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*Sorenson Best Paper Award Recipient*  
**ANALYSIS OF SYSTEM-WIDE INVESTMENT IN  
THE NATIONAL AIRSPACE SYSTEM: A  
PORTFOLIO ANALYTICAL FRAMEWORK AND  
AN EXAMPLE**

**Dipasis Bhadra**  
**The MITRE Corporation**

**Frederick R. Morser**  
**The MITRE Corporation**  
**McLean, Virginia**

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**ABSTRACT**

In this paper, the authors review the FAA's current program investments and lay out a preliminary analytical framework to undertake projects that may address some of the noted deficiencies. By drawing upon the well developed theories from corporate finance, an analytical framework is offered that can be used for choosing FAA's investments taking into account risk, expected returns and inherent dependencies across NAS programs. The framework can be expanded into taking multiple assets and realistic values for parameters in drawing an efficient risk-return frontier for the entire FAA investment programs.

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**Dr. Dipasis Bhadra** is a Principal Economist with the MITRE Corporation's Center for Advanced Aviation System Development (CAASD), a federally-funded research and development corporation. At CAASD, Dipasis works in areas of quantitative modeling of air transportation system including passenger flows between airports, positioning of aircraft, configuration of networks, and program management via portfolio analysis. Prior to joining CAASD in early 2001, Dipasis worked at the World Bank evaluating feasibility of infrastructure projects in South Asia. With a Ph. D. in economics (1991) from the University of Connecticut, Dipasis also teaches introductory economics at the local community college, and holds professional positions at the Transportation Research Board (TRB), American Institute of Aeronautics and Astronautics (AIAA), and Transportation Research Forum (TRF).

**Frederick Morser** is an Outcome Leader at CAASD. Mr. Morser supports forecasting, business case analysis, strategic planning, and performance modeling work. He has been with MITRE for eight years.

## INTRODUCTION<sup>1</sup>

The United States' National Airspace System (NAS) contains a network of air transportation markets linking 485 commercial airports located in and around 363 metropolitan statistical areas. The total number of origin-destination markets in the NAS ranges somewhere between 36,000-40,000 pairs depending upon seasons and economic cycles. There are 315 air traffic control (ATC) facilities that are used to serve these markets meeting the daily travel needs of around 1.5 million passengers. Every day, roughly 40,000 scheduled commercial departures and 13,000 high end<sup>2</sup> general aviation (GA) departures fly in the same controlled airspace. Other GA traffic flying under visual flight rules — perhaps as many as 60,000 departures per day — use terminal facilities services at both commercial and non-commercial airports. In addition, there are military flights that also require terminal and enroute services. This expansive network renders an estimated annual commercial value of around US \$70-110 billion for scheduled GA services and around US \$25-40 billion for unscheduled GA services and an undetermined amount from other services including military (President's Aerospace Commission Report, 2003).

Maintaining this network is expensive. The Federal Aviation Administration (FAA) spends over US \$14 billion annually to fund facilities and equipment (F&E: approx. US \$3B), operations (approx. US \$7B), airports (approx. US \$3B), and research and engineering (approx. US \$0.2B) expenditures. The FAA's NAS modernization program, the impetus behind F&E funding, consists of three elements: the NAS Architecture Plan (i.e., the engineering blueprint),<sup>3</sup> the Capital Investment Plan (CIP),<sup>4</sup> and the

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<sup>1</sup> An earlier version of this paper was presented at the 5th Annual Aviation, Technology, Integration, and Operation of the American Institute of Aeronautics and Astronautics (ATIO/AIAA) conference at Crystal City, VA, Sept. 26-28, 2005. The authors would like to thank those who participated and for their suggestions. They would also like to thank their colleagues, Joseph Sinnott, E. J. Spear, Michele Steinbach, Felipe Moreno-Hines, Debby Pool, Dr. Katherine Harback and Dr. Bill Kuhn, whose comments and suggestions improved the focus of this research.

<sup>2</sup> Turbo fan and turbo prop aircraft flying under instrument flight rules.

<sup>3</sup> This is the comprehensive plan for modernizing the NAS. The plan covers information about architecture concepts, capabilities and plans for development in the future.

<sup>4</sup> The CIP is a 5-year plan that provides details on NAS projects that can be funded within the Office of Management and Budget's future year targets, presently set for 2006-2010. Through the CIP, the FAA fulfills public law obligations (PL 108-447) under which the Agency is "to transmit to the Congress a comprehensive capital investment plan for the Federal Aviation Administration which includes funding for each budget line item for fiscal years 2006 through 2010, with total funding for each year of the plan constrained to the funding targets for those years as estimated and

Operational Evolution Plan (OEP).<sup>5</sup> The FAA has six goals which are the primary focus of their CIP investing strategy:

1. Maintain a high level of safety;
2. Enhance greater mobility throughout the NAS;
3. Promote economic growth;
4. Promote harmony with human and natural environment;
5. Attain a high degree of national security; and
6. Maintain organizational excellence (FAA Flight Plans, 2006).

At present, there are 190 identified programs in the CIP, rolled up into 90 investment programs, designed to serve these broad 6 goals

Currently, CIP programs are evaluated on their individual merits where cost-benefit ratios, net present values, and internal rates of return reflect program effectiveness in meeting the stated FAA goals. While program cost estimates are relatively straight forward, benefits (total benefits) are often hard to quantify. Typically, a mixture of federal cost savings (e.g., higher productivity gains from investments in labor-saving technology) and external social benefits (e.g., better movement of aircraft at the congested airports thus reducing congestion), wherever applicable, are estimated to calculate the net present value of these investments. A combination of Treasury note interest rates (for federal government cost savings) and a real discount rate of 7% (for external social benefits) have been recommended by the Office of Management and Budget (OMB) to evaluate the associated project investments (OMB, 2005).<sup>6</sup> This evaluation criteria and process is fairly common throughout government programs. The A-94 also indicates that “benefit-cost or cost-effectiveness analyses should include comprehensive estimates of the expected benefits and costs to society.” Furthermore, “*possible interactions* between the benefits and costs being analyzed *and other government activities* should be considered.” [see OMB (2005) Section 6; emphasis added].

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approved by the Office of Management and Budget”. See <http://www.faa.gov/asd/> for more details on both the Architecture Plan and CIP.

<sup>5</sup> The OEP is the FAA's 10-year rolling plan to increase both the capacity and efficiency of the NAS while enhancing safety and security. For more details on OEP, see <http://www.gov/programs/oep/index.htm>

<sup>6</sup> The OMB publishes annual discount rates for calculating benefit-cost analysis of federal programs titled “Guidelines and Discount Rates for Benefit-Cost Analysis for Federal Programs”, or what is commonly known as OMB Circular A-94 (see [www.whitehouse.gov/omb](http://www.whitehouse.gov/omb) for more details).

