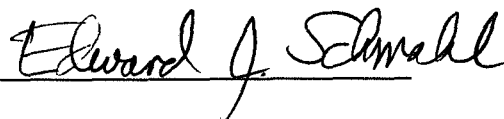*IN-92*
435 182

P. 31

FINAL REPORT
SOLAR FLARE PHYSICS
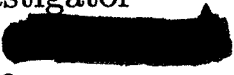### December 14, 1998

NAG 5-2001 (UMD 01-5-26120)
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National Aeronautics and Space Administration

January 1, 1998 to December 31, 1998

Institution: Astronomy Department, University of Maryland
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Solar Flare Physics

We have continued our previous efforts in studies of fourier imaging methods applied to hard X-ray flares. We have performed physical and theoretical analysis of rotating collimator grids submitted to GSFC for the High Energy Solar Spectroscopic Imager (HESSI). We have produced simulation algorithms which are currently being used to test imaging software and hardware for HESSI. We have developed Maximum-Entropy, Maximum-Likelihood, and "CLEAN" methods for reconstructing HESSI images from count-rate profiles. This work is expected to continue through the launch of HESSI in July, 2000.

Section 1 of the following pages shows a poster presentation "Image Reconstruction from HESSI Photon Lists"at the Solar Physics Division Meeting, June 1998; Section 2 shows the text and viewgraphs prepared for "Imaging Simulations" at HESSI's Preliminary Design Review on July 30, 1998 .

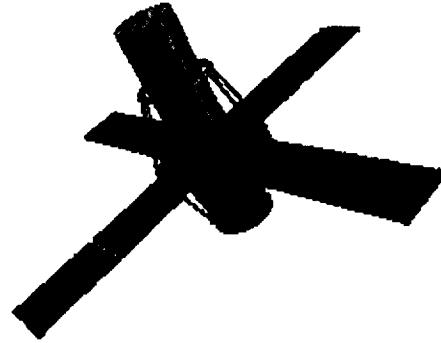# SECTION I

Image Reconstruction from HESSI Photon Lists

Poster presentation at the

June 1998 Solar Physics Division Meeting

Boston, Massachusetts

IMAGE RECONSTRUCTION FROM HESSI

PHOTON LISTS

1. PHOTON LISTS

- O WHY MAKE HESSI SCORES? (.ps)
- O 900-PHOTON SCORE (.ps)
- O PHOTON LIST RECIPE (.ps)
- O ALGORITHM DIAGRAM (.ps)
- O BINNED PHOTON PROFILE, COLLIMATORS 1-4
- O BINNED PHOTON PROFILE, COLLIMATORS 6-9

2. BACK PROJECTIONS

- O Unbinned Back Projection (Caption) (.ps)
- O Unbinned Back Projection maps 0-8 (.ps)
- O Back Projection HOW-TO (.ps)
- O Back-Projection Artifacts (.ps)

3. CHI SQUARE AND ALL THAT

- O Comparing the Model with the Score (.ps)
- O CHI-SQ for Sparse Photons (.ps)
- O Unbinned Back Projection maps 0-8 (.ps)

4. MAXIMUM ENTROPY

- O Max Ent Algorithm (.ps)
- O HOW-TO MEM (.ps)
- O The Gradient Map (.ps)

CONCLUSIONS

This is mainly a progress report and has no final conclusions, but we are able to point out the directions for future work:

- Photon lists must be developed to include:
  - More complex and realistic sources
  - Time variability
  - Internal shadowing and vignetting
  - Grid irregularities
  - Pulse height spectra
  - Aspect Phase Information

- Maximum Entropy Programs must be made:
  - More robust
  - Faster
  - More efficient

- Other Reconstruction Algorithms are needed:
  - CLEAN
  - Maximum Likelyhood
  - Signal Processing

Last updated JUNE 2, 1998.

PHOTON LIST SIMULATION

A photon list is a table of arrival times, detector numbers, and pulse heights for time-tagging telescope such as HESSI. A simulated photon list (also called a "score") can be used for:

- Testing HESSI hardware during development stages
    - Simulated throughput with a "pulser stimulator"
    - Simulation of dead-time effects, pulse pileup, etc.
- Testing HESSI software
    - Proving that mapping routines work
    - Identifying needs/uses/problems in utilities
    - Estimating times and memory required for analysis
- Providing a realistic, simple, model of HESSI output
    - for scientific users
    - and the general public (educational-outreach)

We have provided a simple score file in FITS format, along with an IDL program to read it and create a simple image. The file "score1K0-4.fits" is available on the hesperia ftp site, and can be accessed using anonymous ftp:

```
ftp hesperia.gsfc.nasa.gov
cd pro
binary
get score1K0-4.fits
get score2bp.pro
bye
```

Alternatively, you can use a browser: URL = http://hesperia. gsfc.nasa.gov/hessidoc/pro/, and download the aforementioned .fits and .pro files. The FITS file is displayed with the IDL program score2bp.pro, which back-projects the data onto 64 x 64 arrays. In IDL just type the command ".run score2bp", and you will be prompted for the name of the score file.

