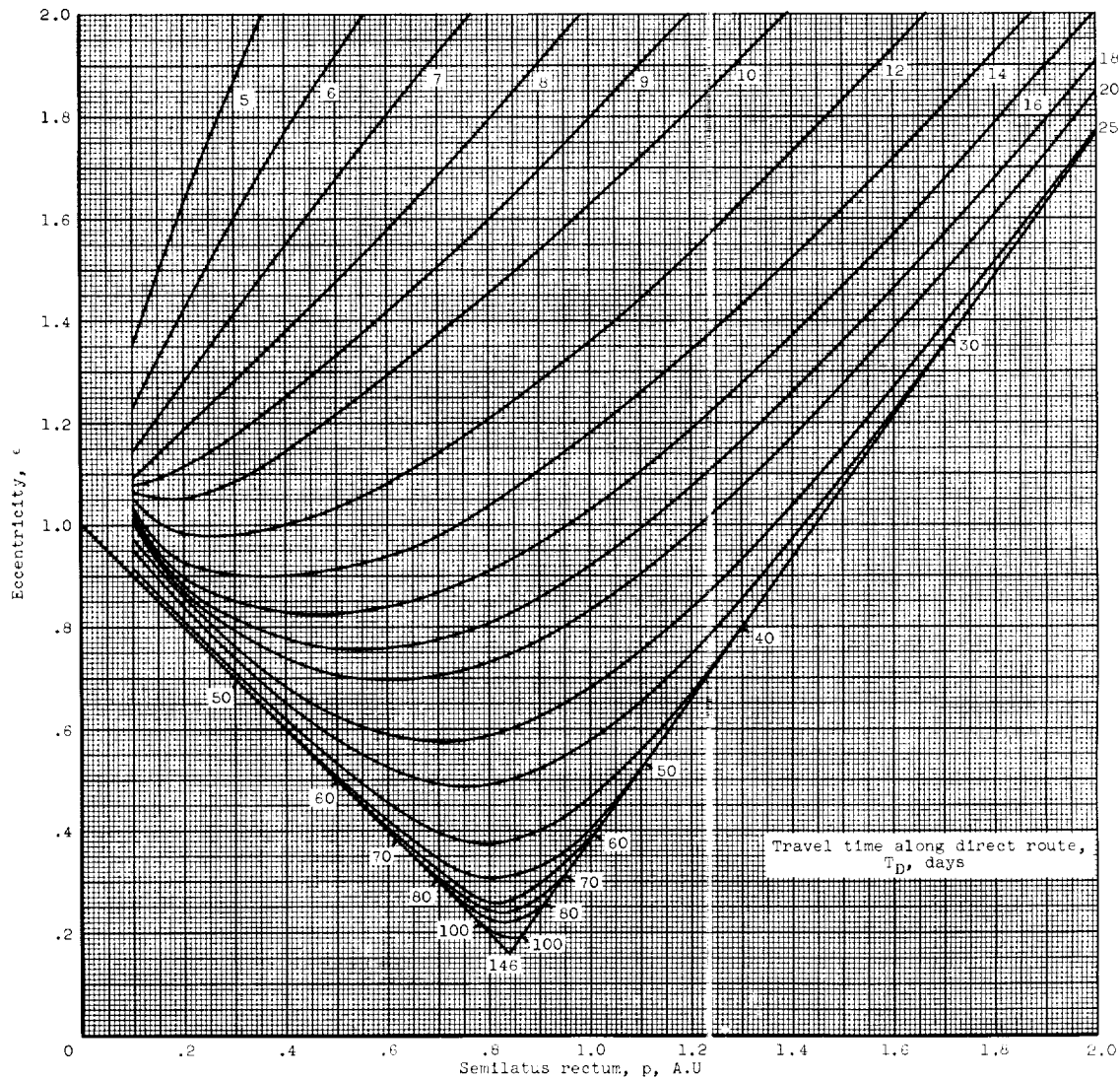
(b) Trajectory angles.

Figure 1. - Concluded. Nomenclature.



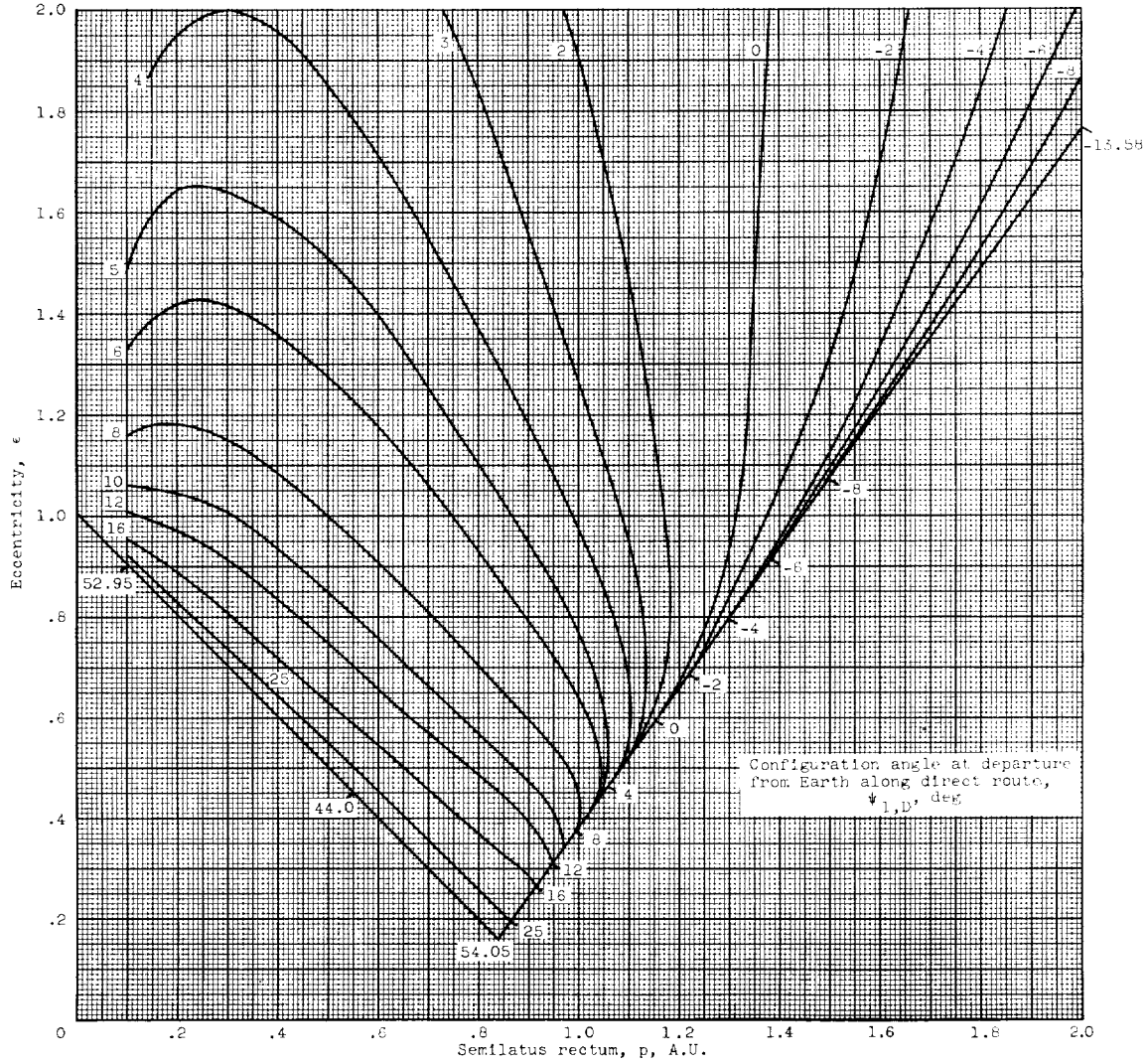
(a) Velocity increment at Earth.

Figure 2. - Trajectory parameters for trip between Earth and Venus. (A set of 22  $\epsilon$ - $p$  charts for Earth-Venus trajectories may be obtained from NASA, Washington 25, D. C.)

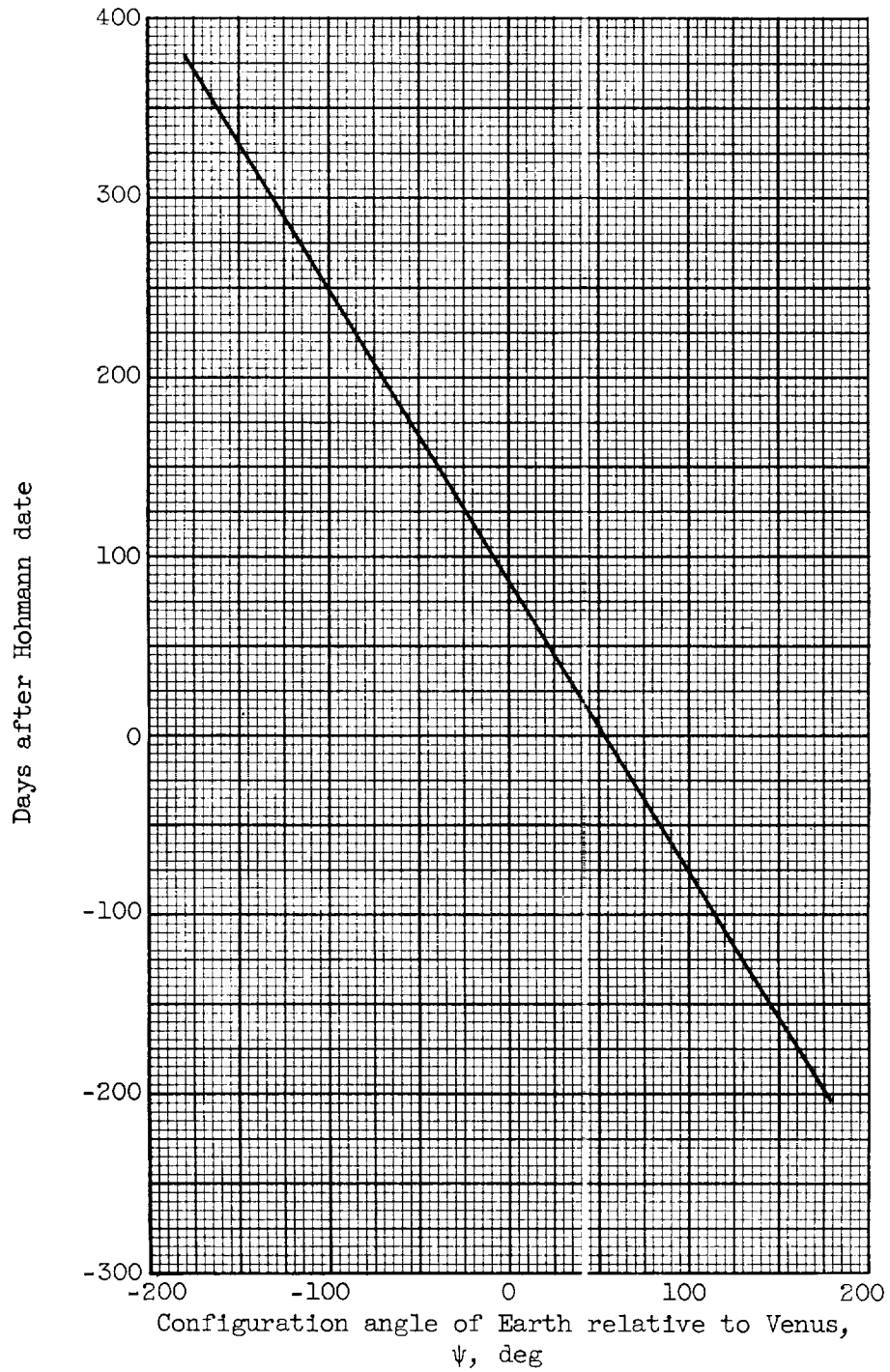


(b) Travel time along direct route.

Figure 2. - Continued. Trajectory parameters for trip between Earth and Venus.