The FAA's current investment analysis (IA) framework/process<sup>7</sup> determines program value and suitability by evaluating performance, lifecycle costs, benefits, program-specific risk, schedule, affordability and compatibility with the overall system architecture for a particular program. This approach is, however, somewhat limited when it comes to incorporating financial and other forms of programmatic interdependencies. The need to fill this gap, that is, the lack of apparent reconciliation between the requirements of system-wide architecture and that of financial requirements, has become even more urgent (GAO, 2005; FAA, 2005) and points to the direction of a "comprehensive strategy for modernizing the NAS (so that) ...major acquisitions are delivered within cost, schedule, and performance milestones" (FAA, 2005, p. 3)

This need leads one to seek alternative methodologies that tie investment programs with potential economies of scope, and benefit that arise from interdependencies among programs. The goal of the engineering architecture and its associated investments is to improve the flow of aircraft in a safe manner that eventually generates economic value in the system. Commercial aviation interests and fiduciary obligations required of the FAA call for system wide financial optimization built alongside the engineering architectural requirements.<sup>8</sup> Broadening the investment evaluation framework may also add new dimensions to understanding true values inherent in the NAS, efficient programs leading to modernization of the

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<sup>7</sup> This was primarily led, until recently, by the Office of System Architecture and Investment Analysis, commonly known as ASD-400. After the Air Traffic Organization [ATO; see <http://www.ato.faa.gov/> for more details] was formed, ASD transitioned into the ATO's Offices of Systems Engineering, Business, Planning and Development, and International (SE BP&D, and International). The Public Law 106-181 (AIR-21) that was passed in April 2000 authorized the FAA to create a Chief Operating Officer (COO) position who would be responsible for overseeing day-to-day traffic control operations, undertaking initiatives to modernize air traffic control (ATC) systems, increasing productivity and implementing cost-saving measures, among other things. In December 2000, the President issued the Executive Order 13180 that authorized the creation of the ATO, headed by the COO (GAO, 2005). The new office leads NAS architecture, system engineering, investment analysis and operations research. The ATO was created in February 2004 by combining FAA's Research and Acquisitions, Air Traffic Services, and Free Flight Offices into one performance-based organization.

<sup>8</sup> The SE BP&D and International of the ATO leads the effort for the investment analysis process and is responsible for formulating investment analysis teams (IATs). By evaluating alternative investment strategies from a broader perspective, these IATs are responsible for putting together investment analysis report and recommendations that are then presented to the Joint Resources Council for the final investment decision. For a selective list of these analyses, see <http://www.faa.gov/asd/ia-or/ia-reports.htm>.

NAS, and interdependencies across programs. Indeed, the ultimate outcome from applying this methodology is to invest in the optimum set of programs, which have embedded interdependencies that maximize return and minimize risk.

FAA investment selection criteria, as with most government investment, require special consideration due to the lack of market signals. In the business world, good investments differentiate themselves from bad investments through measures of return. Markets match consumers with products and services. Bad investments that fail to produce sufficient returns are weeded from the portfolio. FAA investment occurs outside of a market. There are no alternative air traffic service providers with a different portfolio of investments from which consumers can buy air traffic services thus providing market value signals. For this reason other means must be used to measure the value of investments.

In pursuit of a more comprehensive investment strategy we draw upon the literature of financial economics (Bodie et al., 1996; Ross et al., 1999) and offer a portfolio investment framework that is well specified to account for program interdependencies across cost, benefit and risk sharing and accommodate the surrogate market requirements. A Markowitz efficient frontier of risk and return has been built to facilitate the selection of sets of optimal programs within the scope of the FAA's programmatic engineering requirements. Using this framework and applying it to a set of hypothetical program costs, returns and interdependencies, we attempt to demonstrate that choices resulting from how portfolio analysis may provide useful information for optimizing a financial portfolio specified over risks and returns, as opposed to traditionally optimized individual programs.

The paper is organized along the following lines: the next section discusses the structure of the present FAA investments. The third section provides the analytical framework of an experimental approach laying out the empirical underpinnings. The fourth section provides an example of some hypothetical experiments and discusses implications on program implementation. The final section provides conclusions and recommendations for further research.

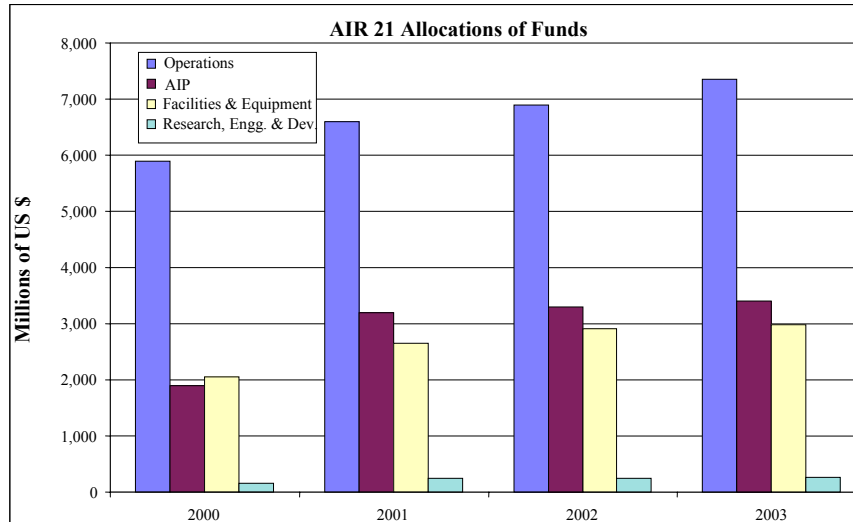
## **BACKGROUND**

The FAA's reauthorization plan, called the AIR-21, aligns the NAS architecture and the CIP with the OMB's five-year budget planning process. The majority of AIR-21 funding was earmarked to improve RADAR modernization and airport construction projects. Under the AIR-21, the total authorized funding for federal aviation programs, starting in 2000, was \$40 billion over three years. An estimated \$33 billion was guaranteed from the

Aviation Trust Fund, while the remaining \$6.7 billion was to be drawn from the General Fund.

Figure 1 provides a broad overview of the allocation of budgetary resources under AIR-21 during the period 2000-2003. Of the total 2003 annual expenditure of \$14 billion, operations consumed the largest share (52%) followed by airport improvement project (AIP) (25%) and F&E (21%). It is interesting to note that since 2001 expenditures for operations have experienced a relatively faster growth rate compared to all other broad expenditures including research, engineering and development (RE&D).

