Turbulence Modeling for the Simulation of

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Abstract

Our research involves study of the behavior of k-ε turbulence models for simulation of bypass-level transition over flat surfaces and turbine blades. One facet of the research has been to assess the performance of a multitude of k-ε models in what we call "natural transition", i.e. no modifications to the k-ε models. The study has been to ascertain what features in the dynamics of the model affect the start and end of the transition. Some of the findings are in keeping with those reported by others (e.g. ERCOFTAC). A second facet of the research has been to develop and benchmark a new multi-time scale k-ε model (MTS) for use in simulating bypass-level transition. This model has certain features of the published MTS models by Hanjalic, Launder, and Schiestel, and by Kim and his co-workers. The major new feature of our MTS model is that it can be used to compute wall shear flows as a low-turbulence Reynolds number type of model, i.e. there is no required partition with patching a one-equation k model in the near-wall region to a two-equation k-ε model in the outer part of the flow. Our MTS model has been studied extensively to understand its dynamics in predicting the onset of transition and the end-stage of the transition. Results to date indicate that it far superior to the standard un-modified k-ε models. The effects of protracted pressure gradients on the model behavior are currently being investigated.

Students involved in this research include Tzong-Huei Chen (Mechanical Engineering, The University of Texas at Austin) and Klaus Sieger (Institute for Thermal Turbomachinery, University of Karlsruhe, Germany). Sponsor for the majority of the research is the NASA-Lewis Research Center, with Mr. Fred Simon as technical monitor.

Turbulence Modeling for the Simulation of Transition in Wall Shear Flows

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Bypass Transition Modeling Program

Objective: Develop turbulence model to account for effect of free stream disturbances on the laminar boundary layer and its transition region

Scope:

- Understand mechanisms of transition in existing 2-eqn models
- Develop improved modeling for bypass transition mechanism, friction and heat transfer

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Governing Equations:

$$\frac{\partial}{\partial x}(\rho \overline{U}) + \frac{\partial}{\partial y}(\rho \overline{V}) = 0$$

$$\underbrace{\rho \overline{U}\frac{\partial \overline{U}}{\partial x} + \rho \overline{V}\frac{\partial \overline{U}}{\partial y}}_{Convection} = \underbrace{\frac{\partial}{\partial y}[\mu \frac{\partial \overline{U}}{\partial y} - \rho \overline{u'v'}]}_{Diffusion} \underbrace{- \frac{dp}{dx}}_{P_u}$$

$$\underbrace{\rho \overline{U}\frac{\partial \overline{i^*}}{\partial x} + \rho \overline{V}\frac{\partial \overline{i^*}}{\partial y}}_{Convection} = \underbrace{\frac{\partial}{\partial y}[\frac{\mathbf{Pr}}{\mu}\frac{\partial \overline{i}}{\partial y} - \rho c \overline{v't'}]}_{Diffusion} \underbrace{+ \frac{\partial}{\partial y}(\mu \overline{U}\frac{\partial \overline{U}}{\partial y} - \overline{\rho u'v'\overline{U}})}_{P_i^*}$$

Turbulence Modeling:

Turbulent Reynolds Stress Model:

$$-\overline{u'v'} = \frac{\mu_t}{\rho}\frac{\partial \overline{U}}{\partial y}$$

Turbulent Heat Flux Model:

$$-\rho c\,\overline{v't'} = \kappa_t \frac{\partial \bar{t}}{\partial y} = \frac{\mu_t c}{\mathbf{Pr}_t}\frac{\partial \bar{t}}{\partial y}$$