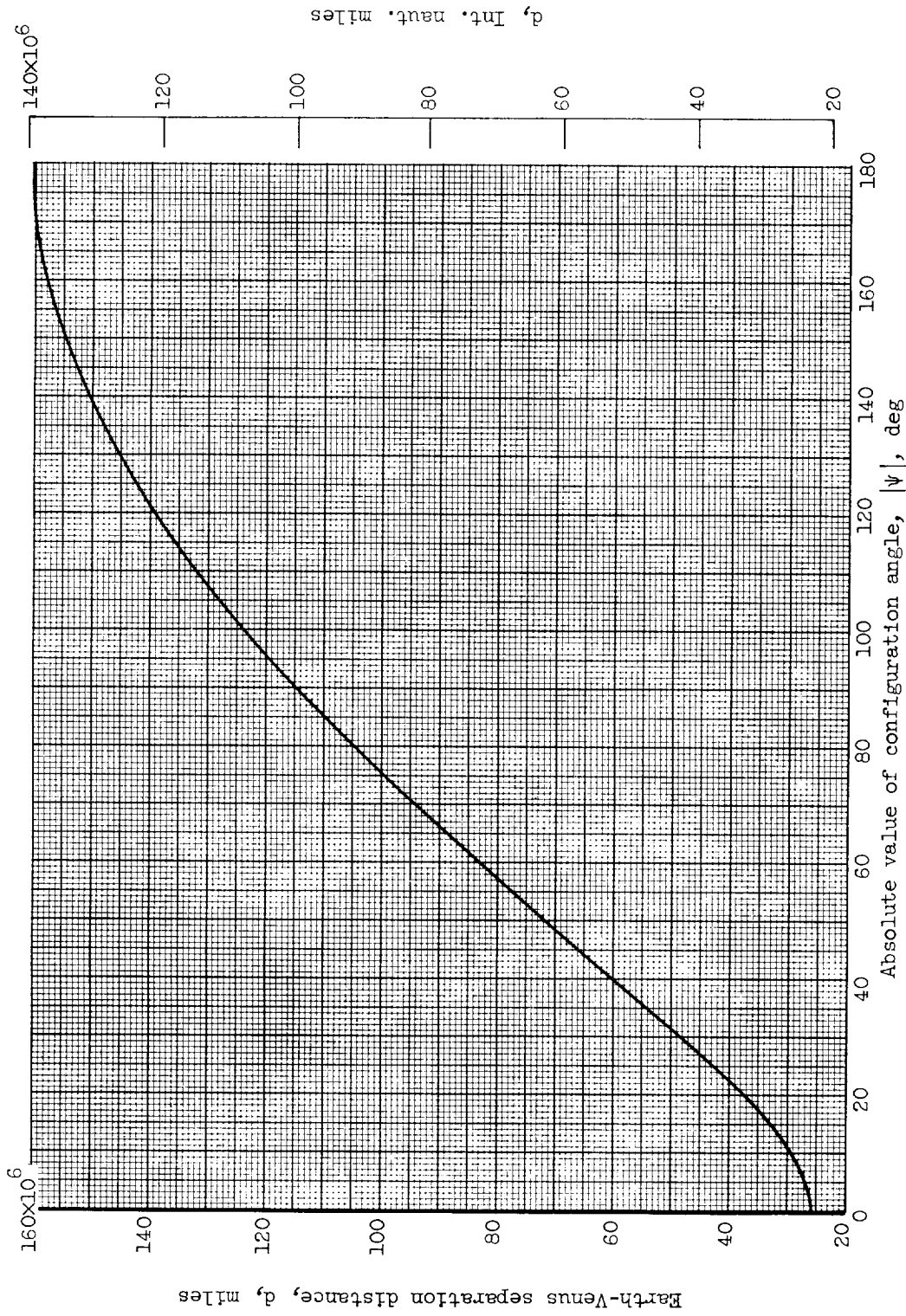
(c) Configuration angle at departure from Earth along direct route.  
Figure 2. - Continued. Trajectory parameters for trip between Earth and Venus.



(d) Relation between date and configuration angle of Earth relative to Venus.

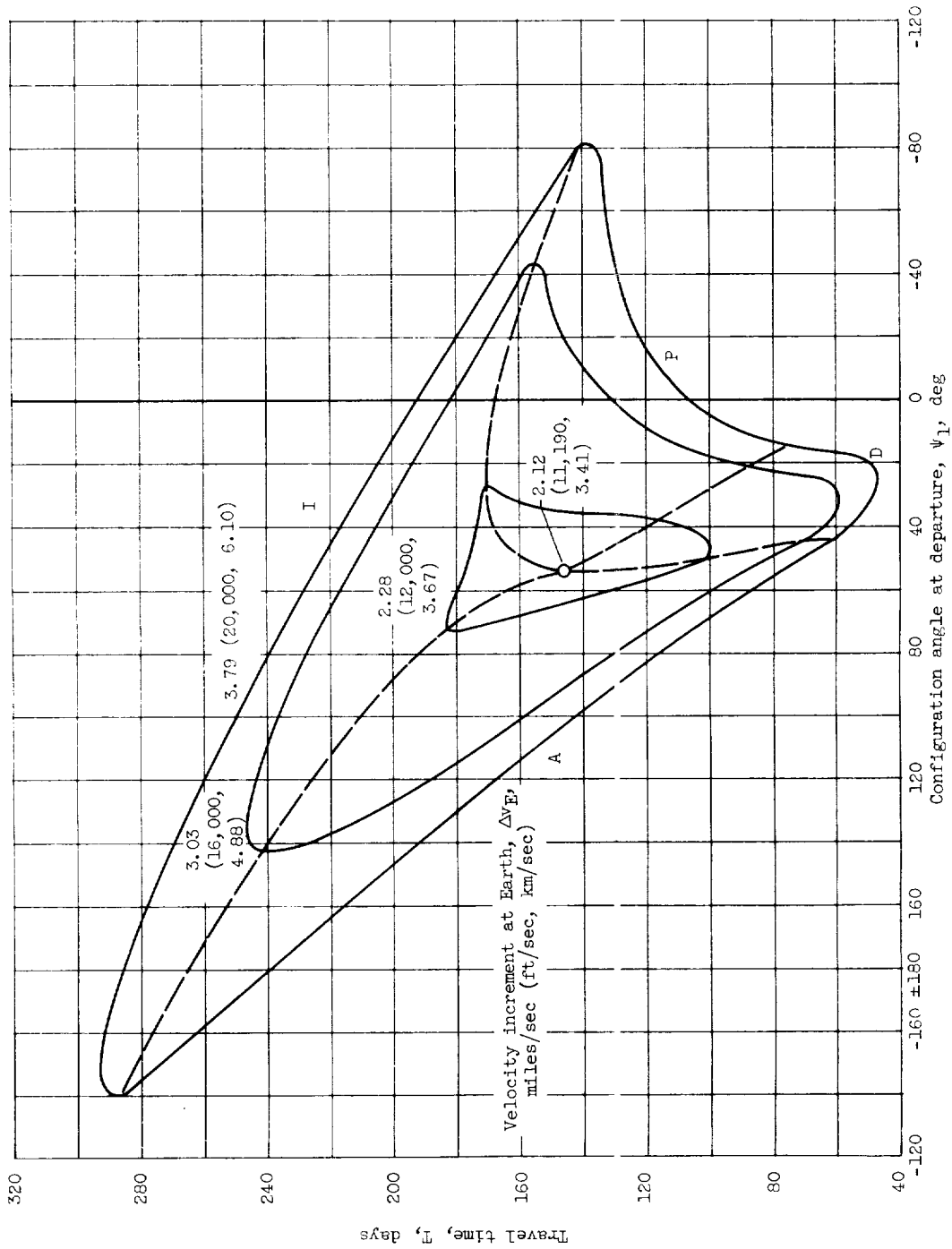
Figure 2. - Continued. Trajectory parameters for trip between Earth and Venus.





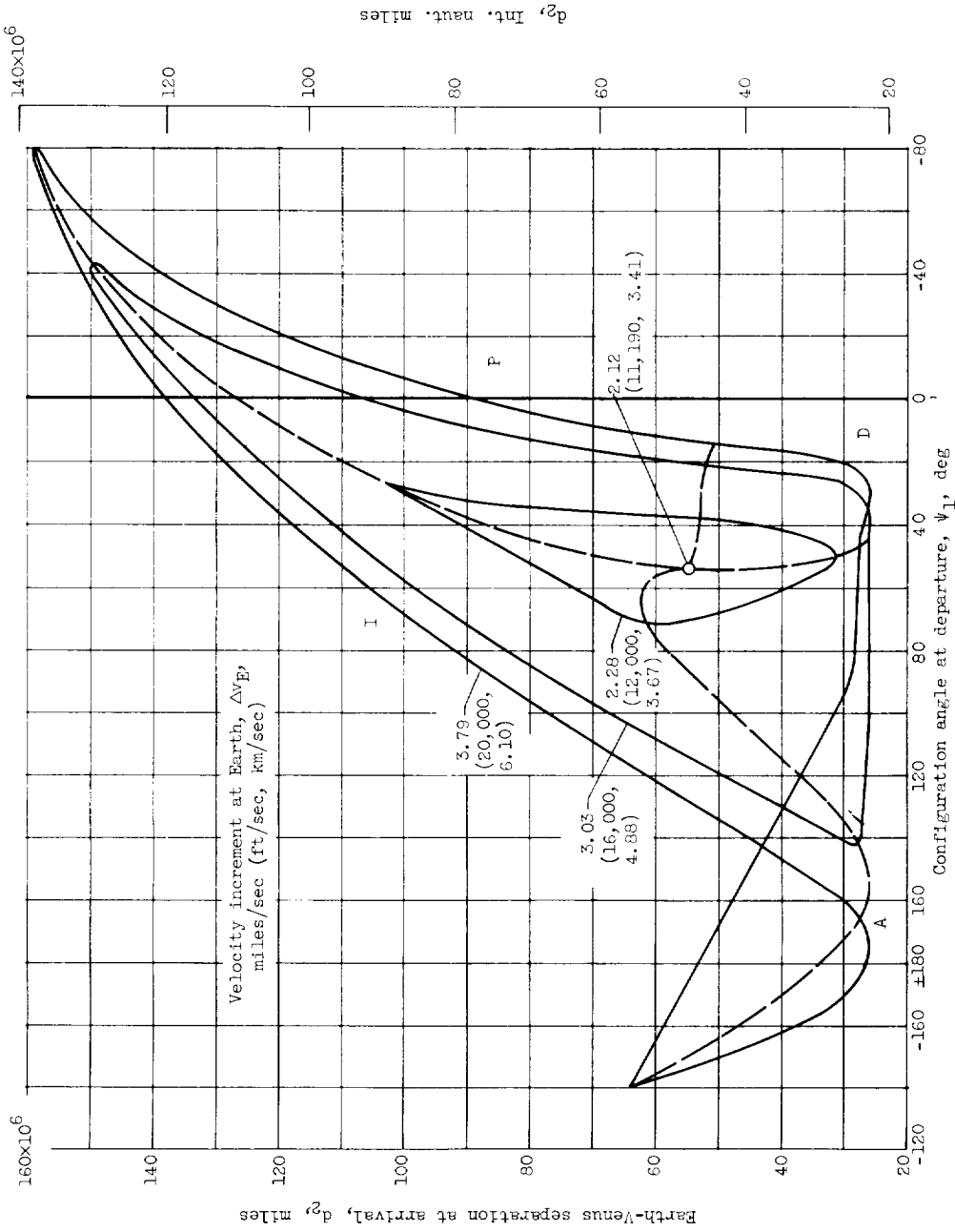
(e) Earth-Venus separation distance.

Figure 2. - Concluded. Trajectory parameters for trip between Earth and Venus.



(a) Travel time, configuration angle at departure, and velocity increment at Earth.

Figure 3. - Path parameters for single-pass Venusian probe launched from an orbit of 1.1 Earth radii.



(b) Earth-Venus separation at arrival, configuration angle at departure, and velocity increment at Earth.

Figure 3. - Concluded. Path parameters for single-pass Venusian probe launched from an orbit of 1.1 Earth radii.