**Figure 1: Allocations of funds under AIR-21, 2000-2003**



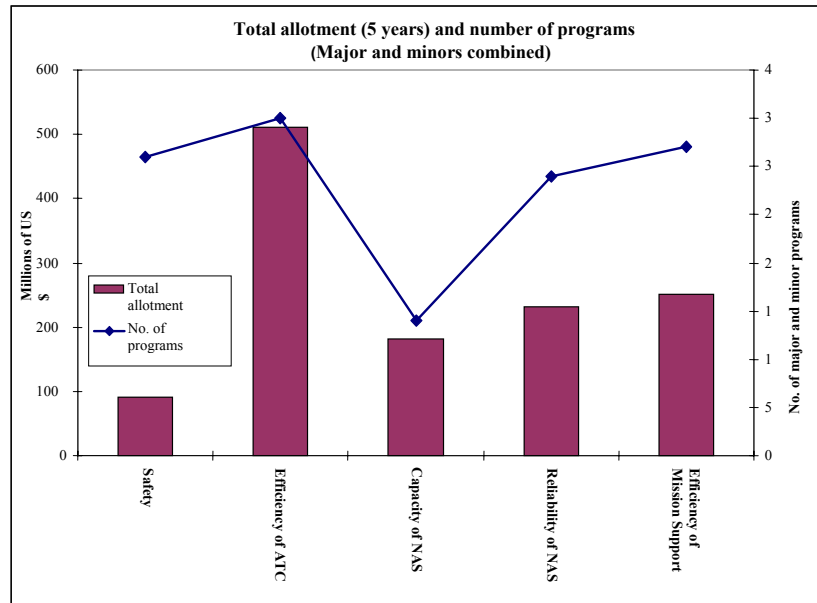
Source: Capital Investment Plan (CIP)/FAA (2004); See [www.faa.gov/asd/](http://www.faa.gov/asd/) for more details.

At present, there are 485 tower facilities (118 of them are towers with RADAR coverage), 185 terminal radar approach control (TRACON) facilities and 21 enroute centers within the continental US (20 in the contiguous US and 1 in Anchorage). In addition, there are five oceanic centers that handle incoming and outgoing traffic beyond the contiguous territories of the US. There are a little over 7,000 air traffic controllers directly associated with terminal facilities, while the rest, around 7,450 are assigned to enroute traffic, both TRACON and enroute.

The FAA uses standard performance measures to establish the suitability and effectiveness of its programs. Generally speaking, programs are designed to support five broad categories: safety, efficiency of ATC, capacity of the NAS, reliability of the NAS, and effectiveness of mission

support. In order to track these more systematically, the FAA collects data on aspects of these categories: average minutes late per flight, percent of flights on time, ground stop minutes, average daily arrival capacity, average daily flights, airport efficiency rate, airport capacity in visual meteorological conditions, airport departure rate, airport arrival rate, airport capacity in instrument meteorological conditions, and airport instrument meteorological conditions index, and other statistics relating to safety.

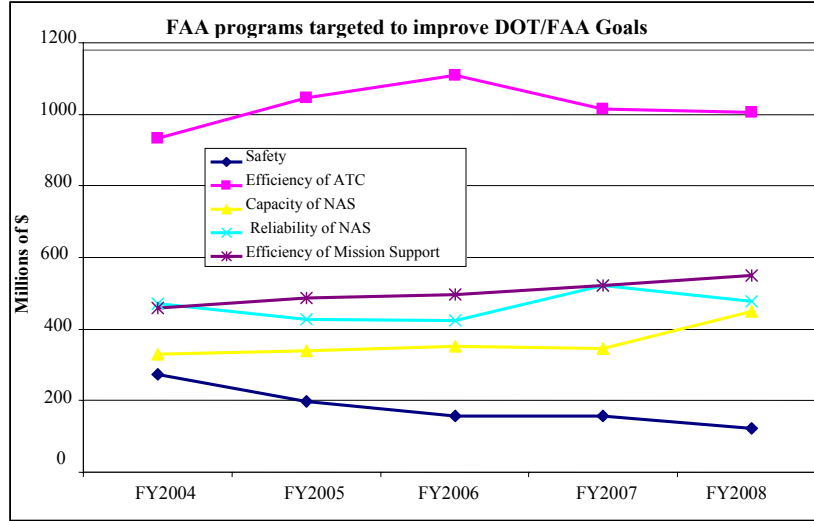
**Figure 2: Allotment of funds for FAA programs, FY 2004-2008**



Source: FAA (2005, May 26). "Status of FAA's Major Acquisitions: Cost Growth and Schedule Delays Continue to Stall Air Traffic Modernization," Report No. AV-2005-061. Washington DC: Office of the Inspector General, Federal Aviation Administration.

A look at the total expenses and program allocations over the five years from FY2004-2008 indicates that the FAA's portfolio of programs are distributed with a major focus on improving the operational efficiencies of the NAS. This focus in programmatic choice is reflected by the FAA's expenditures over time as well (see Figures 2 and 3).

Figure 3: FAA program investments over time, FY2004-2008



Source: CIP/FAA (2004); see [www.faa.gov/asd/](http://www.faa.gov/asd/) for more details.

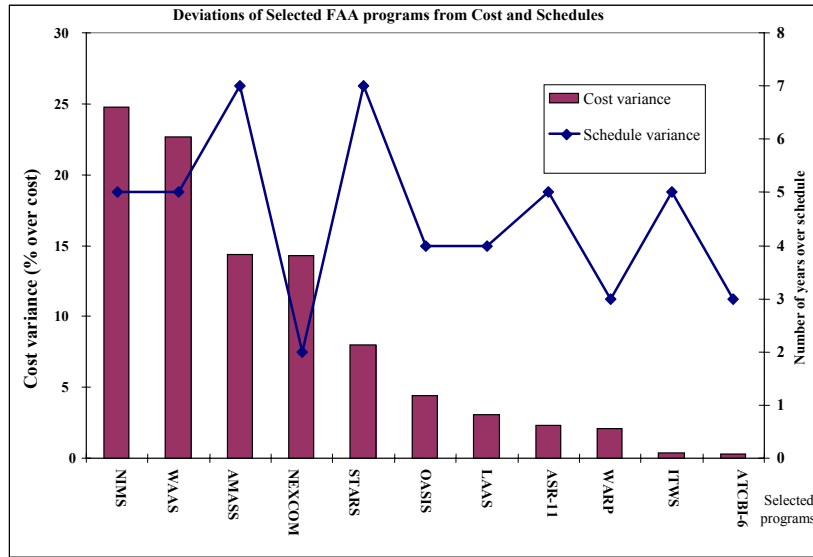
Congressional inquiries (GAO, 2005), an Inspector General (IG) report (FAA, 2005) and independent reviews (Shantz & Hampton, 2005) found that the NAS investment programs are inherently *risky*. Evaluated by the two most commonly used measures, that is, cost and schedule variances, for the FAA’s major programs (16 of them presently), the IG (FAA, 2005) found that 11 of these programs<sup>9</sup> have experienced a total cost growth of over \$5.6 billion (see Figure 4), which is more than twice the FAA’s F&E budget in FY2005.<sup>10</sup> Furthermore, many of these programs have had schedule variances ranging between 2-12 years. Two programs, local area augmentation system and Next Generation Communication, (LAAS and NEXCOM) have been withheld until further evaluation (2008) on the merits of each program (FAA, 2005).

