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On the existence of low-luminosity cataclysmic variables beyond the orbital period minimum

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ABSTRACT

Models of the present-day intrinsic population of cataclysmic variables predict that 99 per cent of these systems should be of short orbital period ($P_{\rm orb} \lesssim 2.5$ h). The Galaxy is old enough that ~ 70 per cent of these stars will have already reached their orbital period minimum (~ 80 min), and should be evolving back toward longer periods. Mass-transfer rates in these highly evolved binaries are predicted to be $\lesssim 10^{-11}$ M$_\odot$ yr^{-1}, leading to M_V of ~ 10 or fainter, and the secondaries would be degenerate, brown dwarf-like stars. Recent observations of a group of low-luminosity dwarf novae (TOADs) provide observational evidence for systems with very low intrinsic M_V and possibly low-mass secondaries. We carry out population synthesis and evolution calculations for a range of assumed ages of the Galaxy in order to study $P_{\rm orb}$ and $\dot{M}$ distributions for comparison with the TOAD observations. We speculate that at least some of the TOADs are the predicted very low-luminosity, post-period-minimum cataclysmic variables containing degenerate (brown dwarf-like) secondaries having masses between 0.02 and 0.06 M$_\odot$ and radii near 0.1 R$_\odot$. We show that these low-luminosity systems are additionally interesting in that they can be used to set a lower limit on the age of the Galaxy. The TOAD with the longest orbital period currently known (123 min), corresponds to a Galaxy age of at least 8.6×10^9 yr.

Key words: binaries: close – stars: low-mass, brown dwarfs – novae, cataclysmic variables – Galaxy: evolution.

1 INTRODUCTION

Cataclysmic variables (CVs) are a class of interacting close binary stars with typical orbital periods ranging from 80 min to ~ 10 h. The two components, a more massive white dwarf (WD) primary and a low-mass secondary, are typically separated by only a few solar radii. CVs include dwarf novae (DN), nova-like, magnetic systems (AM and DQ Hers), and classical novae. Comprehensive reviews of these systems and their evolution are given in Patterson (1984), King (1988) and Warner (1995a).

In the DN, material transferred from the secondary (via Roche-lobe overflow) forms an accretion disc around the primary star that can extend all the way to the WD surface. These stars show outbursts of 2–5 mag, which are widely thought to occur when material stored in the accretion disc is suddenly accreted on to the WD surface due to angular momentum loss caused by thermally unstable viscous heating (cf. Cannizzo, Shafter & Wheeler 1988). Depending on the rate of mass transfer from the secondary, which is related to the orbital period (e.g. Rappaport, Verbunt & Joss 1983, hereafter RVJ; Warner 1995a), the disc can be the dominant light source from high energies to the IR. Typically determined mean values of M_V for DN are 7.5 for systems with orbital periods $\gtrsim 3$ h, and 9.5 for systems with orbital periods $\lesssim 2$ h. CVs with orbital periods in the range of 2–3 h are uncommon, and this interval has been termed the 'period gap'.

Warner (1995a) provides observational information on all CVs for which orbital periods and other detailed information are known. Our current observational knowledge is severely biased towards CVs with orbital periods $\gtrsim 2.5$ h, or those with high mass-transfer rates (i.e., intrinsically bright CVs). Even surveys which covered large areas of the sky searching for UV-excess or blue objects have fallen short of improving on this situation. For example, the PG survey (Green et al. 1982) covered just over 10 000 deg^2 and discovered 29 CVs (Ringwald 1993), but it had an average limiting magnitude of $B_{\rm lim} \sim 16$, so that it discovered only intrinsically bright systems, most with long (≥ 3 h) orbital periods. Our current view of CVs is thus a very skewed one, as the majority of observed CVs are not representative of the actual, or intrinsic, CV population (see Section 2).

During the past several years, Howell and collaborators (Howell & Szkody 1990; Howell, Szkody & Cannizzo 1995; Sproats, Howell & Mason 1996) have provided data to help remedy the problem. They have obtained observations of CVs which are faint, including a subgroup of DN which are *intrinsically* faint, having M_V of 10 to 14, and which show very large-amplitude outbursts (6–10 mag). These tremendous outburst amplitude dwarf novae, or TOADs, have the following properties: infrequent outbursts (months to decades), very low inferred mass-transfer rates ($\dot{M} \lesssim 10^{-11}$ M$_\odot$ yr^{-1}, implying optically thin discs), very low-viscosity disc material in the quiescent state, and short orbital periods ($\lesssim 2.5$ h). The TOADs consist of stars such as WZ Sge and AL Com, two of the shortest period DN known, but also include systems such as TV Crv and EF Peg which have orbital periods near 120 min, just below the period gap (see Howell et al. 1995 for a complete listing).

Absolute magnitudes and inferred mass-transfer rates have been calculated for the faint CVs mentioned above. The inferred mass-transfer rates are based on Smak's (1993) relationship between M_V and $\dot{M}$ as shown in Warner (1995a, fig. 9.8), and do not represent a detailed quantitative relationship (see Section 3). These faint systems are shown in Fig. 1, along with those previously known and catalogued in Warner (1987, 1995a). Two previously known faint systems are now known to be TOADs (WZ Sge and T Leo; the two open squares near $M_V = 11$). The TOADs have calculated M_V of 10 to 14 and inferred $\dot{M}$ in quiescence of 10^{-11} to 10^{-13} M$_\odot$ yr^{-1} (Sproats et al. 1996). From Fig. 1, we see that the TOADs represent an interesting set of stars which have been found to be intrinsically low-luminosity objects. Their observationally derived absolute magnitudes and low $\dot{M}$ values (see Section 4) indicate that these stars represent a class of objects that are not well fitted by the standard CV relations between M_V and orbital period (cf. Warner 1995a).