The following figure shows a 900-photon subset of the score file. Each "staff" shows 9 one-second timelines, with spatial frequency increasing up (collimators 1-9) and time going to the right. The photon arrival times from a point-source burst have been modulated by an analytical version of HESSI grids, and images can be constructed from the arrival times alone, as shown below in this paper.
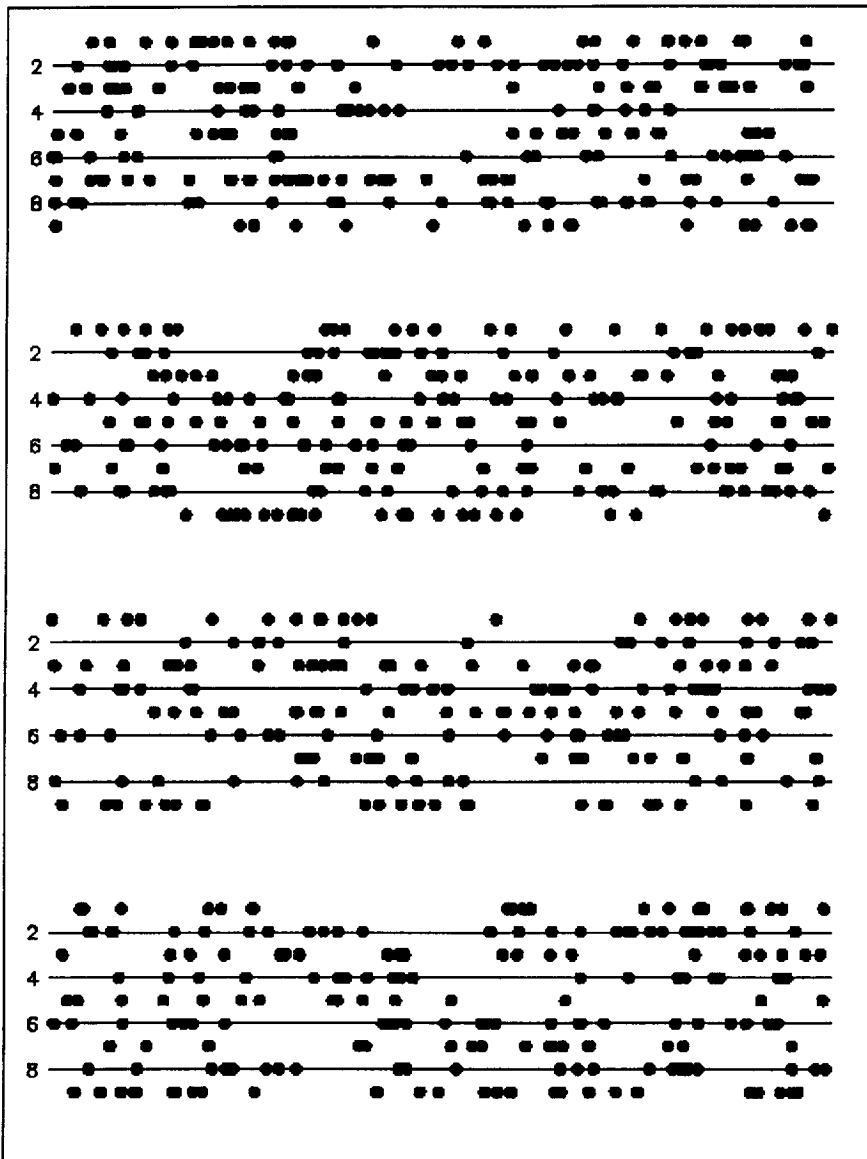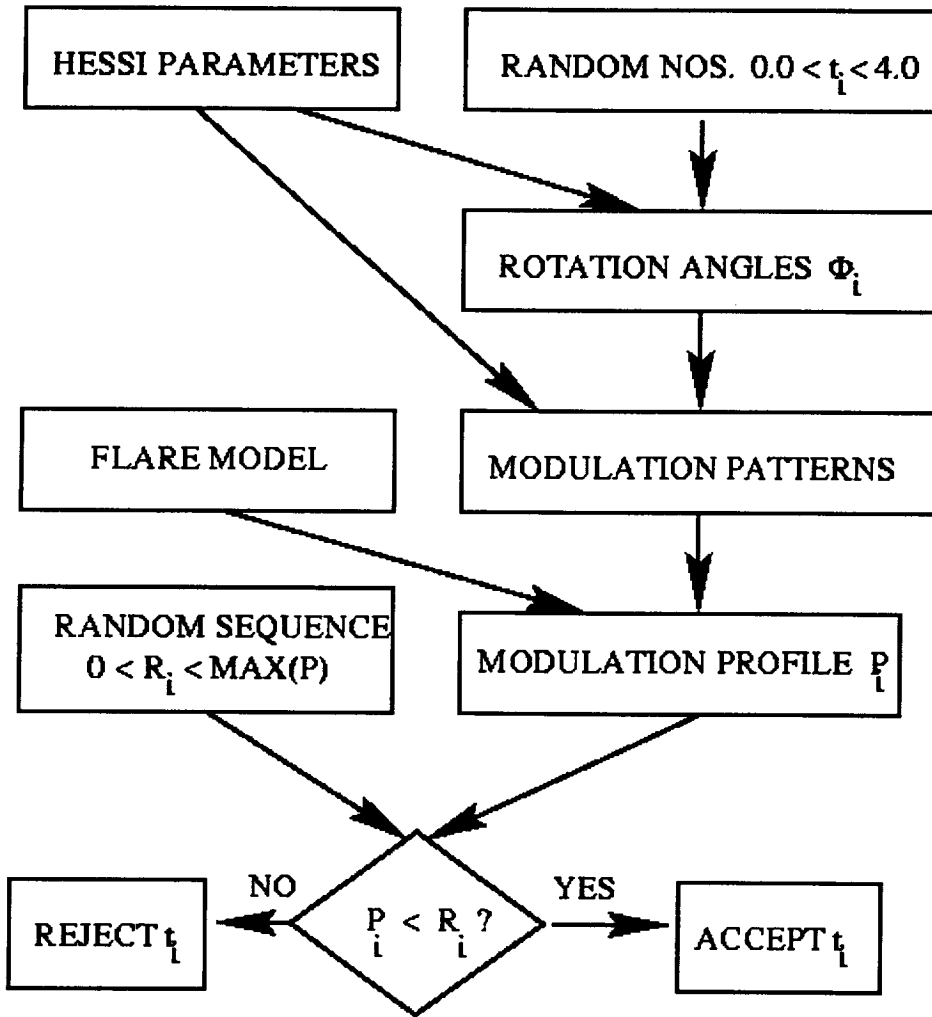
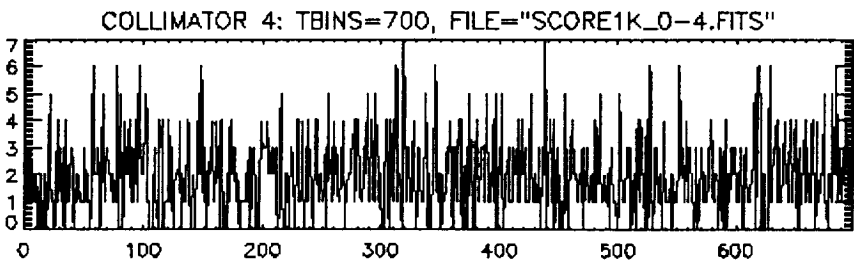900—PHOTON SCORE FROM FILE "score1K0—4.fits"
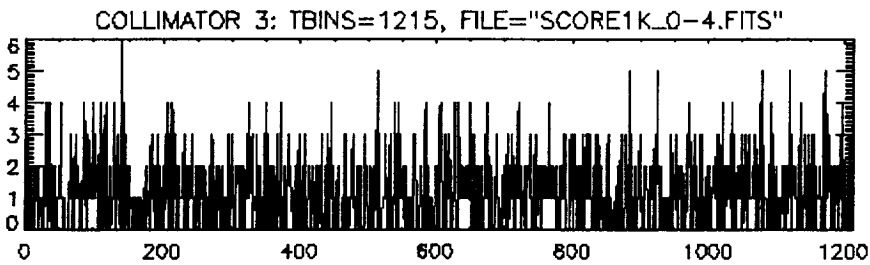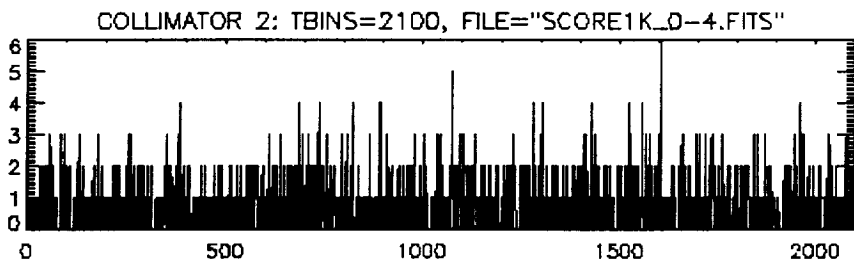
RECIPE FOR A PHOTON LIST

1. Select a flare model in space and time.

2. Create a pseudo-random time sequence t_i within the desired time interval.

3. Compute the rotation angle sequence ϕ_i for one collimator.

4. Calculate the HESSI modulation patterns for each ϕ_i.

5. Generate a sequence of relative probabilities S_i from the scalar product of the model and the modulation pattern at each ϕ_i.

