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NASA DRYDEN FLIGHT RESEARCH CENTER

FINAL REPORT

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1.0 Introduction

The work that follows is concerned with the application of design methodologies for feedback compensation associated with linear systems. In general, the intent is to provide a well behaved closed loop system in terms of stability and robustness (internal signals remain bounded with a certain amount of uncertainty) and simultaneously achieve an acceptable level of performance. The approach here has been to convert the closed loop system and control synthesis problem into the *interpolation* setting. The interpolation formulation then serves as our mathematical representation of the design process. Lifting techniques have been used to solve the corresponding interpolation and control synthesis problems. Several applications using this multiobjective design methodology have been included to show the effectiveness of these techniques. In particular, the mixed H^2–H^∞ performance criteria with algorithm has been used on several examples including an F-18 HARV (High Angle of Attack Research Vehicle) for sensitivity performance.

2.0 An Interpolation Viewpoint

In this section, we will review some techniques that enable the control designer to view the closed loop control synthesis problem as an interpolation type problem. To see this, consider the problem of finding a compensator c to yield an acceptable level of closed loop performance and robustness (for a given open loop plant g) while maintaining internal stability. A typical block diagram for this problem is shown in the following Figure 2.1

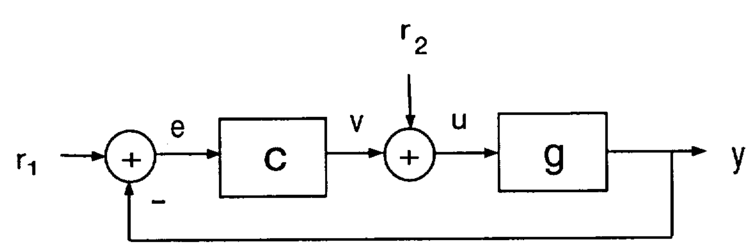


Figure 2.1 Closed Loop System

where r_1 and r_2 are exogenous signals (disturbances), y is the controlled output signal, and the error e and control input with noise u are internal signals. A compensator c is called internally stabilizing if all of the elements of the following transfer matrix

$$\begin{bmatrix} e \\ u \end{bmatrix} = \begin{bmatrix} (1+cg)^{-1} & -g(1+cg)^{-1} \\ c(1+cg)^{-1} & (1+cg)^{-1} \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} \tag{2.1}$$

are in H^∞. By H^∞, we mean the set of all uniformly bounded analytic functions in the open unit disc. In engineering terms, H^∞ is simply the set of all stable transfer functions for discrete time systems. In particular, a rational function g is in H^∞ if and only if all of its' poles are in the complex region $|z| \geq 1$.

Now for simplicity of presentation, assume that all the signals in Figure 2.1 are scalar valued. Then in view of Figure 2.1, a typical controls synthesis problem may be to design an internally stabilizing compensator c to contain the effect of the disturbances r_1 and r_2 on the internal signals e and u or output signal y. For example, consider the signal to noise ratio (transfer function) from r_1 to e in (2.1) given by

$$\frac{e}{r_1} = \frac{1}{1+cg}. \tag{2.2}$$

Then setting a prespecified tolerance γ, the control synthesis problem here is to find an internally stabilizing compensator c that norm bounds the transfer function

$$\left\|\frac{e}{r_1}\right\| = \|(1+cg)^{-1}\| \leq \gamma \tag{2.3}$$

where the norm $\|\cdot\|$ is chosen by the designer. In the H^∞ literature, this is generally called bounding the sensitivity function, see for example [3,28,87]. The solution to this problem yields a stable closed loop system assuring that the signal to noise ratio is bounded by the inequality $\|e\| \leq \gamma \|r_1\|$ when the disturbance r_2 is neglected. Thus, setting γ small decreases the effect of the external disturbance r_1 on the error signal e. As seen by the expression in (2.1), designing a compensator c to internally stabilize the system on all input-output transfer functions and satisfy a performance criteria as in (2.3) is nonlinear and awkward to work with. An alternate way of viewing the control synthesis problem is to first parameterize all stabilizing compensators in terms of one free parameter h in H^∞, and then design h for particular closed loop performance objectives. This latter conversion technique enables us to view the control synthesis problem as an interpolation type problem.

To show how the closed loop control system synthesis problem transcends into an interpolation problem, first assume for simplicity that g is rational. With this assumption g admits a coprime factorization of the form $g = n/d$ where n and d are rational functions in H^∞, and there exists functions p and q in H^∞ that satisfy the following Bezout identity

$$np + dq = 1, \quad \{n,p,d,q\} \in H^\infty . \tag{2.4}$$

Moreover, there exits several efficient algorithms to compute the H^∞ functions n, p, d, and q such that $g = n/d$ and (2.4) is satisfied. Using the factorization $g = n/d$, it has been shown in [85] that the set of all internally stabilizing compensators for the closed loop system in Figure 2.1 is given by

$$c = \frac{p + dh}{q - nh}, \quad h \in H^\infty \tag{2.5}$$

where h is an arbitrary function in H^∞. It can be easily confirmed that substitution of c into (2.1) yields the internal transfer matrix

$$\begin{bmatrix} e \\ u \end{bmatrix} = \begin{bmatrix} (qd-ndh) & (-nq+n^2h) \\ (dp-d^2h) & (qd-ndh) \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} \tag{2.6}$$

Since n,d,p, and q are all in H^∞, equation (2.6) shows that all the components in the transfer matrix in (2.1) are in H^∞ for arbitrary h in H^∞. Therefore, all compensators c of the form (2.5) are internally stabilizing. One can also show that all internally stabilizing compensators are of the form c in (2.5).

Now let us return to the control problem introduced in (2.3). That is, find an internally stabilizing compensator c such that the transfer function from r_1 to e given by $(1 + gc)^{-1}$ is bounded in norm by γ. By substituting the compensator c in (2.5) into the 1-1 component of the transfer matrix in (2.1), we see that

$$(1 + cg)^{-1} = dq - ndh = f - \theta h \tag{2.7}$$

where $f = dq$ and $\theta = nd$ are in H^∞. (Notice that using c in (2.5), every element of the transfer matrix in (2.1) has a similar form as in (2.7) for an arbitrary h in H^∞.) Therefore, the control synthesis problem in (2.3) and other control synthesis problems concerned with norm bounding the elements of the transfer matrix in (2.1) can be converted to the following *Sarason* type interpolation problem.

Sarason Problem *Given a particular* f *and* θ *in* H^∞ *with a tolerance* $\gamma > 0$, *find* h *in* H^∞ *that satisfies the following inequality*

$$\|f - \theta h\| \leq \gamma \quad (h \in H^\infty) \,. \tag{2.8}$$

This problem first appeared in the mathematical literature [68], with one of the first applications to control theory in [18]. From (2.8), one solves for a particular h in H^∞ that satisfies the bound $\|f - \theta h\| \leq \gamma$. Then from this resulting h, the corresponding internally stabilizing compensator c satisfying the inequality in (2.3) is given by $c = (p+dh)/(q-nh)$. Of course, the solution to this interpolation problem depends upon the norm chosen. Some norms are inherently difficult to compute with. For practical and computational reasons, we will concentrate on solving interpolation and control problems using the H^2 and H^∞ norms. Recall that the H^2 norm (respectively, H^∞) for a transfer function g analytic in $|z| < 1$ is defined by

$$\|g\|_2 = \left[\frac{1}{2\pi} \int_0^{2\pi} |g(e^{it})|^2 \, dt \right]^{1/2} \quad \text{and} \quad \|g\|_\infty = \sup_{0 \leq t < 2\pi} |g(e^{it})| \,. \tag{2.9}$$

To relate these norms to some of the results mentioned in the historical review, the H^2 norm was basically used as the criteria for the development of the Weiner filter and the LQG optimization problem, and the H^∞ norm was associated with classical control theory (Bode [9], etc.) and more recent results (for example, see [18,19,20,87,88]) based upon frequency domain techniques. From a practical point of view, the H^2 controllers yield nice closed loop performance properties and H^∞ controllers are robust to uncertainties in the system. In practice, one has to compromise performance for a certain amount of robustness in the synthesis problem. To accomplish this, the synthesis problem here consists of designing controllers for the closed loop system that yield an acceptable level of H^2 performance and inherits the H^∞ robustness. This leads to mixed H^2–H^∞ type control problems. The typical approach in the existing literature is to solve these types of problems using Riccatti equation techniques. Here, we will use commutant lifting techniques to solve these problems. The lifting approach considered here enables us to consider the more general irrational interpolation problems and readily leads to efficient computational techniques in the rational case.