<sup>9</sup> These represent approximately 71% of the funds available for developing and acquiring air traffic control modernization projects (FAA, 2005).

<sup>10</sup> The cost growth is not unique to F&E programs alone. Operational costs from air traffic services, for example, grew by nearly \$1.8 billion in real terms or by 43% during FY 1996-2004 (GAO, 2005).



**Figure 4: Schedule and cost variance of some selected FAA programs (calculated over different periods ranging between 1998-2005; and 2005-2013)**

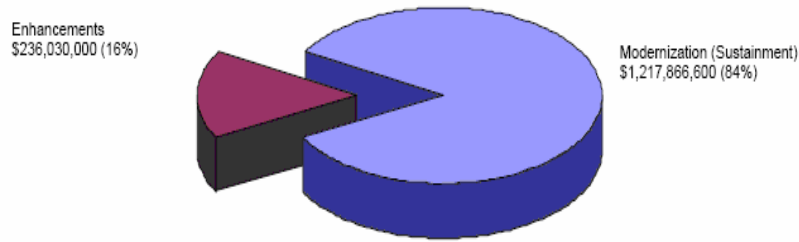


Source: FAA (2005, May 26). “Status of FAA’s Major Acquisitions: Cost Growth and Schedule Delays Continue to Stall Air Traffic Modernization,” Report No. AV-2005-061. Washington DC: Office of the Inspector General, Federal Aviation Administration.

As the industry restructured in the wake of 9/11, the Airways Trust Fund revenue, the main source for F&E expenditures, dwindled considerably. Numerous estimates (Chew, 2005) indicate that the gap between the trust fund collections and cost commitments are expected to widen, potentially affecting FAA program funding. Presently, the FAA spends considerably more on sustaining the NAS, than on enhancing it [see Figure 5 as reported in FAA (2005)].

The program decisions underlying Figure 5 indicate that, under present budgetary arrangements, modernization sustainment programs dominate investment. Most analyses conclude that the FAA in general, and ATO in particular, needs to develop a comprehensive strategy for modernizing the NAS while minimizing risks for all the major acquisition programs. In other words, the FAA should meet the cost, schedule, and performance milestones for all its acquisition programs, especially in this fiscally challenging environment (FAA, 2005; GAO, 2005).

**Figure 5: Breakdown of air traffic control modernization funding by investment area, 2005**



Source: FAA (2005)

A sense of urgency dominates the current budget cycle. The FAA routinely manages unprecedented levels of traffic while maintaining record low fatal accident rates. Studies have repeatedly shown that the level and complexity of traffic and the productivity of controllers and NAS assets is unparalleled. Traffic is projected to grow. Current budgetary pressure and the changing business environment have made prudent investing more important than ever. F&E and AIP budgets growth has lagged behind the operations budget. An aging infrastructure and higher future traffic levels portend the need for an investment approach that extracts the most from the FAA's limited resources.

### THE ANALYTICAL FRAMEWORK

Risk<sup>11</sup> is an essential part of any investment program, private or public. Managing risk is the job of a portfolio manager. Risk exists because the investor can no longer explicitly associate payoff<sup>12</sup> with investment in any asset. In the market place, where return or price provides explicit signals, risks are traded for lower returns and vice versa. Nevertheless, similar trade-offs can be performed in government investments, and hence, an optimal set of public investments can be made if choice sets (i.e., range of possible programs), their interdependencies, and fiscal constraints can be specified adequately.

The analytical framework presented here captures the trade-offs between risks and expected returns in a portfolio consisting of multiple assets. Instead of considering that an investor's preferences are defined over the entire probability distribution of the assets with every possible outcome, this framework supposes that investor's preferences can be described by

<sup>11</sup> Without being too specific, risks in this paper generally represent (a) technical risks; (b) financial risks; and (c) program management risks.

<sup>12</sup> Payoff is described by a set of outcomes each associated by the return distribution (i.e., probability of occurrence).

considering a few summary statistics of the probability distribution<sup>13</sup> of the asset holdings. Mean and variance are two such key statistics that can describe the probability distribution of asset holding fairly well. Originally developed by Harry Markowitz,<sup>14</sup> the mean-variance model has been the foundation of corporate finance for decades.

In the framework presented below, dependencies across assets or projects have been given explicit consideration. This is in contrast with current procedure where capital investment programs are considered to be mutually exclusive. That is, decisions to invest depend primarily on returns (i.e., net present value, internal rate of return, or benefit-cost ratio) from the project alone. The lack of recognition of dependencies across projects often leads to selection bias and leaves very little room for a portfolio manager (i.e., ATO program manager) to compare relative risks versus relative returns in prioritizing projects.

Under the present format of evaluating government projects, risks are considered but only in terms of evaluating cost schedules. Trade-offs between risks and returns—the primary driver for choice in a portfolio—is not present under the current investment analysis framework. The analytical framework presented below is offered as an alternative to evaluate decision rules for selecting programs within the overall capital investment programs.

For demonstrating this framework, it is assumed that investors (or, a manager who decides on investments for NAS improvement, or a NAS portfolio manager) hold a portfolio of assets. Therefore, the focus is on the *expected return and risks from the whole portfolio, not individual assets*. Notice, however, that the financial and economic analysis for individual projects (i.e., standard cost-benefit analysis leading to internal rate of return) may provide important information regarding expected returns, estimated risks, and underlying relations or dependencies between individual assets. Risk is quantified by the standard deviation of the portfolio while returns are evaluated by the probability of events. For example, for a given expected return, different expected standard deviations can be obtained depending on the mix of assets due to varying correlations among the assets. Hence, the authors were able to estimate and predict some form of expected returns along with risks, and correlations among the assets. Furthermore, the underlying preference structure of the portfolio manager, that is, investor's

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<sup>13</sup> Two most commonly used aggregate measures of the probability distribution of asset holdings are average returns (i.e., expected averages over the entire distribution) and standard deviation, a measure of risk or dispersion around the mean.

<sup>14</sup> This research that provided the foundation for portfolio theory in corporate finance earned Markowitz the Nobel Prize in 1990 along with William Sharpe and Merton Miller for developing the theory of price formation for financial assets [Capital Asset Pricing Model (CAPM)] and the theory of corporate finance, respectively.

preference for expected returns against risks, can be postulated by some hypothetical function.