Several authors have discussed the existence of CVs with degenerate secondaries (e.g. Paczyński & Sienkiewicz 1981; Rappaport, Joss & Webbink 1982; Lamb & Melia 1987), and theoretical models of the intrinsic CV population predict that the majority of CVs contain degenerate secondaries (Kolb 1993; see also Section 2). It has also been recently suggested (for observational reasons related to photometric behaviour during outburst) that TOADs may contain low-mass, degenerate secondaries (Howell et al. 1995; Warner 1995b; Howell et al. 1996). Using arguments based on the standard theory of CV formation and evolution, and on the observational properties of the low-luminosity systems presented in Fig. 1, we explore the possibility that at least some TOADs may indeed be the oldest CVs in the Galaxy. If so, they (i) represent the first evidence of the predicted large population of very low-luminosity CVs, (ii) have evolved past the orbital period minimum and are evolving back to periods of ~ 2 h, (iii) should contain very low-mass degenerate (brown dwarf-like) secondary stars, and (iv) may yield a useful constraint on the age of the Galaxy.

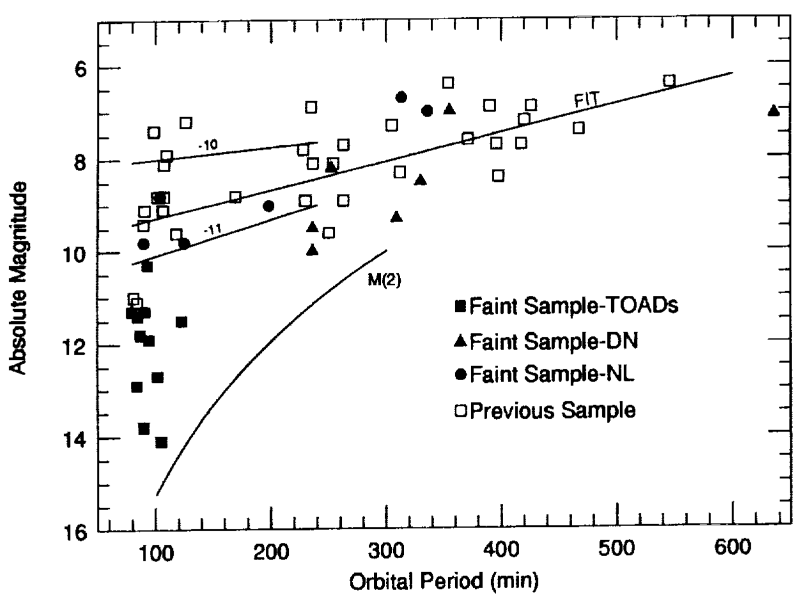

Figure 1. Absolute magnitude versus orbital period for CVs. This plot shows the previous sample for 'typical' systems (Warner 1987: open squares) and the new sample of faint CVs (Sproats et al. 1996: filled symbols) including the TOADs. Typical errors for M_V are ± 1 mag. The two open squares near $M_V = 11$ are previously known DN which are now known to be TOADs. The line marked 'FIT' is the standard relation between orbital period and M_V (using inferred $\dot{M}$ values: Warner 1995a) averaged over all orbital periods, and the two short lines represent the expected M_V values for mass-transfer rates of 1×10^{-10} and 1×10^{-11} M$_\odot$ yr^{-1}. The line labelled M(2) shows the known M_V of single, late-type main-sequence stars (appropriate for Roche-lobe-filling CV secondaries before the period minimum is reached).

2 THE INTRINSIC CV POPULATION

The intrinsic population of CVs has been modelled in detail (e.g. Politano 1988, 1994, 1996; de Kool 1992; Kolb 1993), and a comparison with the observed population clearly illustrates the under-representation of low-luminosity systems prevalent in our current observational picture of CVs. The current percentage of observed CVs with orbital periods greater than 3 h is ~55 per cent, whereas in the intrinsic population this number is expected to be ~1 per cent (Kolb 1993), indicating a *strong* bias toward long-period (bright) systems. Also, the mean WD mass in observed CVs is $\sim 0.8\,M_\odot$ (Ritter & Kolb 1995), whereas the intrinsic mean WD mass is predicted to be $\sim 0.5\,M_\odot$ (Politano 1988, 1996). Selection effects, such as observing CVs with magnitudes of $V=16$ or brighter, have been shown to introduce a severe bias toward systems with high-mass WDs and/or high mass-transfer rates (e.g. Ritter & Burkert 1986; Dünhuber 1993; Howell et al. 1995). The remedy to the current skewed state of affairs in CVs is to reduce these selection effects by observing to fainter (apparent and absolute) magnitudes, and thereby provide a more accurate picture of the actual CV population.