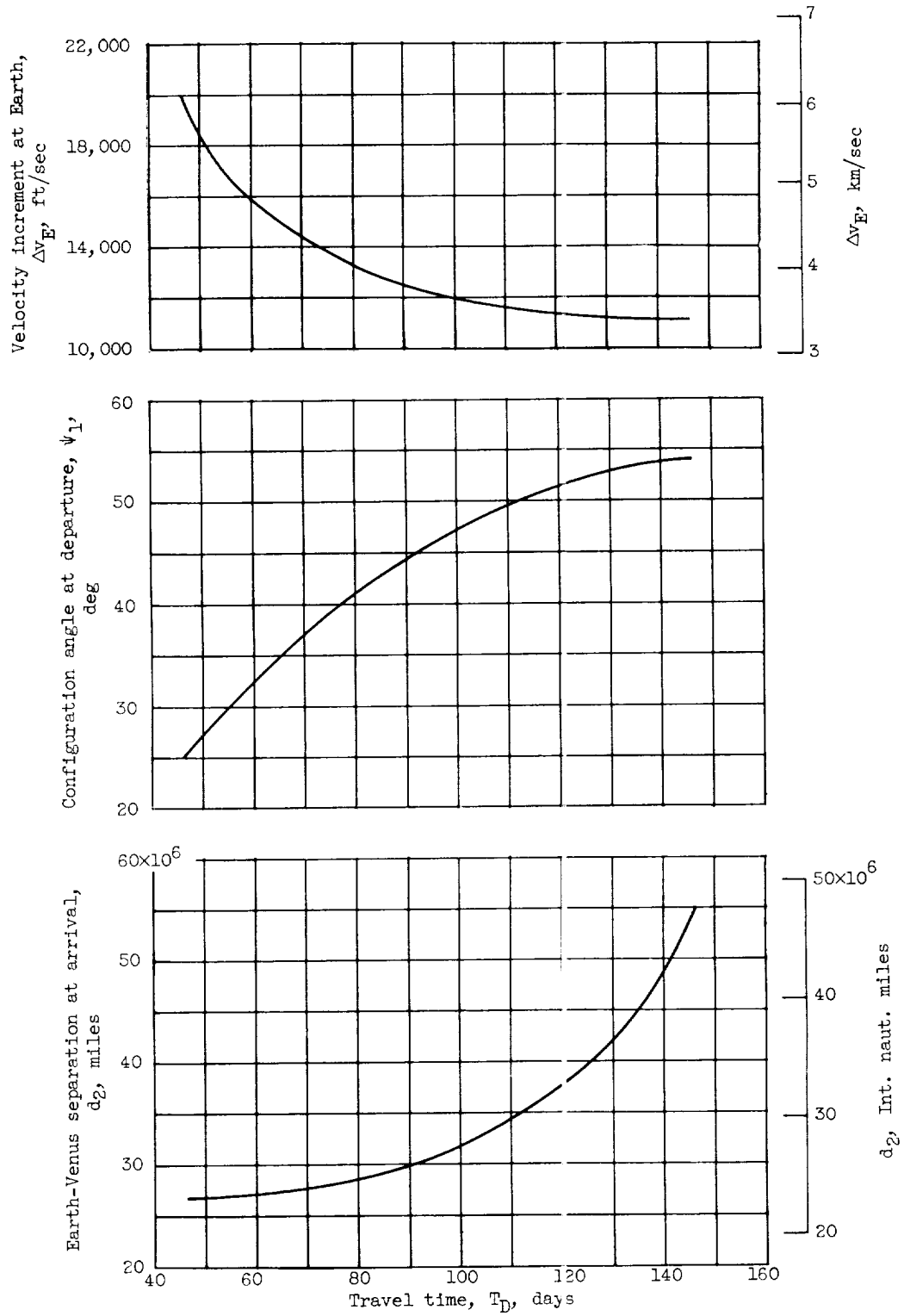
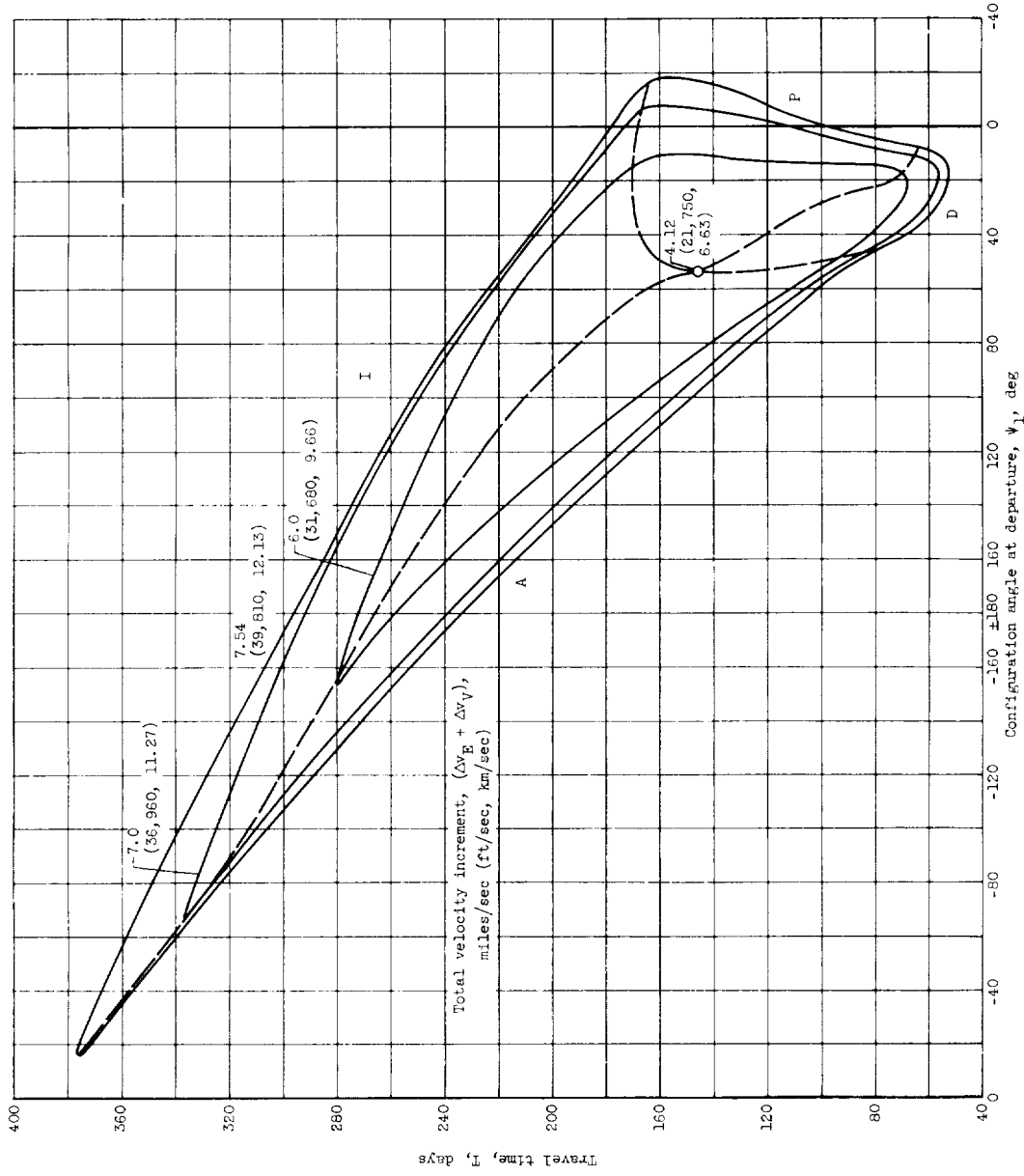
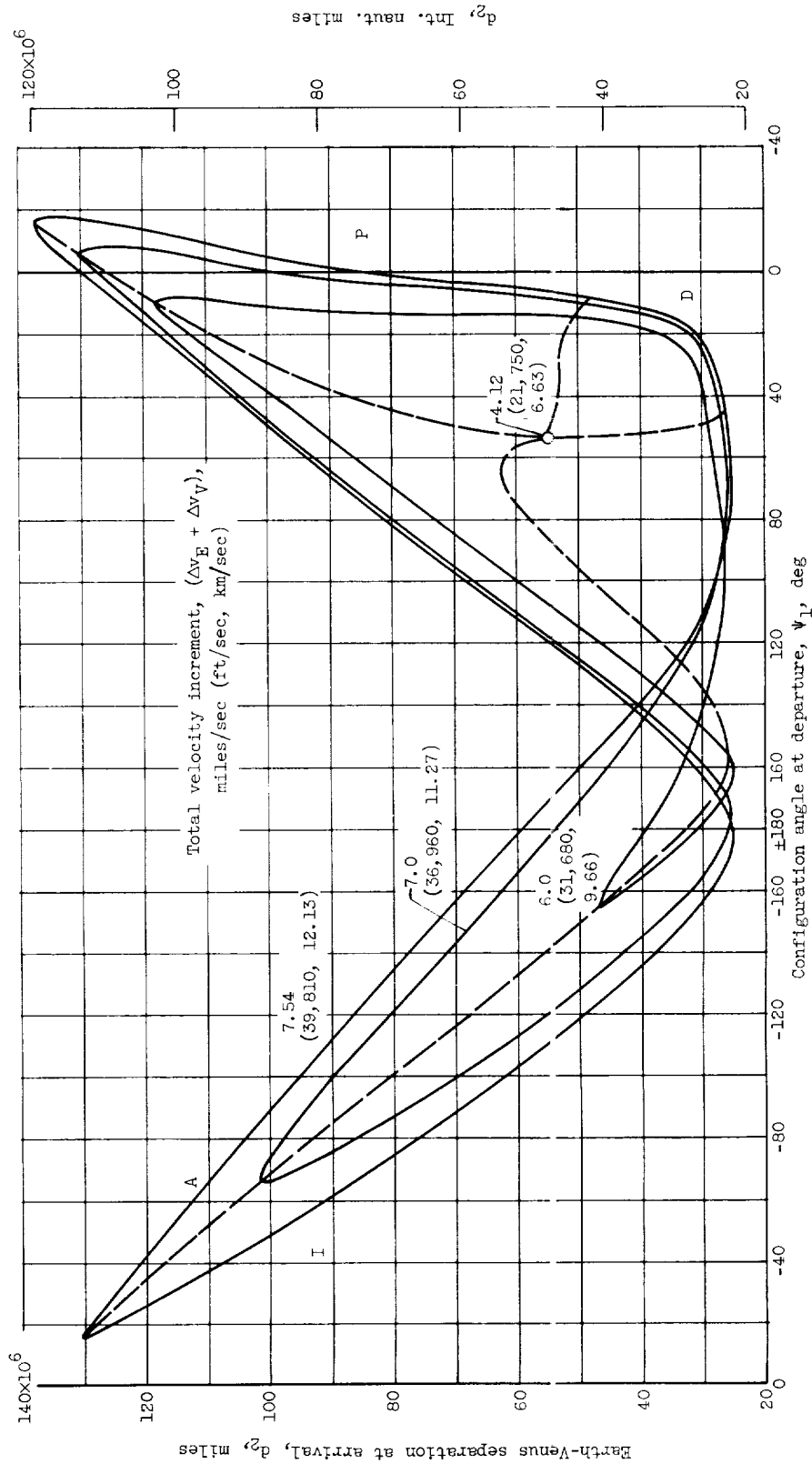


Figure 4. - Path parameters for single-pass Venusian probe launched from an orbit of 1.1 Earth radii for minimum velocity increment.



(a) Travel time, configuration angle at departure, and total velocity increment.

Figure 5. - Path parameters for orbiting Venusian probe launched from an orbit of 1.1 Earth radii. Terminal orbit is at 1.1 Venus radii.



(b) Earth-Venus separation at arrival, configuration angle at departure, and total velocity increment.

Figure 5. - Concluded. Path parameters for orbiting Venusian probe launched from an orbit of 1.1 Earth radii. Terminal orbit is at 1.1 Venus radii.