A portfolio of assets is characterized by two elements: expected return which is computed as weighted average of the return on the individual assets where the weight applied is the fraction of the portfolio invested in the asset. Thus, returns on the portfolio are calculated as the sum of all fractions of the portfolio held in each asset multiplied by the expected return in each asset. The variance, on the other hand, measures the dispersion or the expected value of the squared deviations of the return on the portfolio. In other words, expected return of an asset is a probability-weighted average of its return in all scenarios:  $E(r) = \sum_s P_r(s) r(s)$  where  $P_r(s)$  is the probability of scenario  $s$  and  $r(s)$  is the return in scenario  $s$ . Variance of an asset's return is the expected value of the squared deviations from the expected return, represented by the equation:

$$\sigma^2 = \sum_s P_r(s) [r(s) - E(r)]^2$$

The rate of return on the entire portfolio is a weighted average of the rates of return of each asset comprising the portfolio, with portfolio proportions as weights. This implies that expected rate of return on a portfolio is a weighted average of the expected rate of return on each component asset. When a risky asset is combined with a risk-free asset, the portfolio standard deviation equals the risky asset's standard deviation multiplied by the portfolio proportion invested in the asset.

One way to capture and quantify the effect of hedging and diversification of the portfolio is to construct the covariance and correlations (Ross et al., 1999) across individual items in the portfolio. Covariance measures the degree to which returns on two risky assets move in tandem. A positive covariance thus indicates that asset returns move together. A negative covariance, conversely, means that they vary inversely. Covariance between project  $i$  and  $j$  can be defined as:  $\text{Cov}(r_i, r_j) = \sum_s P_r(s) [r_i(s) - E(r_i)][r_j(s) - E(r_j)]$ .

Often, it is easier to interpret correlation coefficient ( $\rho$ ) than the covariance. The correlation coefficient ( $\rho$ ) is constructed by scaling covariance to assume a value between -1 (perfect negative correlation) and +1 (perfect positive correlation). It is constructed as follows:  $\rho_{i, j} = \text{Cov}(r_i, r_j) / \sigma_i \sigma_j$ . That is, the correlation coefficient between two projects equals their covariance divided by the product of the standard deviations.

When two risky assets with variances  $\sigma_i^2$  and  $\sigma_j^2$ , respectively, are combined into a portfolio ( $p$ ) with portfolio weights  $w_i$  and  $w_j$ , respectively,

the portfolio variance  $\sigma_p^2$  is given by:  $\sigma_p^2 = w_i^2 \sigma_i^2 + w_j^2 \sigma_j^2 + 2 w_i w_j \text{Cov}(r_i, r_j)$ .

Given this background on the structure of assets in terms of their return distribution (i.e., mean, standard deviation or variance; and dependencies within the portfolio that is defined by covariance and/or correlation coefficient), one can postulate the standard investor's choice problem defined over several asset classes comprising the portfolio to maximize utility. Given the inherent property of the portfolio, the utility is also defined as a function of expected returns and standard deviation of return of the selected portfolio. More precisely,

$$u = E(r) - \sigma^2/t(k) \quad (1)$$

where  $u$  is the utility of the portfolio for the investor;  $E(r)$  is the expected return of the portfolio;  $\sigma^2$  is the variance of the portfolio return; and  $t(k)$  is risk tolerance for an investor,  $k$ , that is,  $t(k)$  is the investor's marginal rate of substitution of variance for expected value (i.e., trade-off). Evaluating Equation 1 slightly differently, it is obvious that  $u$  is the measure of portfolio utility that represents risk-adjusted expected return, since it is computed by subtracting a risk penalty [ $\sigma^2/t(k)$ ] from the expected return  $E(r)$ . Thus, for the portfolio as a whole, the utility function can be defined as the following:

$$pu_x(p, k) = E(p) - \sigma^2(p)/t(k) \quad (2)$$

where  $E(p)$  is the expected value or return of portfolio  $p$ ,  $\sigma^2(p)$  is its variance,  $t(k)$  is investor's risk tolerance, and  $pu_x(p, k)$  is the utility of portfolio  $p$  for investor  $k$ . Portfolio utility is measured in the same units as expected returns,  $E(p)$ . Thus, for a given level of utility,  $pu_x$ , all portfolios must satisfy the following condition:

$$pu_x(p, k) = E(p) - \sigma^2(p)/t(k) \quad (3)$$

or

$$E(p) = pu_x + [1/t(k)] * \sigma^2(p)$$

where  $pu_x$  = associated portfolio utility. Different levels of utility associated with higher portfolios can be depicted by a set of indifference curves<sup>15</sup> (see Figure 6).

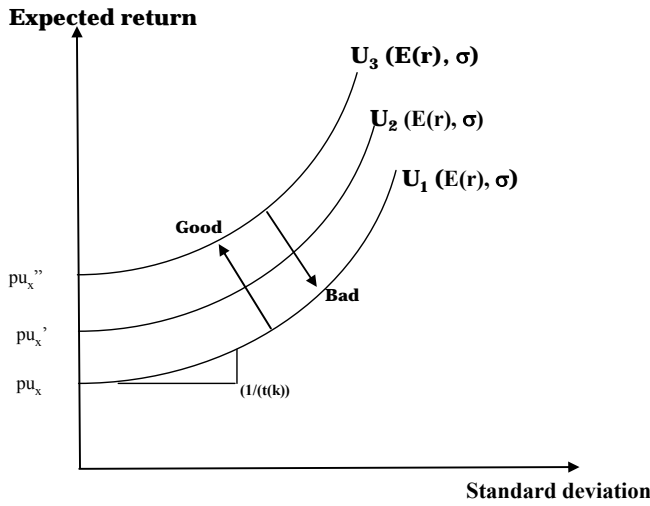
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<sup>15</sup> Indifference curves measures investor's indifference between expected returns and standard deviation (risk). It simply states that higher expected returns have to accompany higher risks in order to provide same levels of utility. Alternatively, given the same expected return, investor prefers less standard deviation (i.e., variability in portfolio) than more. Obviously, the underlying assumption here is that risk is

Finally,  $1/t(k)$  measures the slope along the indifference curve that measures the trade-off ratio of expected return for variance, or marginal rate of substitution of variance for expected value.

Given the above preference structure, how does one determine the choice along the indifference curve or a point on the distribution defining a portfolio? That is, would the investor have \$10,000 for certain or a 50/50 chance of receiving \$0 or \$25,000? While detailed knowledge about the investor's preference structure may be revealing, it is neither necessary nor sufficient to prove that a portfolio choice may exist even without it. The answer to that choice problem, thankfully, may be found through investigating the trade-off that an investor is willing to make in the market place (or at some alternative shadow of such prices), other constraints, and levels of risk tolerance (Varian, 1999).