To be explicit in defining control problems of concern here, lets consider again the problem of designing an internally stabilizing compensator c that norm bounds the sensitivity function $(1 + gc)^{-1}$. That is, search for an internally stabilizing compensator c that yields an acceptable level of performance (H^2 norm bound) and contains a prescribed amount of robustness (H^∞ norm bound) by satisfying the constraints

$$\|(1+gc)^{-1}\|_2 \le \rho_2 \text{ and } \|(1+gc)^{-1}\|_\infty \le \rho_\infty \tag{2.10}$$

where ρ_2 and ρ_∞ are prespecified H^2 and H^∞ norm bounds, respectively. From our previous analysis, we can convert the sensitivity problem into an equivalent Sarason type interpolation problem and search for an h in H^∞ satisfying the bounds

$$\|f - \theta h\|_2 \le \rho_2 \text{ and } \|f - \theta h\|_\infty \le \rho_\infty . \tag{2.11}$$

Thus, setting $c = (p+dh)/(q-nh)$ from h in (2.11) (where $g = n/d$ and the pair p and q satisfy the Bezout identity $np+dq = 1$) gives us a compensator c that satisfies the closed loop bounds in (2.10).

In the sections that follow, we will assume that the reader is familiar with the Youla parameterization previously discussed as applied to a typical controls problem. We will first present some results obtained for the mixed H^2 and H^∞ control synthesis problem with examples including an application to the F-18 HARV model. Finally, in part II we will provide an algorithm for obtaining some mixed H^2–H^∞ bounds for the continuous time control problem.

Part I Discrete Time H^2–H^∞ Compensator Design

1. Summary and Main Theorem

Recall from the introduction that if we are given the plant $g = n/d$, (stable or not), we can parameterize all stabilizing controllers c by the expression $c = (p+dh)/(q-nh)$ where n and d satisfy the Bezout identity $pn + dq = 1$. This leads to the parameterization of all internally stable transfer functions given by various affine functions $f - \theta h$ where f and θ are stable and h is an arbitrary function in H^∞. With this in mind, we can view the sensitivity problem as the equivalent problem of norm bounding the error on a function of the form $f - \theta h$ where h is our design parameter. Now suppose we are given a specified function F in $H^\infty(\mathcal{E}_1, \mathcal{E}_2)$ (the multivariable setting) with the realization $\{A_F, B_F, C_F\}$ and $\{A_\Theta, B_\Theta, C_\Theta, D_\Theta\}$ is a minimal realization for the inner function $\Theta(z)$. Let $Q_{\Theta F}$ to be the solution to the following Lyapunov equation

$$Q_{\Theta F} = A_\Theta^* Q_{\Theta F} A_F + C_\Theta^* C_F \,. \tag{1.1}$$

Also let W_1 be the controllability operator from $l_+^2(\mathcal{U})$ onto $\mathcal{X}$ generated by the pair $\{A_\Theta^*, C_\Theta^*\}$ given by

$$W_1 = \left[C_\Theta^*,\ A_\Theta^* C_\Theta^*,\ A_\Theta^{*2} C_\Theta^*,\ \ldots\right] \tag{1.2}$$

and let W_2 be the controllability operator from $l_+^2(\mathcal{Y})$ into $\mathcal{X}$ generated by the pair $\{A_\Theta^*, \hat{B}\}$ given by

$$W_2 = \left[\hat{B},\ A_\Theta^* \hat{B},\ A_\Theta^{*2} \hat{B},\ \ldots\right] \tag{1.3}$$

where $\hat{B} = Q_{\Theta F} B_F$. Define P_1 and P_2 as the controllabiliy grammians for the pairs $\{A_\Theta^*, C_\Theta^*\}$ and $\{A_\Theta^*, \hat{B}\}$, respectively, where P_1 and P_2 are the solutions to the following discrete time Lyapunov equations

$$P_1 = W_1 W_1^* = A_\Theta^* P_1 A_\Theta + C_\Theta^* C_\Theta \quad \text{and} \quad P_2 = W_2 W_2^* = A_\Theta^* P_2 A_\Theta + \hat{B}\hat{B}^* \,. \tag{1.4}$$

Now define the H^2 and H^∞ optimization problems

$$d_2 = \inf\{\|F - \Theta H\|_2 : H \in H^\infty\} \tag{1.5}$$
$$d_\infty = \inf\{\|F - \Theta H\|_\infty : H \in H^\infty\} \,. \tag{1.6}$$

Then it is well known from the literature that d_2 can be computed from the relation $d_2^2 = \operatorname{trace}(\hat{B}^* P_1^{-1} \hat{B})$ and $d_\infty^2 = \lambda_{max}(P_1^{-1} P_2)$. Next choose $\delta > 1$ and set $\gamma = \delta d_\infty$.

Finally, let

$$\begin{aligned}
A_p &= (\gamma^2 P_1 - P_2)A_\Theta(\gamma^2 P_1 - P_2 + \hat{B}\hat{B}^*)^{-1} \\
B_p &= (\gamma^2 P_1 - P_2)A_\Theta(\gamma^2 P_1 - P_2 + \hat{B}\hat{B}^*)^{-1}\hat{B} \\
C_p &= \gamma^2 C_\Theta(\gamma^2 P_1 - P_2 + \hat{B}\hat{B}^*)^{-1} \\
D_p &= \gamma^2 C_\Theta(\gamma^2 P_1 - P_2 + \hat{B}\hat{B}^*)^{-1}\hat{B}\,.
\end{aligned} \tag{1.7}$$

be the finite dimensional operators. Then we have the following theorem.

Theorem 1

Let the finite dimensional operators $A_p, B_p, C_p,$ *and* D_p *be defined as in* (1.7). *Then* A_p *has all of its eigenvalues inside the unit disc. Furthermore, letting G be the function defined by* $G(z) = D_p + zC_p(I - zA_p)^{-1}B_p$, *there exists a function* H *in* $H^\infty(\mathcal{E}_1, \mathcal{F})$ *satisfying* $F - \Theta H = G$. *In addition, for this particular* H, *the following inequalities are satisfied*

$$\|F - \Theta H\|_\infty \le \delta d_\infty \quad \text{and} \quad \|F - \Theta H\|_2 \le \frac{\delta d_2}{\sqrt{\delta^2 - 1 + \dfrac{d_2^2}{n d_\infty^2}}} \tag{1.8}$$

where n is the dimension of $\mathcal{Y}$. *In particular, by choosing* $\delta^2 = 2 - d_2^2 / n d_\infty^2$, $F - \Theta H$ *satisfies the following bounds*

$$\|F - \Theta H\|_\infty \le d_\infty \sqrt{2 - \frac{d_2^2}{n d_\infty^2}} \quad \text{and} \quad \|F - \Theta H\|_2 \le d_2 \sqrt{2 - \frac{d_2^2}{n d_\infty^2}} \tag{1.9}$$

The results presented in Theorem 1 are very useful in solving control synthesis problems in H^∞, and this theorem guarantees that an H can be found to obtain the bounds given in (1.8). Furthermore, with a realization for G given in (1.7),H can be found by simply taking the adjoint of Θ to obtain

$$H(z) = \Theta^*(1/\bar{z})(F(z) - G(z)). \tag{1.10}$$

We will conclude this section by summarizing the algorithm for computing an H in the previous Theorem when F and Θ are given.

2. H^2–H^∞ Discrete Time Algorithm

In this section, we will summarize the algorithm to construct an H satisfying the norm bounds in (1.8) and apply the algorithm to several aircraft examples.

(1). Compute a minimal realization $\{A_\Theta, B_\Theta, C_\Theta, D_\Theta\}$ for Θ.

(1.. Compute a realization $\{A_F, B_F, C_F\}$ for F.

(2.. Solve the Lyapunov equation $Q_{\Theta F} = A_\Theta^* Q_{\Theta F} A_F + C_\Theta^* C_F$

(4). Form the finite dimensional operator $\hat{B} = Q_{\Theta F} B_F$

(5). Compute the controllability grammians

$$P_1 = A_\Theta^* P_1 A_\Theta + C_\Theta^* C_\Theta \quad \text{and}$$
$$P_2 = A_\Theta^* P_2 A_\Theta + \hat{B}\hat{B}^* .$$

(6). Compute $d_2^2 = \text{trace}(\hat{B}^* P_1^{-1} \hat{B})$ and $d_\infty^2 = \lambda_{max}(P_1^{-1} P_2)$.

(7). Choose $\delta > 1$ and find the realization $\{A_p, B_p, C_p, D_p\}$ for G.

(8). Solve for $H(z) = \Theta^*(\frac{1}{\overline{z}})(F(z) - G(z))$ where $\Theta^*(\frac{1}{\overline{z}}) = D_\Theta^* + B_\Theta^*(zI - A_\Theta^*)^{-1} C_\Theta^*$.