6. Create a uniform random number sequence R_i of the same length, scaled to the range $[0, \max(p_i)]$.

7. Accept each time/angle value associated with those of S_i which are smaller than the corresponding R_i. (I.e, "modulate" the photons.)

8. Append the accepted times, along with their collimator numbers, to the photon list.

9. Repeat until done.

PHOTON LIST ALGORITHM

```
┌─────────────────────────┐        ┌──────────────────────────────┐
│   HESSI PARAMETERS      │        │  RANDOM NOS.  0.0 < tᵢ < 4.0  │
└─────────────────────────┘        └──────────────────────────────┘
```

$$\text{HESSI PARAMETERS} \qquad \text{RANDOM NOS. } 0.0 < t_i < 4.0$$

$$\text{ROTATION ANGLES } \Phi_i$$

$$\text{FLARE MODEL} \qquad \text{MODULATION PATTERNS}$$

$$\text{RANDOM SEQUENCE } 0 < R_i < \text{MAX}(P) \qquad \text{MODULATION PROFILE } P_i$$

NO $\qquad P_i < R_i \,? \qquad$ YES

$$\text{REJECT } t_i \qquad\qquad \text{ACCEPT } t_i$$

BINNED PHOTON LISTS

COLLIMATOR 1: TBINS=3600, FILE="SCORE1K_0-4.FITS"

COLLIMATOR 2: TBINS=2100, FILE="SCORE1K_0-4.FITS"

COLLIMATOR 3: TBINS=1215, FILE="SCORE1K_0-4.FITS"

COLLIMATOR 4: TBINS=700, FILE="SCORE1K_0-4.FITS"

BINNED PHOTON LISTS

COLLIMATOR 6: TBINS=240, FILE="SCORE1K_0−4.FITS"

COLLIMATOR 7: TBINS=135, FILE="SCORE1K_0−4.FITS"

COLLIMATOR 8: TBINS=75, FILE="SCORE1K_0−4.FITS"

COLLIMATOR 9: TBINS=45, FILE="SCORE1K_0−4.FITS"

UNBINNED BACK PROJECTIONS

The following figures show full-disk (note limb circle) and small-field maps made directly from the score. Reading from left to right, these show projections from the coarsest 4 collimators and their sum (map 5). A box in the latter map shows the field mapped by the finest four collimators. These are shown individually for collimators 1-3, followed by their sum with the 4th collimator in the last map.

COLLIMATOR 6   COLLIMATOR 7   COLLIMATOR 8

COLLIMATOR 9   COLLIMATOR 6+7+8+9   COLLIMATOR 1

COLLIMATOR 2   COLLIMATOR 3   COLLIMATOR 1+2+3+4

```
┌─────────────────────┐       ┌─────────────────────────┐
│    HESSI SCORE      │       │    HESSI PARAMETERS     │
└─────────────────────┘       └─────────────────────────┘
           │                            │        │
           ▼                            ▼        │
┌──────────────────────────────────────────┐    │
│    ROTATION ANGLES        Φ_i            │    │
└──────────────────────────────────────────┘    │
                    │                        │
                    ▼                        ▼
┌──────────────────────────────────────────┐
│    MODULATION PATTERN M_i                │
└──────────────────────────────────────────┘
                    │
                    ▼
           ┌─────────────────────┐
           │    Add to Dirty Map │
           └─────────────────────┘
```
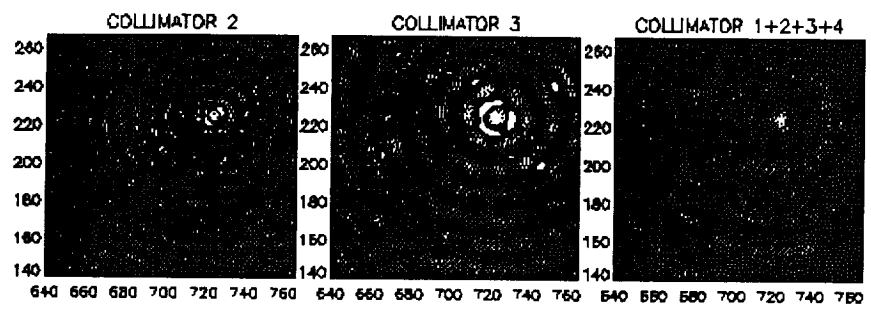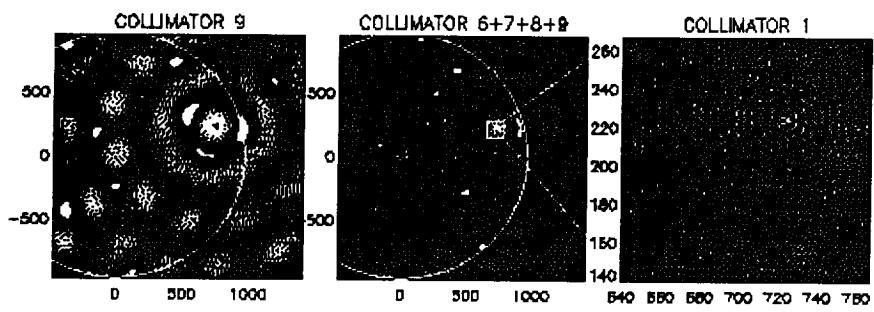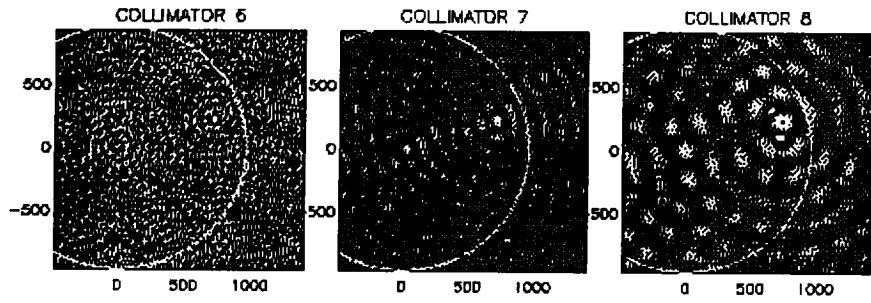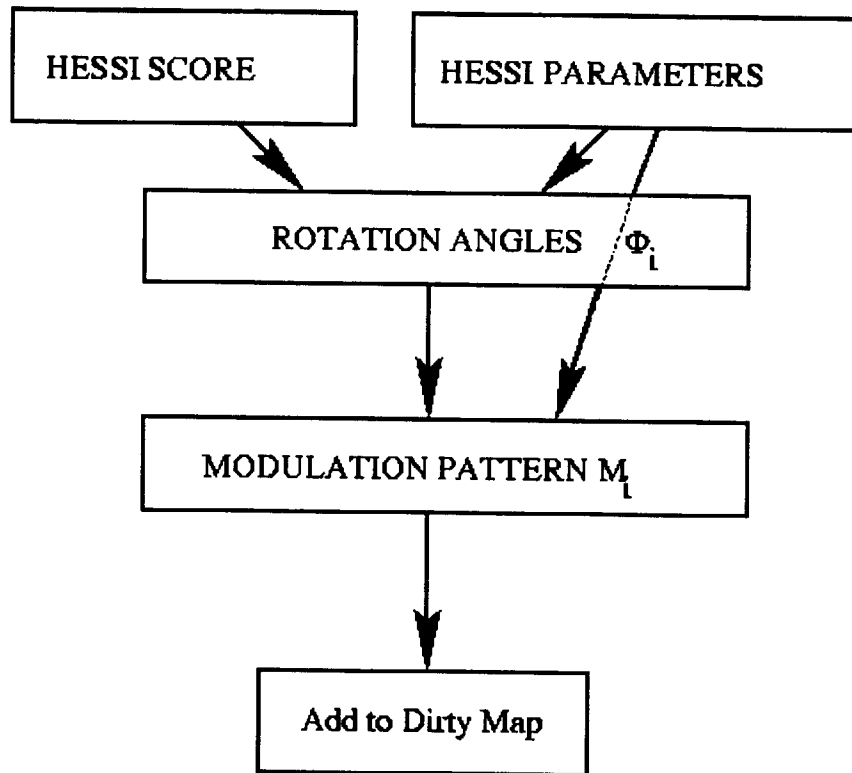
Artifacts in Back-Projection Maps