The data in Fig. 1 provide us with an observational sample of CVs that may possibly represent systems belonging to the intrinsic CV population, especially at short orbital periods, and therefore is a sample that can potentially provide meaningful tests of theoretical models. Theoretical models of the intrinsic, present-day CV population predict that ~99 per cent of all CVs have orbital periods $\lesssim 2.5$ h (Kolb 1993). These systems are expected to be intrinsically faint ($M_V \gtrsim 8$), and to have low mass-transfer rates ($\dot{M} \lesssim 10^{-10}\,M_\odot\,yr^{-1}$). In addition, as a typical CV evolves, it reaches a minimum orbital period near 80 min (see, e.g., Paczyński & Sienkiewicz 1981 and Rappaport et al. 1982). The Galaxy is old enough that ~70 per cent of all CVs are predicted to have reached this period minimum and to be currently evolving towards longer orbital periods (Kolb 1993). CVs in this latter 70 per cent are predicted to have very low mass-transfer rates ($\dot{M} \lesssim 10^{-11}\,M_\odot\,yr^{-1}$, $M_V \gtrsim 10$) and to contain very low-mass ($\leq 0.06\,M_\odot$), degenerate (brown dwarf-like) secondaries (e.g. Rappaport et al. 1982; RVJ; this paper, Section 3).

The systems in Fig. 1, taken at face value, can be used to begin to test the validity of the above predictions. Of the 26 observed systems with orbital periods below 3 h, 14 of them, or 54 per cent, have $M_V \geq 10$ and estimated values of $\dot{M} \lesssim 10^{-11}\,M_\odot\,yr^{-1}$. As the sample of low-luminosity CVs is increased and more quantitative determinations of the observed and inferred parameters can be made, we will be able to provide important (and long-awaited) constraints on theoretical models of the intrinsic CV population.

3 SECULAR EVOLUTION AND CVs WITH DEGENERATE SECONDARIES

In the conventional picture of CV evolution (see, e.g., RVJ and Hameury et al. 1988), the early phases are expected to be dominated by angular momentum losses due to magnetic braking via a magnetically constrained stellar wind from the donor star. Mass-transfer rates are typically $\sim 10^{-8}$ to $10^{-9}\,M_\odot\,yr^{-1}$ for these systems, and typical orbital periods range from ~10 to ~3 h, just at the upper edge of the period gap. At some point in the evolution, the secondary becomes completely convective (at $\sim 0.3\,M_\odot$) and, in the currently accepted view, magnetic braking is assumed to be greatly reduced. The near cessation of magnetic braking reduces the mass-transfer rate and allows the secondary to shrink toward its thermal equilibrium radius. This causes a period of detachment (in which $\dot{M}$ drops to essentially zero) which lasts until the Roche lobe shrinks sufficiently to bring the secondary back into contact with it, at an orbital period of ~2 h. This is the commonly accepted explanation for the observed period gap at 2–3 h in CVs (RVJ; Spruit & Ritter 1983).

When mass transfer recommences at $P_{orb} \sim 2$ h, it is then driven largely by gravitational radiation losses at rates of $\sim 10^{-10}\,M_\odot\,yr^{-1}$. As the orbit shrinks and the mass of the donor star decreases, the mass-loss time-scale increases, but the thermal time-scale, τ_{KH}, increases much faster, due to the $\sim M^{-3}$ dependence of τ_{KH}. Therefore, at some point the thermal time-scale grows larger than the mass-transfer time-scale. When this occurs, the donor star is unable to adjust to the mass-loss on its thermal time-scale, and it therefore starts to expand upon further mass-loss, in accordance with its adiabatic response, i.e., $[d\ln(R)/d\ln(M)]_{ad} < 0$. Somewhat before this point is reached, the orbital period begins to increase with further mass transfer. The orbital period at this point is typically ~80 min, and the mass of the donor star is $\sim 0.06\,M_\odot$. From this point on, the mass of the donor star will continue to decrease (but with longer and longer time-scales), the orbital period will increase back up to periods approaching ~2 h (within a Hubble time), and electrons in the interior of the donor star will become increasingly degenerate.

To make some of these evolutionary descriptions somewhat more quantitative, we show in Fig. 2 the secular evolution of a CV under the influence of magnetic braking and gravitational radiation. The evolution code used to generate these results is very nearly the same as was used by RVJ, except that the treatment of the secondary (donor star) has been improved. To calculate the evolution of the secondary, we used a version of our code that has been used previously to follow the evolution of brown dwarfs (Nelson, Rappaport & Joss 1986, 1993). The results of these brown dwarf calculations are in excellent accord with those of Lunine, Hubbard & Marley (1986), who utilized a more sophisticated evolution code.

The initial constituent masses of the system whose evolution is shown in Fig. 2 were $M_{WD}=0.8\,M_\odot$ and $M_{donor}=0.5\,M_\odot$. Fig. 2(a) shows both the orbital period and the mass-transfer rate as functions of evolution time, for an assumed donor star with solar composition. The calculations have been carried out beyond the oldest plausible age for such a binary. All of the evolutionary phases and features discussed above are clearly present in Fig. 2. We note that the value of P_{min} is not substantially influenced by the prior evolution either with or without magnetic braking.