We will conclude this section with two examples; the first is a drone aircraft and the second is the NASA F-18 HARV (High Angle of Attack Research Vehicle) fighter aircraft.

Example 1 Drone Aircraft

In this section we will show by example how the results in section 1 can be applied to a typical aircraft controls problem. We borrowed the open loop plant data from [19] for the lateral attitude dynamics of a drone aircraft. The open loop plant is given by the state space model

$$\dot{x} = Ax + B(u + w)\,, \quad y = Cx \tag{2.1}$$

where

$$A = \begin{bmatrix} -.0853 & -0.0001 & -0.9994 & 0.0414 & 0 & 0.1862 \\ -46.86 & -2.757 & 0.3896 & 0 & -124.3 & 128.6 \\ -0.4248 & -0.0622 & -.0671 & 0 & -8.792 & -20.46 \\ 0 & 1 & 0.0523 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -20 & 0 \\ 0 & 0 & 0 & 0 & 0 & -20 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 20 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad D = 0 \tag{2.2}$$

The states in this model represent, in order: sideslip angle, roll rate, yaw rate, roll angle, elevon surface deflection, rudder surface deflection, elevon servo command, and rudder servo command. For simplicity, we will consider the single input to single output map from the elevon servo command to sideslip angle, resulting in a non-minimal realization of order 6. The input output map of the open loop plant g is then given by the following transfer function

$$g = \frac{176.09s^2 + 226.55s - 10.47}{s^5 + 22.91s^4 + 58.21s^3 + 4.08s^2 + 74.21s + 2.67}\,. \tag{2.3}$$

This resulting transfer function is non minimum phase with two unstable complex poles. The following paragraph summarizes the results of our simulations using the H^2–H^∞ algorithm in section 2 for the open loop plant g given in equation (2.3).

To begin, lets consider again the H^2–H^∞ sensitivity problem as presented in section 1 with the given open loop plant g in (2.3). Recall that we want to find an internally stabilizing compensator c that bounds the errors

$$\|(1 + gc)^{-1}\|_2 \leq \gamma_2 \text{ and } \|(1 + gc)^{-1}\|_\infty \leq \gamma_\infty \tag{2.4}$$

where γ_2 and γ_∞ are bounds given as functions of the parameter $\delta > 1$ in equations (1.8). The first step in our design process was to discretize the given plant g in (2.3) using the Bilinear transformation in the Matlab subroutine. We then applied the algorithm from section 1 for the construction of the compensator c with four different values for δ. Using fast Fourier transform (FFT) techniques, the H^2 and H^∞ norms were computed for the resulting closed loop transfer functions for values $\delta = 1.0001$, 1.05, and 1.1. Recall from section 1 that for each particular $\delta > 1$ chosen, the main Theorem guarantees that the closed loop transfer function obtains the given bounds in (1.8). In our example, the actual closed loop H^2 and H^∞ norms of the closed loop transfer function were computed for each δ.

For example, fo the three values of δ, the actual H^2 norm was computed and compared to the bounds as shown in the following Table 2.1.

Table 2.1 (H-2 Norm)

δ	$\inf\|(1 + gc)^{-1}\|_2$	$\|(1+gc)^{-1}\|_2$	γ_2
1.0001	0.4202	1.0288	1.0294
1.05	0.4202	0.7484	0.8507
1.1	0.4202	0.6395	0.7533

Notice from the data that as $\delta \to 1$, the actual H^2 norm $\|(1 + gc)^{-1}\|_2$ increases because the theorem predicts H^∞ optimality as $\delta \to 1$ and hence no guarantee on the H^2 performance. As δ increases, for example, for $\delta = 1.1$, Table 2.1 shows that the closed loop has better resulting H^2 performance. To show how the H^∞ performance varies with δ, we have listed values of $\|(1 + gc)^{-1}\|_\infty$ as a function of the same three values of δ as shown in the following Table 2.2.

Table 2 (H-infinity Norm)

δ	$\inf\|(1+gc)^{-1}\|_\infty$	$\|(1+gc)^{-1}\|_\infty$	γ_∞
1.0001	1.0299	1.0300	1.0300
1.05	1.0299	1.0777	1.0814
1.1	1.0299	1.1229	1.1329

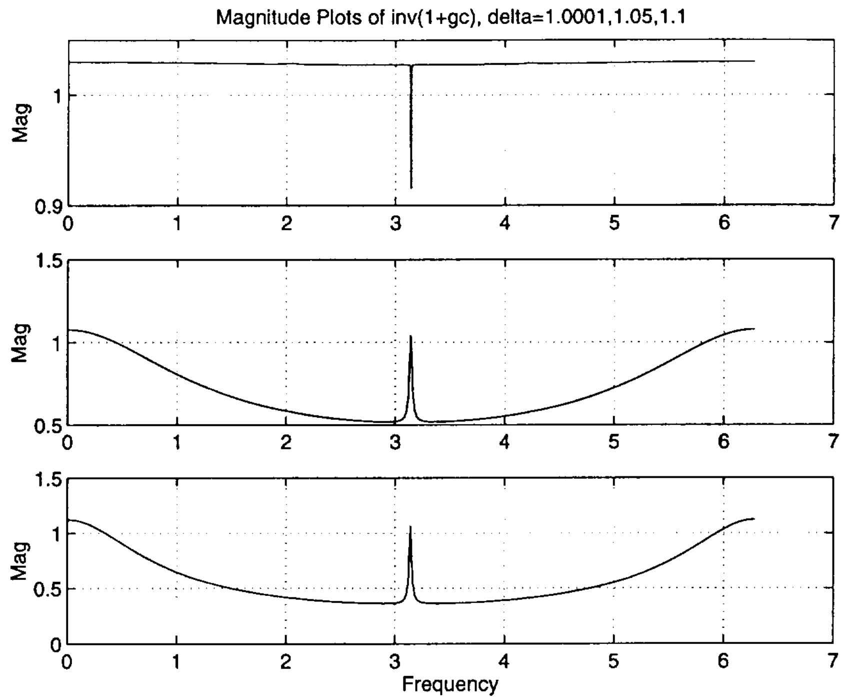
The optimal and computed norms are listed to show that they actually fall within the predicted bounds given in equations (1.8). It is also interesting to note that the actual closed loop poles approach unity as δ approaches unity (H^∞ optimal) although this is the design for least sensitivity to uncertainty. The spectral radius of the closed loop system and compensator order (complexity) is given in Table 2.3 below.

Table 2.3 (Continuous Time)

δ	Spectral Radius	Controller Order	No. Unstable Controller Poles
1.0001	0.9995	11	3
1.05	0.9976	10	3
1.1	0.9974	10	3

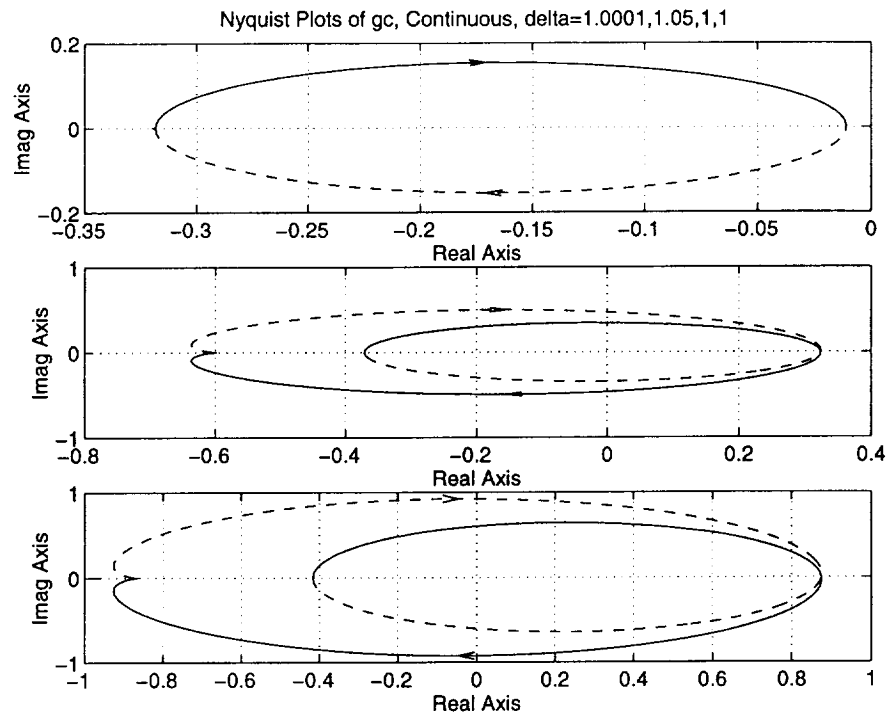
As expected, the peak on the Bode plot of the closed loop transfer function $(1 + gc)^{-1}$ becomes smaller as δ approaches 1. In addition, the following plots in Figure 2.4 show that the infinity norm bound is successfully achieved.