- It has been shown that "Back-Projection" produces the statistically best estimate of the source position.

- The Back-Projection map, of course, is not the "cleanest" or simplest possible map representing the true source.

- If the collimator phase is constant during rotation, the Back-Projection map shows a ficticious source at the spin axis position due to the lack of modulation at that point.

- The collimator phase will almost certainly not be constant for HESSI, so the spin axis will be washed out. Thus Back-Projection maps may be better in this respect than shown here.

- There is a fictitious "mirror" source seen in Back-Projection maps at a position symmetric to the true source relative to the spin axis. The sign of the mirror source depends on the collimator phase, and its variations will also wash it out with respect to the true source.

- The rings around the source in Back-Projection maps have radii and spacing that depend on the collimator pitch. Adding the maps from several collimators therefore tends to smooth out the rings, while reinforcing the true source.

- The above being said, there is no substitute for a truly "clean" map, and Back-Projection maps will provide only the first look, after which reconstruction will take over.

COMPARING MODELS WITH THE SCORE

- The score can be thought of as a time profile with all zeroes and ones $\{n_i\}$, the zeroes being as significant as the ones.

- In general, the model count rate profile $\{e_i\}$ will be non integral and non zero.

- The χ^2 statistic can be used as a measure of fit of $\{e_i\}$ to $\{n_i\}$ but it will not follow the χ^2 distribution if $n_i < 10$.

- Binning can mitigate this problem, but can result in loss of real temporal structure.

- Bin sizes should be determined by the collimator geometry, the burst location and the rotation rate, not the number of counts.

- Alternates exist to the χ^2 statistic, such as the correlation coefficient or the C statistic.

- For low count rates we use the $C = -2 \log(P)$ statistic (Cash 1979, Ap.J. 228, 939), where:

$$P = \prod_{i=1}^{N} \frac{e_i^{n_i} e^{-e_i}}{n_i!}$$

- This is the probability that a Poisson-distributed sample $\{n_i\}$ arises from the measured (or modeled) sample $\{e_i\}$.

The Chi-squared Statistic in the Poisson Limit

In comparing a model e_i of the observed photon count rate n_i, it is common to employ the statistic:

$$S = \sum_{i=1}^{N} \frac{(n_i - e_i)^2}{e_i}.$$

In the limit of large values of n_i, S follows the χ^2 distribution, but it does not when the values of the countrate n_i are all ones and zeroes, or if many values of n_i are very small. Since we expect that many gamma-ray events observed by HESSI will have only a few photons, whose distribution follows Poisson statistics, we must consider the best way to find confidence limits of models.

Following Cash (1979), we adopt the statistic:

$$C = -2(n_i \ln(e_i/n_i) - e_i + n_i).$$

It is easily shown that $C \to S$ as $n_i \to \infty$. In practice, $C \approx S$ if $n_i > 10$, and like S, C vanishes if $e_i = n_i$.

For the case where there are p parameters and p-q constraints, one obtains confidence limits by letting all p parameters vary, finding the mimimum $(C_p)_{min}$, and by varying p-q parameters to find the minimum $(C_{p-q})_{min}$. Then the statistic

$$\Delta C = (C_{p-q})_{min} - (C_p)_{min}$$

obeys the χ^2 probability distribution with q degrees of freedom. In particular, if one wants to find confidence limits for a particular pixel of a Maximum Entropy map, it is necessary to perform two minimizations:

1. Compute the quantity $(C_p)_{min}$ from the $\{e_i\}$ found by the Maximum Entropy search through parameter (pixel) space.
2. Select one of the pixels to test for goodness of fit, then determine $(C_{p-1})_{min}$ by searching for the minimum of C using MEM for all but that one pixel, whose flux is held constant.

Finally compute ΔC, and determine the confidence level from the χ^2 distribution with one degree of freedom. By Wilk's theorem, this is valid as long as the total number of photons is large ($>> 10$), even if individual count rates are all zeroes and ones.