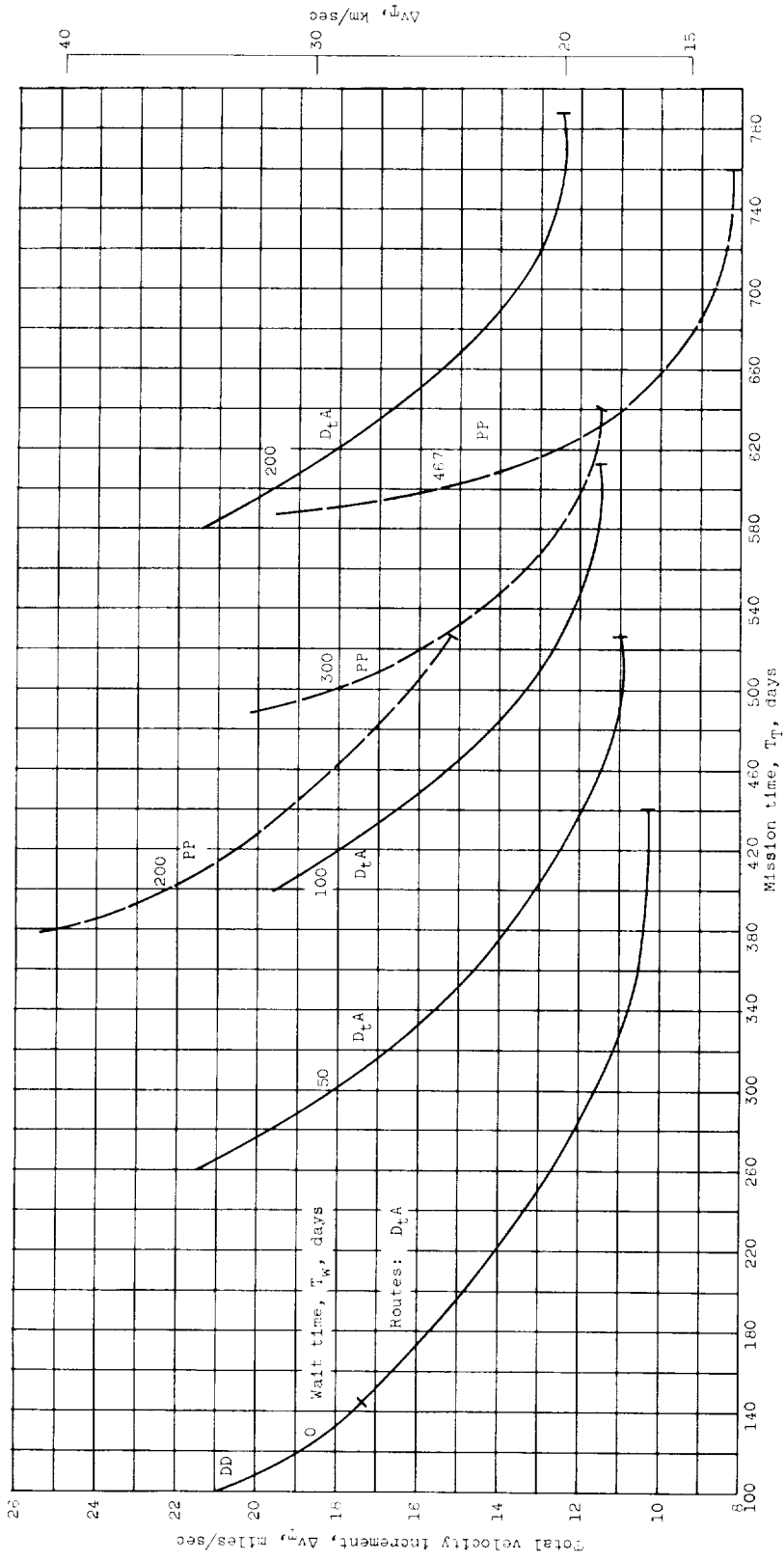
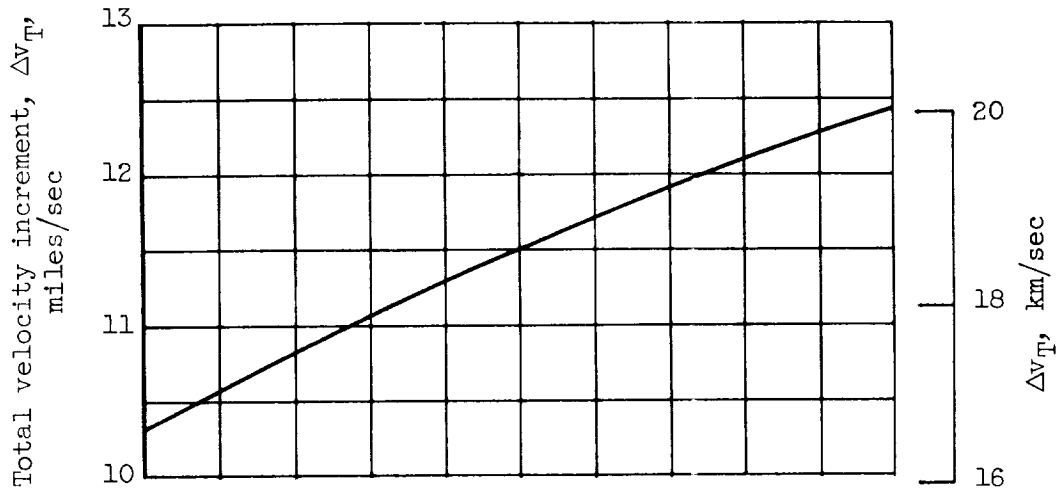
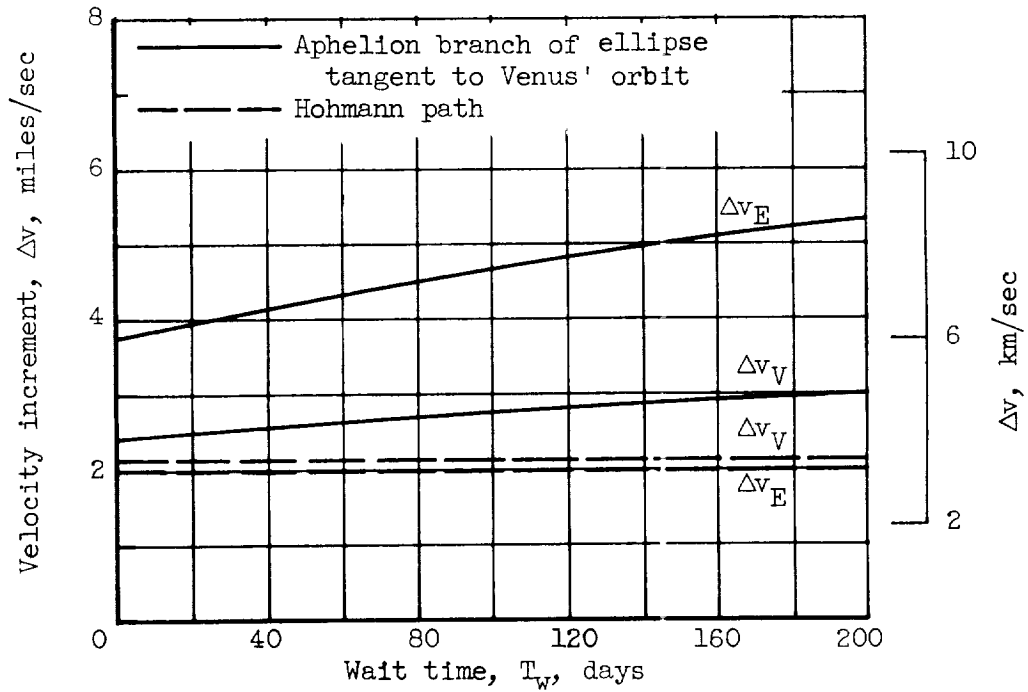


Figure 8. - Effect of wait time at Venus on total velocity increment and mission time for round trips to Venus, starting, waiting, and ending in circular orbits at 1.1 planet radii.



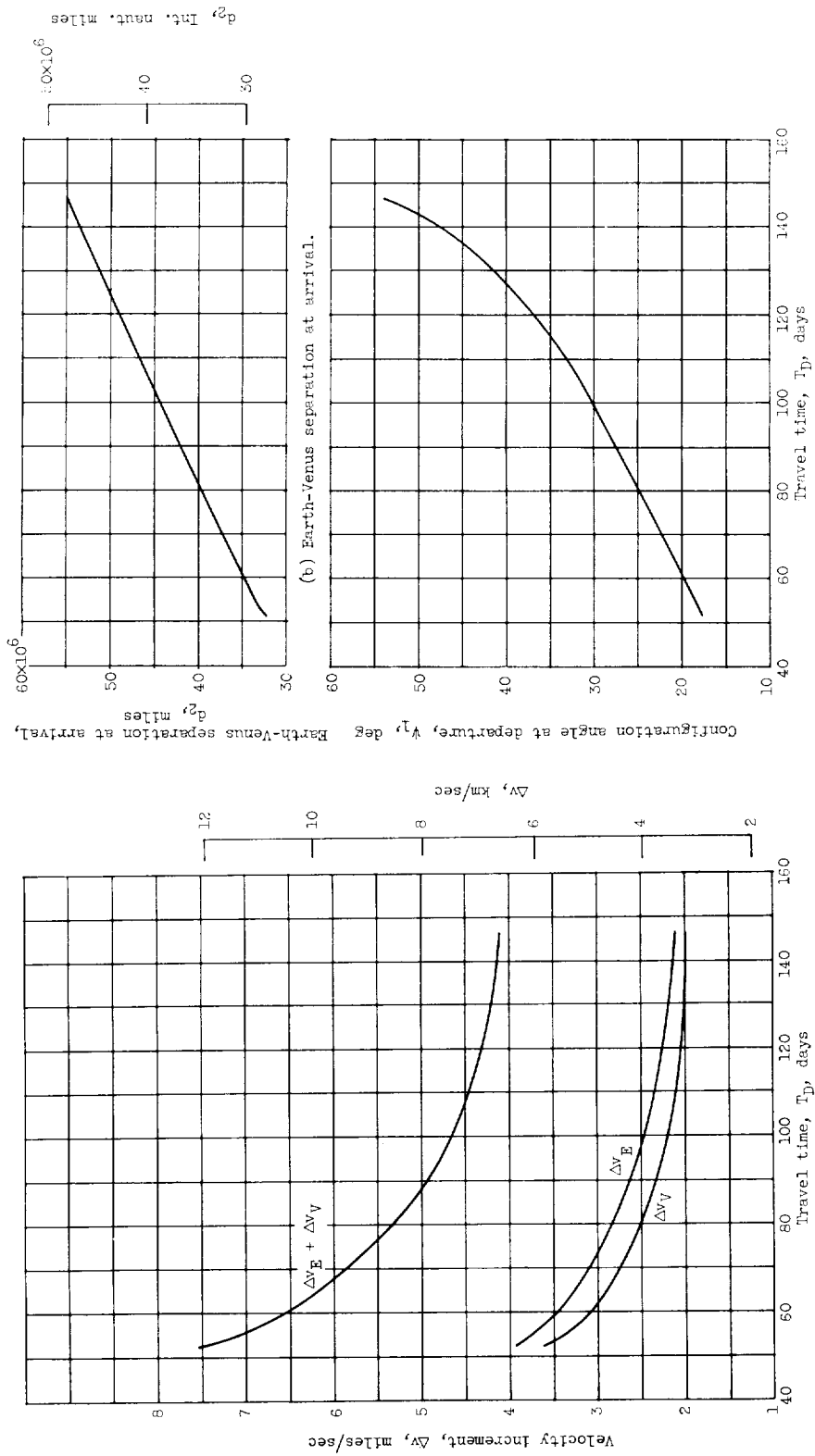
(a) Total velocity increment.



(b) Velocity increments at Earth and Venus.

Figure 9. - Path parameters for direct route tangent to Earth's orbit and aphelion round trips to Venus starting, waiting, and ending in circular orbits at 1.1 planet radii. Data for minimum total velocity increment. Either outward or return trip must be taken along Hohmann ellipse.





(a) Velocity increment.

(b) Earth-Venus separation at arrival.

(c) Configuration angle at departure.

Figure 6. - Path parameters for orbiting Venusian probe launched from an orbit of 1.1 Earth radii. Terminal orbit is at 1.1 Venus radii. Data for minimum total velocity increment.

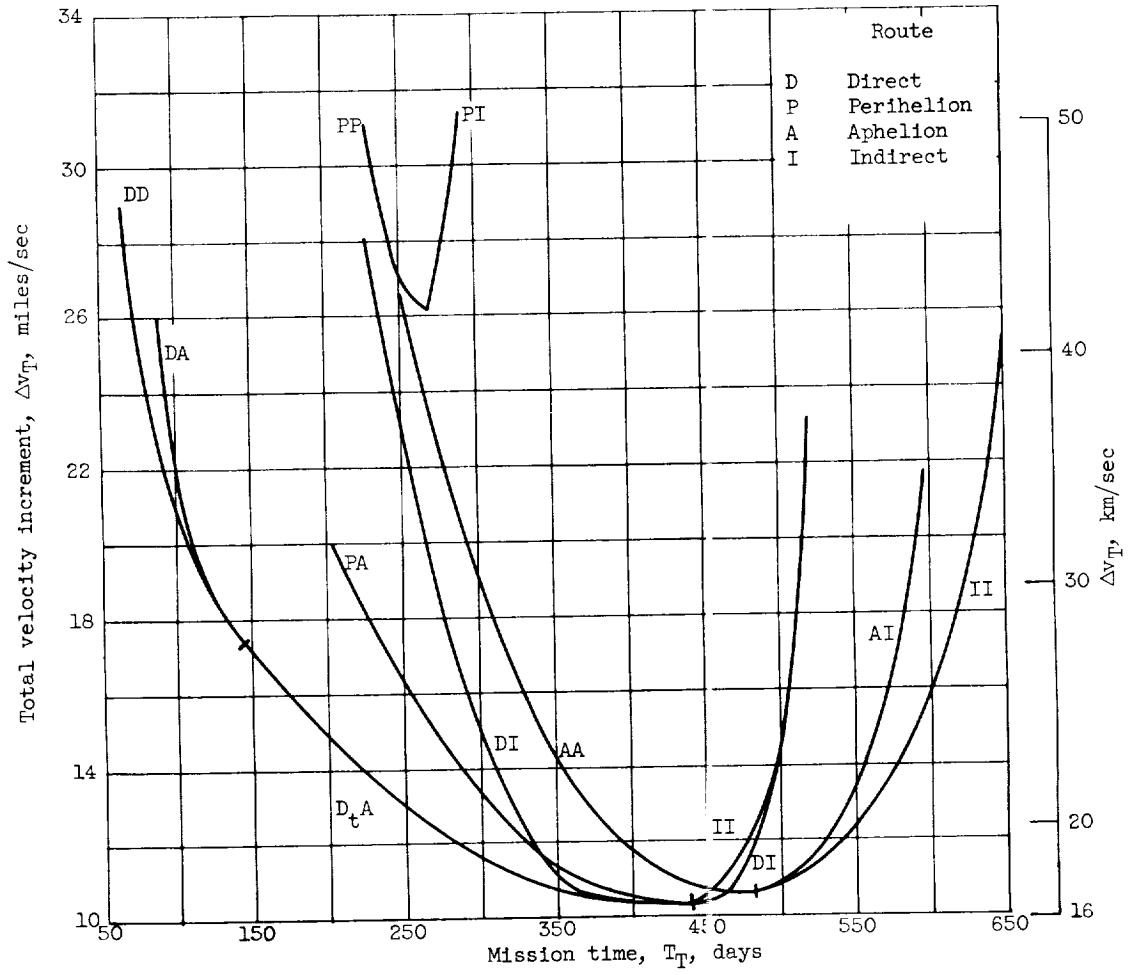
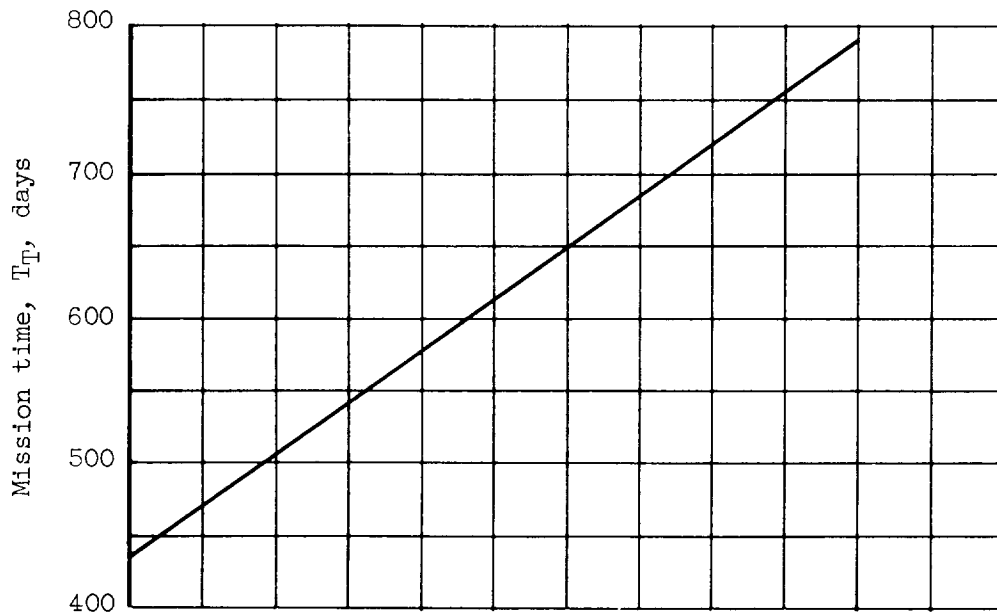