Figure 2.4



Since the idea behind minimization of the sensitivity function $(1+gc)^{-1}$ using H^{∞} as a performance measure is to increase robustness, we have plotted the resulting magnitude plots of gc. By transforming the discrete time closed loop system to the continuous time via the bilinear transformation, the resulting Nyquist plots of gc in Figures 2.5 on the following page clearly varify the expected increase in robustness as $\delta \rightarrow 1$.

Figure 2.5



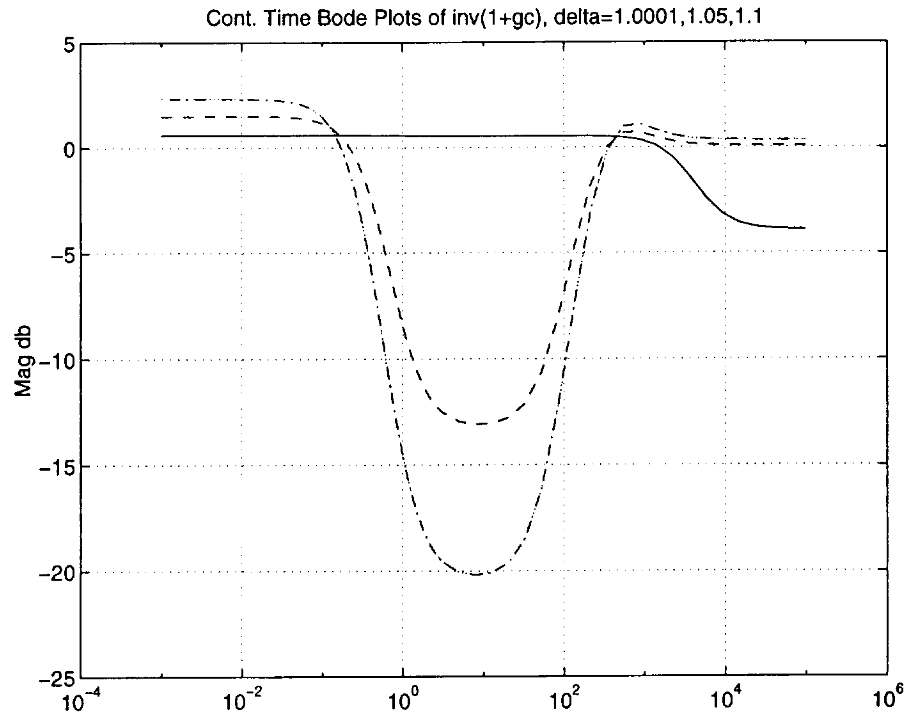
The resulting gain margins were computed for each δ chosen and the results are given in Table 2.6 below.

Table 2.6 (Continuous Time)

δ	Gain Margin
1.0001	3.13
1.05	2.70
1.1	1.16

Finally, Figure 2.7 is a Bode plot of the resulting continuous time closed loop transfer function $(1 + gc)^{-1}$ for the various values of $\delta > 1$.

Figure 2.7



This plot varifies that the peak magnitude on the Bode plot is reduced with decreasing δ as anticipated.

The results and example in this section have shown to be a successful methodology for multiple objective control system synthesis. In particular, if the designer is interested in compromising objectives with respect to the H^2 and H^∞ norms, the resulting algorithm is numerically pleasing and does not require a search for obtaining reasonable results. The next example is a direct application of these results to the F-18 HARV fighter aircraft model.

Example 2 F-18 HARV Fighter Aircraft

We have successfully demonstrated a methodology for the construction of compensators that achieve closed loop bounds on both the H^2 and H^∞ performance problems. The previous results extend to the multivariable case and to the continuous time problem. In this section, we will show how these results can be implemented on the F-18 model that NASA Dryden has generously provided.

In the example that follows, we are given a 151 order state space model of the F-18 HARV (High Angle of Attack Research Vehicle) aircraft at flight conditions of Mach .9 with 10 inputs and 10 outputs. This model includes 102 states specifically for high fidelity actuator dynamics augmented with the modal data of the aircraft structure itself. The actuator dynamics eith order of complexity (dynamical order) are as follows;

aileron	8
stabilitator	11
rudder	7
leading edge flap	8
trailing edge flap	9
vanes	8

Since there are two sets of actuators, we have 51x2 = 102 states for 12 actuators inclusive in the model. For the main plant model of the aircraft structure, we have the states ϕ, theta, and ψ, for bank angle, pitch, and heading angle. For displaced degrees of freedom, we have u, v, w for forward, side, and vertical displacements. Finally, p, q, and r are the roll, pitch, and yaw degrees of freedom in that order. This model also includes 10 symmetric modes and 10 antisymmetric modes and their rates which make up 40 states. The interested reader can consult NASA TM 4493 for details concerning the actuator dynamics and the augmented state space model. In order to demonstrate the effectiveness of our compensator synthesis methodology, we have taken the input-output map from thrust vectoring in the pitch mode to the true angle of attack α. This model includes 8 states for the actuator. In the aircraft modal data we have kept the lowest four symetric and lowest four antisymetric modes. Hence, our plant model consists of 25 states from which we will apply our design methodology.

In the control synthesis problem that follows, our design goal is to achieve good closed loop mixed H^2–H^∞ performance using the sensitivity function (i.e. $(1+gc)^{-1}$) as our resulting closed loop input output map. Here we have applied the mixed H^2–H^∞ control synthesis algorithm as listed in this section to this single input - single output plant. For various $\delta > 1$ we have constructed compensators to achieve closed loop H^2 and H^∞ performance bounds as given in Theorem 1. For values of $\delta = 1.0001$, 1.5, 3, and ∞ we have constructed a state space solution for the closed loop transfer function g_{cl} and computed the resulting H^2 norms. The following Table 2.8 lists these values, and show in particular that as $\delta \to \infty$ we obtain the optimal H^2 solution as shown by the last row of numbers in Table 2.8.

Table 2.8 H^2 Norm Data

<table>
<tr><th>δ</th><th>$\inf \|(1+gc)^{-1}\|_2$</th><th>$\|(1+gc)^{-1}\|_2$</th><th>γ_2</th></tr>
<tr><td>1.0001</td><td rowspan="4">275.3202</td><td>304.5828</td><td>304.5921</td></tr>
<tr><td>1.05</td><td>297.8435</td><td>301.4759</td></tr>
<tr><td>3.0</td><td>275.5405</td><td>278.1628</td></tr>
<tr><td>∞</td><td>275.3202</td><td>275.3203</td></tr>
</table>

The bound γ_2 as guaranteed from Theorem 1 is obviously satisfied for all three values of $\delta > 0$. We have also computed the resulting H^∞ norm of g_{cl} for four values of δ and have listed these values in the following Table 2.9.

Table 2.9 H^∞ Norm Data

<table>
<tr><th>δ</th><th>$\inf \|(1+gc)^{-1}\|_\infty$</th><th>$\|(1+gc)^{-1}\|_\infty$</th><th>$\gamma_\infty$</th></tr>
<tr><td>1.0001</td><td rowspan="4">304.5989</td><td>304.6121</td><td>304.6293</td></tr>
<tr><td>1.05</td><td>310.8247</td><td>319.8288</td></tr>
<tr><td>3.0</td><td>371.3927</td><td>913.7967</td></tr>
<tr><td>∞</td><td>381.0382</td><td>∞</td></tr>
</table>

Notice that as $\delta \rightarrow 1$, the resulting bound γ_∞ is the optimal H^∞ bound and is successfully achieved as the first row of Table 2.9 suggests.

The resulting Bode plots for $\delta = 1.0001$, 1.05, and ∞ are shown in the following Figure 2.10.

