Orbit Modification of Earth-Crossing Asteroids/Comets Using Rendezvous Spacecraft and Laser Ablation[7]

SANG-YOUNG PARK
Swales Aerospace

DANIEL D. MAZANEK
NASA Langley Research Center

Introduction

Fast space trips are important to intercept and rendezvous with an impacting asteroid or comet, particularly those not detected many years in advance. Fast trajectories can shorten space flight times and allow orbit modification efforts to begin earlier. The earlier the effort begins, the less change in velocity (ΔV) required to alter the object's trajectory. However, shorter trip times require more propellant to provide enough thrust if the travel distance is fixed. This additional propellant mass can be a burden to the structural architecture of spacecraft. Thus, it is necessary to trade off between flight time and propellant mass.

Background

This report describes the approach and results of an end-to-end simulation to deflect a long-period comet (LPC) by using a rapid rendezvous spacecraft and laser ablation system. The laser energy required for providing sufficient deflection ΔV and an analysis of possible intercept/rendezvous spacecraft trajectories are studied in this analysis. These problems minimize a weighted sum of the flight time and required propellant by using an advanced propulsion system. The optimal thrust-vector history and propellant mass to use are found in order to transfer a spacecraft from the Earth to a targeted celestial object. One goal of this analysis is to formulate an optimization problem for intercept/rendezvous spacecraft trajectories.

One approach to alter the trajectory of the object in a highly controlled manner is to use pulsed laser ablative propulsion (ref. 1). A sufficiently intense laser pulse ablates the surface of a near-Earth object (NEO) by causing plasma blowoff. The momentum change from a single laser pulse is very small. However, the cumulative effect is very effective because the laser can interact with the object over long periods of time. The laser ablation technique can overcome the mass penalties associated with other nondisruptive approaches because no propellant is required to generate the ΔV (the material of the celestial object is the propellant source). Additionally, laser ablation is effective against a wide range of surface materials and does not require any landing or physical attachment to the object. For diverting distant asteroids and comets, the power and optical requirements of a laser ablation system on or near the Earth may be too extreme to contemplate in the next few decades. A hybrid solution would be for a spacecraft to carry a laser as a payload to a particular celestial body. The spacecraft would require an advanced propulsion system capable of rapid rendezvous with the object and an extremely powerful electrical generator, which is likely needed for the propulsion system as well. The spacecraft would station-keep with the object at a "small" standoff distance while the laser ablation is performed.

[7]Chapter nomenclature available in chapter notes, p. 217.

Trajectory Optimization Algorithm

For intercept and rendezvous trajectories, optimization problems (ref. 2) in three dimensions are formulated to minimize flight time with moderate propellant mass. Many problems in the design of modern guidance and control systems require optimization of the trajectory, which minimizes (or maximizes) some performance criterion. Using the theory of the calculus of variations, the formulation of these problems yields a two-point boundary-value problem (TPBVP). The resulting optimal trajectory also satisfies the physical constraints and the given differential equations. The open-loop optimal trajectory can be used as a reference trajectory for intercept or rendezvous with Earth-crossing asteroids or comets. To simplify the presentation and focus more on the inequality constraint, we first present the necessary conditions for an optimal control problem without the inequality constraint and then discuss the inequality constraint separately. A general optimal control problem can be stated as follows:

Given the performance index, J, and radial velocity, *u*,

$$J(u) = \phi[x(t_f), t_f] + \int_{t_0}^{t_f} L(x, u, t)\, dt \qquad (1)$$

subject to the dynamic equations, $\dot{x}$, and boundary conditions

$$\dot{x} = f(x, u, t),\ x(t_0) \equiv x_0,\ t_0 \text{ given} \qquad (2)$$

and with free final time t_f, find the control history $u(t)$ to minimize J(u) with the prescribed terminal constraints

$$\Phi[x(t_f), t_f] = 0 \qquad (3)$$

Here $x(t) \in R^n$ are the state variables, $u(t) \in R^l$ are the control components, and $\Phi \in R^k$. The Hamiltonian function is defined with Lagrange multipliers $\Lambda(t) \in R^n$ as

$$H \equiv L + \Lambda^T f \qquad (4)$$

The performance index in equation (1) is augmented and rewritten as

$$J' = \phi[x(t_f), t_f] + \nu^T \Phi[x(t_f), t_f] + \int_{t_0}^{t_f} (H - \Lambda^T \dot{x})\, dt \qquad (5)$$

where ν is a constant multiplier vector of the dimension of the constraint Φ. The Minimum Principle requires that the optimal controls minimize the Hamiltonian function, H:

$$u^*(t) = \min_{u \in \Omega} \arg H(x^*, \Lambda^*, u, t) \qquad (6)$$

where Ω is the set of admissible controls and x^*, Λ^*, and u^* are the extremal states, costates, and controls, respectively. The initial time and the initial states are known. The conditions to be satisfied to minimize J' are found by taking the first variation of J' and setting relations equal to zero. From this, the states, costates, and the Hamiltonian function satisfy the following conditions:

$$\dot{x}^T = H_\Lambda \qquad (7a)$$

$$\dot{\Lambda}^T = -H_x \tag{7b}$$

$$\Lambda^T(t_f) = \left(\phi_x + \nu^T \Phi_x\right)_{t=t_f} \tag{7c}$$

$$H(t_f) = -\left(\phi_t + \nu^T \Phi_t\right)_{t=t_f} \tag{7d}$$

$$H_u = 0 \tag{7e}$$

Equation (7e) can be solved for the control, so that the control is removed from equations (7a) and (7b). There are n + k + 1 unknown values; n Λ's; k ν's; and the final time. These values can be solved by using the k terminal constraints, equation (3), the n equations, equation (7c), and equaton (7d).