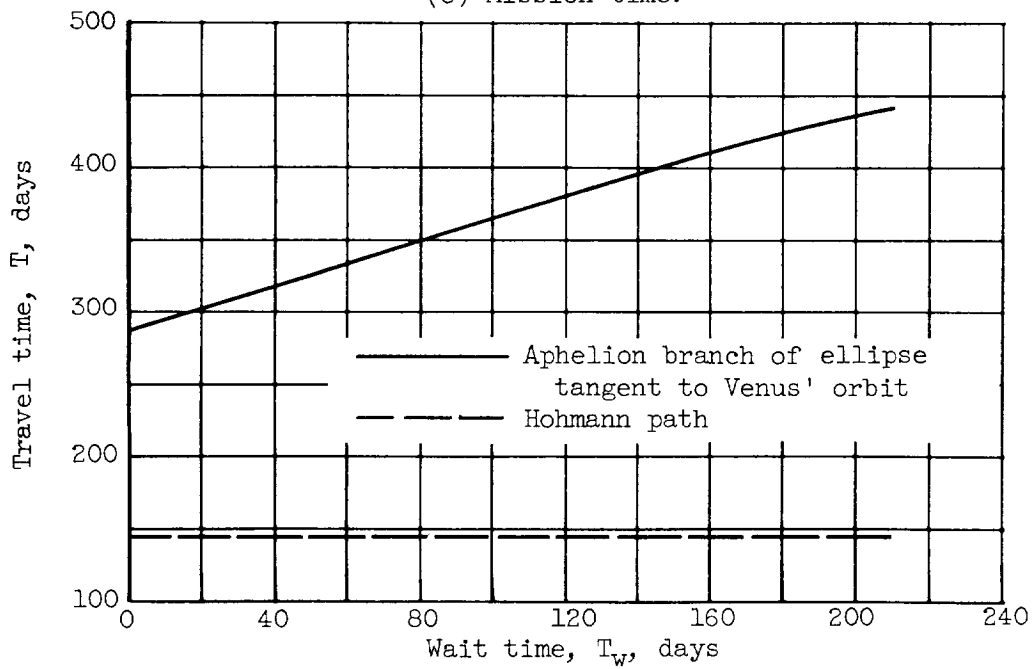
Figure 7. - Total velocity increments and mission times for round trips to Venus starting, waiting, and ending in circular orbits at 1.1 planet radii. Wait time, 0 days.

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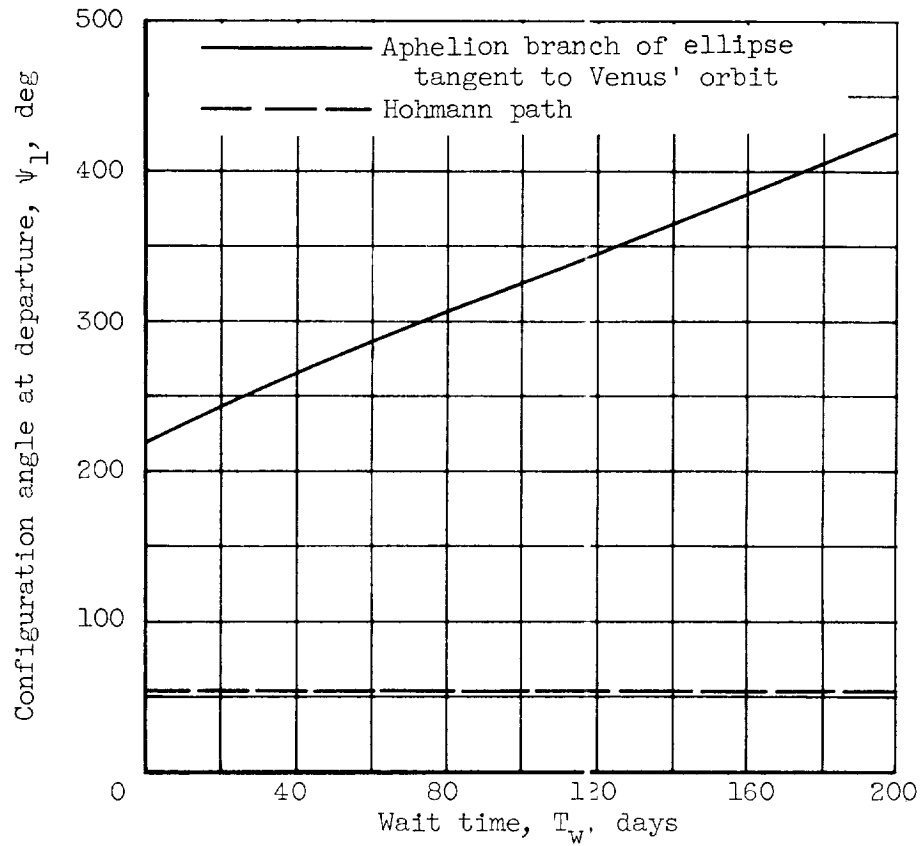


(c) Mission time.



(d) Travel time.

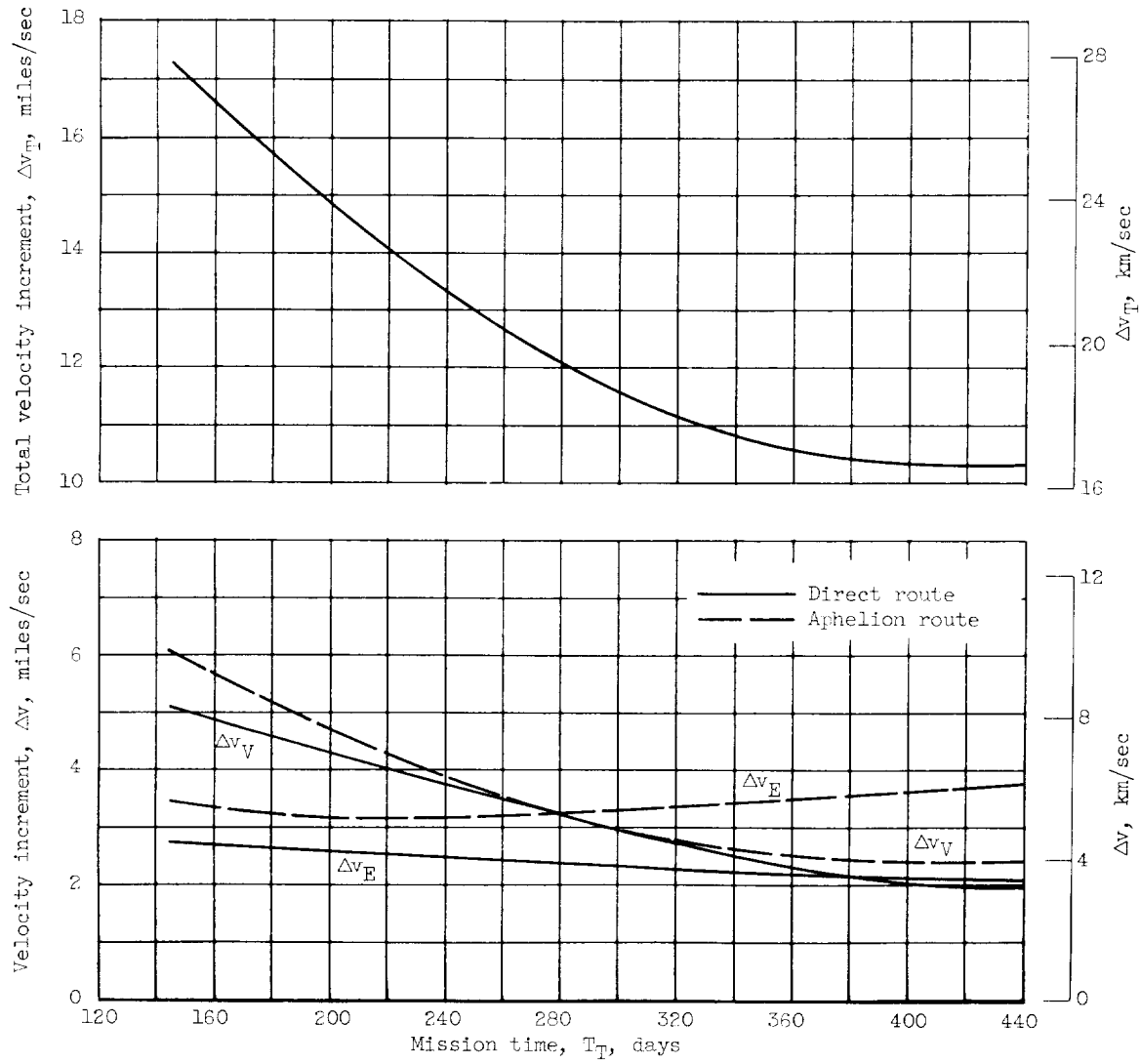
Figure 9. - Continued. Path parameters for direct route tangent to Earth's orbit and aphelion round trips to Venus starting, waiting, and ending in circular orbits at 1.1 planet radii. Data for minimum total velocity increment. Either outward or return trip must be taken along Hohmann ellipse.



(e) Configuration angle at departure from Earth.

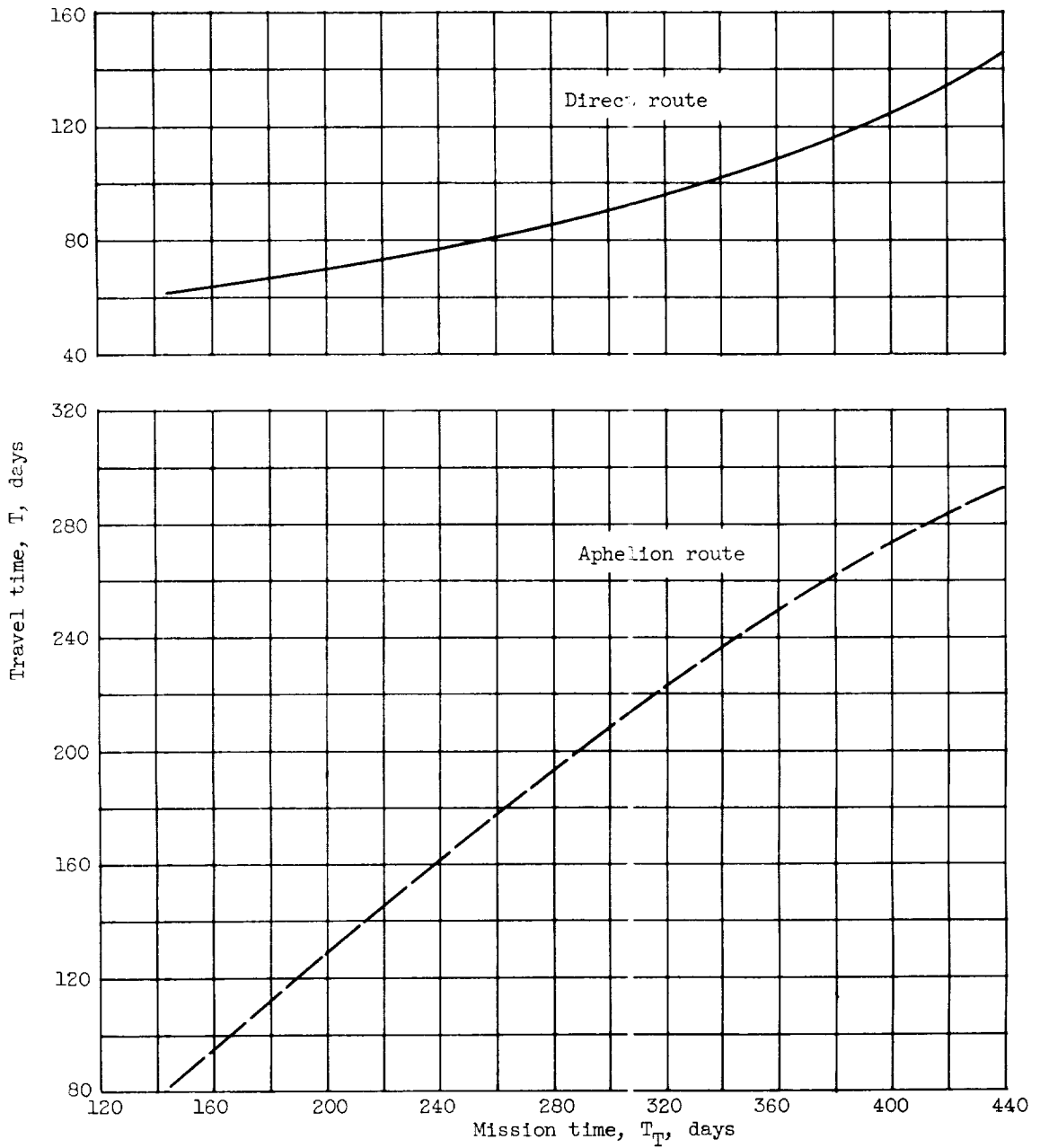
Figure 9. - Concluded. Path parameters for direct route tangent to Earth's orbit and aphelion round trips to Venus starting, waiting, and ending in circular orbits at 1.1 planet radii. Data for minimum total velocity increment. Either outward or return trip must be taken along Hohmann ellipse.