Figure 6: Structure of preferences for a portfolio of asset choices



Suppose that risky assets and risk-free assets can be traded at the market place. This hypothetical exercise (i.e., trading risk for expected returns) allows us to construct the investor's affordability set for a portfolio with risk to a risk-free investment.<sup>16</sup> The weighted average of the expected return ( $R_p$ )

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inherently *bad*, and therefore, has to be compensated by some *good* which is higher returns (Varian, 1999).

<sup>16</sup> Defining risk-return trade-off in the market, not the actual return in a particular month or year, is the foundation of CAPM.

on two assets, one risky return ( $R_m$ ) and one not-risky ( $R_f$ ), therefore, can be written as:

$$R_p = bR_m + (1-b)R_f$$

where  $b$  is the fraction of investment on these two assets, or,  

$$= R_f + b(R_m - R_f) \quad (4)$$

Since  $R_f$  is risk-free, therefore, standard deviation of the portfolio (with one risky and one risk-free asset) is the fraction of the portfolio invested in the risky asset ( $b$ ) times the standard deviation of the asset ( $\sigma_m$ ):<sup>17</sup>

$$\sigma^2(p) = b^2\sigma^2(m)$$

or

$$\sigma(p) = b\sigma(m)$$

and

$$b = \sigma(p)/\sigma(m)$$

Therefore, Equation 4 can be rewritten as:

$$R_p = R_f + [(R_m - R_f)/\sigma(m)] * \sigma(p) \quad (5)$$

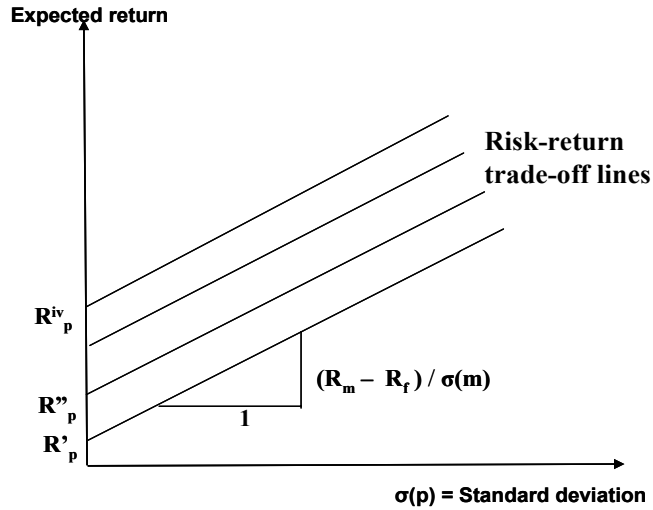
which is the affordability line because it describes the market trade-off between risk [ $\sigma(p)$ ] and expected return ( $R_p$ ). Note,  $R_m$  could be any portfolio, but is considered here as a single choice for simplicity.

Thus, for a given level of portfolio returns or  $R_p$ , the iso-affordability line (i.e., trade between  $R_m$  and  $R_f$  yielding the same portfolio return of  $R_p$ ) or security-market line can be described by the following figure.

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<sup>17</sup> In other words,  $b$  measures the responsiveness of expected return ( $R_p$ ) to movements in the market portfolio ( $R_m$ ). If the portfolio were to expand to include multiple risky assets, then,  $b$  would be equal to covariance between the return on asset  $i$  and the return on the market portfolio divided by the variance of the market. This statistic (also known as *beta* from the portfolio theory) can reveal a great deal of information regarding the effectiveness of the portfolio.

Figure 7: Affordability set



Notice that when the portfolio consists of no risk (i.e., standard deviation = 0), then,  $R_p = R_f$  (i.e., vertical intercept). The slope of the iso-affordability line is equal to  $(R_m - R_f)/\sigma(m)$  which measures the price of risk, that is, extra risk an investor must incur to enjoy a higher expected return. In other words, the line will be upward sloping as long as the expected return on the market is greater than the return on the risk free asset.

Affordability is incomplete without the constraints and boundaries on portfolio choice selection. Thus, investment choices are constrained by the following two conditions:

$$\sum(x) = 1;$$

or, more generally,

$$\sum(x) = L \quad (6)$$

where  $L$  is a constant.

That is, sum of all portfolio investments exhaust the entire budget, that is, full-investment constraint (i.e., no slack left in budget constraint). In addition, project investments may require that some parts of the budget sets may be outside the feasibility bounded from lower and upper ends,<sup>18</sup> that is:

<sup>18</sup> For many of the capital investment projects, too low an investment solution may be trivial, while too high a solution may be budget busting.



$$\begin{aligned}
 & x \leq ub \\
 & x \geq lb; \\
 \text{or} \\
 & lb \leq x \leq ub \qquad (7)
 \end{aligned}$$

Now that we have defined both the choice set and the constraints, the goal is to find the best portfolio, that is, the one with the maximum possible utility. The decision variables are the asset holdings, that is, the elements of vector  $x$  that form the portfolio  $p$ .

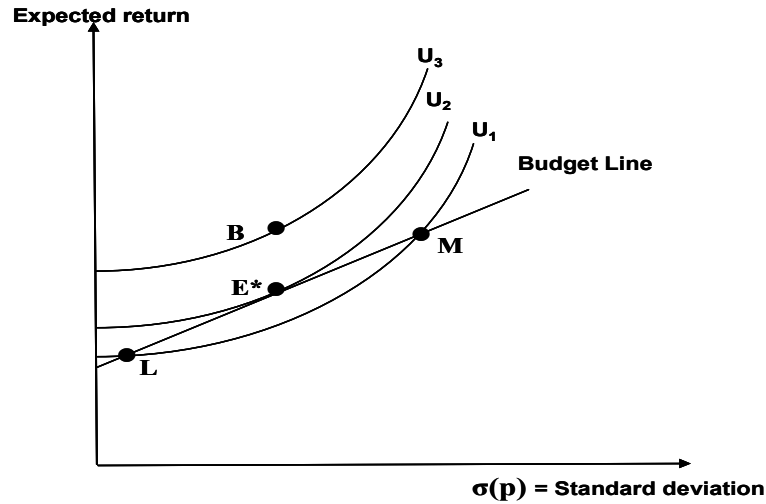
Notice, as these elements are varied, the utility of the associated portfolio will change. The authors wish to vary those choices (i.e., elements of  $x$ ) until the maximum possible utility is attained. Finally, the allowable combinations of  $x$  choice sets are typically constrained by other factors (i.e., investment and boundary constraints). Therefore, the standard asset allocation problem (i.e., trade-off between expected return and risk that give rise to an efficient solution in elements of  $x$ ) can be stated as:<sup>19</sup>

$$\begin{aligned}
 & \text{Select:} && x(i) \\
 & \text{to maximize:} && u = E(p) - \sigma^2(p)/t(k) \qquad (8) \\
 & \text{where:} && E(p) = x' * e \\
 & && \sigma^2(p) = x' * C * x \\
 & \text{subject to:} && \text{sum}(x) = L : \text{Fully Invested} \\
 & && lb \leq x \leq ub : \text{ceiling and floor conditions.}
 \end{aligned}$$

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<sup>19</sup> The dual of this primal problem is: Minimize variance subject to fixed utility,  $u = \underline{u}$ .