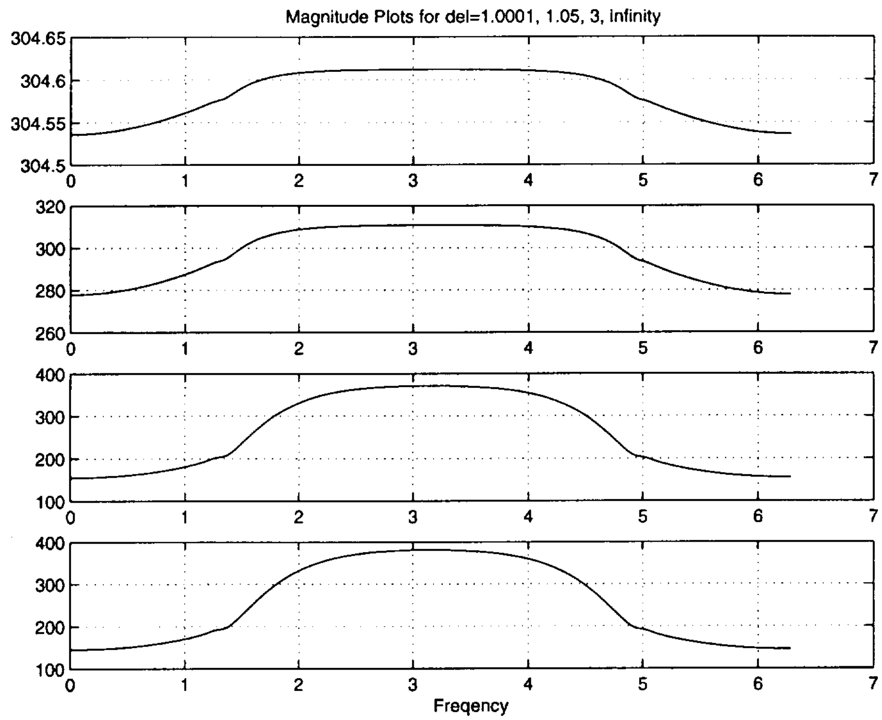


Figure 2.10

Its clear from this plot that as δ approaches 1, the resulting magnitude plot of $\|(1+gc)^{-1}\|$ appears flat across the spectrum. In general, one can place weighting transfer functions in the optimization problem to shape these magnitude plots as desired for high frequency roll off, etc.

In some problems, the time domain characteristics are more critical to the designer such as the impulse response of the resulting closed loop system. For pure interest, we have plotted the impulse response of the resulting closed loop sensitivity function for two values $\delta = 1.0001$ and $\delta = \infty$ and superimposed them to show the difference in responses as shown in the following Figure 2.11.

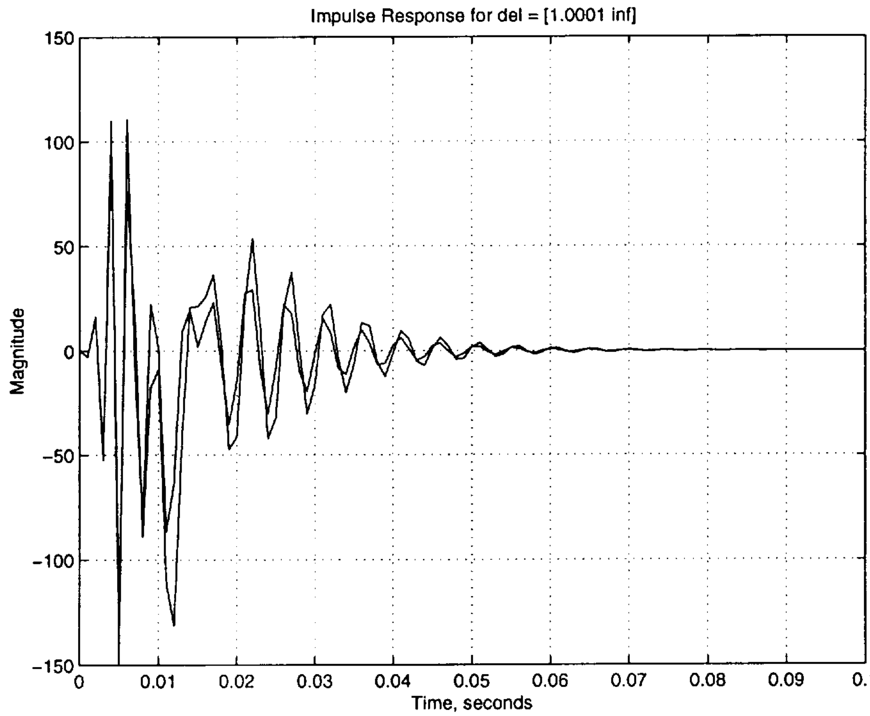


Figure 2.11

As one would expect, the H^2 optimal solution ($\delta = \infty$) has a better damped response, but in general the H^2 is much less robust to uncertainty.

In general, one dissadvantage of using interpolation type results for control design methodologies is that the resulting controller is in general high order. This can cause implementation problems due to the complexity of the controller structure. In this example, the resulting compensator was typically of order greater than 70-80 states, which makes it difficult for implementation. We had success in computing these compensators due to the numerical efficiency using fast Fourier transform techniques. For multivariable applications, one can use the discrete Riccatti equation for obtaining the compensator c via the Youla parameterization.

In conclusion, this example was a more realistic problem that one encounters in industry. We have shown for the F-18 HARV single input single output system from pitch vectoring to angle of attack that the algorithm for mixed H^2 and H^∞ bounds has successfully achieved these bounds anticipated by Theorem 1.

Part II Continuous Time H^2–H^∞ Compensator Design

1. Summary and Main Theorem

In the previous section, solutions were presented that solved variations the Sarason optimization problem. Performance was measured in terms of the H^2 and H^∞ norms. Thus far, everything has been presented in terms of the H^2 and H^∞ spaces on the unit disc. In this section, we will solve these same types of problems on the spaces associated with the H^2 and H^∞ problems on the half planes. It can be shown that an appropriate unitary bilinear transformation from the unit disc to the half plane indeed allows us to consider similar $H^2 - H^\infty$ type interpolation problems as done in section I. In particular, we will solve the continuous time Sarason problem and present an example to show how the algorithm we have developed can be applied to a typical controls problem.

2. H^2 and H^∞ Optimization Problems

In this section, we have made use of the commutant lifting theorem to extend and tighten the Kaftal-Larson-Wise bounds [47] to the right half plane setting. Throughout, $L^2(\mathcal{E})$ is the space of all square integrable Lebesgue measurable functions f(iw) for $-\infty < w < \infty$ with values in the Hilbert space $\mathcal{E}$, under the inner product $(f,g) = 1/(2\pi)\int(f,g)dw$. The space $H^2(\mathcal{E})$ is the Hardy space of all analytic functions in the right half s-plane (Re(s) > 0) with values in $\mathcal{E}$ whose norm

$$\|f\|_2^2 = \sup_{\sigma>0} \frac{1}{2\pi} \int_{-\infty}^{\infty} \|f(\sigma+iw)\|^2 \, dw \qquad (1.1)$$

is finite. By passing limits to the iw axis a.e., $H^2(\mathcal{E})$ is viewed as a subspace of $L^2(\mathcal{E})$. Throughout, $K^2(\mathcal{E})$ is the orthogonal complement of $H^2(\mathcal{E})$ in $L^2(\mathcal{E})$. If $\mathcal{E}$ is the Hilbert space generated by the set of all Hilbert-Schmitt operators from $\mathcal{U}$ into $\mathcal{Y}$, then $L^2(\mathcal{E})$ is denoted by $L^2(\mathcal{U},\mathcal{Y})$ and $H^2(\mathcal{E})$ is denoted by $H^2(\mathcal{U},\mathcal{Y})$. As expected, $L^\infty(\mathcal{U},\mathcal{Y})$ is the Banach space of all uniformly essentially bounded Lebesgue measurable functions on the interval $(-\infty,\infty)$ whose values are linear operators from $\mathcal{U}$ into $\mathcal{Y}$. The space $H^\infty(\mathcal{U},\mathcal{Y})$ is the subspace of $L^\infty(\mathcal{U},\mathcal{Y})$ corresponding to the set of all uniformly bounded analytic functions in Re(s) > 0 whose values are linear operators from $\mathcal{U}$ into $\mathcal{Y}$.

To expedite the main results of this section, we will summarize the proof of the optimal H^2 solution and state the results of the optimal H^∞ solution. To begin, let α be a scalar in $\mathcal{C}^1$ with $\mathrm{Re}(\alpha) > 0$ and let ϕ be the function in H^2 denoted by $\phi = \mathrm{Re}(\alpha)/(s+\bar{\alpha})$. Also let $\|\cdot\|_2$ denote the $L^2(\mathcal{U},\mathcal{Y})$ norm and assume $\mathcal{U}$ is finite dimensional. Let F be a specified function in $L^\infty(\mathcal{U},\mathcal{Y})$, and Θ an inner function in $H^\infty(\mathcal{E},\mathcal{Y})$, that is, $\Theta(iw)$ is a.e. an isometry. Now consider the weighted H^2 optimization problem,

$$d_{2,\alpha} = \inf\{\|(F - \Theta H)\phi\|_2 : H \in H^\infty(\mathcal{U},\mathcal{E})\} . \tag{1.2}$$

For the solution to this problem, let $P_{\mathcal{H}'}$ denote the orthogonal projection onto the space $\mathcal{H}' = L^2(\mathcal{Y}) \ominus \Theta H^2(\mathcal{E})$. Then an application of the projection theorem yields $d_{2,\alpha} = \|P_{\mathcal{H}'}F\phi\|_2$.

