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# FINAL REPORT

## Hydrodynamic Stability and Magnetic Reconnection in Disks and Stars
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During the period of this grant (12/01/94-1/31/99), the PI has studied nonmagnetic stability, waves, and tidal forcing in disks and stars. The CoI has studied mechanisms for fast magnetic reconnection.

## Parametric tidal instabilities in disks

An important theme of the PI's work under this grant has been applications of parametric instability.

The simplest example of parametric instability is a harmonic oscillator with a periodic modulation of the spring constant. If the modulation frequency is close to twice the natural frequency of the oscillator, the amplitude of oscillation tends to grow exponentially ([1]). The growth rate is proportional to the strength of the modulation, but it also depends upon the closeness to resonance of the two frequencies, and upon natural damping rate or "$Q$" of the oscillator. Parametric instabilities are very common in physics. A familiar example is a jogger's ponytail—normally a very strongly damped pendulum, it can be destabilized by the variation in effective gravity during the jogger's stride. Observation confirms that the period of the pendulum is half that of the jogger's vertical motion. In astrophysics, parametric instability may occur by external tidal forcing, or by interaction among eigenmodes. In the latter case, an energetic eigenmode may destabilize modes of half its frequency, provided some weak nonlinearity exists to couple them.

Under a previous Astrophysical Theory grant (NAGW-2419), the PI discovered a parametric instability of tidally forced disks such as the accretion disks in cataclysmic variables and Xray binaries [2]. The destabilized modes are tightly-wound, incompressible, three-dimensional waves analogous to g-modes and r-modes in stars. Later work has confirmed our analysis [4]. It was hoped that these modes might provide a source of turbulence and angular momentum transport in accretion disks. However, a follow-up investigation of this instability by local numerical simulations, although confirming the analytically estimated growth rates, found negligible angular momentum flux [3]. Other work, partly supported by the ATP, now strongly indicates that the transport mechanism in such disks is magnetohydrodynamic turbulence [6]. Nevertheless, the parametric mechanism may truncate the outer edges of disks in close binaries [2], and it may be important in disks of very low ionization such as protostellar disks, or even cataclysmic-variable disks in quiescence where the MHD mechanism may be ineffective [5].

All analyses up to 1996 were done in a local approximation where the orbital frequency, shear rate, and tidal field were treated as constants. The locally computed growth rate turns out to depend strongly on radius, and it was unclear how to average these local rates to obtain the correct global rate. This is a critical issue for accretion disks in close binaries, because the local growth rate is comparable to the orbital frequency towards the outer edge of the disk but decreases rapidly inwards. Paper #1 examined this issue in a simplified global model where the destabilizing terms vary with position. We found that the global growth rate is essentially equal to the maximum local rate, provided that the latter is smoothed over a radial range equal to the distance that the destabilized wave propagates at its group speed in one growth time. Thus, in an accretion disk, waves would grow rapidly in the outer parts but would propagate both inwards and outwards at a maximum group speed of order the disk thickness divided by the orbital period. It can be shown that these waves carry negative angular momentum. Thus the ingoing waves carry a positive angular momentum flux, but in a purely local approximation, this is exactly balanced by the negative angular momentum flux of the outgoing waves. In a disk, the ingoing waves would dominate at small radii because they would be excited farther out, and the net angular momentum flux would be outward. It is not clear, however, how far the ingoing waves can propagate before damping through nonlinear interactions. Unfortunately, because of the short wavelength and three dimensional nature of these waves, global nonlinear numerical simulations will probably remain out of reach for several years to come.

## Tidal dissipation in stars

The discovery of parametric tidal instabilities in disks naturally led us to ask whether a similar mechanism might not operate within the interiors of detached close binary stars. The strongest tidal forces occur in contact binaries. But these systems are expected to have circular orbits and synchronous spins; the tide is steady in a corotating frame and therefore cannot be resonant with any stellar mode of finite frequency.