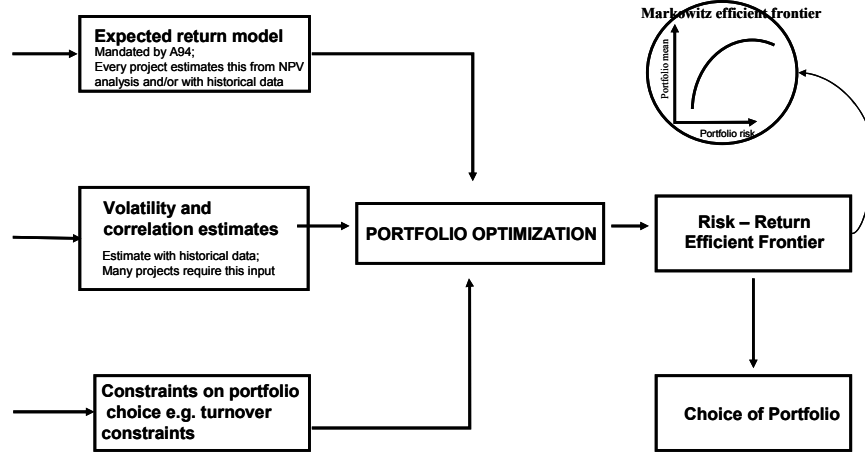
Figure 8: Equilibrium portfolio choice resulting from optimal choice of expected return and risk



The solution to the above problem can be best summarized in Figure 8. The process of finding optimal solution ( $E^*$ ) is reached by varying levels of risk and/or alternatively, by offering minimum risk for varying levels of expected return. Thus, from a point such as  $L$ , an investor would prefer to accept higher risk for more returns thus attaining a higher utility from the portfolio choice. Alternatively, starting from point  $M$ , the investor would do just the opposite and attain a higher level of utility. Thus, the point  $E^*$  at which, the slope of the indifference curve is equal to the slope of the budget line, that is,  $(t/k) = (R_m - R_f)/\sigma(m)$ , would represent the optimal choice of expected return and risk for a particular portfolio. Thus, to the northeast of  $E^*$  is the efficient frontier for the choice set while to the southeast is feasible.

By iteratively finding the optimal choice points varying the parameters of the above choice problem, different portfolios can be derived while maintaining the most efficient risk-return frontier, also known as Markowitz efficient frontier. The figure below summarizes the entire choice problem described above (Markowitz, undated).

Figure 9: Summary of the portfolio choice in investment process



Source: Markowitz (undated).

Notice that the above choice problem involves the maximization of a quadratic (utility) function of the decision variables, subject to a set of linear constraints (i.e., fully invested), some of which are inequalities (i.e., floor and ceilings). This non-differentiable non-linear problem is termed as a quadratic programming (QP) problem. It may be solved with a general QP algorithm; or with a procedure designed to deal only with problems that have similar structures. However, solutions to this problem can also be parametrically approximated by piece-wise linear programs, but it is somewhat limited.

In this exercise, the authors demonstrate an algorithm<sup>20</sup> that can solve the standard asset allocation problem in a simple and intuitive way keeping the QP structure. More complexities can be added on later, both in terms of expanding assets and recasting the problem in different ways altogether. While somewhat limited in its range of application, the standard problem is easy to program for illustrating key economic principles that may also apply to a very broad range of optimization problems involving project investment analysis.

<sup>20</sup> The example constructed here is based on Sharpe's *Gradient Method* solution to a standard three-asset allocation problem [see [http://www.stanford.edu/~wfsarpe/mia/opt/mia\\_opt1.htm](http://www.stanford.edu/~wfsarpe/mia/opt/mia_opt1.htm) for more details]. While his original algorithm was written on MATLAB, other software can be used to replicate this or other allocation problems.

### SOLUTION TO A COMPLEX PROBLEM: AN EXAMPLE OF AN ALGORITHM USING GRADIENT QUADRATIC PROGRAMMING METHOD

Applying the described investment portfolio methodology, the authors offer the following hypothetical example using hypothetical distributions and names for programs, for example, Program A, B, and C evaluated against holding cash. Accordingly, the numbers in this exercise are imaginary. They are not intended to reflect actual program costs and benefits, but rather as an illustration of this form of comparative analysis. A complete benefit cost analysis would be required with estimated risks and interdependencies for actual investment decisions. While this should be done, for this example it has not been done.

The authors assume the following functional forms and other associated inputs for the QP:

Assumed Utility function:

$$U(p) = E(p) - [(\sigma^2)/tk]$$

where:  $U(p)$  = the utility of the portfolio;  $E(p)$  = the portfolio's expected return;  $\sigma^2$  = the portfolio's standard deviation of return; and  $tk$  = parametric risk tolerance for investor. The following table provides the parameters of the choice problem along with other constructs.

Table 1: Parameters for the choice problem<sup>21</sup>

INPUTS	Correlation Matrix <sup>22</sup>								
	MIN	INIT	MAX	ExpRet	StdDev	c:cash	c:progA	c:progB	c:progC
cash	0.000	0.200	1.000	2.800	1.000	1.000	0.100	0.150	0.170
ProgramA	0.000	0.050	1.000	6.300	7.400	0.100	1.000	0.350	0.223
ProgramB	0.000	0.400	1.000	10.800	15.400	0.150	0.350	1.000	0.425
ProgramC	0.000	0.350	1.000	17.280	27.320	0.170	0.223	0.425	1.000

Source: Sharp (2006), The Gradient Method, Available at [http://www.stanford.edu/~wfsjarpe/mia/opt/mia\\_opt1.htm](http://www.stanford.edu/~wfsjarpe/mia/opt/mia_opt1.htm); Accessed on April 26, 2006.

<sup>21</sup> All numbers in this demonstration here are hypothetical and for illustration purposes only.

<sup>22</sup> Correlation coefficient is an easier statistic to interpret than covariance, +1 representing perfect correlation and -1 representing perfectly negative. Correlation coefficient between 2 variables equals their covariance divided by the product of the standard deviations.