The control inequality constraint is represented as

$$u_{min} \le u(t) \le u_{max} \qquad 0 \le t \le t_f \tag{8}$$

Control variable inequality constraint is augmented to the cost function and additional necessary conditions are obtained as a result (refs. 3 and 4). The optimal trajectory is composed of two types of control—nominal control $[u_{min} < u(t) < u_{max}]$ and boundary control $[u(t) = u_{min}$ or $u_{max}]$. The nominal control satisfies the same necessary conditions as the unconstrained problem. For boundary control, the inequality constraint becomes an equality constraint. Many classical problems in the calculus of variations treat constraints of this form very well. A new Hamiltonian function with control variable inequality constraint is redefined as

$$\tilde{H} = H + \mu_1(u - u_{max}) + \mu_2(u_{min} - u) \tag{9}$$

where

$$\left.\begin{aligned} \mu_1(u - u_{max}) = 0 \quad \mu_1 \ge 0 \\ \mu_2(u_{min} - u) = 0 \quad \mu_2 \ge 0 \end{aligned}\right\} \tag{10}$$

The necessary conditions and controls for active constraints are

$$\dot{\mathbf{x}}^T = \tilde{\mathbf{H}}_{\boldsymbol{\Lambda}} \tag{11}$$

$$\dot{\boldsymbol{\Lambda}} = -\tilde{\mathbf{H}}_{\mathbf{x}} \tag{12}$$

$$\tilde{H}_u = 0 \tag{13}$$

We use the control u(t) from the condition $H_u = 0$ when the control constraints are not active. For the problem with active inequality constraints, equations (11) and (12) are the equations of state and costate variables. For the Lagrange multiplier μ_i, i = 1,2 must necessarily satisfy that

$$\mu_i = 0 \quad \text{if the associated constraint is not active}$$

$$\mu_i \ge 0 \quad \text{if the associated constraint is active}$$

and μ_i can be obtained from solving $\tilde{H}_u = 0$ for μ_i. Hence, when u(t) < u_{min}, the control and the Lagrange multipliers are u(t) = u_{min}, $\mu_1 = 0$, $\mu_2 = H_u$. When u(t) > u_{max}, the control and the Lagrange multipliers are u(t) = u_{max}, $\mu_2 = 0$, $\mu_1 = -H_u$.

Many numerical algorithms to solve optimal control problems have been developed. Indirect methods are theoretically based on the Minimum Principle, which characterizes the set of optimal states and controls in terms of the solution of a boundary value problem. One indirect method is the shooting method, which yields solutions of high precision. The shooting method is a second order method and hence is very sensitive to small changes of costate initial conditions. Shooting methods have the associated difficulties caused by instability of the initial value problem for the system of differential equations and by the requirement for good initial guesses for the iterative solutions of nonlinear problems. In this analysis, a shooting method is used to solve the comets or asteroids intercept or rendezvous trajectory problems.

Problem Statement

Propulsion

Many future propulsion systems have been proposed and analyzed. One potential propulsion approach that has been examined for a Comet/Asteroid Protection System (CAPS) deflection capability is the Variable Specific Impulse Magnetoplasma Rocket (VASIMR). VASIMR is a high power magnetoplasma rocket that gives continuous and variable thrust at constant power (ref. 5). Hydrogen plasma is heated by radio frequency (RF) power to increase exhaust velocity up to 300 km/s. The power output of the engine is kept constant, thus thrust and specific impulse, I_{sp}, are inversely related. Thrust is increased proportional to the power level. The engine can optimize propellant usage and deliver a maximum payload in minimum time by varying thrust and I_{sp} (ref. 6). Therefore, VASIMR can yield the fastest possible trip time with a given amount of propellant by using constant power throttling (CPT). A10-kW space demonstrator experiment has been completed, and a VASIMR engine with 200-MW power could be available around the year 2050. The specific impulse range of the engine would be 3000 s to 30 000 s, and the corresponding thrust range would be approximately 13 600 N to 1360 N (assuming 100 percent power efficiency of 200 MW). To calculate acceleration, *a*, and spacecraft mass flow rate, the following relationships are used. The thrust, $T = |\dot{m}| v_e$, and exhaust velocity, $v_e = I_{sp} g_o$, are described by specific impulse, I_{sp}, and the acceleration due to gravity at the Earth's sea level, g_o. Mass flow rate, $\dot{m}$, is itself negative value.

The power, p, required to expel mass at the mass flow rate, $\dot{m}$, is $\varepsilon p = \frac{1}{2} |\dot{m}| v_e^2$. ε is the efficiency of the propulsion system. Thus, we know

$$\varepsilon p = \frac{1}{2} T v_e \Rightarrow T = \frac{2\varepsilon p}{v_e} \Rightarrow T = \frac{2\varepsilon p}{I_{sp} g_o} \tag{14}$$

Using equation (14), acceleration due to VASIMR can be derived:

$$a = \frac{T}{m} = \frac{2\varepsilon p}{m g_o} \frac{I}{I_{sp}} \tag{15}$$

where m is spacecraft mass at any time. Finally, the mass flow rate is calculated as

$$\dot{m} = \frac{-T}{v_e} = -\frac{2\varepsilon p}{g_o^2} \frac{1}{I_{sp}^2} \tag{16}$$

Equations of Motion

The spacecraft is considered to fly in three-dimensional interplanetary space. The three-degree-of-freedom equations of motion are the following:

$$\dot{r} = u \tag{17a}$$

$$\dot{u} = \frac{v^2}{r} + \frac{w^2}{r} + a \sin \alpha \cos \beta - \frac{1}{r^2} \tag{17b}$$

$$\dot{v} = -\frac{uv}{r} + \frac{vw \sin \phi}{r \cos \phi} + a \cos \alpha \cos \beta \tag{17c}$$

$$\dot{w} = -\frac{uw}{r} - \frac{v^2 \sin \phi}{r \cos \phi} + a \sin \beta \tag{17d}$$

$$\dot{\theta} = \frac{v}{r \cos \phi} \tag{17e}$$

$$\dot{\phi} = \frac{w}{r} \tag{17f}$$

$$\dot{m} = -\frac{2\varepsilon p}{g_o^2 I_{sp}^2} \tag{17g}$$

where r is the radial distance from the Sun to spacecraft, u is the radial velocity, v is the tangential velocity, w is the normal velocity, θ is the angle measured from the x-axis (defined as vernal equinox) in the x-y plane, ϕ is the angle measured from x-y plane, m is the mass of spacecraft, *a* is acceleration of spacecraft, p is the power of spacecraft, g is the gravitational parameter at Earth's sea level, and I_{sp} is the specific impulse of spacecraft engine. The control variables are thrust direction angle in plane (α), thrust direction angle of out-of plane (β), and the specific impulse (I_{sp}).