Now consider the H^∞ Sarason optimization problem;

$$d_\infty = \inf\{\|F - \Theta H\|_\infty : H \in H^\infty(\mathcal{U},\mathcal{E})\} \tag{1.3}$$

where F is a specified function in $L^\infty(\mathcal{U},\mathcal{Y})$ and Θ is an inner function in $H^\infty(\mathcal{E},\mathcal{Y})$ (all solutions to a more general 4-block problem are given in [36]). By using standard commutant lifting techniques [4], it follows that the optimal error is given by $d_\infty = \|\Gamma\|$ where Γ is the operator from $H^2(\mathcal{U})$ into $\mathcal{H}'$ defined by $\Gamma = P_{\mathcal{H}'}M_F \,|\, H^2(\mathcal{U})$. (Here, M_G denotes the multiplication operator from $L^2(\mathcal{U})$ into $L^2(\mathcal{Y})$ defined by $M_G u = Gu$ for u in $L^2(\mathcal{U})$ where G is in $L^\infty(\mathcal{U},\mathcal{Y})$). In fact, this result will be a by product of our analysis. The optimization problems in (1.2) and (1.3) occur naturally in engineering applications and control theory , see [4,8,28,39].

The above nomenclature and identifications enable us to solve a right half plane version with respect to the main theorem of section I in concern with H^2–H^∞ optimization problems. Here, we have assumed that the typical controls problem has been converted to a Sarason type interpolation problem. Now we can state the main results of this section.

Theorem 2 *Let* F *be an element of* $L^\infty(\mathcal{U},\mathcal{Y})$, $\delta > 1$, *and n the dimension of* $\mathcal{U}$. *Also let* Θ *be an inner function in* $H^\infty(\mathcal{E},\mathcal{Y})$. *Then there exists an* H_α *in* $H^\infty(\mathcal{U},\mathcal{E})$ *satisfying* $\|F - \Theta H_\alpha\|_\infty \leq \delta d_\infty$ *and*

$$\|(F - \Theta H_\alpha)\phi\|_2 \leq \delta d_{2,\alpha}\left[\delta^2 - 1 + 2d_{2,\alpha}^2/(n\mathrm{Re}(\alpha)d_\infty^2)\right]^{-1/2} . \tag{1.4}$$

In particular, by choosing $\delta^2 = 2-2d_{2,\alpha}^2/(nRe(\alpha)d_\infty^2)$*, there exists an* H_α *satisfying the following bounds*

$$\|(F-\Theta H_\alpha)\phi\|_2 \le d_{2,\alpha}\left[2-2d_{2,\alpha}^2/(nRe(\alpha)d_\infty^2)\right]^{1/2} \tag{1.5}$$

$$\|(F-\Theta H_\alpha)\|_\infty \le d_\infty\left[2-2d_{2,\alpha}^2/(nRe(\alpha)d_\infty^2)\right]^{1/2}. \tag{1.6}$$

Notice in the previous Theorem 2 that our results depend upon the α chosen. This α can be used as a weight in typical H^2 and H^∞ controls problems. Our proof of Theorem 2 is based upon the central solution to the commutant lifting theorem [26,27,73,74]. The following paragraph discusses implementation of the results in Theorem 2.

To show the practical aspects of Theorem 2, we will present some explicit formulas that enable us to construct an H satisfying the inequalities in (2.4). Finally, we will present some examples of of these results.

We say that F is a rational function in $K^\infty(\mathcal{U},\mathcal{Y})$ if F(s) is a proper rational function with all of its poles in the open right half complex plane. Let A be an operator on a finite dimensional vector space $\mathcal{X}$, B the operator from $\mathcal{U}$ into $\mathcal{X}$, and C the operator from $\mathcal{X}$ into $\mathcal{Y}$. Then {A,B,C} is a minimal anticausel realization for F if the pair {A,B} is controllable, {C,A} is observable, and

$$F(s) = C(-sI-A)^{-1}B, \tag{2.7}$$

(see [48,66] for further details on realization theory). Since F is in $K^\infty(\mathcal{U},\mathcal{Y})$, all of the eigenvalues of A are in the open left half plane. Since Θ is two sided inner, we can assume without loss of generality that $\Theta = I$, hence the operator Γ previously defined is now the Hankel operator mapping $H^2(\mathcal{U})$ into $\mathcal{H}' = K^2(\mathcal{Y}) = L^2(\mathcal{Y}) \ominus H^2(\mathcal{Y})$ defined by $\Gamma = P_{\mathcal{H}'}M_F \mid H^2(\mathcal{U})$, where $P_{\mathcal{H}'}$ is the orthogonal projection onto the space $K^2(\mathcal{Y})$. It is well known (see [26,28,35,39,88]) that this Hankel operator Γ admits a decomposition of the form $\Gamma = W_o W_c^*$, where W_o is the operator from $\mathcal{X}$ into $K^2(\mathcal{Y})$ defined by $W_o = C(-sI-A)^{-1}$ and W_c is the operator from $\mathcal{X}$ to $H^2(\mathcal{U})$ defined by $W_c = B^*(sI-A^*)^{-1}$. The controllability grammian P and the observability grammian Q are defined by $P \triangleq W_c^* W_c$ and $Q \triangleq W_o^* W_o$. Because {A,B,C} is a minimal realization, both P and Q are strictly positive definite operators and are the unique solutions to the following Lyapunov equations

$$AP+PA^*+BB^* = 0 \quad \text{and} \quad A^*Q+QA+C^*C = 0\,. \tag{2.8}$$

The minimal realization $\{A,B,C\}$ along with its controllability and observability grammians P and Q, will play a key role in presenting state space solutions for the function $F - H_\alpha$. Finally, recall that $\|\Gamma\|^2 = d_\infty^2$ equals the largest eigenvalue of QP, (see [4,26,28,35,39,88]).

Using the previous definitions, the H^2 optimization problem is solved trivially by decomposing $F\Phi$ into orthogonal components. Notice from the following expansion,

$$\begin{aligned} F\Phi &= C(-sI-A)^{-1}B(\mathrm{Re}(\alpha)/(s+\bar{\alpha})) \\ &= \mathrm{Re}(\alpha)\left[C(\bar{\alpha}I-A)^{-1}(s+\bar{\alpha})^{-1}B+C(-s-A)^{-1}(\bar{\alpha}I-A)^{-1}B\right], \end{aligned} \tag{2.9}$$

that the first and second terms of (2.9) are contained in $H^2(\mathcal{U},\mathcal{Y})$ and $K^2(\mathcal{U},\mathcal{Y})$, respectively. So if

$$\hat{H} = C(\bar{\alpha}I-A)^{-1}B\ , \tag{2.10}$$

then $\hat{H}$ is the solution to the following optimization problem

$$\begin{aligned} d_{2,\alpha} &= \inf\{\|(F-H)\phi\| : H \in H^\infty\} \\ &= \{\|(F-\hat{H})\phi\| : \hat{H} = C(\bar{\alpha}I - A)^{-1}B\}\ . \end{aligned} \tag{2.11}$$

Moreover, the resulting optimal H^2 error is

$$d_{2,\alpha}^2 = \|\mathrm{Re}(\alpha)W_o(\bar{\alpha}I-A)^{-1}B\|_2^2 = \frac{(\mathrm{Re}(\alpha))^2}{2\pi}\,\mathrm{trace}\left[B^*(\bar{\alpha}I-A^*)^{-1}Q(\alpha I-A)^{-1}B\right] \tag{2.12}$$

Since F is rational, a realization for $G_r = F - \Theta H_\alpha$ can be constructed as shown in the following Theorem 3.

Theorem 3 *Let* $\{A \text{ on } \mathcal{X},B,C\}$ *be a minimal anticausel realization for the function* F *in* $K^\infty(\mathcal{U},\mathcal{Y})$. *Let* P *and* Q *be the Lyapunov solutions defined in* (2.8) *for the pairs* $\{A,B\}$ *and* $\{A,C\}$, *respectively. Set* $\delta > 1$ *and* $\gamma = \delta d_\infty$ *where* $d_\infty^2 = \lambda_{max}(QP)$. *Let* N_r *in* $K^\infty(\mathcal{U},\mathcal{Y})$ *and* D_r *in* $H^\infty(\mathcal{U},\mathcal{U})$ *be the rational functions given by*

$$N_r = \gamma^2 W_o(\gamma^2 I-PQ)^{-1}(\bar{\alpha}I-A)^{-1}B \quad \text{and} \tag{2.13}$$

$$D_r = \frac{1}{s+\bar{\alpha}} + W_c Q(\gamma^2 I-PQ)^{-1}(\bar{\alpha}I-A)^{-1}B\ . \tag{2.14}$$

Then $G_r = N_r D_r^{-1}$ *is a function in* $L^\infty(\mathcal{U},\mathcal{Y})$ *satisfying the bounds in Theorem* 2 *and admits a decomposition of the form* $G_r = F - H_\alpha$ *for some* H_α *in* $H^\infty(\mathcal{U},\mathcal{Y})$.

