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Summary Report

on

**STRUCTURAL SIMILITUDE AND  
SCALING LAWS  
(RESEARCH GRANT NAG-1-1280)**

Submitted by

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## INTRODUCTION

Aircraft and spacecraft comprise the class of aerospace structures that require efficiency and wisdom in design, sophistication and accuracy in analysis and numerous and careful experimental evaluations of components and prototype, in order to achieve the necessary system reliability, performance and safety.

Preliminary and/or concept design entails the assemblage of system mission requirements, system expected performance and identification of components and their connections as well as of manufacturing and system assembly techniques. This is accomplished through experience based on previous similar designs, and through the possible use of models to simulate the entire system characteristics.

Detail design is heavily dependent on information and concepts derived from the previous steps. This information identifies critical design areas which need sophisticated analyses, and design and redesign procedures to achieve the expected component performance. This step may require several independent analysis models, which, in many instances, require component testing.

The last step in the design process, before going to production, is the verification of the design. This step necessitates the production of large components and prototypes in order to test component and system analytical predictions and verify strength and performance requirements under the worst loading conditions that the system is expected to encounter in service.

Clearly then, full-scale testing is in many cases necessary and always very expensive. In the aircraft industry, in addition to full-scale tests, certification and safety necessitate large component static and dynamic testing. Such tests are extremely difficult, time consuming and definitely absolutely necessary. Clearly, one should not expect that prototype testing will be totally eliminated in the aircraft industry. It is hoped, though, that we can reduce full-scale testing to a minimum.

Full-scale large component testing is necessary in other industries as well. Ship building, automobile and railway car construction all rely heavily on testing.

Regardless of the application, a scaled-down (by a large factor) model (scale model) which closely represents the structural behavior of the full-scale system (prototype) can prove to be an extremely beneficial tool. This possible development must be based on the existence of certain structural parameters that control the behavior of a structural system when acted upon by static and/or dynamic loads. If such structural parameters exist, a scaled-down replica can be built, which will duplicate the response of the full-scale system. The two systems are then said to be structurally similar. The term, then, that best describes this similarity is structural similitude.

Similarity of systems requires that the relevant system parameters be identical and these systems be governed by a unique set of characteristic equations. Thus, if a relation or equation of variables is written for a system, it is valid for all systems which are similar to it. Each variable in a model is proportional to the corresponding variable of the prototype. This ratio, which plays an essential role in predicting the relationship between the model and its prototype, is called the scale factor.

### **THE RESEARCH PROGRAM**

A research program was initiated which dealt with the implementation of structural similitude and scaling laws in designing scaled down models. Laminated beam plates and laminated plates and cylindrical shells were employed as a demonstration tool. Particular emphasis is placed on the cases of free vibration and buckling behavior of shells and panels. The results of this investigation are outlined in the following annual report:

1. Structural Similitude and Scaling Laws for Laminated Beam-Plates (March 1992)
2. Structural Similitude and Design of Scaled Down Laminated Models (December 1993)
3. Design of Scaled Down Models for Laminated Shells and Plates (March 1995)

4. Design of Scaled Down Models for Laminated Cylindrical Shells (March 1996)
5. Scaled Down Models for Laminated Shells and Size Effects (November 1997).

The Major accomplishments of the research project can be summarized as follows:

1. Similarity conditions (scaling laws) were derived for the buckling and vibration analysis of laminated plates and shells. In addition, similarity conditions were derived for transversely loaded beam-plates.
2. Complete and partial similarity were studied and geometric constraints were derived for complete similarity.
3. Partial similarity, through distorted models, was found to be more general and was used in the design of scaled down models.
4. Scaled down models were designed for a variety of geometries and load conditions.
5. The applicability of structural similitude was demonstrated analytically through the following steps:
  - (a) The behavior of the designed scaled down model was obtained through analysis (theoretical predictions).
  - (b) These data were assumed to be experimentally obtained.
  - (c) Through the use of the similarity conditions the data of step (b) were used to predict the behavior of the prototype.
  - (d) The predictions of step (c) were compared to the analytical results for the prototype. The comparison was excellent, which clearly supported the effectiveness of structural similitude.
  - (e) The effect of initial geometric imperfections was assessed, when dealing with buckling and postbuckling of cylindrical shells under axial compression.

In all of the above, it was assumed that the material behavior was linearly elastic and therefore no size effects were considered. This assumption was revisited towards the end of the program and studies dealing with strength size effects (scale effects) in laminated composite materials were initiated.