The Hamiltonian function is

$$H = w_t + \lambda_r u + \lambda_u \left(\frac{v^2}{r} + \frac{w^2}{r} + a \sin \alpha \cos \beta - \frac{1}{r^2} \right) + \lambda_v \left(-\frac{uv}{r} + \frac{vw \sin \phi}{r \cos \phi} + a \cos \alpha \cos \beta \right)$$
$$+ \lambda_w \left(-\frac{uw}{r} - \frac{v^2 \sin \phi}{r \cos \phi} + a \sin \beta \right) + \lambda_\theta \frac{v}{r \cos \phi} + \lambda_\phi \frac{w}{r} - \lambda_m \frac{2\varepsilon p}{g_o^2 I_{sp}^2} \tag{18}$$

The costate equations are

$$\dot{\lambda}_r = \lambda_u\left(\frac{v^2+w^2}{r^2}-\frac{2}{r^3}\right)+\lambda_v\left(\frac{vw\sin\phi}{r^2\cos\phi}-\frac{uv}{r^2}\right)+\lambda_w\left(-\frac{v^2\sin\phi}{r^2\cos\phi}-\frac{uv}{r^2}\right)+\lambda_\theta\frac{v}{r^2\cos\phi}+\lambda_\phi\frac{w}{r^2} \tag{19a}$$

$$\dot{\lambda}_u = -\lambda_r+\lambda_v\frac{v}{r}+\lambda_w\frac{w}{r} \tag{19b}$$

$$\dot{\lambda}_v = -\lambda_u\frac{2v}{r}+\lambda_v\left(\frac{u}{r}-\frac{w\sin\phi}{r\cos\phi}\right)+\lambda_w\frac{2v\sin\phi}{r\cos\phi}-\lambda_\theta\frac{1}{r\cos\phi} \tag{19c}$$

$$\dot{\lambda}_w = -\lambda_u\frac{2w}{r}-\lambda_v\frac{v\sin\phi}{r\cos\phi}+\lambda_w\frac{u}{r}-\lambda_\phi\frac{1}{r} \tag{19d}$$

$$\dot{\lambda}_\theta = 0 \tag{19e}$$

$$\dot{\lambda}_\phi = -\lambda_v\frac{vw}{r\cos^2\phi}+\lambda_w\frac{v^2}{r\cos^2\phi}-\lambda_\theta\frac{v\sin\phi}{r\cos^2\phi} \tag{19f}$$

$$\dot{\lambda}_m = \lambda_u\frac{2\varepsilon p}{m^2 g I_{sp}}\sin\alpha\cos\beta+\lambda_v\frac{2\varepsilon p}{m^2 g I_{sp}}\cos\alpha\cos\beta+\lambda_w\frac{2\varepsilon p}{m^2 g I_{sp}}\sin\beta \tag{19g}$$

Performance Index

Optimal control theory is concerned with finding the control history to optimize a measure of the performance index of the following general form:

$$J(u) = -m_f + \int_{t_0}^{t_f} w_t\,dt \tag{20}$$

where m_f represents the final mass of spacecraft, and w_t is weight for flight time (w_t is set as 10 in this analysis). For the problem at hand, it is required to find the optimal trajectory that maximizes the final mass of spacecraft and minimizes the flight time.

Initial Conditions and Terminal Conditions

The spacecraft departs from the Earth with the following initial conditions at t = 0 s:

$r(t_0)$ = 1 au, $u(t_0) = 0$, $v(t_0)$ = Earth's velocity, $w(t_0) = 0$,

$\theta(t_0)$ = obtained from spacecraft departure time before collision,

$\theta(t_0) = 0°$, $m(t_0)$ = free (unknown), t_f = free (unknown)

The initial mass of spacecraft is a free parameter to include propellant mass. Thus, initially the costate of mass is set as $\lambda_m(t_0) = -1$. The other costate values $[\lambda_r(t_0), \lambda_u(t_0), \lambda_v(t_0), \lambda_w(t_0), \lambda_\theta(t_0), \lambda_\phi(t_0)]$ at t_0 are unknown.

Final boundary conditions are specified to satisfy the position and velocity of asteroids/comets. The spacecraft must intercept or rendezvous with the targeted asteroid/comet with specified orbit. To achieve the desired trajectory, final conditions should be satisfied. These are the positions of spacecraft for intercept trajectory and the positions and velocities of spacecraft for rendezvous trajectory. Hence, for intercept trajectory, the terminal state conditions are

$$\Phi\left[x(t_f),t_f\right] \equiv \begin{Bmatrix} r(t_f) - r_{target}(t_f) \\ \theta(t_f) - \theta_{target}(t_f) \\ \phi(t_f) - \phi_{target}(t_f) \\ m(t_f) - m_{dry} \end{Bmatrix} = 0 \tag{21}$$

Rendezvous trajectory has the following terminal state conditions:

$$\Phi\left[x(t_f),t_f\right] \equiv \begin{Bmatrix} r(t_f) - r_{target}(t_f) \\ u(t_f) - u_{target}(t_f) \\ v(t_f) - v_{target}(t_f) \\ w(t_f) - w_{target}(t_f) \\ \theta(t_f) - \theta_{target}(t_f) \\ \phi(t_f) - \phi_{target}(t_f) \\ m(t_f) - m_{dry} \end{Bmatrix} = 0 \tag{22}$$

where subscript "target" denotes the state of targeted celestial object, and m_{dry} is the spacecraft dry mass. From transversality conditions, we obtain the following costate terminal conditions for intercept trajectory:

$$\lambda_r(t_f) = \nu_1 \tag{23a}$$

$$\lambda_u(t_f) = \lambda_v(t_f) = \lambda_w(t_f) = 0 \tag{23b}$$

$$\lambda_\theta(t_f) = \nu_2 \tag{23c}$$

$$\lambda_\phi(t_f) = \nu_3 \tag{23d}$$

$$\lambda_m(t_f) = \nu_4 \tag{23e}$$

where ν_i is the Lagrange multiplier. From transversality conditions, we obtain the following costate terminal conditions for rendezvous trajectory:

$$\lambda_r(t_f) = \nu_1 \tag{24a}$$

$$\lambda_u(t_f) = \nu_2 \tag{24b}$$

$$\lambda_v(t_f) = \nu_3 \tag{24c}$$

$$\lambda_w(t_f) = \nu_4 \tag{24d}$$

$$\lambda_\theta(t_f) = \nu_5 \tag{24e}$$

$$\lambda_\phi(t_f) = \nu_6 \tag{24f}$$

$$\lambda_m(t_f) = \nu_7 \tag{24g}$$

Furthermore, from equation (7d) the following condition is also satisfied at t_f:

$$H(t_f) = 0 \tag{25}$$

Equation (25) becomes another boundary equation at the final time. For the intercept problem, there are 14 differential equations describing the states and costates, with 15 unknowns [$m(t_0)$, $\lambda_r(t_0)$, $\lambda_u(t_0)$, $\lambda_v(t_0)$, $\lambda_w(t_0)$, $\lambda_\theta(t_0)$, $\lambda_\phi(t_0)$, t_f, ν_1, ν_2, ν_3, ν_4, $u(t_f)$, $v(t_f)$, and $w(t_f)$] and 15 boundary conditions [$r(t_0)$, $u(t_0)$, $v(t_0)$, $w(t_0)$, $\theta(t_0)$, $\phi(t_0)$, $\lambda_m(t_0)$, eqs. (21), $\lambda_u(t_f)$, $\lambda_v(t_f)$, $\lambda_w(t_f)$, and eq. (25)]. For the rendezvous problem, there are 14 differential equations describing the states and costates, with 15 unknowns [$m(t_0)$, $\lambda_r(t_0)$, $\lambda_u(t_0)$, $\lambda_v(t_0)$, $\lambda_w(t_0)$, $\lambda_\theta(t_0)$, $\lambda_\phi(t_0)$, t_f, ν_1, ν_2, ν_3, ν_4, ν_5, ν_6, and ν_7] and 15 boundary conditions [$r(t_0)$, $u(t_0)$, $v(t_0)$, $w(t_0)$, $\theta(t_0)$, $\phi(t_0)$, $\lambda_m(t_0)$, eqs. (22), and eq. (25)]. Thus, the two-point boundary problem can be completely solved with these boundary conditions.

Controls

The control variables are thrust direction angle in-plane (α), thrust direction angle of out-of-plane (β), and specific impulse (I_{sp}). A second-order necessary condition, the Legendre condition, states that the second derivative of the Hamiltonian, with respect to the controls, must be greater than or equal to zero for the performance index to be at a minimum. Thus, $H_{\alpha\alpha}$, $H_{\beta\beta}$ must be greater than or equal to zero for the performance index to be at a minimum. The first derivative of H with respect to α and convexity condition yields a control variable of α as follows:

$$\sin\alpha = \frac{-\lambda_u}{\sqrt{\lambda_u^2 + \lambda_v^2}} \tag{26a}$$

$$\cos\alpha = \frac{-\lambda_v}{\sqrt{\lambda_u^2 + \lambda_v^2}} \tag{26b}$$

The first derivative of H with respect to β and convexity condition yields a control variable of β as follows:

$$\sin\beta = \frac{-\lambda_w}{\sqrt{\lambda_u^2 + \lambda_v^2 + \lambda_w^2}} \tag{27a}$$

$$\cos\beta = \frac{\sqrt{\lambda_u^2 + \lambda_v^2}}{\sqrt{\lambda_u^2 + \lambda_v^2 + \lambda_w^2}} \tag{27b}$$

The first derivative of H with respect to I_{sp} and convexity condition yields a control variable of I_{sp} as follows:

$$I_{sp} = \frac{-2m\lambda_m}{g_o\sqrt{\lambda_u^2 + \lambda_v^2 + \lambda_w^2}} \tag{28}$$

$$I_{sp\,min} \leq I_{sp} \leq I_{sp\,max} \tag{29}$$

If the I_{sp} constraint is not active and $\lambda_m < 0$, the solution would be optimal. If the unconstraint control $I_{sp} < I_{sp\ min}$, then $I_{sp} = I_{sp\ min}$, whereas $I_{sp} = I_{sp\ max}$ if the unconstraint control $I_{sp} > I_{sp\ max}$. For $\lambda_m > 0$, the optimal solution is going to be $I_{sp} = I_{sp\ min}$.

Interplanetary optimal trajectories have been computed to maximize the final mass of the vehicle and to minimize the flight time. The optimal problem is a free final time problem to find the three controls satisfying the state and costate equations.

End-to-End Simulation

Here we consider a fictitious impacting LPC whose orbital parameters are given by aphelial distance r_a = 100 au and perihelion distance r_p = 0.7 au with inclination i = 50°. These orbital parameters yield semimajor axis a = 50.35 au, eccentricity e = 0.98609732, and orbital period of 357.27 years. It is assumed that the LPC has its density as ρ = 1000 kg/m^3. The minimum required impulses for deflecting an impactor by 3 Earth radii are solved in this analysis, and the calculation is always performed to move the LPC's trajectory from crossing the Earth's orbit at the Earth's center. The solutions represent impulse vectors that can be described by the magnitude of the minimum impulse and the optimal impulse angle. The gravitation effects of Earth are considered by using a three-dimensional optimization problem to calculate the impulse vectors. Figure 1 includes the minimum ΔV with respect to the impulse time that is defined as time before collision when ΔV is applied. Figure 2 includes the optimal impulse angle with respect to the impulse time. The impulse angle is described as being in an asteroid's/comet's orbital plane and is defined as the angle from the asteroid's/comet's original velocity vector to the impulse vector toward the Sun-asteroid/comet line. The dotted line in these figures explains the preperihelial collision case (a collision occurs before an asteroid/comet passes its perihelion), whereas the solid line in these figures explains the postperihelial collision case (a collision occurs after an asteroid/comet passes its perihelion). Figure 3 shows an estimate of the typical energy required for laser ablation to deflect the 1-km comet by 3 Earth radii, when preperihelial collision with Earth is considered. The required laser energy is also a function of the object's density and the required ΔV, which varies depending on the object's orbit and when the deflection occurs. It is easy to estimate the required laser energy for any size comet (or asteroid) because the minimum ΔV is linearly proportional to the cube of its diameter. Figure 4 explains a detail of figure 3 for an LPC, using fixed impulse times, and describes the required energy for laser ablation to deflect the given size of a comet by 3 Earth radii. The estimated energy is calculated assuming that the cumulative energy generated by the laser is applied as an equivalent impulsive ΔV at some time before collision. Because the laser ablation occurs over a significant period of time, the laser interaction must be complete prior to the time specified for each curve in order to assure that the deflection could be accomplished. Figure 5 shows a preliminary estimate of the achieved energy for a given laser power and operation period. Figures 4 and 5 can be used to estimate the nominal laser power required for a deflection mission. To illustrate a preperihelial case, figure 4 shows that approximately 5×10^4 GJ of energy is required to deflect a 0.2-km comet by 3 Earth radii if applied 1 year before