Mixing Length Model:

$$\frac{\mu_t}{\rho} = \ell^2 \frac{\partial \overline{U}}{\partial y}$$

Two-Equation Model:

$$\frac{\mu_t}{\rho} = C_\mu f_\mu \frac{k^2}{\varepsilon}$$

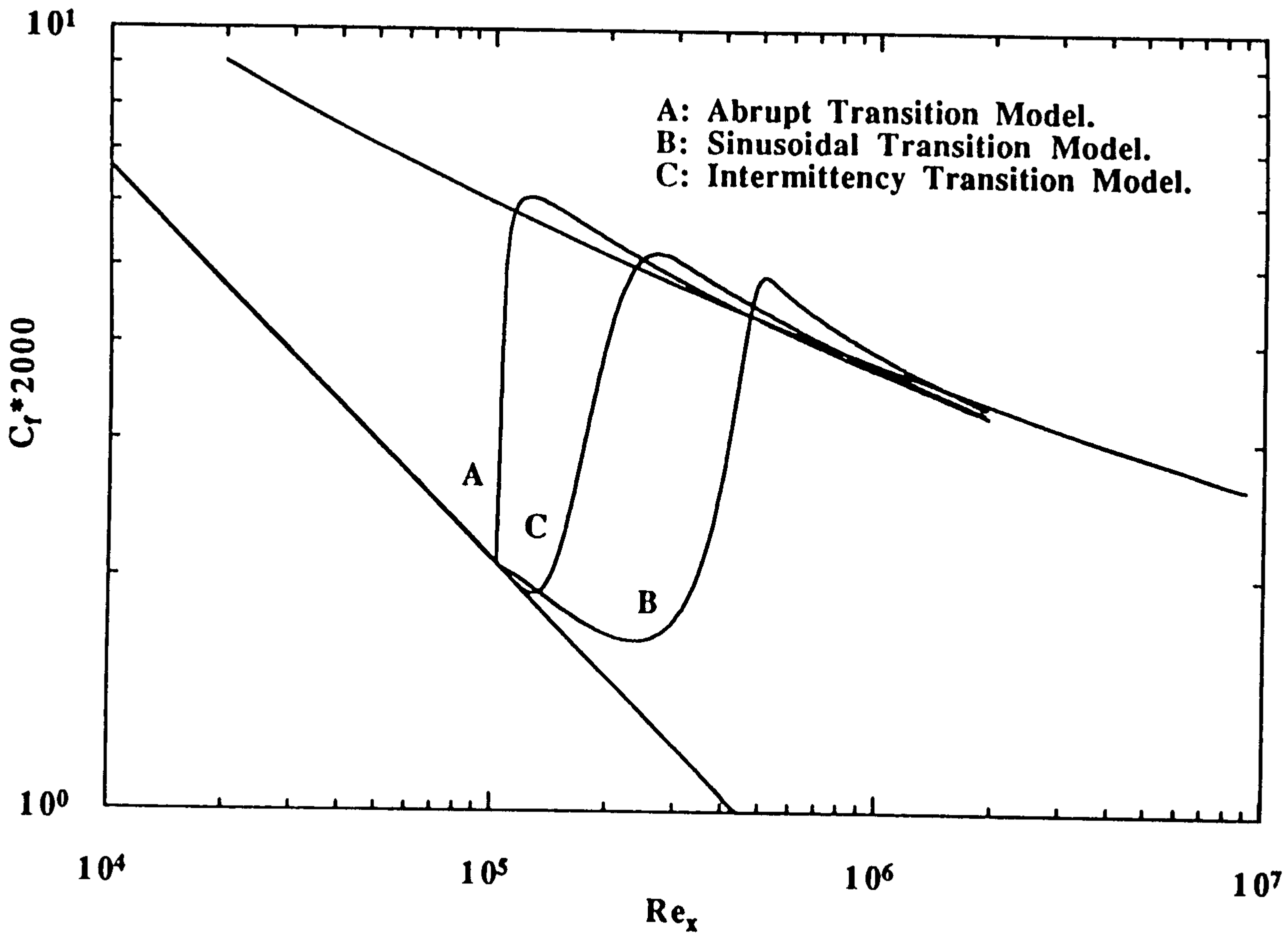