MAXIMUM ENTROPY ALGORITHM
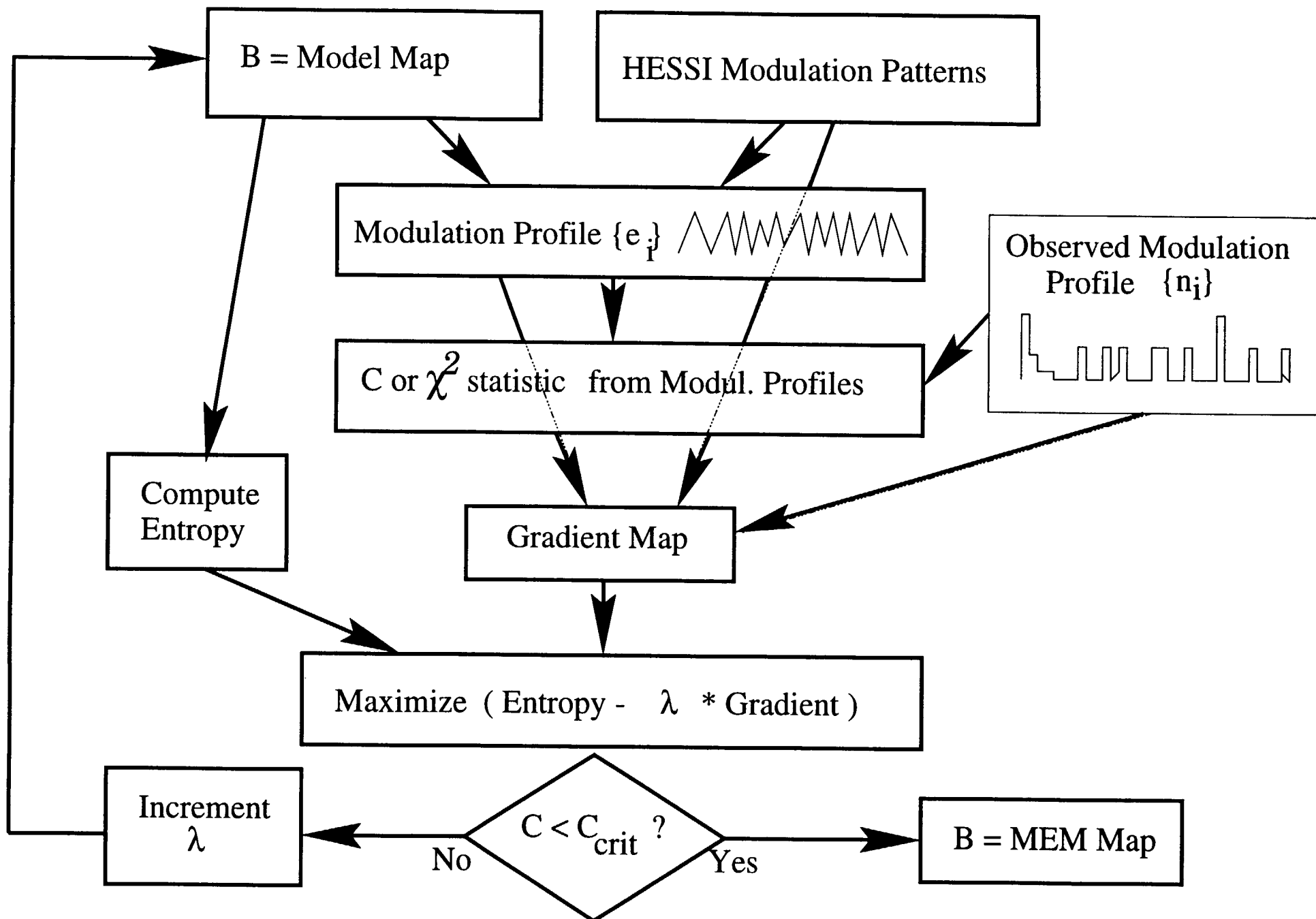
MAXIMUM ENTROPY ALGORITHM

1. Start with a blank map B, and a small positive value of parameter λ.

2. From B, compute the model modulation profile $\{e_i\}$.

3. Calculate the C (or χ^2) statistic measuring the difference between the observed modulation profile $\{n_i\}$ and $\{e_i\}$.

4. From the modulation pattern, $\{n_i\}$ and $\{e_i\}$, compute the gradient of C (or χ^2) with respect to the free parameters (the map pixels). This is the "gradient map".

5. Using the fixed-point method (Sakao, 1992), find the map B which yields a maximum of the difference of the entropy and λ times the gradient map.

6. If C (or χ^2) has not fallen below a pre-determined limit, increase λ by a few percent and repeat from step 2.

7. B is the "Maximum Entropy" map.

GRADIENT MAPS

One of the most important parts of the Maximum Entropy algorithm is the computation of the grad χ^2 map. This is the gradient of the "goodness-of-fit" function χ^2 or, in our case, C with respect to the map pixels. It is a reasonably good representation of the true map, and therefore deserves mention.

The gradient of C with respect to the model modulation profile is very simple:

$$\frac{\partial C}{\partial e_i} = 1 - n_i^2/e_i^2$$

Its derivitatives with respect to the map pixels f_i are then given by the chain rule:

$$\frac{\partial C}{\partial f_i} = \sum_k \frac{\partial C}{\partial e_k} \frac{\partial e_k}{\partial f_i}$$

The last derivatives on the right hand side are simply the modulation patterns, because the e_k are simply the inner product of the modulation patterns with the model map f_i.

The figure below shows the first gradient map of the Maximum Entropy for the same case as the back-projections above. Note the absence of the spurious spin-axis source.

SECTION II
Imaging Simulations
Presentation at
HESSI's Preliminary Design Review
### July 30, 1998
### College Park, Maryland

IMAGING SIMULATIONS

WHAT WE HAVE DONE

Modulation Patterns

- Analogous to HXT, but for HESSI will vary according to application.
    - HXT has only 64 modulation patterns, one per subcollimator.
    - HESSI will have hundreds to thousands.
    - FOV size and location are arbitrary in HESSI.
    - Center of rotation not constant, so pattern phases will vary with time.
    - The number of patterns equals the number of time bins.
- Essential to simulations and imaging software
    - Initial maps are weighted sums of the modulation patterns.
    - Even pure Fourier methods have to return to image space and add patterns together.
    - Modulation patterns exploit HXT, HEIDI heritage.
- Patterns derived from empirical measurements
    - Presently are based on theoretical grid models.
    - By April 1999 will be derived from OGCF and XGCF measurements.
    - Grid imperfections will be incorporated using phase and amplitude matrices.
- In general, patterns need to be constructed quickly.
    - They form an "inner loop" in most reconstruction algorithms.
    - Impractical to rotate fixed patterns--slower than on-the-fly computation.
    - Every pattern must be phase shifted using the aspect solution.
    - Memory/disk constraints may mandate "on-the-fly" computation.
        - 1000 (say) 128 x 128 patterns require 65 MB of memory;
        - A slow computation usually beats all but fastest disks;
        - On-the-fly computation provides more freedom of action.
- Recent breakthrough: Order of magnitude speedup of computation:
    - Method involves copying from a 1-d profile of the subcollimator profile
    - A graphic explanation
    - Makes it possible to back-project scores in < 4 s per collimator.
- Free parameters of the modulation pattern
    - Slit/Slat ratio, a function of distance from spin axis,
        - Internal shadowing important -- can be a 10-20% effect
    - Harmonic content: Fundamental and any subset of harmonics 1 to ∞,
    - Arbitrary phase offsets given by aspect solution,
   - Phase and amplitude correction matrices.