Tidal capture, whereby a near-grazing encounter between two single stars results in a bound system [7], makes extremely eccentric and nonsynchronous binaries. Tidal capture between neutron stars and main-sequence stars, probably in the course of three-star interactions, may account for the superabundance of low-mass Xray binaries (LMXBs) in globular clusters [10]. By the time they are seen as LMXBs, the tidal-capture systems have circularized and synchronized, and it is important to understand how this comes about. The initial encounter, at a pericentral separation less than 3 stellar radii, tidally excites upwards of $10^{44}$ ergs in a few global oscillation modes of the nondegenerate star. If this energy is not rapidly dissipated, it may return to the orbit and unbind the system [8, 9], or else more distant encounters may torque the orbit and increase the pericenter. (The initial orbital period is hundreds or thousands of years.) But if the dissipation is too rapid and occurs mainly in the outer parts of the star, then the star will swell up, resulting in a physical collision at a subsequent pericentral passage [11, 12].

Paper #2 examined parametric instability in the context of tidal capture. Initially, most of the energy is deposited into a few large-scale quadrupolar modes of oscillation, because only these couple directly to the tidal potential. But these modes are weakly nonlinear, and the three-mode coupling between a large-scale mode and parametrically resonant pairs of small-scale modes is significant. By explicitly evaluating these couplings, we estimated that the large-scale modes should damp in $\sim 4E_{44}^{-1/2}$ days by transfer of energy to large numbers of short-wavelength, high-order g-modes. (Note that this process is negligible for the five-minute oscillations of the Sun, where the average energy per mode is only $\sim 10^{28}$ erg.) This is much shorter than the initial binary orbital period and the conventional linear damping time. Furthermore, although the dissipation rate of the large-scale modes exceeds the luminosity of the star by orders of magnitude, the star may not swell up, because the high-order g-modes reside in the radiative core, whose total thermal energy is large ($\sim 10^{48}$ erg) compared to the tidal increment. Thus, parametric instability is probably an important component of the tidal-capture mechanism.

There is a long-standing problem with the circularization of normal binaries (*i.e.*, not formed by tidal capture or other exotic processes) in which both components lie on the main-sequence. Since the transition between circularized and eccentric orbits appears to occur at longer binary periods (larger stellar separations) for older binaries, circularization probably occurs by gradual dissipation of the nonstationary tide during the main-sequence lifetime [13]. The standard mechanism for dissipation is turbulent viscosity in stellar convection zones [14]. Paper #3 re-examined this mechanism for binaries consisting of two solar-type stars. Our best estimate is that turbulent viscosity does not circularize such binaries during their main-sequence lifetime beyond a period of about 3 days; with very conservative assumptions, this could be extended to 6 days. The difference between our "best" and "conservative" assumptions is the degree to which turbulent dissipation is suppressed when the tidal period is shorter than the typical turnover time for convective eddies. Calculations in Paper #3 indicate strong suppression, supporting earlier claims [15].

Observationally, however, Population II main-sequence binaries circularize out to periods of at least 11 days, and possibly as long as 19 days (see [16] and Paper #3). Because the time required for circularization is predicted to be a strong function of binary period ($\propto P^{16/3}$), this is a serious conflict between observation and theory, and it suggests that the dominant tidal dissipation mechanism has not yet been identified. This has implications for other types of close binaries, for example extrasolar planets [17].

Paper #4 explores an alternative circularization process: excitation of g-modes at the interface between the radiative core and the convection zone. This mechanism is well known but had been applied only to early-type stars ([18]). We found that the g-mode process is competitive with turbulent convection for solar-type binaries. Apparently this is a coincidence, since the two mechanisms scale differently with orbital period. There are some subtleties involving the damping of these high-order g-modes within the core (nonlinearities may be important) and the evolution of the g-mode eigenfrequencies during the main sequence lifetime. Despite these theoretical uncertainties, an upper bound can be placed on the circularization rate by this mechanism. Unfortunately, even in combination, turbulent convective viscosity and g-mode excitation are unable to explain the observations cited above unless circularization occurs prior to the main sequence. (The latter possibility cannot yet be ruled out, but the observational evidence is against it.) An independent group has recently reached similar conclusions [19].

The PI and Princeton graduate student Michael Blanton spent considerable effort studying whether parametric instabilities could be responsible for circularization. In order to predict the secular effect of these (or any other) instabilities, it is necessary to understand how they saturate. This is an intrinsically nonlinear question, and we attempted to address it by three-dimensional numerical simulations using a fourier-based spectral code. We found that sufficiently strong tides can indeed give rise to a statistically steady, weakly nonlinear cascade of g modes. We have estimated, however, that the linear parametric instability will be suppressed by radiative diffusion at tidal periods longer than about three days. This work has not been published.