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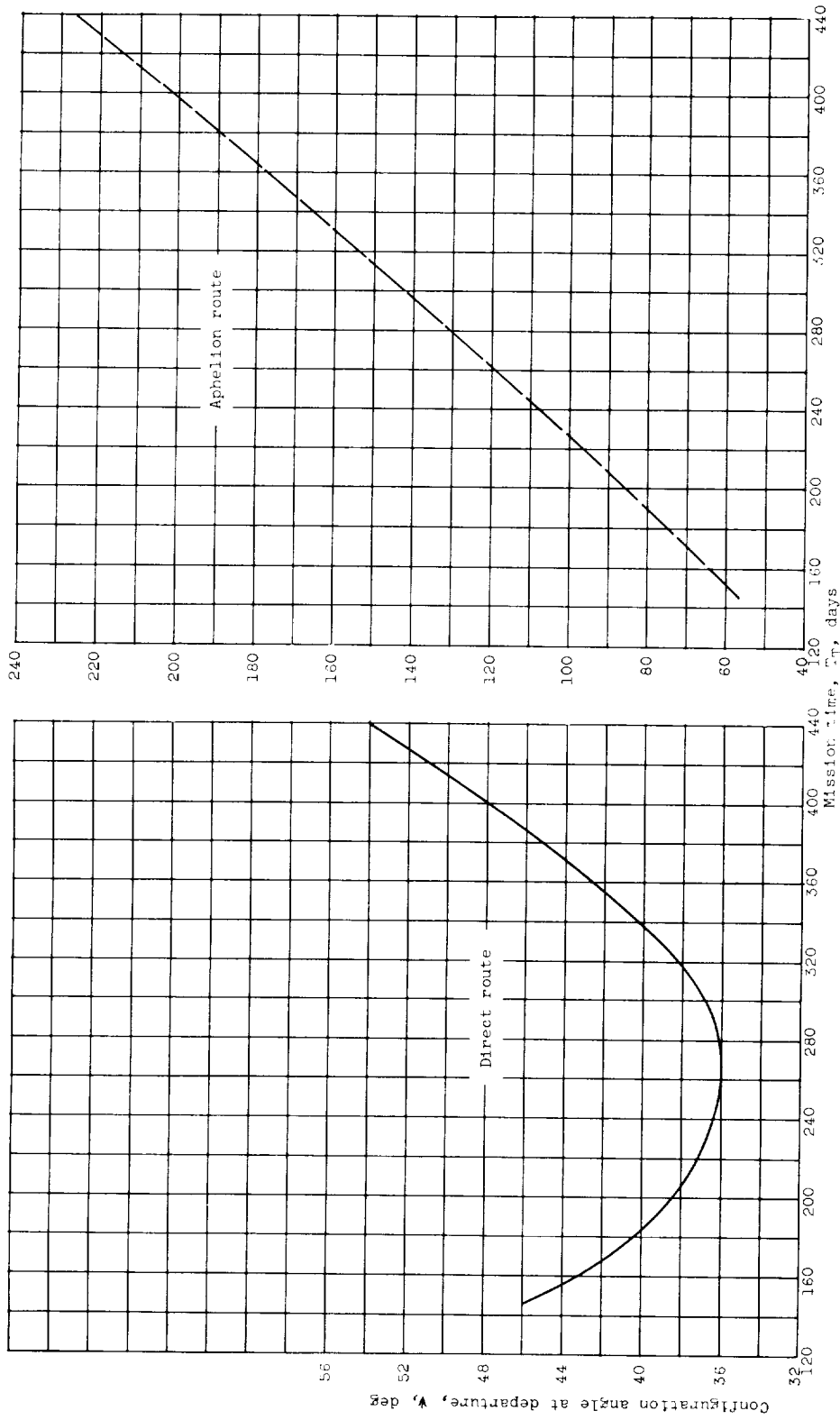
(a) Velocity increments.

Figure 10. - Path parameters for direct route tangent to Earth's orbit and aphelion round trips to Venus starting, waiting, and ending in circular orbits at 1.1 planet radii. Wait time in parking orbit at Venus, 0 days.



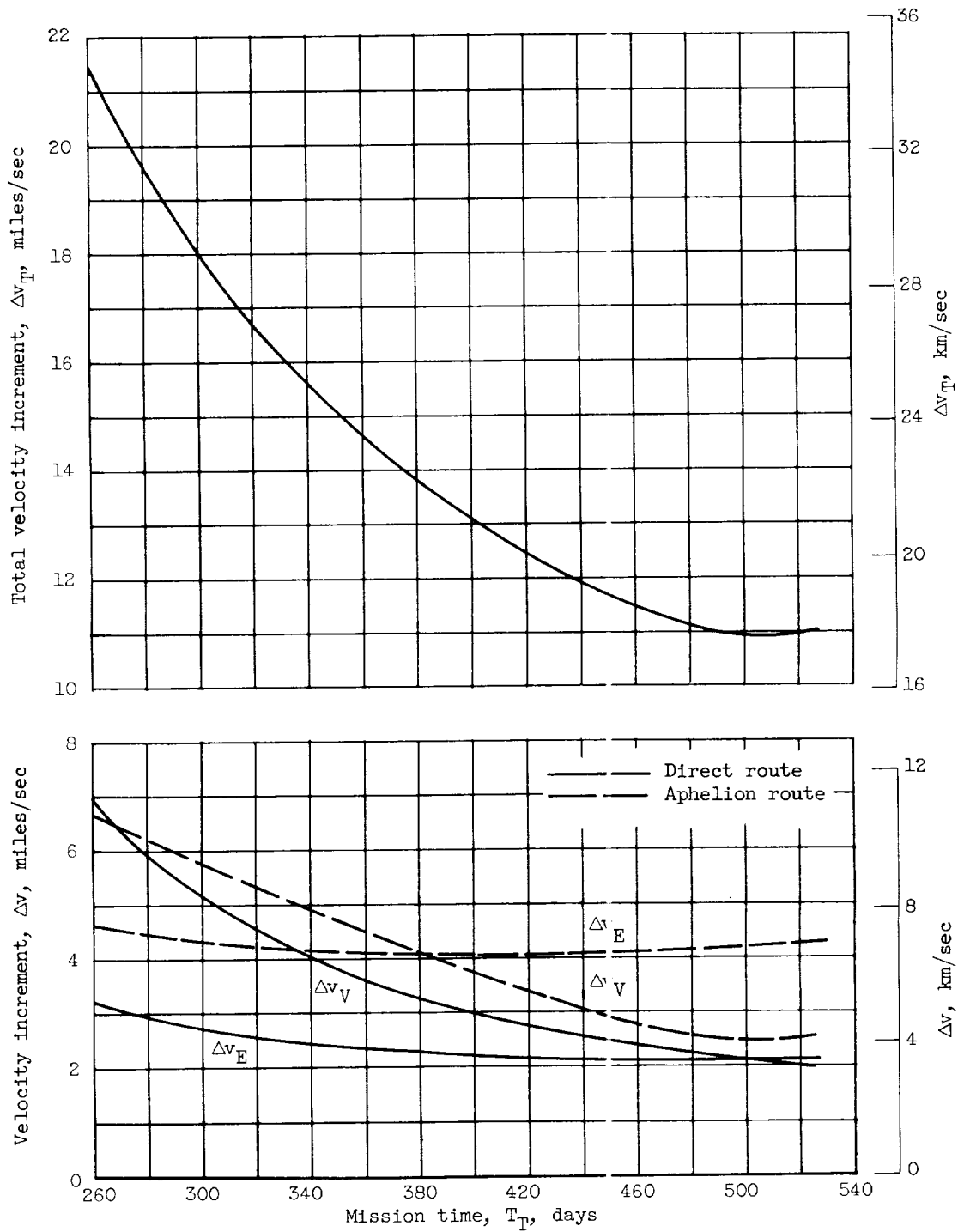
(b) Travel time.

Figure 10. - Continued. Path parameters for direct route tangent to Earth's orbit and aphelion round trips to Venus starting, waiting, and ending in circular orbits at 1.1 planet radii. Wait time in parking orbit at Venus, 0 days.



(c) Configuration angle.

Figure 10. - Concluded. Path parameters for direct route tangent to Earth's orbit and apheilion round trips to Venus starting, waiting, and ending in circular orbits at 1.1 planet radii. Wait time in parking orbit at Venus, 0 days.

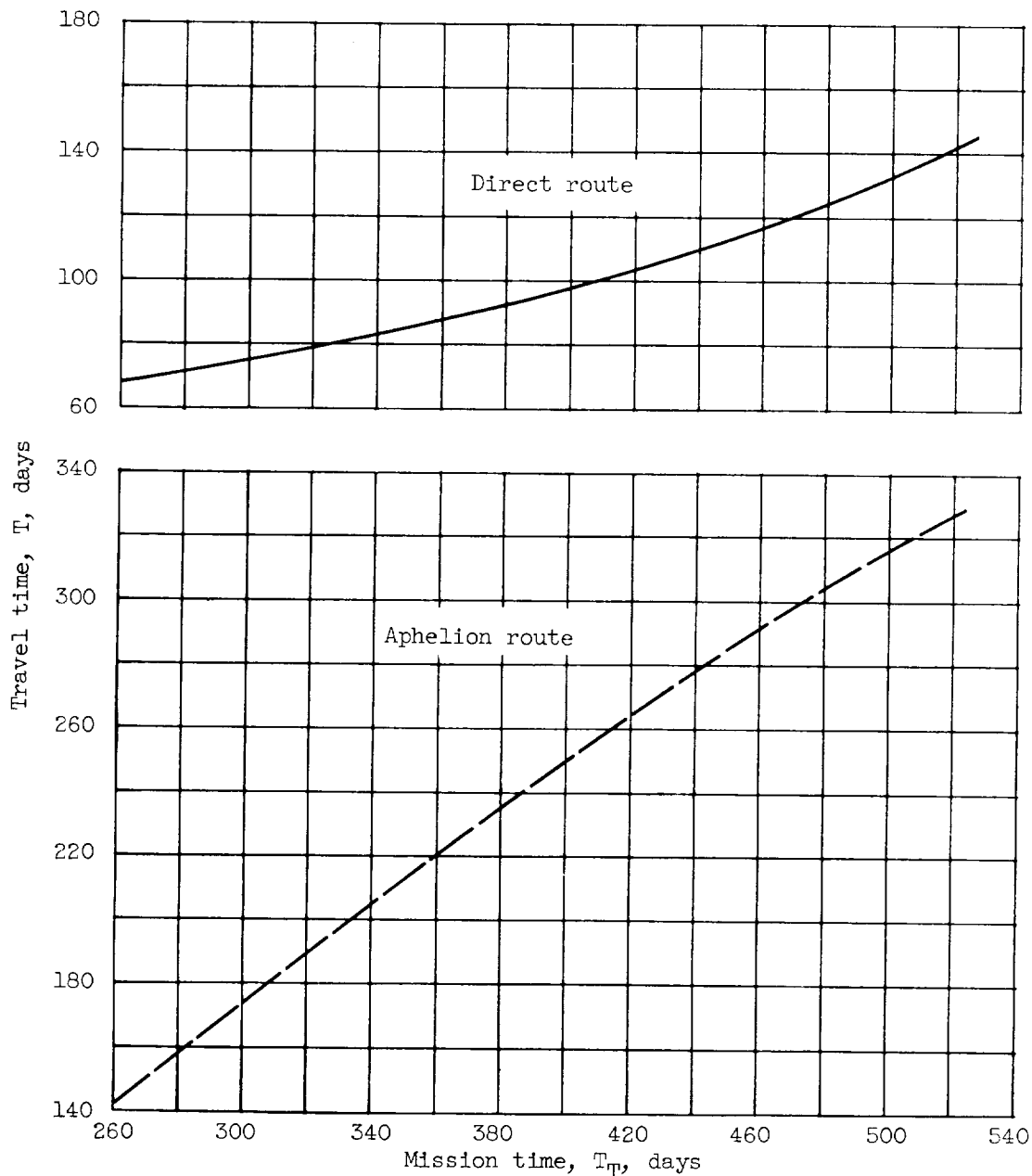


(a) Velocity increments.

Figure 11. - Path parameters for direct route tangent to Earth's orbit and aphelion round trips to Venus starting, waiting, and ending in circular orbits at 1.1 planet radii. Wait time in parking orbit at Venus, 50 days.