Notice if we let $\alpha \to \infty$ along the real axis, then (2.13) and (2.14) show that $G_r = N_r D_r^{-1}$ approaches $G = ND^{-1}$ where

$$N = \gamma^2 W_o(\gamma^2 I - PQ)^{-1} B \text{ and } D = I + W_c Q(\gamma^2 I - PQ)^{-1} B . \qquad (2.15)$$

Moreover, the function $G = ND^{-1} = (F-H)$ for H in $H^\infty(\mathcal{U},\mathcal{Y})$ solves the Sarason problem with the bounds

$$\|G\|_\infty \le \delta d_\infty \text{ and } \|G\|_2 \le \frac{\delta\|P_{\mathcal{H}'}F\|_2}{\sqrt{\delta^2-1}} \qquad (2.16)$$

for $\delta > 1$. We conclude this section with a remark concerning the factorization of H satisfying $G = F - H$ when $\Theta = I$.

Remark Assuming α is positive real, the identity

$$(sI-A^*)\phi = (sI-A^*)\alpha/(s+\alpha) = \alpha\left[I-(s+\alpha)^{-1}(\alpha I+A^*)\right] \qquad (2.17)$$

along with some tedious manipulations can be used to show that $H_\alpha = F - G_r$ has a realization $H_\alpha = D_\alpha + C_\alpha(sI-A_\alpha)^{-1}B_\alpha$ where

$$\begin{aligned}
A_\alpha &= -\{(\gamma^2Q^{-1}-P)+(\alpha I-A)^{-1}BB^*\}^{-1}\{(I-A/\alpha)^{-1}BB^*-(\gamma^2Q^{-1}-P)A^*\} \qquad (2.18)\\
B_\alpha &= \{(\gamma^2Q^{-1}-P)+(\alpha I-A)^{-1}BB^*\}^{-1}(\alpha I-A)^{-1}B\\
C_\alpha &= -\gamma^2CQ^{-1}\left\{\begin{array}{l}\{(\gamma^2Q^{-1}-P)+(\alpha I-A)^{-1}BB^*\}^{-1}\{(I-A/\alpha)^{-1}BB^*-(\gamma^2Q^{-1}-P)A^*\}\\ \qquad + (I-\gamma^{-2}QP)A^*-\gamma^{-2}\alpha QP\end{array}\right\}\\
D_\alpha &= \gamma^2CQ^{-1}\{(\gamma^2Q^{-1}-P)+(\alpha I-A)^{-1}BB^*\}^{-1}(\alpha I-A)^{-1}B .
\end{aligned}$$

Furthermore, setting

$$A_H = A^*-(\gamma^2I-QP)^{-1}QBB^*, \; B_H = (\gamma^2I-QP)^{-1}QB \text{ and } C_H = CP , \qquad (2.19)$$

it can be easily shown that as $\alpha \to \infty$, $H_\alpha \to H = C_H(sI-A_H)^{-1}B_H$ uniformly over the complex right half plane. As a matter of fact, this is the solution corresponding to the unweighted H^2–H^∞ problem due to the fact that as $\alpha \to \infty$, $\alpha/(s+\alpha) \to 1$. Notice if F is purely anticausel, the H satisfying the optimal H^2 unweighted problem ($\gamma,\alpha \to \infty$) is the trivial solution $H = 0$. The following section shows how these formulas are useful for rational functions with an application to control system synthesis.

2. H^2–H^∞ Discrete Time Algorithm

In this section, we will summarize the algorithm to construct an H_α satisfying the norm bounds in (2.4) and apply the algorithm to an example.

(1). Compute a minimal realization $\{A_F, B_F, C_F\}$ for F.

(2). Compute the Lyapunov equations for P and Q in (2.8)

(3). Compute d_α and d_∞

(4). Set $\delta > 1$ and $\alpha > 0$

(5). Compute $\{A_\alpha, B_\alpha, C_\alpha, D_\alpha\}$ for H_α

We will now apply the previous algorithm to a typical controls problem associated with disturbance attenuation. Suppose we are given an open loop plant g and consider the design of a filter c that achieves a preset performance criteria on the resulting closed loop system. The following block diagram shows pictorially the plant g, filter c, and exogenous disturbance w and controlled output signal z.

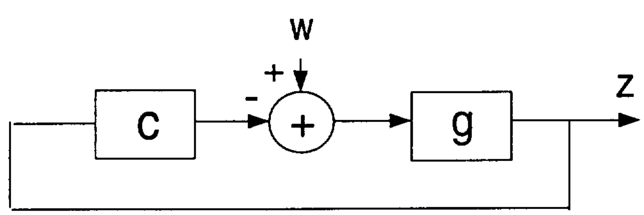


Figure 6.3 Closed Loop System

From the diagram, the input-output map from w to z is given by

$$\frac{z}{w}(s) = g_{cl} = \frac{g}{1 + gc} . \tag{6.3}$$

For simplicity, we will assume $g = n/d$ is rational and stable with an inner-outer factorization given by $g = g_i g_o$. With open loop stability in mind, a parameterization of all stabilizing controllers is given by $c = h/(1 - gh)$ for arbitrary h in H^∞. Using this expression for c and substituting into the closed loop expression in (6.3) we obtain the following norm expressions for the closed loop system g_{cl},

$$\|g_{cl}\|_{2,\infty} = \left\|\frac{g}{1 + g(h/(1 - gh))}\right\|_{2,\infty} = \|g - g^2h\|_{2,\infty} = \|g_i^* g_o - g_o^2 h\|_{2,\infty} . \qquad (6.4)$$

Thus, setting $g_o^2 h = P_+(g_i^* g_o) + h'$, we can equivalently consider the control problem in the Nehari setting with a search for h' in H^∞ and norms

$$\|g_{cl}\|_{2,\infty} = \|P_+(g_i^* g_o) + P_-(g_i^* g_o) - g_o^2 h\|_{2,\infty} = \|P_-(g_i^* g_o) - h'\|_{2,\infty} . \qquad (6.5)$$

Now consider the following stable open loop plant g with inner-outer factorization $g = g_i g_o$ given by

$$g(s) = \frac{w_n^2(s^2-2\zeta_n w_n s+w_n^2)}{(s+1)(s+2)} = \left[\frac{s^2-2\zeta_n w_n s+w_n^2}{s^2+2\zeta_n w_n s+w_n^2}\right]\left[\frac{w_n^2(s^2+2\zeta_n w_n s+w_n^2)}{(s+1)(s+2)}\right] = g_i g_o . \qquad (6.6)$$

Using various values for δ and α with $\zeta_n = .005$ and $w_n = 10$, we have computed the rational function $P_-(g_i^* g_o) - h'$ and simulated the Bode plots.

The magnitudes in decibels for $P_-(g_i^* g_o) - h'$ (which has the identical Bode plot as g_{cl}) with values of $\delta = 1.001$, $\delta = \sqrt{2}$, and $\delta = \infty$ with various $\alpha > 0$ have been plotted and shown in the following figures.

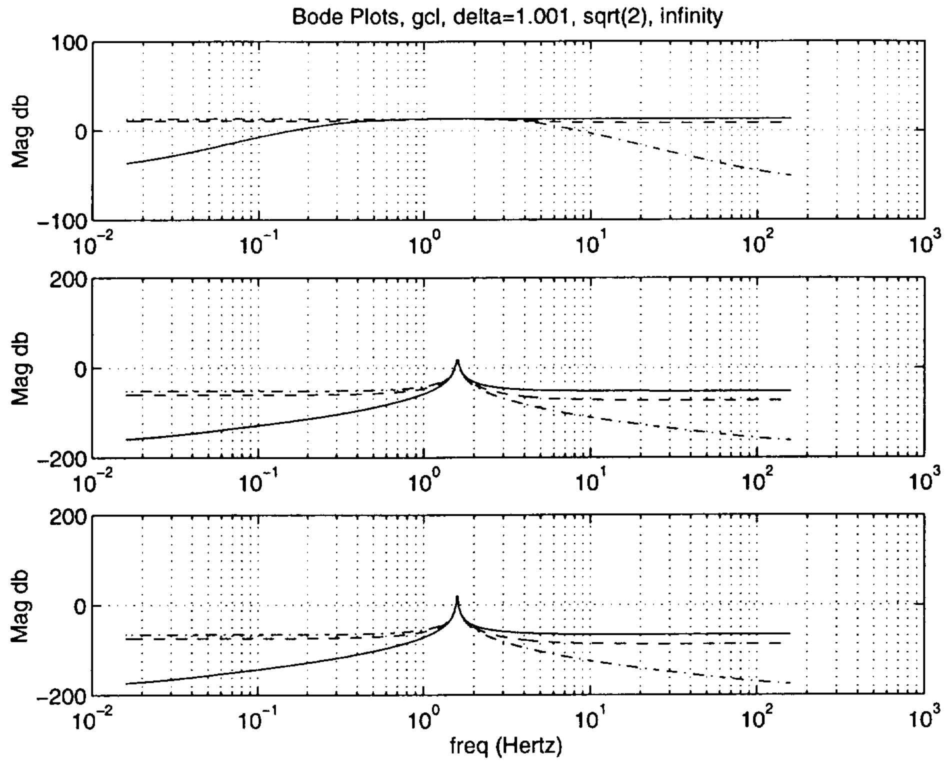


Figure 6.4 Magnitude Plots

The following Table 6.5 gives the numerical values with $\alpha = 10$ from the resulting computations.