»
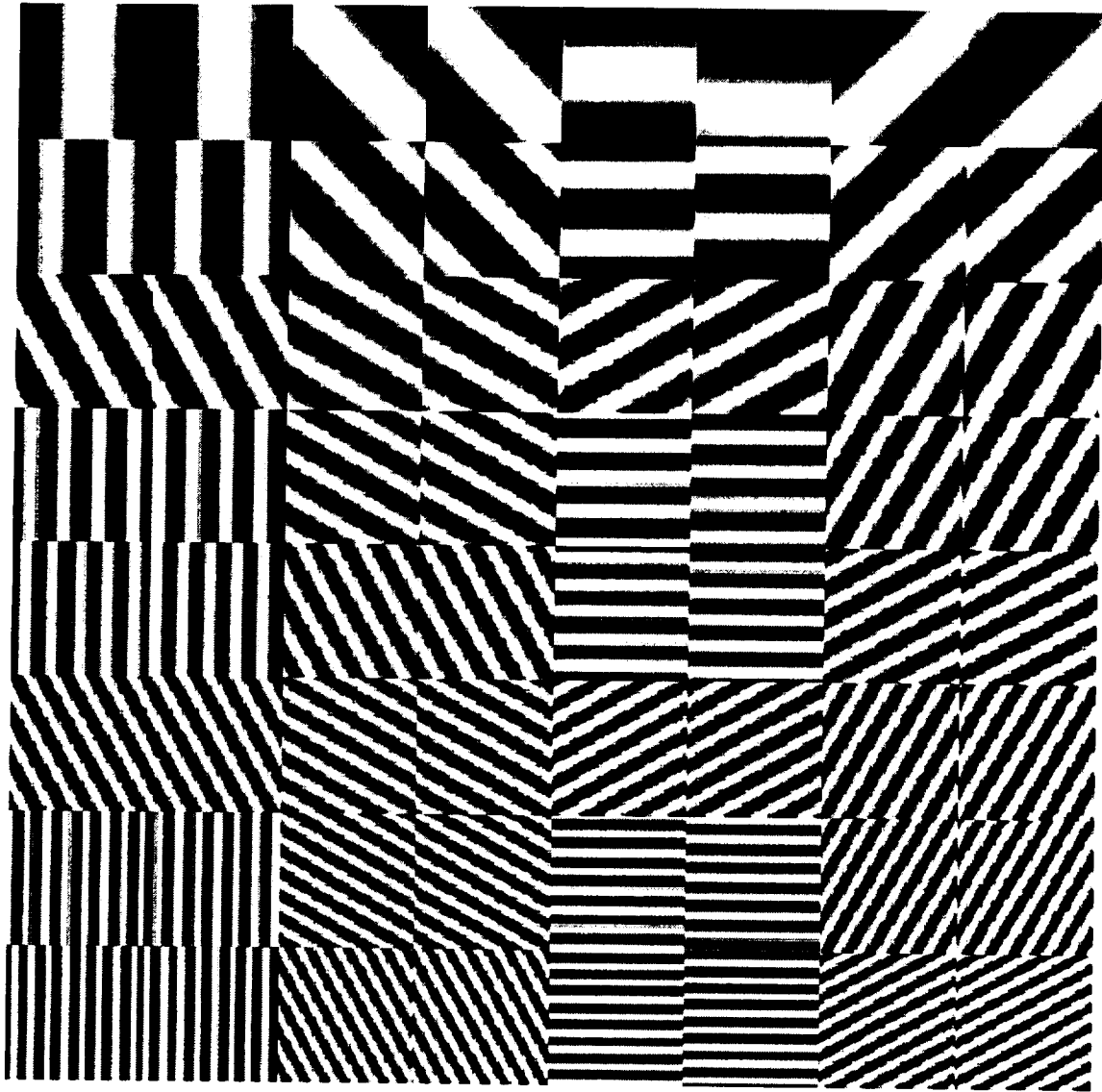
This figure shows the 64 modulation patterns of the Hard X-ray Telescope on the Yohkoh satellite. Each square of black and white bars is the visibility pattern (Fourier pattern) for a single sub-collimator. The finest patterns have a spatial resolution of 8".
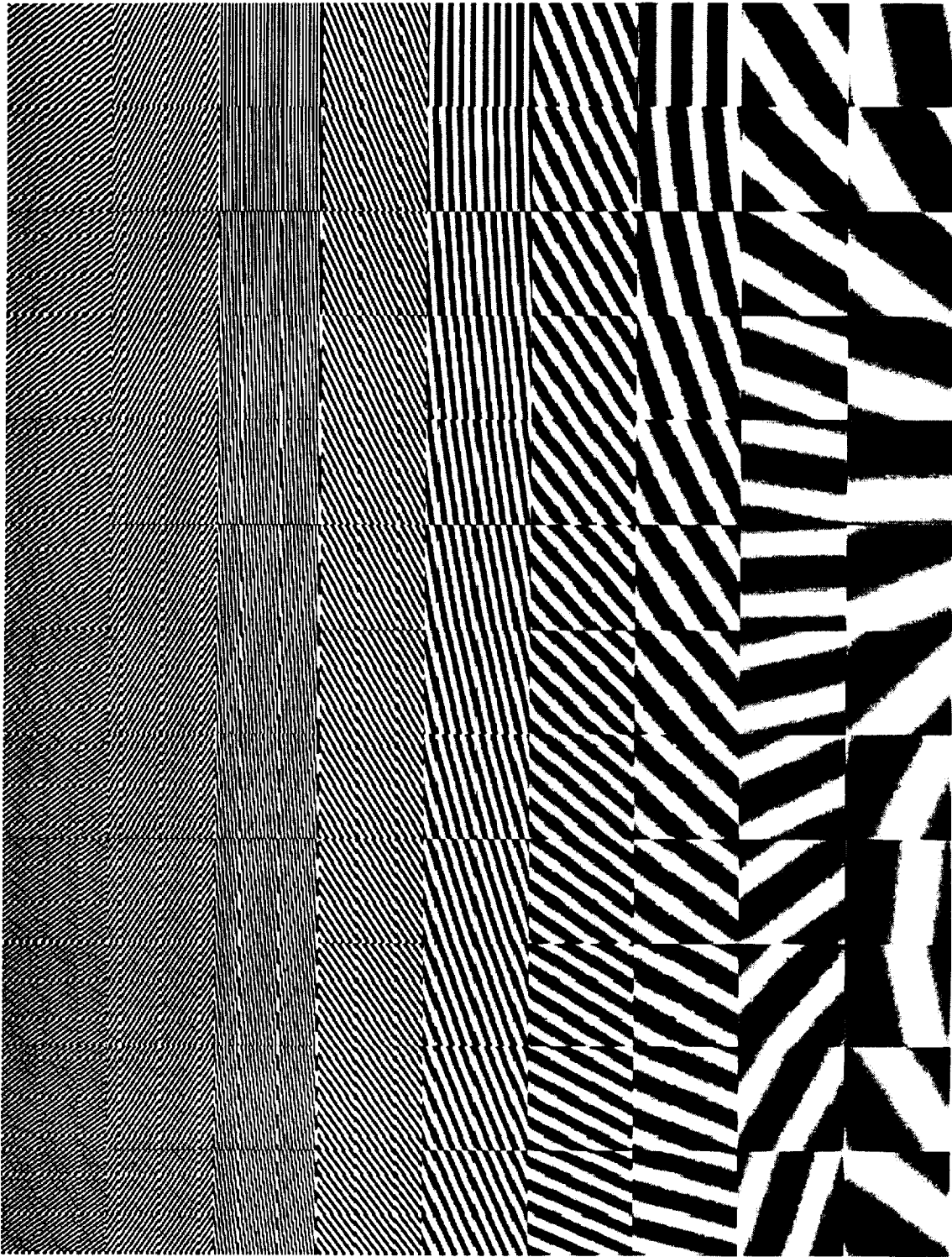
This figure shows a small subset of the modulation patterns for the HESSI Telescope. Each column of squares is a subset of the visibility pattern (Fourier pattern) for a single sub-collimator during one telescope rotation. The counts are binned for 0.5-40 millisec to provide sufficient statistics for one Fourier component. The finest patterns, which are binned for about 2.5 ms, have a spatial resolution of 2.3". Only the first 12 of 1620 are shown. The coarser collimators are binned for progressively longer