As the system reaches the minimum period and evolves to longer orbital periods, $\dot{M}$ decreases from $\sim 10^{-10}\,M_\odot\,yr^{-1}$ to about $\sim 10^{-12}\,M_\odot\,yr^{-1}$ and the orbital period increases from its minimum value of ~80 min to ~2 h within an evolution time of $\sim 10^{10}$ yr. In Fig. 2(b) we show the corre-

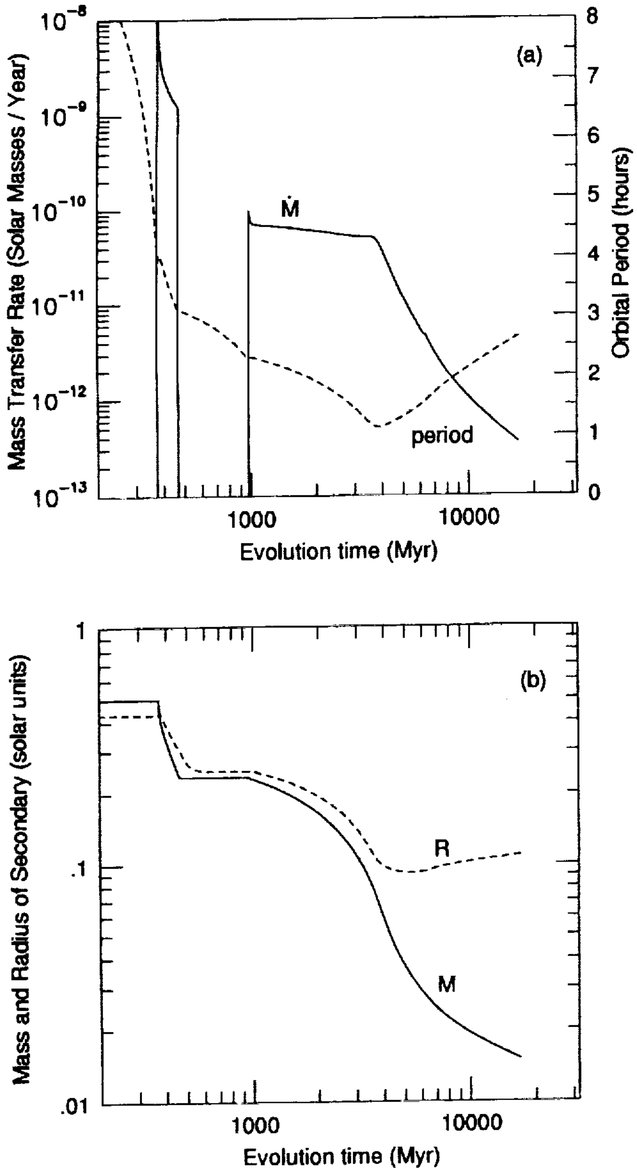

Figure 2. Model of the secular evolution of a CV driven by magnetic braking and gravitational radiation. The figure shows the evolution over time of (a) the orbital period and mass-transfer rate, and (b) the radius and mass of the secondary (donor) star.

sponding evolution of the mass and the radius of the donor star. Note that as the donor star becomes increasingly degenerate, for masses below $\sim 0.06\,M_\odot$, the combination of adiabatic expansion with mass-loss, and the loss of thermal energy from the star, keep the radius nearly constant at about $\sim 0.1\,R_\odot$. This is, in fact, very close to the radius of a completely degenerate star of mass $0.01\,M_\odot$ with a solar composition, and only ~ 40 per cent larger than the radius of a degenerate $0.05\text{-}M_\odot$ star.

While systems with such low $\dot{M}$ may, at first glance, appear to be unobservable, in fact it appears that we may have already observed a number of such short-period systems with $\dot{M} \leq 10^{-12}\,M_\odot\,\mathrm{yr}^{-1}$ ($M_V \sim 12$–14; see Fig. 1). The data are consistent with those systems having already reached their minimum period, evolving towards longer periods, and containing substantially degenerate secondaries.

Thus we see that in the lowest luminosity CVs, the secondary stars are brown dwarf-like objects with masses equal to 20–60 Jupiter masses and radii that are all very close to $0.1\,R_\odot$, similar to 'field' brown dwarfs. While they used to be normal hydrogen-burning stars (i.e., red dwarfs), they should now have similar effective temperatures to field brown dwarfs. One difference, however, is that the optically thin accretion disc will *not* effectively shield the secondary, and X-ray heating is likely to be important. We also would *not* expect the presence of Li spectral features (as are expected in field brown dwarfs; e.g. Nelson, Rappaport & Chiang 1993). The TOAD brown dwarfs will continue to lose mass and, like 'field' brown dwarfs, cool down and become increasingly degenerate.

