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Laboratory Study of Magnetorotational Instability and Hydrodynamic Stability at Large Reynolds Numbers

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ABSTRACT

Two plausible mechanisms have been proposed to explain rapid angular momentum transport during accretion processes in astrophysical disks: nonlinear hydrodynamic instabilities and magnetorotational instability (MRI). A laboratory experiment in a short Taylor-Couette flow geometry has been constructed in Princeton to study both mechanisms, with novel features for better controls of the boundary-driven secondary flows (Ekman circulation). Initial results on hydrodynamic stability have shown negligible angular momentum transport in Keplerian-like flows with Reynolds numbers approaching one million, casting strong doubt on the viability of nonlinear hydrodynamic instability as a source for accretion disk turbulence.

1. Introduction

Accretion disks are the most efficient energy source known to astrophysics. Whereas hydrogen fusion has a maximum efficiency for converting rest-mass to radiation $\lesssim 1\%$, black-hole or neutron-star accretion can achieve $\sim 5-40\%$. Accretion disks power many of the most luminous and violent astrophysical sources, including X-ray binaries, quasars, and perhaps gamma-ray bursts. Protostellar accretion disks, though energetically much less efficient, are nevertheless of great interest as sites of planet formation.

To support the rapid accretion process, an efficient mechanism to transport angular momentum outward is required. Molecular viscosity is completely inadequate. Since Keplerian flow profiles are linearly stable in hydrodynamics, there exist two plausible mechanisms for turbulent transition: nonlinear hydrodynamic instability or linear magnetorotational instability (MRI). The latter is favored for ionized accretion disks ranging from quasars to cataclysmic variables (Balbus & Hawley 1998). Protoplanetary disks, which may be too resistive for MRI, may require the former.

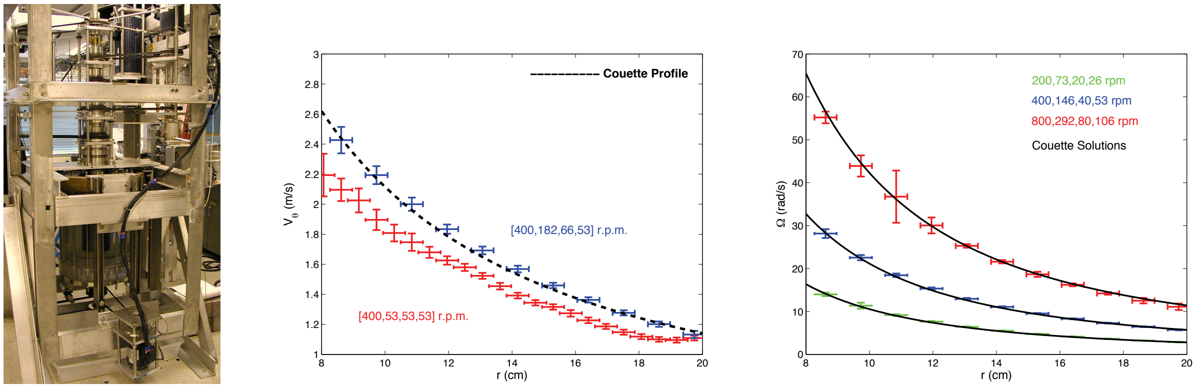

Fig. 1.— *(Left)* A photograph of the assembled apparatus. *(Center)* Flow profiles measured by Laser Doppler Velocimetry with (upper symbols, $\Omega_1 > \Omega_3 > \Omega_4 > \Omega_2$) and without (lower symbols, $\Omega_1 > \Omega_3 = \Omega_4 = \Omega_2$) differentially rotating endcaps. *(Right)* MRI-favorable profiles are produced at 3 different overall speeds at fixed ratios $\Omega_1 : \Omega_3 : \Omega_4 : \Omega_2$.