## Self-gravitating disks.

During calendar 1998, the PI returned to problems involving the stability of gaseous disks, this time with self-gravity.

Paper #5 studies nonlinear spiral density waves in weakly selfgravitating disks where the potential is dominated by a central mass. There are two natural astrophysical contexts: protostellar disks, and molecular disks around supermassive black holes in galactic nuclei. Because the potential is nearly keplerian, the single-armed spirals have very special dynamics and may be able to propagate over large radial distances with much less dissipation than spirals of any other multiplicity. By variational methods, we derive nonlinear versions of the dispersion relation, angular momentum flux, and propagation velocity in the tight-winding limit. The pitch angle increases with amplitude until the tight-winding approximation breaks down. By other methods, we find a series of nonlinear logarithmic spirals which is exact in the limit of small disk mass and which extends to large pitch angle. This paper has been submitted to MNRAS and has received a favorable referee's report.

Paper #6 is a rather formal study of the linear stability of disks with power-law surface density profiles. Although highly idealized, such disks can be useful "test beds" for theoretical studies, and one would think that their stability properties should be easily characterized. Since power-law disks have no preferred length or time scale, however, there has been confusion in the literature whether such disks have a continuum of unstable linear modes or perhaps no unstable modes at all. Paper #6 resolves this paradox by analysing the particular case of a gaseous, isentropic disk with a completely flat rotation curve (the Mestel disk) exactly. It turns out that the linear stability problem is ill-posed. Instabilities exist, but their pattern speeds depend upon an undetermined phase with which waves are reflected from the origin. For any definite choice of this phase, there is an infinite but discrete set of growing modes. The complex eigenfrequencies fall at the intersections of a logarithmic spiral with a radial ray in the complex frequency plane. The pitch angle of the spiral and the position of the ray are independent of the central phase; they depend upon the ratio of sound speed to circular velocity in the disk, the degree of selfgravity, and the multiplicity of the arms. But the orientation of the spiral, and hence the locations of the eigenfrequencies along the fixed ray, vary with the choice of central phase. Exact, closed-form results are obtained for non self-gravitating normal modes and is shown to agree with approximate results obtained from the shearing sheet in the short-wavelength limit. *This provides the first exact, analytically solved stability analysis for a differentially rotating disk.* For self-gravitating normal modes, numerical results are obtained by solving recurrence relations in Mellin-transform space. This paper has been submitted to MNRAS and received a favorable referees report.

Paper# 7, a more phenomenological paper, attempts to explain the young massive stars seen in the inner 0.1 parsec of the Galaxy as the result of gravitational instabilities in a compact gaseous disk. (Like Paper# 5, this will form part of Princeton graduate student Erick Lee's Ph.D. dissertation, completed under the PI's advice and supervision.) The evidence for a black hole in the Galactic Center with a mass $\approx 2.5 \times 10^6 M_\odot$ is now overwhelming [20, 21]. Because of the extremely strong tidal fields due to this black hole, conventional star formation seems difficult [22]. Paper# 7 demonstrates, however, that formation from a disk is possible in this environment and naturally explains some of the salient observed characteristics of this young stellar population: in particular, its inferred total mass and its retrograde rotation. This paper has gone through several drafts and will probably be submitted to MNRAS within a week or two of this writing.

## Magnetic reconnection

A systematic investigation of magnetic reconnection was pursued under this grant by the CoI and his collaborators.

In Paper #8, magnetic reconnection was treated by a combination of global and local techniques. The global approach showed that the geometry of the magnetic reconnection layer and the separatrix layer separating the reconnected and unreconnected regions is determined globally independent of the physics in the reconnection layer. It was also shown that in the limit of infinite magnetic Reynold's number the plasma flows from the reconnection layer into the separatrix layer through a Y structure rather than through an X structure [Paper #11].

In addition, the global solution also determined the proper boundary conditions to apply to the reconnection layer. an asymptotic boundary layer analysis of the reconnection layer, [#13,#15] led to a unique reconnection rate. If one attempts a Petschek structure for a faster rate inside the layer one finds that this immediately reverts back to the unique structure which is close to that proposed by Sweet and Parker. The reason for the failure of the Petschek model (under the assumption of constant resistivity MHD was uncovered [#12,#15]. It was further found that inside the reconnection layer itself the osculating solution proposed by Cowling and Priest was not unique and generally does not occur [#14].