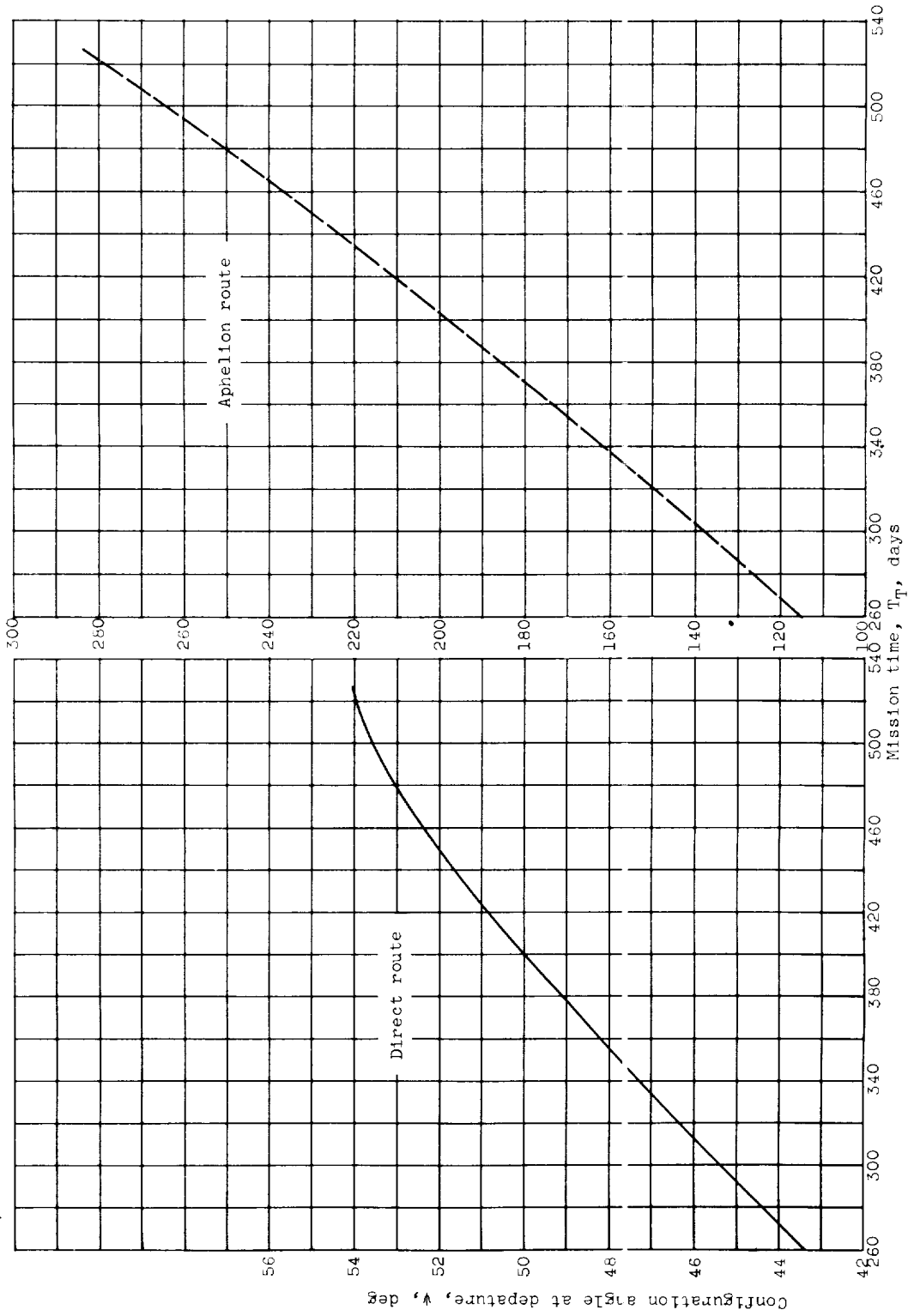


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(b) Travel time.

Figure 11. - Continued. Path parameters for direct route tangent to Earth's orbit and aphelion round trips to Venus starting, waiting, and ending in circular orbits at 1.1 planet radii. Wait time in parking orbit at Venus, 50 days.

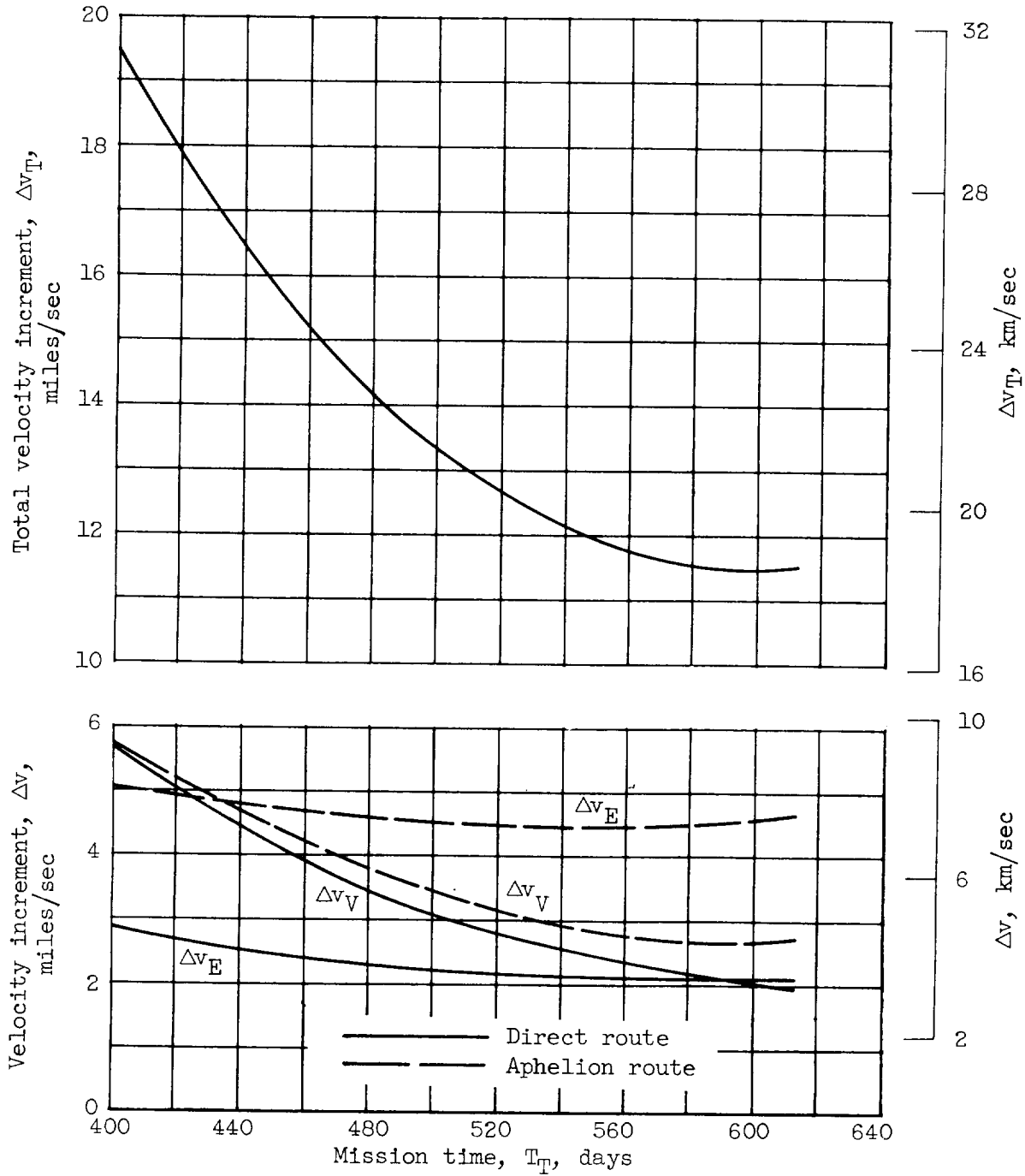


(c) Configuration angle.

Figure 11. - Concluded. Path parameters for direct route tangent to Earth's orbit and aphelion round trips to Venus starting, waiting, and ending in circular orbits at 1.1 planet radii. Wait time in parking orbit at Venus, 50 days.

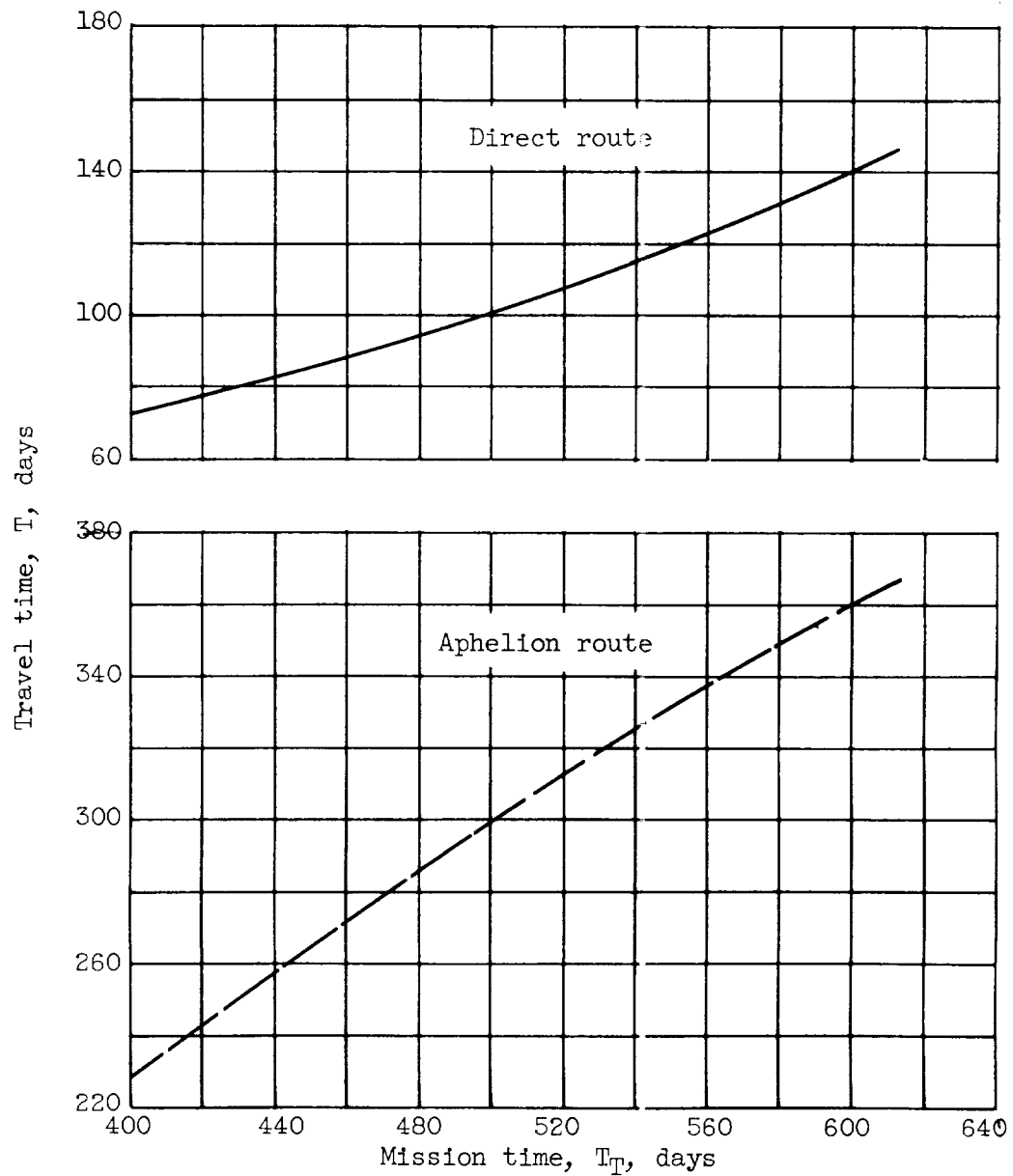
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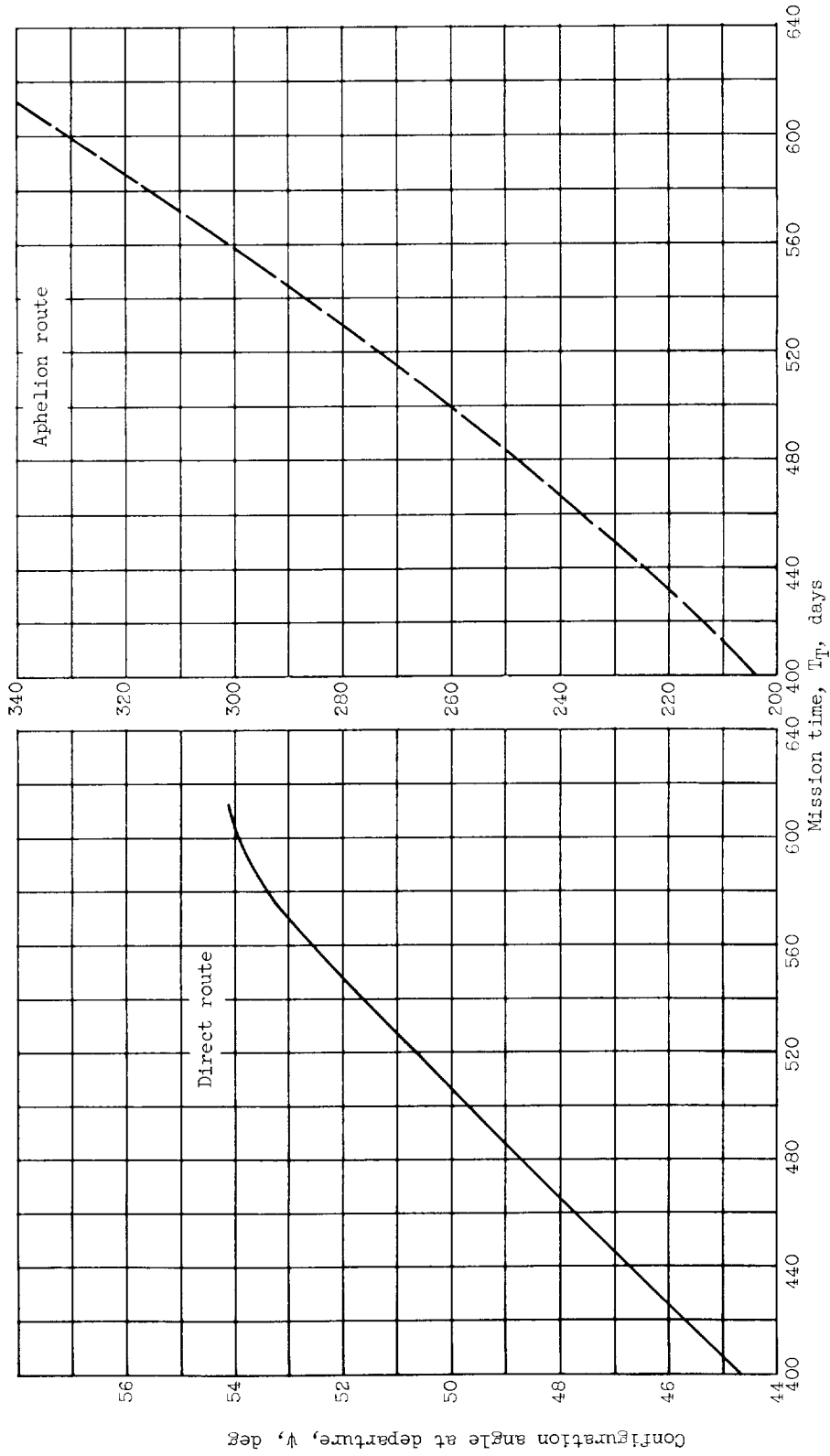
(a) Velocity increments.