Linear MRI is simple and robust. MRI saturation and turbulence, however, are known only from simulations, which differ in 2D and 3D; none resolve all active lengthscales [see Balbus & Hawley (1998) and references therein]. Nonrotating hydrodynamic shear flows are typically turbulent at $Re \gtrsim 10^3$. (Richard & Zahn 1999, (RZ)) argue for hydrodynamic turbulence in disks based on old Couette-flow experiments (Wendt 1933; Taylor 1936), where at large enough Re, the flow became turbulent with the inner cylinder at rest—a strongly Rayleigh-stable case. RZ deduce a turbulent viscosity for disks of the form

$$\nu_{\rm T} = -\beta r^3 \frac{\partial \Omega}{\partial r}\ , \qquad \beta = (1-2)\times 10^{-5}, \qquad Re_{\rm crit} \sim \beta^{-1}. \tag{1}$$

The minimum Re for turbulence is sensitive to gap width $[(r_2 - r_1)/r_2]$ and probably to the ratio $R_\Omega \equiv 2(d\ln\Omega/dr)^{-1}$ of rotation to shear. There have been very few experiments relevant to RZ's hypothesis for disk-like R_Ω (Richard 2001). Local 3D simulations find purely hydrodynamic flows to be stable for angular-momentum gradients as strongly positive as in disks [see e.g., Balbus et al. (1996); Lesur & Longaretti (2005)].

To study these proposed mechanisms, a novel laboratory apparatus in a short Taylor-Couette flow geometry has been built in Princeton (Ji et al. 2001; Goodman and Ji 2002), among other proposed or existing experiments (Noguchi et al. 2002; Sisan et al. 2004; Hollerbach & Rüdiger 2005). We briefly describe the apparatus in the next Section, followed by initial hydrodynamic results. A short summary will be given in the last Section.

2. The Apparatus

To minimize Ekman circulation, differentially rotating endcaps are used (Kageyama et al. 2004). The annular area between the inner and outer cylinders has been modified to

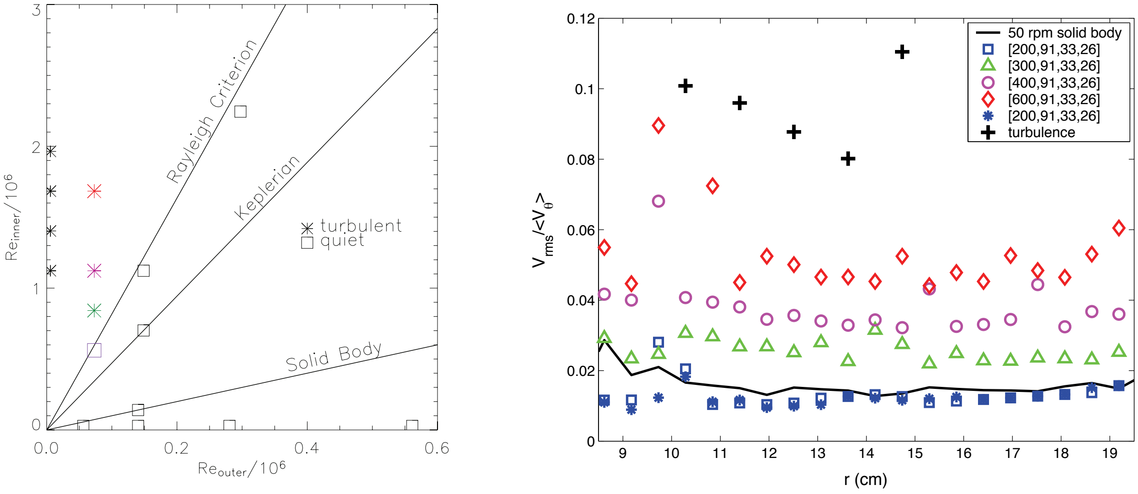

Fig. 2.— *(Left)* A diagram of initial experiments on hydrodynamic stability in the plane of $(Re_{\mathrm{outer}}, Re_{\mathrm{inner}})$. Square symbols indicate quiet flows (fluctuations $\lesssim 1\%$) while stars indicate fluctuations $> 2\%$. *(Right)* The measured fluctuation level. Triangles, circles, and diamonds correspond to the three points above the Rayleigh line, from bottom to top, at $Re_{\mathrm{outer}} \approx 0.05 \times 10^6$ in the left panel; squares and asterisks both correspond to the point just below the Rayleigh line at same Re_{outer}. Solid-body ($\Omega_1 = \Omega_3 = \Omega_4 = \Omega_2$) and Rayleigh-unstable ($\Omega_2 = 0$) cases are shown for reference.