In the the magnetic reconnection experiment carried out at the Princeton Plasma Physics Laboratory, the MRX the plasma resistivity was measured and was found to be much larger than the predicted Spitzer resistivity [#11]. This is believed to be due to a lower hybrid instability i. e. and ion acoustic instability above the ion cyclotron frequency. The thickness of the reconnection layer was measured and found to be consistent with the marginal condition for this instability. The electron ion drift velocity was found to always be about a factor three times the ion acoustic speed the value for onset of this instability. If one makes the same assumption. that the reconnection layer thickness is such that one is at the marginal limit in solar flares and applies the Sweet Parker model, then one obtains a reconnection time that is nearly in the range observed in solar flares [#12]. The possibility also exists that such an anomalous resistivity would be space dependent which would reestablish the even faster Petschek mechanism and close the gap between solar observations and theory [#15].

An analysis of the reconnection experiment applying the larger measured anomalous resistivity shows that the experimental rate is consistent with the Sweet Parker rate for a variety of experiments [#11].

## Use of funds

Most of the expenditures have supported the tuition and stipend of graduate students. Students who collaborated with the PI and were supported from this grant Siang Peng Oh, Arielle Phillips, Serge Dobrovolsky, Leonid Malyshkin, and Bart Pindor (the latter two are continuing their collaboration with the PI with other funds). Students who collaborated with the CoI include Victoria Dorman and Alexander Schekochihin.

Funds were also used in partial support of summer salary for the PI and CoI, travel to a small number of conferences, computer support, publication costs, and minor expenses such as photocopying and mailing.

## Publications supported by this grant

#1 Ryu, D., Goodman, J., & Vishniac, E. T. 1996, ApJ 461, 805.
"Global aspects of elliptical instability in tidally distorted accretion disks."

#2 Kumar, P. & Goodman, J. 1996, ApJ, 466, 946.
"Nonlinear damping of oscillations in tidal-capture binaries."

#3 Goodman, J. & Oh, S. P. 1997, ApJ, 486, 403.
"Fast Tides in slow stars: the efficiency of eddy viscosity."

#4 Goodman, J., & Dickson, E. S., 1998, ApJ, 507, 938.
"Dynamical Tide in Solar-Type Binaries."

#5 Lee, E. & Goodman, J. 1999, MNRAS, submitted.
"Nonlinear Single-armed Spiral Density Waves in Nearly Keplerian Disks."

#6 Goodman, J. & Evans, N. W. 1999, MNRAS, submitted.
"The paradox of the scale-free disks."

#7 Lee, Erick 1999, in preparation. "A Star Formation Scenario for the Central One-Tenth Parsec of the Milky Way"

#8 Uzdensky, D., Kulsrud, R., & Yamada, M. 1996, Physics of Plasmas, 3, 1220.
"Theoretical Analysis of Driven Magnetic Reconnection in a Laboratory Plasma"

#9 Yamada, M., Ji, H., Hsu, S., Carter, T., Kulsrud, R., Bretz, N., Jobes, F., Ono, Y., & Perkins, F. 1997, Physics Of Plasmas, 4, 1936.
"Study of Driven Magnetic Reconnection in a Laboratory Plasma"

#10 Ji, H., Yamada, M., Hsu, S., & Kulsrud, R. 1998, Phys. Rev. Lett., 80, 3256.
" Experimental Test of the Sweet Parker Model of Magnetic Reconnection"

#11 Uzdensky, D. & Kulsrud, R. 1997, Physics Of Plasmas, 4, 3960.
"Cusp and Y-Type Magnetic Structures and Velocity Fields at the Endpoint of the Reconnection Layer"

#12 Kulsrud, R., 1998, Physics of Plasmas, 5, 1599.
Magnetic Reconnection in a Magnetohydrodynamic Plasma

#13 Uzdensky, D. M., 1998, PhD dissertation for the Department Astrophysical Sciences, Princeton University.
"Theoretical Study of Magnetic Reconnection"

#14 Uzdensky, D. & Kulsrud, R. 1998, Physics of Plasmas 5, 3249.
"Viscous Boundary Layer near the Center of the Resistive Reconnection Region"

#15 Uzdensky, D. & Kulsrud, R. 1999, submitted to Phys. Rev. Lett.
"2D Numerical Study of the Resistive Reconnection Layer"

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