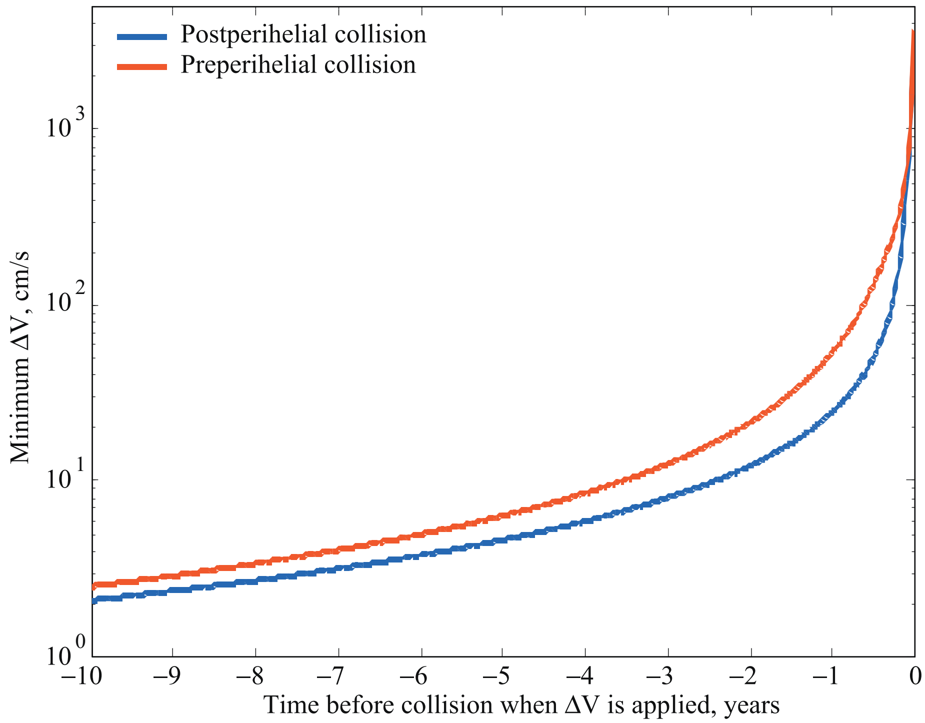

Figure 1. Minimum ΔV to deflect 1-km LPC by 3 Earth radii.

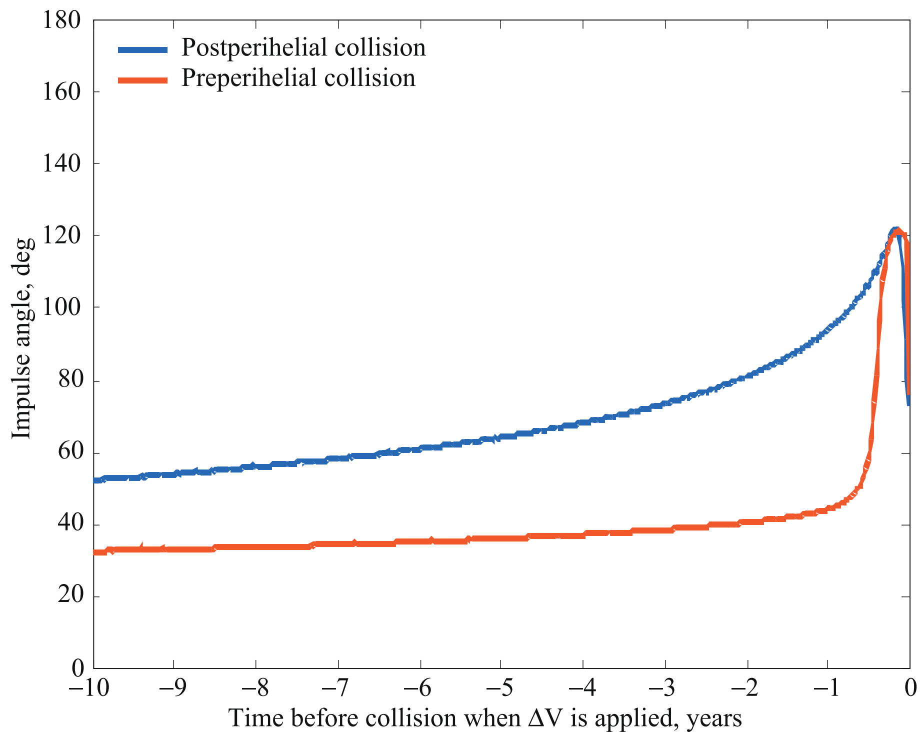

Figure 2. Optimal impulse angle with respect to impulse time.