accommodate two intermediary rings at both endcaps (Burin et al. 2006). The dimensions are $r_1 = 7.06$cm, $r_2 = 20.30$cm, and $h = 27.86$cm. All dimensions are accurate to ±0.02cm. A picture of the apparatus is shown in Fig. 1(left).

The suppression of Ekman circulation has been demonstrated using water. Figure 1 shows radial profiles of angular velocity measured by Laser Doppler Velocimetry (LDV). When the rings corotate with the outer cylinder, the flow deviates substantially from the ideal Couette profile. When the rings are activated, the Couette profile is accurately restored [Fig. 1(left)]. Furthermore, the Couette profiles are consistently reproduced at different speeds with the same speed ratios between rotating components [Fig. 1 (right)]. This is especially encouraging because MRI-favorable profiles can be produced at a range of speeds with confidence (Burin et al. 2006).

3. Initial Results of Hydrodynamic Stability at Large Reynolds Numbers

With the newly available apparatus, some initial results on hydrodynamic stability have been obtained using the LDV technique. These are summarized in Fig. 2, where the data are plotted in the plane of $Re_{\mathrm{outer}} \equiv \Omega_2 r_2 (r_2 - r_1)/\nu$ and $Re_{\mathrm{inner}} \equiv \Omega_1 r_2 (r_2 - r_1)/\nu$. Note that the Reynolds numbers achieved so far are already larger by a factor of 30 than those in the

Richard's experiment; also our gap is much wider ($r_2/r_1 \approx 2.9$ *vs.* 1.3). (Richard also used split endcaps, but with rings affixed to the inner and outer cylinders.) Solid-body rotation, which should be laminar, serves to calibrate systematic offsets in the velocity measurements and their statistical errors, $\sim 1\%$ per sample.

Three types of experiments have been performed to study flow stability at large Reynolds numbers. Those with the outer cylinder at rest ($\Omega_2 = 0$) are highly turbulent with fluctuation levels as large as 10% (see Fig. 2). Those with the *inner* cylinder at rest ($\Omega_1 = 0$) are as quiescent as solid-body rotation ($\Omega_1 = \Omega_2$), with fluctuations much smaller than those reported by Richard. The third type have $1 < \Omega_1/\Omega_2 < (r_2/r_1)^2$ as in keplerian disks. The latter flows are robustly quiet, showing maximum fluctuations indistinguishable from the solid-body case. Hysteresis, a hallmark of subcritical instability, was reported by Richard. A series of experiments was performed to test this. Ω_1 was raised (at constant $\Omega_2, \Omega_3, \Omega_4$) into the linearly unstable regime, then returned to the original linearly stable value. Significant fluctuations were found only in the linearly unstable regime (Fig. 2); no hysteresis was detected.

This results are further supported by direct measurements of Reynolds stress by a dual LDV, and it is found (Ji et al. 2006) that the measured transport coefficients (β) are below the proposed values by 3σ, and in addition, they are almost identical to those measured in solid-body rotation, which has no free energy to support turbulence. Thus these results have shown, for the first time, that purely hydrodynamic instabilities are unlikely to explain rapid accretion in cold disks.

4. Summary

Mechanisms for efficient angular momentum transport in accretion disks are studied in a laboratory experiment in a short Taylor-Couette flow using water and liquid gallium. Initial results on hydrodynamic stability at large Reynolds numbers show, for the first time, that the nonlinear pure hydrodynamic instabilities are very unlikely to sustain the required turbulence. Experiments using liquid gallium to study MRI are planned for the near future.

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