intervals in proportion to their spatial resolution (2.3, 4.0, 6.9, 12.0, 20.7, 35.9, 62.1, 108, and 186 arc sec, resectively). The number of modulation patterns depend on the degree of binning chosen, but in this case, if all were shown, the number of materns in each column would be 1620, 935, 540, 312, 180, 104, 60, 35, and 20, respectively.

LOOK–UP TABLE ARRAY

F(S)

S

θ

MODULATION
PATTERN
ARRAY

(X1, Y1)

0
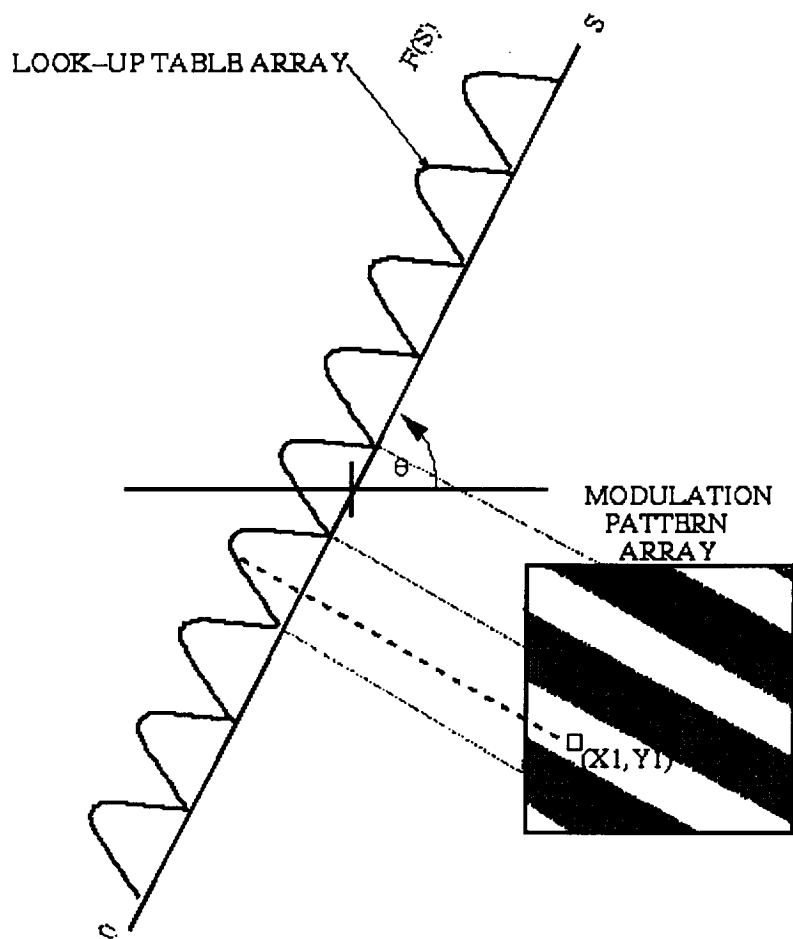
Each pixel of the modulation pattern array is created by reading a
value out of the lookup table F(s), which is obtained from the
collimator pre–flight measurements. F(s) is shown above as the
curve to the left of the thick line 0S, which lies at an angle   θ
measured CCW from the x axis.

General points in the array are taken by projecting
the perpendicular to the line 0S and reading F(S). This method
replaces a cosine by a lookup, which is an order–of–magnitude
faster.