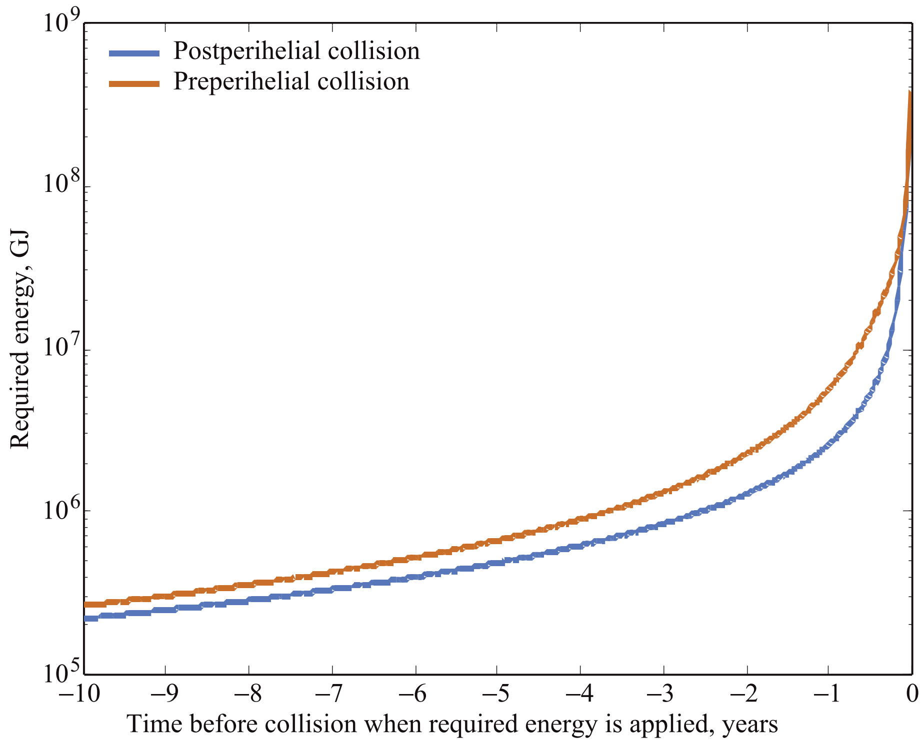

Figure 3. Required laser energy to deflect 1-km LPC by 3 Earth radii.

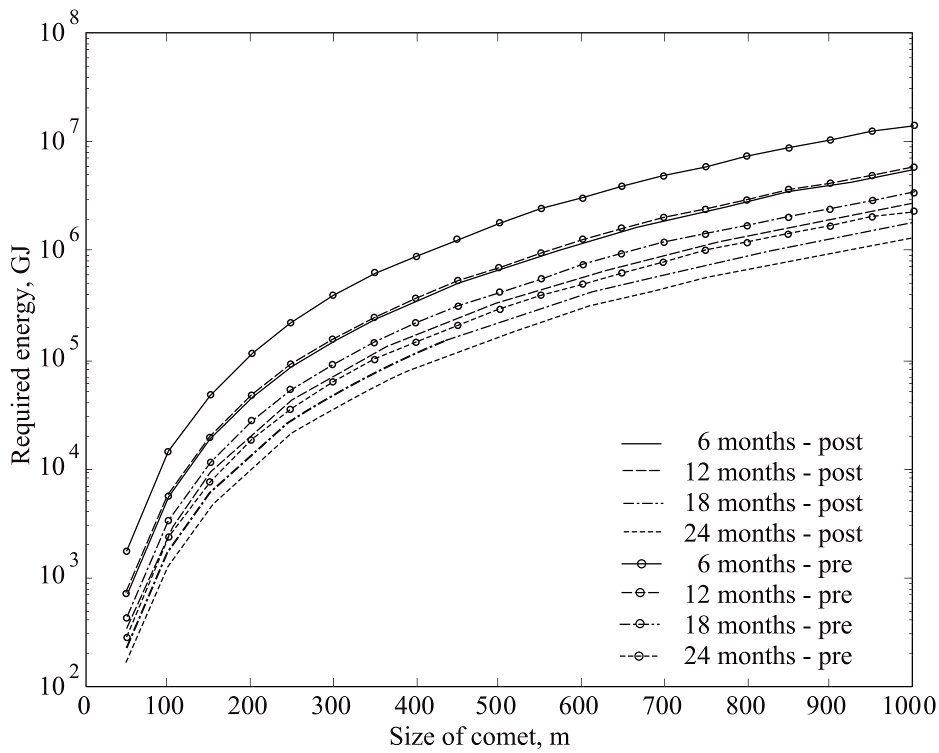

Figure 4. Estimated energy required for laser ablation versus diameter for LPCs (ρ = 1000 kg/m³).

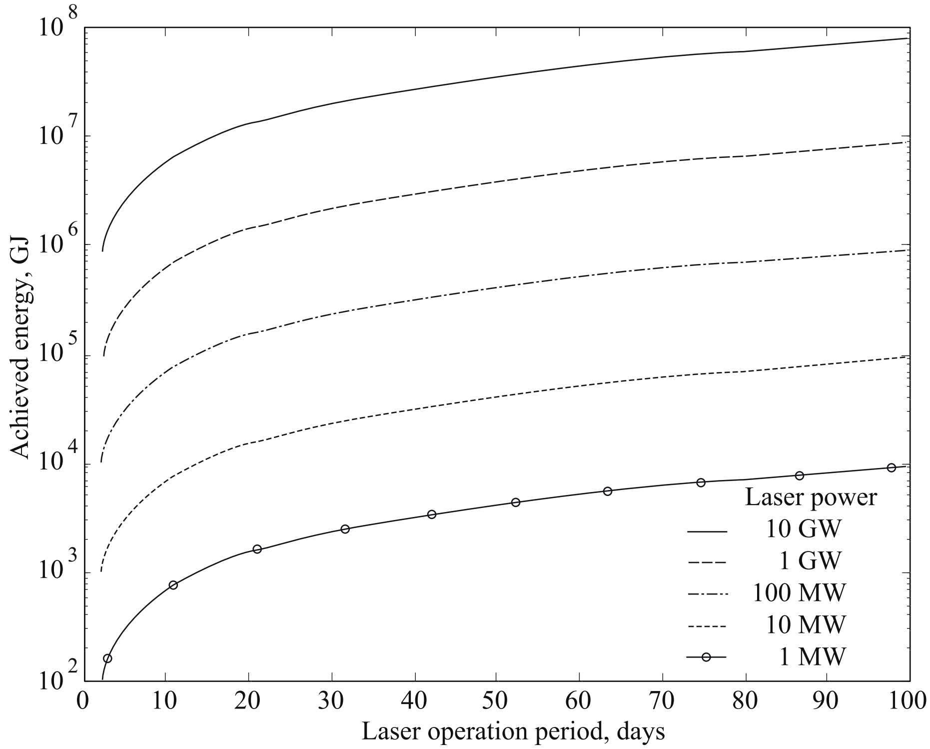

Figure 5. Estimate of achieved energy for given laser power and operation period.

collision. Figure 5 shows that a 100-MW laser (10 000-kJ pulse and 10-Hz laser repetition frequency) would be required to operate continuously for approximately 5 days in order to achieve this cumulative energy. Figure 6 illustrates the trajectories and thrust vector for a 100-metric ton (t) spacecraft (including 10-t payload) to intercept or rendezvous with this particular LPC for a departure time of 7 months before a preperihelial collision with Earth.