Table 6.5 Norm Data

δ	α	d_∞	$\|G_{cl}\|_\infty$	$\|\cdot\|_\infty$ bound	$d_{2,\alpha}$	$\|G_{cl}\phi\|_2$	$\|\cdot\|_2$ bound
1.001			1.9599	1.9599		3.9819	4.1764
$\sqrt{2}$	10	1.9579	2.4887	2.7689	0.6159	0.7088	0.8626
∞			2.8677	∞		0.6159	0.6159

Notice that for each δ chosen the resulting norms are within the bounds given from the theoretical results. The following paragraph discusses the computations required to obtain the compensator c.

Recall that the compensator c is given by $c = h/(1 - gh)$. Hence, using the expression $g_o^2 h = P_+(g_i^* g_o) + h'$ in c we obtain

$$c = \frac{P_+(g_i^* g_o) + h'}{g_o^2 - g(P_+(g_i^* g_o) + h')} . \qquad (6.7)$$

In particular, we computed the compensator using the formula in (6.7) and the resulting closed loop transfer function for the cases that $\delta = \sqrt{2}$ and $\alpha = 0.1, 10$ and 1000 in the previous example. For instance, setting $\delta = \sqrt{2}$ and $\alpha = 10$, the resulting compensator was computed from the formula in (6.7) and given by

$$c = \frac{-124.8s^2 - 50.76s - 12357.22}{s^2 - 44.83s - 548.25} . \qquad (6.8)$$

It is of particular interest that the resulting closed loop g_{cl} computed from the actual c in (6.7) for $\delta = \sqrt{2}$ and $\alpha = 0.1$, 10, and 1000 has the expected Bode plot as shown in Figure 6.6.

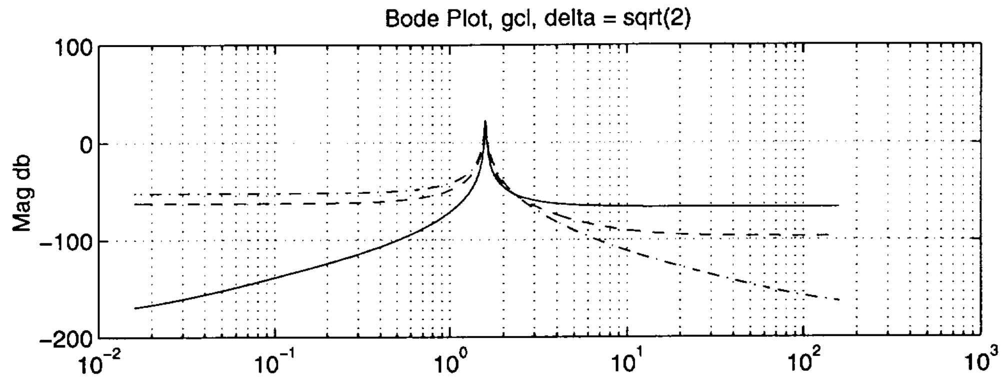


Figure 6.6 Magnitude Plot

For applications and actual implementation purposes, it is usually desirable to have low order compensators for minimizing computation time. We have listed the dynamical orders of the compensator c and associated closed loop system g_{cl} and the resulting number of unstable compensator poles in the following Table 6.7.

Table 6.7 Compensator Data

α ($\delta = \sqrt{2}$)	compensator order	closed loop plant order	no. of unstable compensator poles
0.1	3	5	1
10	2	4	1
1000	2	4	0

Notice from Table 2.5 that for low values of α, the resulting compensator has one unstable pole, and as $\alpha \to \infty$ the compensator is stable.

Conclusions

The previous theorems and examples show that the multiobjective H^2–H^∞ control problem can be viewed from a Sarason (model matching) point of view. Here, we have used a lifting approach to obtain our results once in the interpolation setting. State space algorithms have been prescribed for the construction of these solutions and resulting compensator. We have also shown that our solution satisfies some interesting H^2–H^∞ bounds. Future work would involve generalizing these results to 2-block and

4-block settings (we have implicitly solved the 1-block problem). Finally, we hve successfully applied these algorithms to the F-18 HARV fighter aircraft in a design example.

References

[1] P. Dorato, C. Abdallah, and V. Cerone, *Linear Quadratic Control: An Introduction,* Prentice Hall, 1995.

[2] M. Athens (Ed.), "Special issue on linear-quadratic-gaussian problem," *IEEE Trans. Automat. Contr.* AC-16 (Dec. 1971).

[3] J. C. Willems, "Least squares stationary optimal control and the algebraic Riccati equation," *IEEE Trans. Automat. Contr.* AC-16 (Dec. 1971): 621-634.

[4] J. C. Doyle and G. Stein, "Robustness with observers," *IEEE Trans. Automat. Contr.* AC-24 (Aug. 1979): 607-611.

[5] D. C. Youla, J. J. Bongiorno, Jr., and H. A. Jabr, "Modern Wiener-Hopf design of optimal controllers: Parts I, II," *IEEE Trans. Automat. Contr.* AC-21 (1976): 3-14, and AC-21 (1976): 319-330.

[6] G. Zames, "Feedback and optimal sensitivity: Model reference transformations, multiplicitive seminorms, and approximate inverses. *IEEE Trans. Automat. Contr.,* AC-26 (1981): 301-320.

[7] Doyle, J. C., "Lecture notes in advances in multivariable control," *ONR/Honeywell Workshop,* Minneapolis, 1984.

[8] Glover, K., "All optimal Hankel-norm approximations of linear multivariable systems and their L_∞ error bounds," *Int. J. Control,* vol. 39, pp. 1115-1193, 1984.

[9] Doyle, J. C., K. Glover, P.P. Khargonekar, B.A. Francis, "State-Space Solutions to standard H_2 and H_∞ control problems," *IEEE Trans. Auto. Contr.,* AC-34 (1988): 831-847.

[10] Boyd, S., *Linear Matrix Inequalities in System and Control Theory,* SIAM, 1994.

[11] K. Zhou, J. C. Doyle, and K. Glover, *Robust and Optimal Control,* Prentice Hall, 1995.

[12] Doyle, J. C., K. Zhou, K. Glover, and B. Bodenheimer, "Optimal control with mixed H_2 and H_∞ performance objectives", *IEEE Trans. Autom. Contr.,* AC-39 (1994): 1575-1587.

[13] C. Foias and A. E. Frazho, "The Commutant Lifting Approach to Interpolation Problems, Operator Theory: Advances & Applications," Vol. 44, Birkhäuser Verlag, Basel, 1990.

[14] C. Foias, A. E. Frazho and W. S. Li, "The Exact H^2 Estimate for the Central H^∞ Interpolant," *Operator Theory; Advances of Applications*, Vol. 64, 1993, Birkhauser Verlag, Basel.

[15] M. Smith and A. Frazho, "Strong Stabilization and H-infinity Optimization for SISO Systems", *Proc. Aller. Conf. Communications, Control, and Computing,* To be published, 1996.

[16] K. Hoffman, *Banach Spaces of Analytic Functions*, Englewood Cliffs, Prentice Hall, New Jersey, 1962.

[17] D. Sarason,"Generalized Interpolating in H^∞," *Trans. American Math. Soc.*, 127 (1967), pp. 179-203.

[18] J. C. Doyle, R. A. Francis and A. R. Tannenbaum, *Feedback Control Theory*, McMillan, New York, 1992.

[19] D. Brett Rideley, Siva S. Banda, AFWAL-TR-85-3102, "Introduction to Robust Multivariable Control", Feb 1986.

[20] R. E. Kalman and R. S. Bucy, "New Results in linear filtering and prediction theory," *Trans. ASME Ser. D. (J. Basic Engr.)* 83 (1961): 95-107.