Figure 12. - Path parameters for direct route tangent to Earth's orbit and aphelion round trips to Venus starting, waiting, and ending in circular orbits at 1.1 planet radii. Wait time in parking orbit at Venus, 100 days.



(b) Travel time.

Figure 12. - Continued. Path parameters for direct route tangent to Earth's orbit and aphelion round trips to Venus starting, waiting, and ending in circular orbits at 1.1 planet radii. Wait time in parking orbit at Venus, 100 days.



(c) Configuration angle.

Figure 12. - Concluded. Path parameters for direct route tangent to Earth's orbit and aphelion round trips to Venus starting, waiting, and ending in circular orbits at 1.1 planet radii. Wait time in parking orbit at Venus, 100 days.

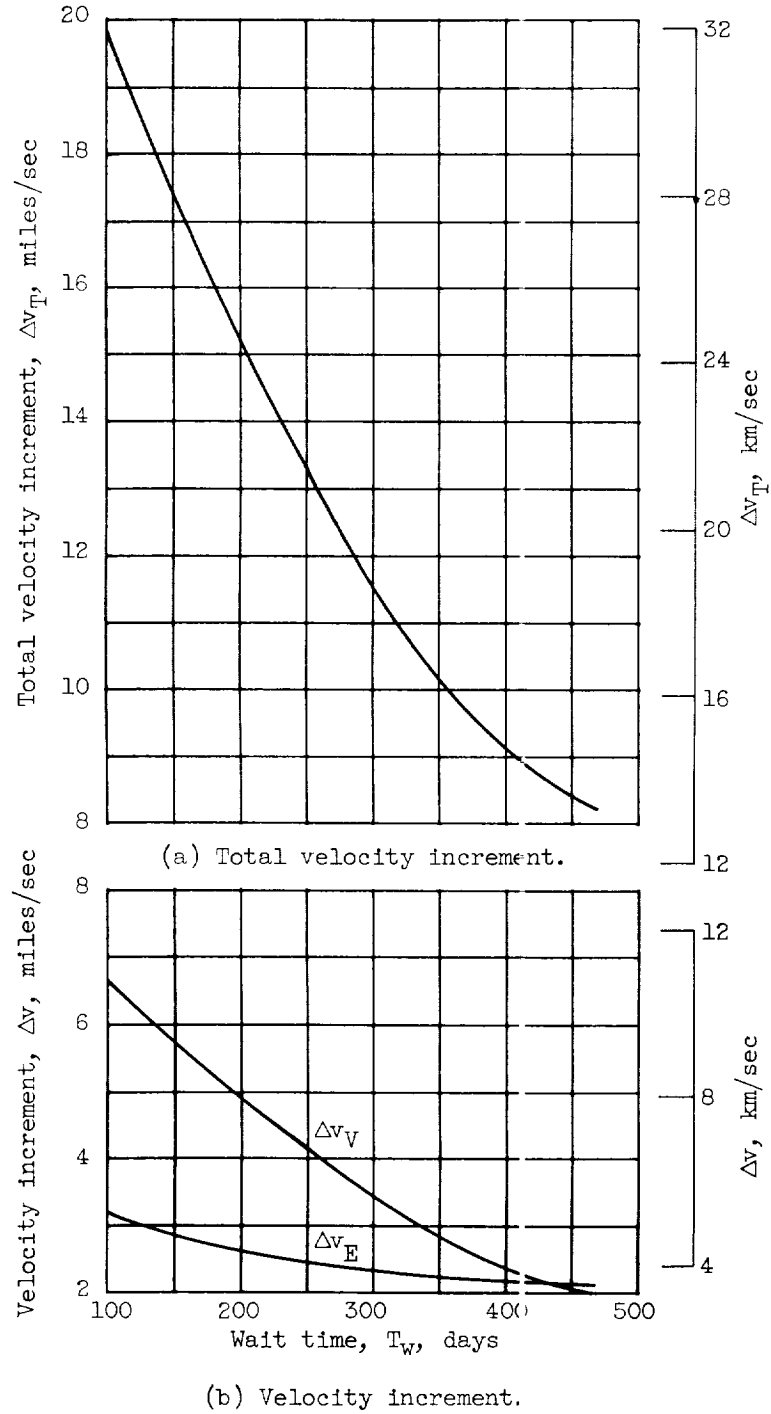
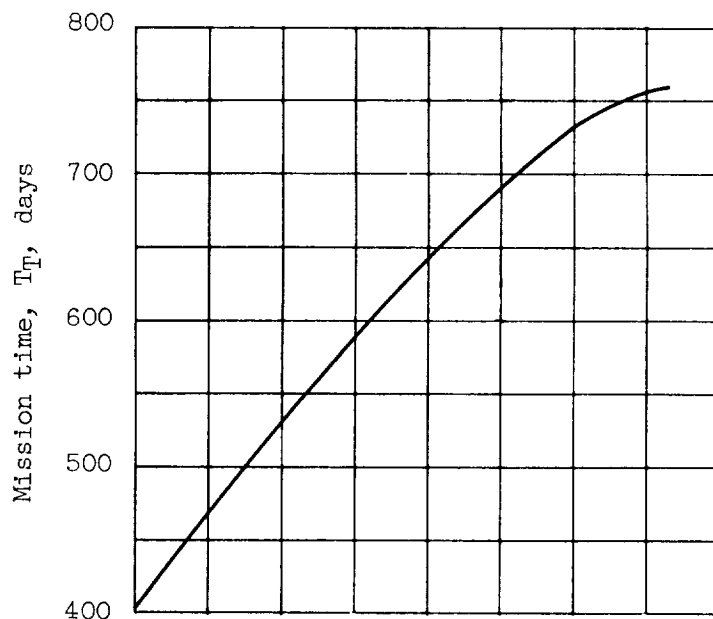
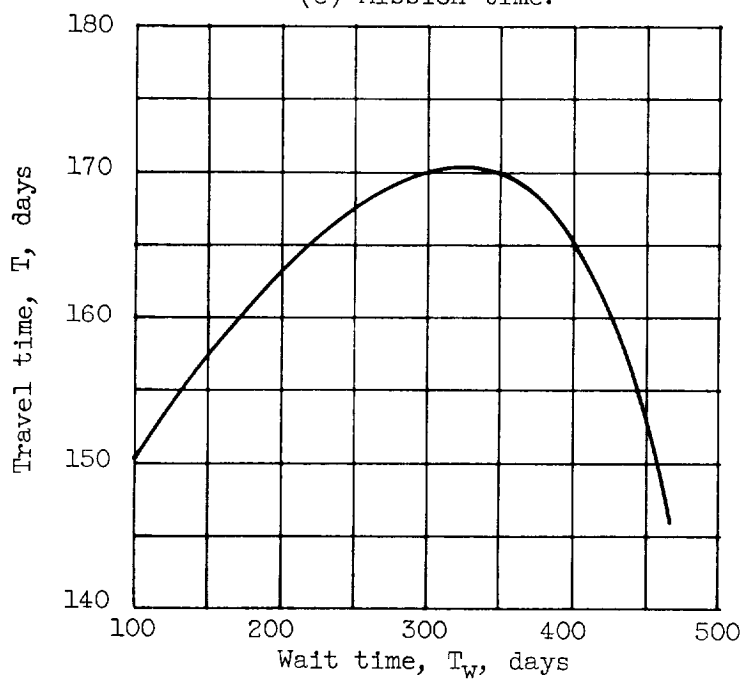


Figure 13. - Path parameters for perihelion-perihelion round trips to Venus starting, waiting, and ending in circular orbits at 1.1 planet radii. Data for minimum velocity increment. Outward and return paths are the same.

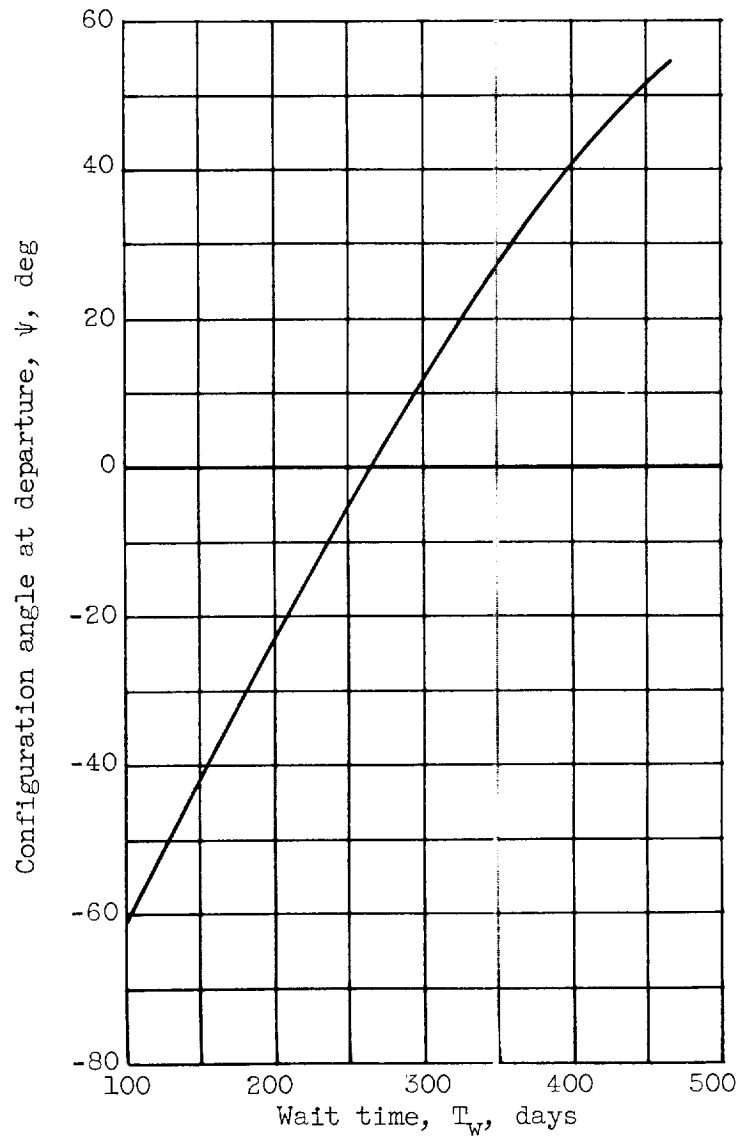


(c) Mission time.



(d) Travel time.

Figure 13. - Continued. Path parameters for perihelion-perihelion round trips to Venus starting, waiting, and ending in circular orbits at 1.1 planet radii. Data for minimum velocity increment. Outward and return paths are the same.



(e) Configuration angle at departure.

Figure 13. - Concluded. Path parameters for perihelion-perihelion round trips to Venus starting, waiting, and ending in circular orbits at 1.1 planet radii. Data for minimum velocity increment. Outward and return paths are the same.