The propulsion system is assumed to operate at 90-percent power efficiency. Figures 7 and 8 show flight time and required propellant for intercept and rendezvous trajectories for various departure times when a 100-t spacecraft with 1 GW of power is assumed. For this specific LPC, there is a peak at 11 months departure time because the spacecraft must fly in the reverse direction with respect to Earth's orbital velocity. Because there are local minima and maxima in the propellant required and flight time, as shown in the figures, it can be concluded that the values are dependent upon the orbital geometry relationship as well as distance between the Earth and the comet. For a given departure time, the rendezvous trajectory requires more propellant and longer flight time than the intercept trajectory. This is because the terminal velocity of rendezvous spacecraft must be matched with the target's velocity, which is not required for the intercept trajectory. Asteroids and comets with different orbital elements will have different flight times and propellant requirements. Even for the same celestial object, a postperihelial impact would have different results from those of a preperihelial impact.

Once the spacecraft has rendezvoused with the LPC, a laser ablation system makes use of the same electrical power system that the propulsion system uses for the orbital transfer. For example, if the laser ablation operation can be completed 12 months before collision, approximately 3×10^6 GJ of energy would be required for a 0.8-km LPC to be deflected by 3 Earth radii. If we choose a 500-MW laser

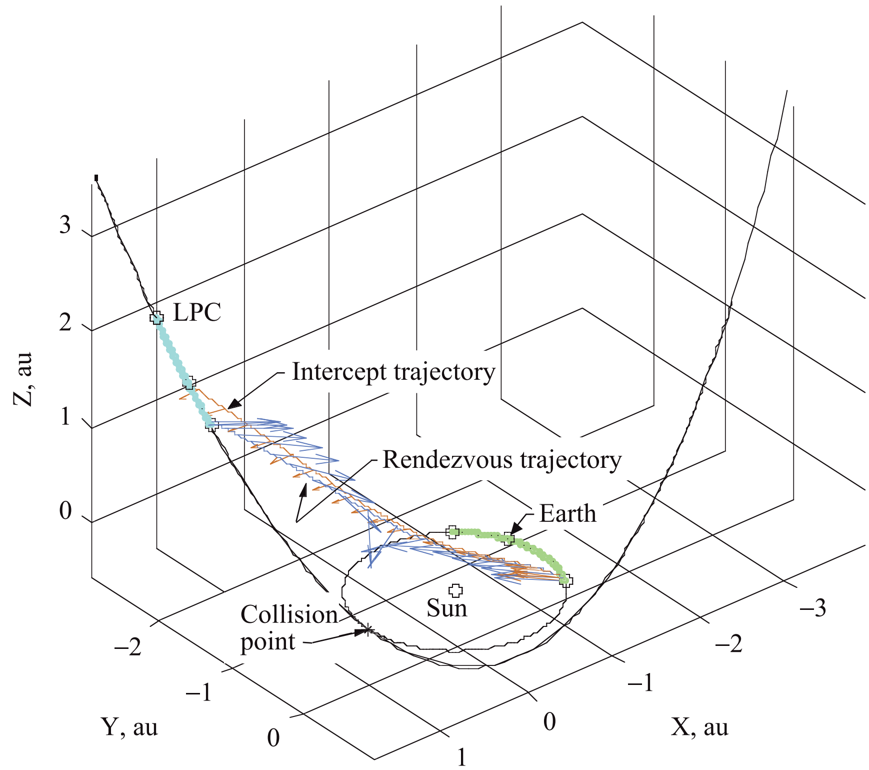

Figure 6. Example long-period comet intercept and rendezvous trajectories.

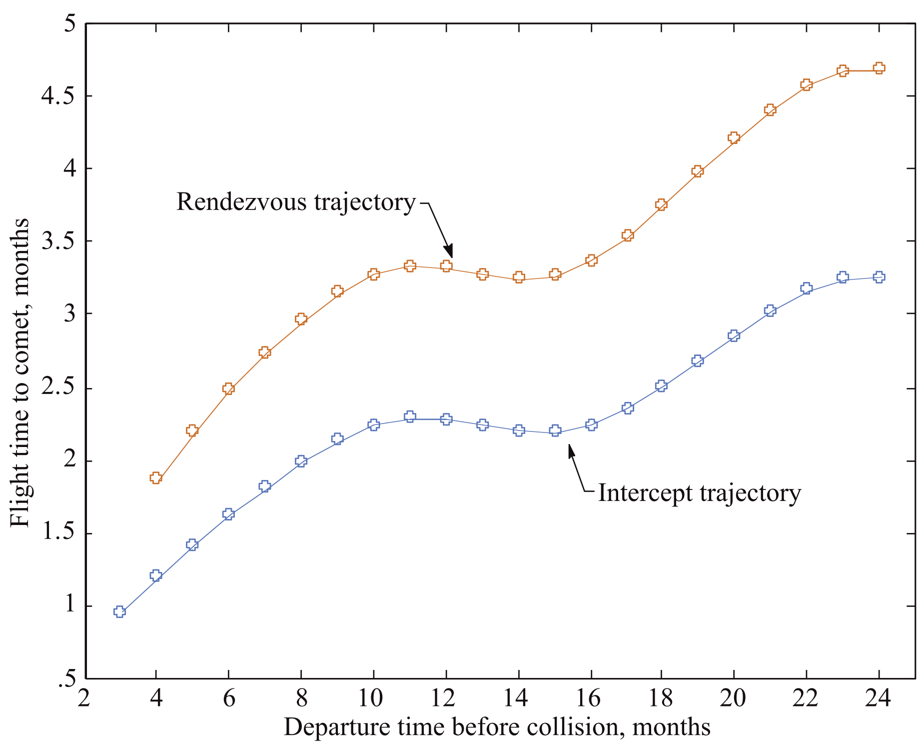

Figure 7. Flight time for each departure time.