To investigate further the expected distribution and properties of the systems we are tentatively identifying with TOADs, we have carried out a population synthesis calculation of CVs, with emphasis on systems below the period gap. (For earlier population synthesis studies of CVs see, e.g., Politano 1996, de Kool 1992, Kolb 1993 and Di Stefano & Rappaport 1994.) For the present study we utilize a Monte Carlo approach with most of the same input assumptions as were used in the population synthesis study of supersoft X-ray sources carried out by Rappaport, Di Stefano & Smith (1994, hereafter RDS). We briefly review the procedure here, but refer the reader to the cited paper for more details. We start by assuming an age for the Galaxy, t_G. Then, 10^7 primordial binaries are chosen; for each, the primary mass, secondary mass, and orbital period are chosen in accordance with the 'standard model' detailed in RDS (see their table 2 and equation 1). The time of birth for each binary, t, was chosen from a uniform random distribution in the range: $0 < t < t_G$. Each primordial binary was 'followed' (see RDS for the prescriptions used) to see if mass transfer from the primary to the secondary would occur, and if such mass transfer would result in a common-envelope phase – the end-product of which would be a low-mass star in orbit with a WD. The assumed energetics that govern the end-point of the common-envelope phase are described by equation (2) of RDS. The evolution time, up to and including the common-envelope phase, was taken to be just the main-sequence lifetime of the primary.

If a particular primordial binary was 'successful' in evolving into a WD main-sequence binary, then the subsequent evolution was followed in detail with the same binary evolution code used to carry out the calculation shown in Fig. 2. As in most other related binary evolution calculations, we made the assumption that magnetic braking is active when, and only when, the donor star has a radiative core (see RVJ, Spruit & Ritter 1983 and Hameury et al. 1988). The magnetic braking was specified by the Verbunt & Zwaan (1981) model with parameter values of $\gamma = 3$ and $f = 2$ (see RVJ for definitions). The evolution was stopped at the current epoch, i.e., when the sum of the time to form the progenitor, the time for the binary to become a CV, and the subsequent evolutionary time as a CV equals the present age of the Galaxy, t_G. At the end of the evolution, we stored the properties of the binary for subsequent statistical analyses. Many systems, especially those with low-mass main-sequence companions (potential donor stars which are com-

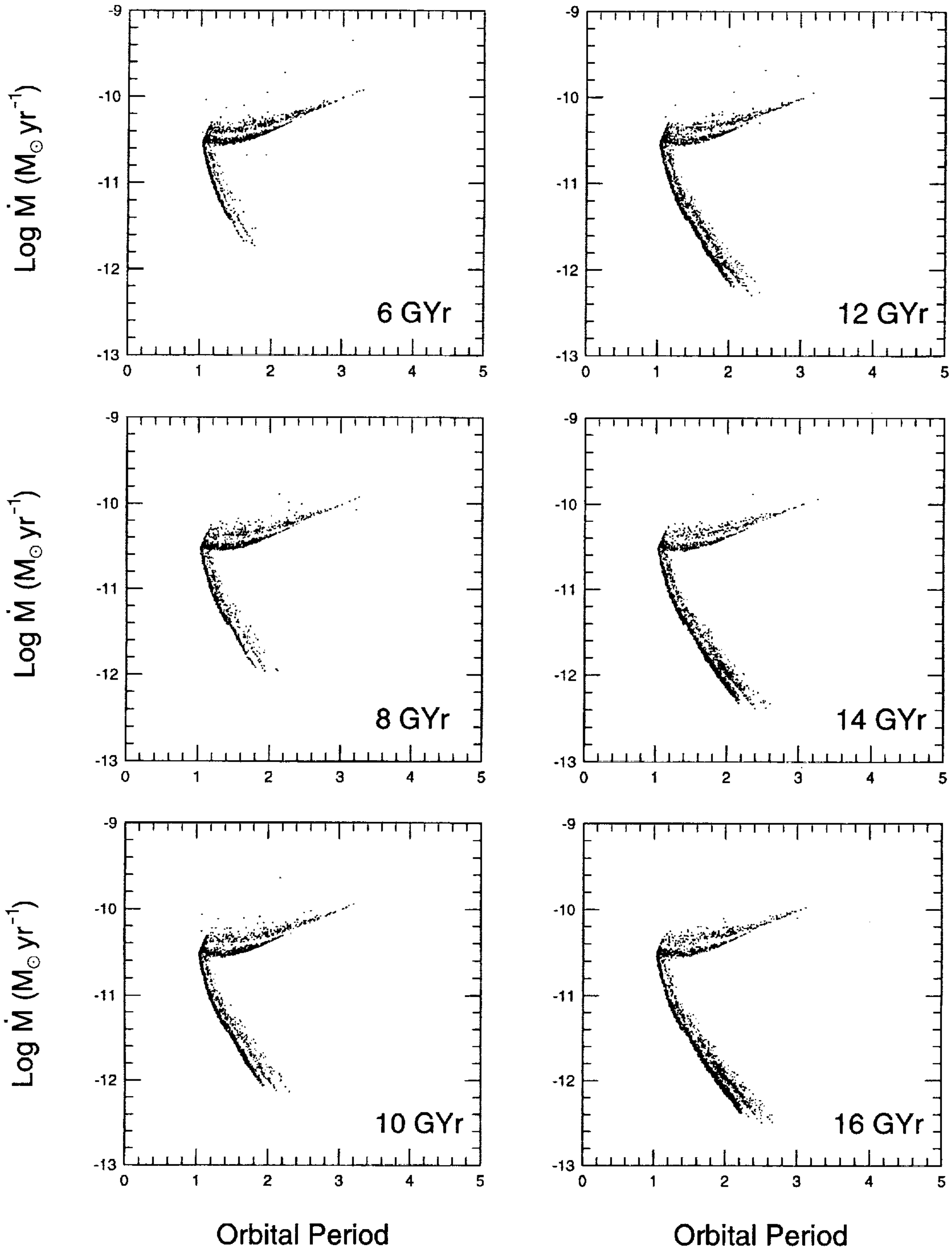

Figure 3. Population synthesis simulations of low-luminosity CVs for different ages of the Galaxy. The number labelling each panel is the assumed Galaxy age in Gyr. Each dot represents one system evolved in the computer. The normalization is such that the plots should contain approximately the number of CVs predicted in a 300 × 300 pc^2 region of the Galaxy.

pletely convective), never experience mass transfer because the orbit cannot shrink sufficiently by the present epoch for the donor star to fill its Roche lobe. We find that, of all the primordial binaries that we start with, only $\sim 2 \times 10^{-4}$ of them become semi-detached CVs with mass transfer.