The MIN and MAX, or lb and ub from the above choice problem, represent lower (all zero) and upper bounds (all 1) of proportion of investment on four investment choices, cash, PROGRAM A, PROGRAM B and PROGRAM C.<sup>23</sup> ExpRet [i.e.,  $E(p)$ ] and StdDev (i.e.,  $\sigma^2$ ) represent, respectively, expected returns and standard deviations of the assets stated in terms of percent return per year.<sup>24</sup> Correlation matrix estimates correlations among the asset classes which can be calculated on the basis of the covariance matrix,  $C$ .<sup>25</sup>

Finally, three more inputs are required. For simplicity, we assume  $L = \text{sum}(x) = 1$ , that is, sum of all allocations equal to 1; somewhat moderate risk-taking attitude, and hence,  $R_t = t/k = 50$  (100 would be complete risk taking while 0 representing complete risk averse) and finally, trading decisions (i.e., swapping one investment for another) has been set at marginal utility cut-off ( $MU_{\text{buy}} - MU_{\text{sell}}$ ) at 0.0001. In other words, if there is a possibility of slight change (0.0001) in utility, through buying and selling (also known as swapping) and hence cumulative impacts through marginal utilities, then, the investor would alter his portfolio to realize the potential gain.

Notice that our example involves four assets, that is, cash, PROGRAM A, PROGRAM B, and PROGRAM C. With optimized utility, the solution space is five-dimensional. With added restrictions (i.e., full-investment constraint) imposed, we are able to present the allocations in three dimensions (since the fourth asset is the residual sum). This makes it possible to graph the relationship between three of the assets (not with the utility). The resulting surface will have some of the attributes of a hill. Notice, however, that only a portion (not all) of this utility hill is feasible given the constraints. We must therefore restrict our search to coordinates in which the sum of the amounts invested in all assets to be 1.0 or less and both upper and lower boundaries have been met.

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<sup>23</sup> Similarly, these hypothetical names have been used to represent different investment choices that the portfolio manager may have.

<sup>24</sup> Notice that for real applications, as opposed to the hypothetical example presented in this paper, expected returns from similar projects (or those which have been estimated by individual project's cost-benefit analyses) can replace these values. Similarly, the standard deviations and correlations among their returns can best be estimated from projects' financials and/or from expert opinions. In absence of these parameters, one can experiment with range of expected values (e.g., expected returns with range of values from 5-30% annually) with corresponding assumptions regarding correlations in order to derive the solutions.

<sup>25</sup> As discussed earlier, the correlation coefficient between 2 variables equals their covariance divided by the product of the standard deviations.

Optimizing the four-asset portfolio requires climbing to the highest feasible point, given restrictions, on the hill by swapping assets. This can be performed in multiple stages. First, we start with a feasible portfolio that satisfies all the conditions stated above. Second, we find the feasible direction in which we can move upward at the greatest rate. More specifically, we select the direction that will result in the greatest increase in altitude (utility) per step (change in portfolio holdings)—that is, the steepest gradient<sup>26</sup>. Third, having selected a direction, we continue climbing until a new peak or a boundary line have been reached and no more gain can be had from further climbing. That is, given the restrictions on the portfolio, a climb through swap/buy is feasible when the following conditions have been met: (a) the asset to be sold is below the upper bound (ub); (b) the asset to be bought is above its lower bound (lb); and, finally, (c) marginal utility gain from this swap is higher. Then we determine the feasible direction of steepest ascent again and repeat the process. When no feasible direction leads upward, we stop. These rules together also give optimal amount to swap when the process of improvement stops yielding the equilibrium. Given the nature of the terrain in a standard problem, this procedure will place us on the highest allowable point, that is, provide the portfolio with the greatest possible utility.

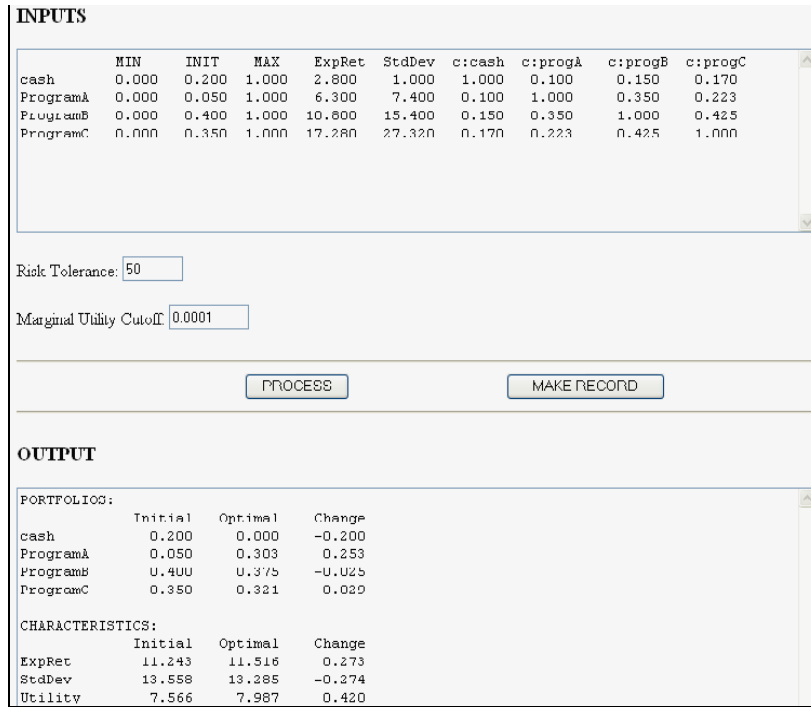
Figure 10 presents the output of the portfolio choice simulation that we performed using Sharpe's algorithm. Starting with baseline distribution of asset holding, clearly, there is a scope for reallocation that may improve the investor's utility. For example, given the parameters, the optimal portfolio allocation indicates that investor's welfare can be improved by moving away from cash holding altogether.

A large beneficiary of the portfolio realignment, given the assumed hypothetical parameters, appears to be Program A. The results of these reallocations are reflected in the market portfolio as a whole via the increase in expected returns (from 11.243% to 11.516%) and a reduction in risk (from 13.558% to 13.285%). The increase in expected returns and reduction in risk exposure, in our hypothetical example, increased utility (from 7.566 to 7.987, or 5.56%) in the investor's market asset holding—clearly an optimal move.

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<sup>26</sup> This method is called Gradient Method.

Figure 10: Output of the portfolio choice simulations



### CONCLUSION AND FURTHER RESEARCH

In this paper, the FAA's current investment methodology and budget allocations were reviewed. A preliminary investment portfolio analytical framework that may address some of the noted deficiencies was laid out. By drawing upon the well developed theories of corporate finance, the authors have offered an investment framework that takes into account risk, expected returns, and inherent dependencies across NAS programs. The authors present an algorithm in this paper and apply it to a hypothetical four-asset allocation problem. By iteratively solving the QP problem, the authors demonstrate that reallocation may in fact result in improvement in investor's welfare.

This proposed framework is relatively simple and has been used for demonstration purpose only. It can be improved in numerous ways. For example, the framework can be expanded to include multiple assets and realistic values for parameters to include expected returns, standard

deviations, and interdependencies, in particular, tasks that may be dealt with in future research.

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