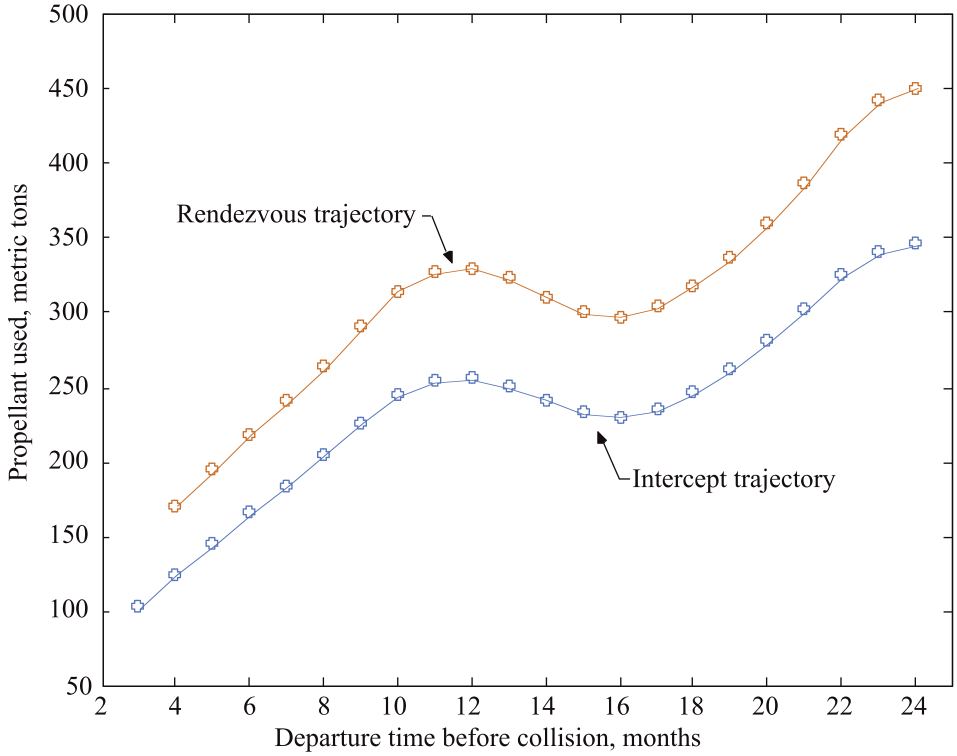

Figure 8. Propellant used for each departure time.

system (assuming 1 GW of supplied power and 50-percent laser system efficiency), it would take about 90 days of continuous operation to provide this amount of energy. From figure 7, the rendezvous spacecraft with a laser ablation system should depart from Earth approximately 19 months before the collision, with a trip time of about 4 months. For the mission, approximately 340 t of propellant would be required, assuming 100-t total dry mass of the spacecraft and payload. Depending on the payload mass capability of the propulsion system, multiple deflection devices could be delivered to the impactor, including a fallback option to the laser ablation system. In case the rendezvous deflection mission was unsuccessful, a similar spacecraft with a different payload (e.g., nuclear explosive device) could be sent to deflect the target using an intercept trajectory. If we assume that the intercept spacecraft departs from Earth 9 months before the collision with a 10-t payload, the spacecraft can arrive at the target approximately 7 months before impact and may require less than 230 t of propellant (as shown in figs. 7 and 8).

A multimegawatt nuclear electric propulsion (NEP) system using a VASIMR engine is currently estimated to have a maximum overall specific mass of 1.0 kg/kW (ref. 7). For a 1-GW system, this would result in a total spacecraft dry mass of 1000 mt (neglecting payload mass). This is ten times the total spacecraft mass assumed for this analysis, and the mass of a future laser ablation payload is presently not well understood. The assumed power efficiency of 90 percent is also optimistic. More capable power generators (gigawatt class) with lower specific masses could provide the power needed to reduce trip times and provide more powerful lasers. Deflecting an impactor by only 3 Earth radii would be sufficient for a deflection effort. More powerful lasers would be capable of providing a greater miss distance, and thus more margin for uncertainty in the object's orbit. NEOs with greater densities would also require a more capable laser ablation system. Longer warning times would reduce the requirements on the orbit modification system but would make the CAPS detection system more challenging to implement. For the 0.8-km LPC assumed in this analysis, the detection system would need to determine the comet's trajectory at least 19 months prior to its preperihelial collision with the Earth. For impactors with extremely short warning times, an intercept trajectory may be the only feasible scenario for diverting the object.

Technological advances that can significantly reduce the specific mass of the rendezvous spacecraft and laser payload may permit this type of deflection approach to become a reality. A tiered planetary defense approach using rapid rendezvous and intercept spacecraft could provide a feasible scenario to protect the Earth from an impacting LPC, as well as other classes of impacting NEOs. A rendezvous spacecraft with a laser ablation payload could also provide a capable and robust orbit modification approach for altering an NEO's orbit for resource utilization.

Concluding Remarks

This report presents intercept/rendezvous trajectories for an advanced spacecraft that is designed to deliver laser ablation energy to an Earth-crossing long-period comet. The trajectory optimization problem is solved using the shooting method, which yields highly accurate solutions. The open-loop optimal solutions can be used as reference spacecraft trajectory for the deflection problem. The end-to-end simulation in this report demonstrates a conceptual approach to altering the orbit of an Earth impacting long-period comet, particularly one which represents an immediate threat.

References

1. Phipps, C. R.: Lasers Can Play an Important Role in the Planetary Defense. *Proceedings of the Planetary Defense Workshop*, Lawrence Livermore National Laboratory, Livermore, CA, May 1995.
2. Bryson, A. E.: Dynamic Optimization, Addison-Wesley, Menlo Park, CA, 1999.
3. Maurer, H.; and Pesch, H. J.: Solution Differentiability for Parametric Nonlinear Control Problems With Control-State Constraints. *J. Optim. Theory & Appli.*, vol. 86, no. 2, 1995, pp. 285–309.
4. Kreim, H.; Kugelmann, B.; Pesch, H. J.; and Breitner, M. H.: Minimizing the Maximum Heating of a Re-Entering Space Shuttle: An Optimal Control Problem With Multiple Control Constraints. *Opt. Control Appli. & Methods*, vol. 17, 1996, pp.45–69.
5. Chang-Diaz, F. R., et al.: The Physics and Engineering of the VASIMR Engine. 36th AIAA/ASME/SAE/ASEE Joint Propulsion Conference, 17–19 July 2000, Huntsville, AL, AIAA 2000-3756.
6. Chang-Diaz, F. R.: The VASIMR Rocket. *Sci. American*, Nov. 2000, pp. 90–97.
7. Smith, B.; Knight, T.; and Anghaie, S.: Multimegawatt NEP With Vapor Core Reactor MHD. Space Technology and Applications International Forum (STAIF 2002), Albuquerque, NM, Feb. 3–6, 2002.