The population synthesis calculations described above were repeated for a sequence of assumed ages for the Galaxy; i.e. for $t_G = 6$, 8, 10, 12, 14 and 16 Gyr. The results are shown in Fig. 3, as a sequence of plots showing $\dot{M}$ versus P_{orb} for the simulated CVs. No observational selection effects have been included; these results are taken directly from the outputs of our population synthesis and evolution calculations, and thus describe properties of the intrinsic CV population. Note, first, that there are hardly any systems (< 2 per cent) above the period gap (i.e., with $P_{orb} > 3$ h). This is consistent with previous population studies such as those of Kolb (1993), and is due to the relatively short lifetimes that CVs are expected to have during this phase of their evolution (a consequence of the much higher rates of mass transfer that are experienced for systems above the period gap). The relatively large number of *observed* CVs above the period gap is largely due to selection effects stemming from the much higher luminosities for these systems, and hence their greater detectability out to larger distances (Ritter & Burkert 1986; Dünhuber 1993). We also see that there is a reduced, but finite, number of systems with orbital periods within the 'gap' region (i.e., with $2 < P_{orb} < 3$ h). Again, we have made no attempt to correct for observational selection in this region, nor to analyse statistically the properties of the gap in our population synthesis results, as this has been well studied in earlier works (see, e.g., Kolb 1993).

Below the period gap, the population synthesis results for all Galactic ages show the same characteristic 'elbow'-shaped region in the $\dot{M}$–P_{orb} plane (see Fig. 3). The ensemble of evolutionary tracks clearly show the migration to the minimum orbital period and back up toward longer periods – with the concomitant dramatic decline in the mass-transfer rates. It is important to note that the evolutionary age of any given system (the time since the primordial binary was formed) in these plots is not uniquely specified by its position along the evolutionary track. This is due to (i) the range of main-sequence lifetimes for the progenitor primaries, (ii) the range of orbital separations of the systems that emerge from the common-envelope phase, and (iii) the differences in mass of the donor star when the CV phase commences.

For each assumed age of the Galaxy, the number of CVs in our populations synthesis sample is about 10^3. This is only a small fraction of the $\sim 10^6$–10^7 such systems that are expected in the Galaxy at the present time; however, the statistical significance of the results is sufficient to allow us to make the following statements. First, we find that for a Galactic age of 10^{10} yr, the numbers of systems above the period gap, in the gap, below the gap but above orbital period minimum, and past orbital period minimum are in the ratio of 2:30:100:200, respectively (with substantial statistical uncertainty in the first of these). While these rates are model dependent (especially in the choice of the initial mass ratio distribution in primordial binaries), they do provide the reader with a rough sense of the relative populations in the different phases of evolution. These numbers are in general agreement with other population studies of CVs (e.g. Kolb 1993). From these ratios, we conclude that there should be 2 systems that we would associate with the TOADs for every CV with more classical properties. Secondly, we find that there is a clear trend in the maximum orbital period that can be attained in the post-minimum-period evolution during the history of our Galaxy, which depends on the assumed age of the Galaxy. Thirdly, within the evolutionary sequences in Fig. 3 one can easily see two distinct sets of tracks above the period minimum, and perhaps three sets for systems below period minimum, especially among the oldest systems.

The most populous track is the one lying at the smallest values of $\dot{M}$ (for a given value of P_{orb}). These are the systems with low-mass He WDs ($\lesssim 0.45\ M_{\odot}$).[1] The tracks with the next highest values of $\dot{M}$ are systems with CO WDs ($\gtrsim 0.55\ M_{\odot}$) which started out with more massive donor stars ($\gtrsim 0.05\ M_{\odot}$) and evolved through the period gap in the manner shown in Fig. 2. Finally, the systems with the highest values of $\dot{M}$, and which reach the longest orbital periods, are systems with CO WDs which started out with *less* massive donor stars ($\lesssim 0.3\ M_{\odot}$) in a relatively close orbit after the common-envelope phase ($\lesssim 6$ h), and were thus able to form (come into Roche lobe contact) below the period gap. These systems were thereby able to avoid the early part of the evolution shown in Fig. 2 (as were the systems with He WDs). The systems which reach the longest orbital periods are therefore those (a) whose progenitor binaries were formed very early in the Galaxy's history, (b) that formed below the gap, relatively close to the orbital period minimum, and (c) that had relatively massive progenitor primaries so that both the time of WD formation was short and the mass of the white dwarf was sufficient to allow gravitational radiation losses to be competitive with systems that evolved through the gap.