Effect of shadowing on slit/pitch ratios

RADIAL OFFSET= 960.000
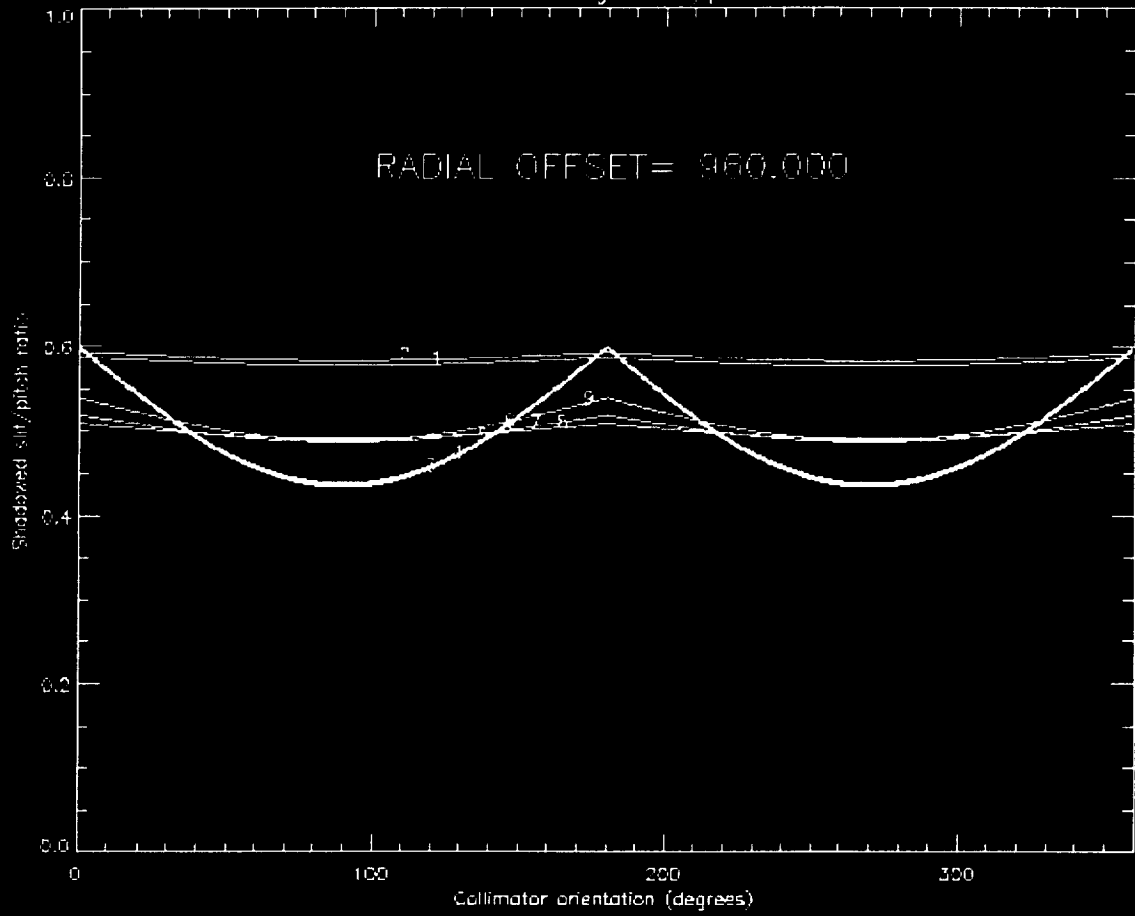
Infinite photon case (> 10000 events/coll/s)

- ● Maximum Entropy
  - O Poisson noise was added to the model maps before reconstruction.
  - O Gives nice results BUT...
  - O Has NOT been tested using a score with realistic statistics.
  - O x^2 statistic was not used in the earliest mapping (pre 1997)
  - O Later (1997) x^2 statistic was used. (example)
- ● Max Likelihood (Richardson-Lucy) --x^2 statistic was not used
  - O To get convergence, the number of data points and map pixels must be approximately equal.
  - O Not appropriate for low countrate regime.
  - O Nevertheless, non-binning algorithm shows promise.
- ● CLEAN
  - O x^2 statistic was not used
  - O But it can be incorporated to provide termination test.
- ● Analogy with radio astronomy methods is good, BUT has its limits:
  - O Radio techniques are not designed for Poisson-distributed photons.
  - O In those routines which use statistical tests, $\sigma^2 \neq$ count
- ● All working programs will be revised to incorporate event lists so that they can work with real HESSI data.

»

MAXIMUM LIKELIHOOD RECONSTRUCTION

Given a model count rate time profile $\{D_i\}$, and an observed count-rate profile $\{N_i\}$, we wish to determine the likelihood of the observed profile. (The N_i are all integers assumed to be Poisson distributed, and the D_i are not necessarily integers, but they are all positive.) From Poisson statistics, the probability of getting N counts when c are expected is:

$$\rho(N \mid D) = \frac{exp(-D)D^N}{N!}$$

and so the joint likelihood L of getting the observed counts N(i) in the time bin i, given the expected counts D(i) is given by:

$$\ln L = \sum_i N(i)\, \ln D(i) - D(i) - \ln N(i)!$$

The maximum likelihood map solution occurs when all partial derivatives with respect to the map pixels $\{O_i\}$ vanish:

$$\frac{\partial \ln L}{\partial O(j)} = \sum_i \frac{\partial \ln L}{\partial D(j)}\frac{\partial D(j)}{\partial O(j)} = 0$$

$$= \sum_i [\frac{N(i)}{D(i)} - 1]P(i,j)$$

The quantity P(i,j) is the modulation pattern, where the index j is the pixel number and the sum is over the time index i. The Richardson-Lucy iteration for the solution is:

$$O_{new}(j) = O(j)\sum_i P(i,j)\frac{D(i)}{N(i)} \bigg/ \sum_i P(i,j)$$

1

Finite photon case (< 1000 events/coll/s)

- Scores
  - O Simulations of the raw data structure--time, subcollimator, pulseheight
  - O Important for testing software
  - O And for running a "pulser stimulator" through the flight electronics.
  - O Very simple Monte-Carlo program
  - O Photon list algorithm diagram
  - O See hessidoc ftp site: hesperia.gsfc.nasa.gov/hessidoc/pro.
  - O Or WWW site: http://hesperia.gsfc.nasa.gov/~schmahl
  - O Binned Photon Lists
- Back projection--the basic imager
  - O Back projection: How it works
  - O Quick-look maps of flares
  - O Useful to localize flare center for subsequent reconstruction
  - O Can be used as "dirty maps" for "cleaning"
- Reconstruction
  - O Maximum Entropy, problems
  - O Maximum Likelyhood, possibilities
  - O In all methods, must define a count-rate *based goodness of fit*
- Confidence limits:
  - O Must define the appropriate statistic for count-rate profiles and maps.
  - O x^2 Statistic not x^2 distributed
  - O Webster Cash (Ap.J. 1979, *228*, 939) showed the way.
  - O Likelihood Function
    - ■ $L = \Pi \, e^{-C} C^N / N!$
    - ■ Recipe for confidence testing of a pixel:
      1. Run optimization/reconstruction using all map pixels as free parameters => $L = L_0$
      2. Run optimization/reconstruction holding a map pixels at some test level => $L = L_1$
      3. Then $\ln (L_1 / L_0)$ is x^2 -distributed.
    - ■ Details in SPD poster page

»

WHAT WE PLAN TO DO

Who is doing what?

- More complex scores (Schmahl)
  - Exponentially increasing source
  - Multiple, extended sources
  - Incorporate simulated grid errors
- MaxEnt, Max Likelihood, CLEAN (Schmahl)
- Forward Model (Hurford)
- PIXONS (McTiernan?)
- Fourier (Vilmer?)
- Self-Calibration (Aschwanden)
- Full-sun mapping (Kosugi?)
  - Needed for lowest energies
  - Big problems with over resolution for extended features
  - Internal shadowing varies across disk--How to incorporste it?
- Spatial-Spectral
  - HXT sometimes shows dips in bremsstrahlung spectra (impossible)
  - Points out the need for spectral/spatial reconstruction
  - Can use penalty function (ala' Gary 1996) or 3-D entropy