### **PUBLICATIONS AND PRESENTATIONS**

Several refereed journal papers resulted from these studies, in addition to the five reports listed in the previous section. Moreover, the principal investigator presented five seminars at various universities and government labs (Ohio State University, Georgia Institute of Technology, Technion-Israel Institute of Technology and at the Naval Surface Warfare Center-Carderock Division). Finally, ten papers were presented at national and international professional meetings. In all of these, the financial support provided by NASA through this Grant was gratefully acknowledged.

The publications and presentations are listed below:

#### (a) Publications

1. Simites, G.J. and Rezaeepazhand, J., "Structural Similitude for Laminated Structures", Composites Engineering, Vol. 3, Nos. 7-8, 1993, pp. 751-765.
2. Simites, G.J., Rezaeepazhand, J. and Starnes, J.H., Jr., "Use of Scaled-Down Models for Predicting Vibration Response of Laminated Plates", Composite Structures, Vol. 30, 1995, pp. 419-426.
3. Simites, G.J. and Rezaeepazhand, J., "Structural Similitude and Scaling Laws for Buckling of Cross-Ply Laminated Plates", J. Thermoplastic Composite Materials, Vol. 8, July 1995, pp. 240-251.
4. Rezaeepazhand, J., Simites, G.J. and Starnes, J.H., Jr., "Design of Scaled Down Models for Stability of Laminated Plates", AIAA J., Vol. 33, No. 3, 1995, pp. 515-519.

5. Simites, G.J. and Rezaepazhand, J., "Scale Models for Laminated Cylindrical Shells Subjected to Axial Compression", Composite Structures, Vol. 34, No. 4, 1996, pp. 371-379.
6. Rezaepazhand, J., Simites, G.J. and Starnes, J.H., Jr., "Design of Scaled Down Models for Predicting Shell Vibration Response", J. of Sound and Vibration, Vol. 195, No. 2, 1996, pp. 301-311.
7. Rezaepazhand, J. and Simites, G.J., "Structural Similitude for Vibration Response of Laminated Cylindrical Shells with Double Curvature", Composites, Part B: Engineering, Vol. 28B, No. 3, 1997, pp. 195-200.
8. Simites, G.J., Rezaepazhand, J. and Sierakowski, R.L., "Scaled Models for Laminated Cylindrical Shells Subjected to External Pressure", Mechanics of Composite Materials and Structures, Vol. 4, 1997, pp. 267-280; also Proceedings, of the seventh Japan-US Conference on Composite Materials, Kyoto, June 1995.
9. Tabiei, A., Sun, J. and Simites, G.J., "On the Scaling Laws of Cylindrical Shells Under Lateral Pressure", AIAA J., Vol. 35, No. 10, 1997, pp. 1669-1671.

#### **PRESENTATIONS**

1. "Structural Similitude and Scaling Laws for Laminated Beam-Plates", 1992 ASME Applied Mechanics Summer Meeting, Scottsdale, AZ, April 28-May 1, 1992.
2. "Structural Similitude and Scaling Laws for Cross-Ply Laminated Plates", 8th Technical Conference of the American Society for Composites at Ohio Aerospace Institute, Cleveland, OH, October 19-21, 1993.
3. "Design of Scaled Down Structural Models" at the Workshop on Scaling Effects in Composite Materials and Structures", sponsored by the UCSD Institute for Mechanics and Materials and NASA Langley Research Center, Hampton, VA, November 15-16, 1993.

4. "Design of Scaled Down Models for Stability and Vibration Studies", 35th AIAA/ASME.../ASC SDM Conference, Hilton Head, SC, April 18-20, 1994.
5. "Scaled Down Models for Stability Analysis of Laminated Cylindrical Shells", 9th Technical Conference of the American Society for Composites, at the University of Delaware, Newark, DE, September 20-22, 1994.
6. "Design of Scaled Down Models for Predicting Shell Vibrational Response", 36th AIAA/ASME.../ASC SDM Conference, New Orleans, LA, April 10-13, 1995.
7. "Structural Similitude for Vibration Response of Laminated Cylindrical Shells with Double Curvature", ASME Winter Annual Meeting, San Francisco, CA, November 12-17, 1995.
8. "Scaled Models for Laminated Cylindrical Shells Subjected to External Pressure", 7th Japan-US Conference on Composite Materials, Kyoto, Japan, June 19-22, 1995.
9. "Structural Similitude and Scaling Laws for Laminated Shells", 46th Int'l. Astronautical Congress, Helsinki, September 30-October 5, 1995.
10. "Scaled Down, Imperfection Sensitive Composite Cylindrical Shells Under Axial Compression and Lateral Pressure", 38th AIAA/ASME.../ASC, SDM Conference, Kissimee, FL, April 12-16, 1997.