In order to quantify the end-points of the evolution tracks shown in Fig. 3, we adopted the following statistical analysis. Our current sample of known TOADs is only ~ 20. We therefore chose a random sample of 20 systems from our population synthesis results below orbital period maximum, and found both the longest orbital period and the minimum value of $\dot{M}$ in that sample. This was repeated 1000 times, and the distributions of $P_{orb,max}$ and $\dot{M}_{min}$ were constructed. From these distributions we determined the 95 per cent confidence limits for $P_{orb,max}$ and $\dot{M}_{min}$ that are likely to be found in a sample of 20 low-luminosity CVs. (No weighting for observational selection effects was included; thus our 95 per cent confidence limits should be conservative, since the longest period systems should have the lowest luminosities.) We can represent how this maximum period and minimum $\dot{M}$ depend on t_G by the following simple fitting formulae:

$$P_{orb,max} \simeq 81 + 4.8(t_G/\mathrm{Gyr}) \qquad (\mathrm{min}) \tag{1}$$

$$\log_{10}(\dot{M}/\mathrm{M}_{\odot}\ \mathrm{yr}^{-1}) \simeq -11.3 - 0.080(t_G/\mathrm{Gyr}). \tag{2}$$

Finally, we caution that the use of equation (1) to set a constraint on the age of the Galaxy is based on the assumption that the only angular momentum loss mechanism in post-period-minimum CVs is that due to gravitational radiation. The existence of significant additional angular momentum loss mechanisms would obviously weaken the constraint set by equation (1). In future work we plan to investigate systematically the modifications to equation (1) that would result if, for example, magnetic braking does not cease when the donor star becomes completely convective. A preliminary set of evolution runs that we have carried out in this regard (i.e., continuous magnetic braking with parameters $\gamma = 4$ and $f = 1$) for a Galactic age of 10 Gyr yields a maximum orbital period of ~ 2.9 h. This is to be compared with the value of 2.2 h obtained from equation (1). However, such continuous magnetic braking is at least inconsist-

[1]The predicted existence of a significant population of CVs containing He WDs is not new, and has been discussed in several previous studies (Politano 1988, 1996; de Kool 1992; Kolb 1993). However, these studies did not focus on the post-minimum-period systems, nor did they present $\dot{M}$–P_{orb} plots for their entire synthetic populations.

ent with the current conventional explanation for the period 'gap' in CVs between 2 and 3 h. We also expect that future refinements to the population synthesis calculations described in this paper, that is, use of an improved stability criterion for rapid mass loss, inclusion of mass loss due to winds on the giant branches, and consideration of a Population II composition (in light of the extreme age of the longest-period systems), may also affect the terminal values discussed here.

4 DISCUSSION

The idea that TOADs may have very low mass transfer rates over long time-scales seems fairly compelling and has motivated us to consider their association with CVs that have evolved past the orbital period minimum. However, a number of theoretical and observational uncertainties remain before such an association can be made unambiguously. We discuss the most significant of these uncertainties here.

First, we have assumed in this paper that the mass-transfer rates inferred for the TOADs are a long-term phenomenon and, furthermore, that they represent time-averaged values of $\dot{M}$. Taking into account the high accretion rates that may occur during outburst along with the low $\dot{M}$ values that occur during the long inter-outburst cycles, we recognize that the TOAD mean mass-transfer rates may be different than those discussed here. Sproats et al. (1996) have shown, however, that for three or four TOADs with sufficient observational data to yield an indication of their recurrence time and outburst duration, the mean $\dot{M}$ values are not out of line with that expected for mass transfer driven by gravitational radiation alone. We emphasize here that the type of observational information used by Sproats et al. is necessarily sparse, since the needed long-term studies do not exist at all for many of the TOADs (see Howell et al. 1995). It is also possible that our limited 'view' of the TOADs (i.e., during only the past 5–10 yr) has indeed revealed systems with low $\dot{M}$ and low luminosity, but these properties may be transitory and occur only on astrophysically short time-scales.

Secondly, we also cannot be certain of the scaling of the M_V–$\dot{M}$ relations for the TOADs. As yet, no accretion disc models have been calculated for mass-transfer rates lower than $\sim 10^{-11}$ M$_\odot$ yr^{-1} and, as we mentioned above, our scaling is based only on approximate empirical calibration. Howell et al. (1995) discussed accretion disc models which provided a good match to observational outburst data for the TOADs. However, as with all such models published to date, the lowest mass-transfer rates used were $\sim 10^{-11}$ M$_\odot$ yr^{-1}. If the M_V values of the TOADs are indeed even close to those shown in Fig. 1, the mass-transfer rates during minimum are much lower than 10^{-11} M$_\odot$ yr^{-1}, possibly lower than 10^{-12} M$_\odot$ yr^{-1}. At such low values of $\dot{M}$, one may consider that these systems would be permanently on the lower *stable* branch of the accretion disc limit cycle, and that outbursts could never occur. Cannizzo et al. (1988), in their work on the accretion disc limit cycle and the relation between the local disc column density (Σ) and $\dot{M}$, show via analytic scaling arguments that the minimum $\dot{M}$ which can just produce outbursts is about 10^{-13} M$_\odot$ yr^{-1}. Thus, while actual models at these low rates have not been produced (but are in progress), it appears that outbursts are still possible even at the low rates inferred for some of the TOADs (Cannizzo 1996, private communication). Studies of the TOADs during outburst and at minimum light are thus crucial to provide input for realistic accretion disc modelling using the realistic very low $\dot{M}$ values.

Lastly, mass determinations of secondary stars in TOADs are essentially non-existent. This is due to the faintness of the TOADs, the existence of only one known partially eclipsing system (allowing a fair determination of the system inclination), and the fact that no spectral features from the secondary stars have been seen. Radial velocity studies using optical emission lines (from the accretion discs) have been performed for a few TOADs. However, when mass determinations have been attempted, they started with the usual assumptions of choosing a secondary radius (based on a *main-sequence* Roche-lobe-filling secondary), assigning the secondary a mass (based on the same assumption), using the measured value of K_1 and guessing the orbital inclination to obtain a value for the mass ratio, and then finally solving for the primary mass. Thus, in these cases, the initial assumptions used to solve the problem nullified any efforts at determining the true secondary mass.

For the one partially eclipsing TOAD mentioned above, WZ Sge, a relatively direct determination of the secondary mass is possible. Smak (1993) found a value for M_2 of 0.06 ± 0.02 M$_\odot$. WZ Sge has an orbital period of 81.6 min (just at the bend in the elbow in Fig. 3), and its secondary star, if past the period minimum and degenerate, is predicted by the results presented here to have a mass near 0.06 M$_\odot$. We note that it is also the case that the secondary star in WZ Sge would be predicted to have nearly this same mass even if still *approaching* the orbital period minimum near 80 min. We mention here that WZ Sge has been deduced to have a relatively high mass-transfer rate from observations of the hotspot amplitude near the time of outbursts; however, Osaki (1996) and Howell et al. (1995) have shown that the long-term outburst behaviour can be reproduced by use of a low mean mass-transfer rate. WZ Sge represents a system in need of further detailed study due to its partially eclipsing nature. Further observational work is badly needed in order to determine secondary masses for the TOADs, particularly for systems with orbital periods further from the period minimum (e.g., $\gtrsim 100$ min), where clear evidence for or against the brown dwarf-like nature of the secondary may be provided.

In spite of the above theoretical and observational uncertainties, it is nevertheless interesting to speculate about the association of TOADs with post-minimum-period CVs containing degenerate secondaries. If this idea is correct, it allows us to (i) confirm a number of our theoretical understandings concerning the evolution of CVs, (ii) augment our exploration of brown dwarfs, and (iii) derive an interesting constraint on the age of the Galaxy.

If the secondary stars in TOADs are indeed shown to be degenerates, then they would represent an interesting complement to studies of brown dwarfs in open clusters (where there is some age information), and those in wider binary orbits with nearby low-mass, high proper motion stars. For brown dwarfs in low-luminosity CVs, there is potentially valuable information to be gleaned about their properties: (i) the mass/radius relation obtained from the Roche lobe filling criterion; (ii) the possibility of measuring the mass directly if the system inclination and K_1 and K_2 can be determined; and (iii) age information (or limits thereon) that can be inferred from the evolutionary status of the binary (i.e., its orbital period).

Finally, we return to the idea that the longest period TOADs may provide information about the age of the Galaxy. The TOAD with the longest known orbital period is EF Peg ($P_{\rm orb}=123$ min) and there are a number of others with periods near 110 min. The orbital period of EF Peg in conjunction with equation (1) provides a tentative lower limit to the age of the Galaxy of 8.6×10^9 yr. This corresponds to an upper limit to the Hubble constant of 76 km s^{-1} Mpc^{-1} for assumed values of the cosmological parameters $\Omega=1$ and $\Lambda=0$. The TOADs with orbital periods near 110 min yield less interesting limits on the age of the Galaxy of $\sim 6 \times 10^9$ yr. Clearly, a statistically enhanced orbital period distribution for TOADs would be of great interest in this regard.

5 SUMMARY

Theory predicts that, at the present epoch, ~ 99 per cent of all CVs should have orbital periods below the period gap. Of these, ~ 70 per cent should have already reached the period minimum near 80 min and be evolving back towards longer periods, the maximum of which is given by equation (1) above. These low-luminosity CVs are likely to contain degenerate secondaries of mass ~ 0.02–$0.06\,M_\odot$ (~ 20–60 Jupiter masses) and radii near $0.1\,R_\odot$. Recognizing the uncertainties discussed above and the lack of detailed observational information for the low-luminosity DN presented in Fig. 1, we have discussed the possibility that some of the TOADs may represent CVs that are the oldest members of their class. As such, they would be the observational counterpart of the long-sought-after, theoretically predicted systems containing degenerate (brown dwarf) secondaries. If this idea can be verified, the TOADs can be used to set an interesting constraint on the age of the Galaxy and provide an important complement to our knowledge of brown dwarfs.

It is clear that further observations of low-luminosity CVs are needed. These should include observations at minimum light, during outburst, and over long temporal scales. The latter are needed in order to provide measures of the inter-outburst time-scale and the outburst durations for a much larger sample of TOADs, thus allowing better estimates of the overall $\dot{M}$ and M_V. These observations may also be expected to provide important and much needed inputs and constraints on theoretical models of the intrinsic CV population at short orbital periods.

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