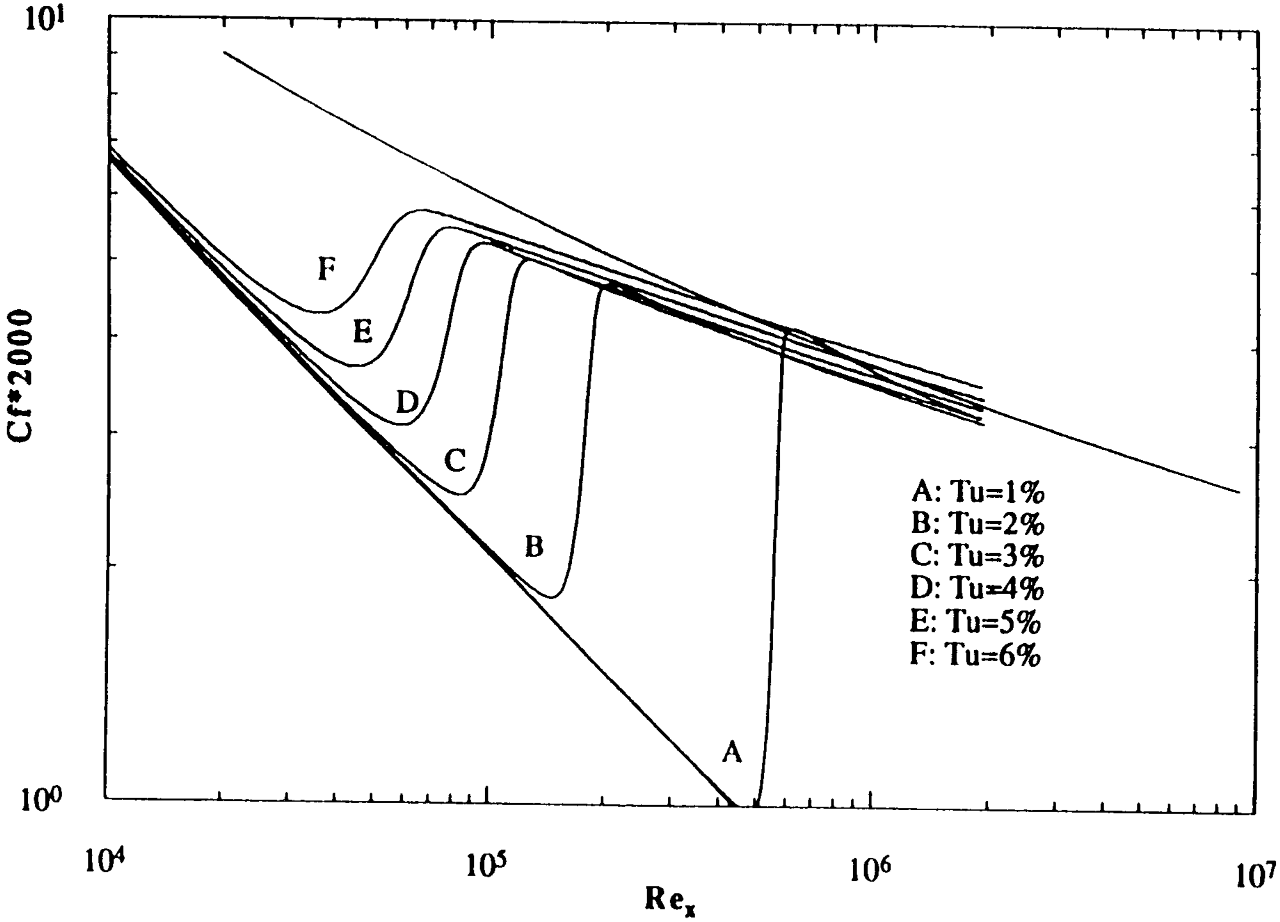
Figure 2: Examples of A^+ and γ calculations of transitional flow over a flat plate with a free stream turbulence level, Tu, of 3%.

Two-Equation Models:

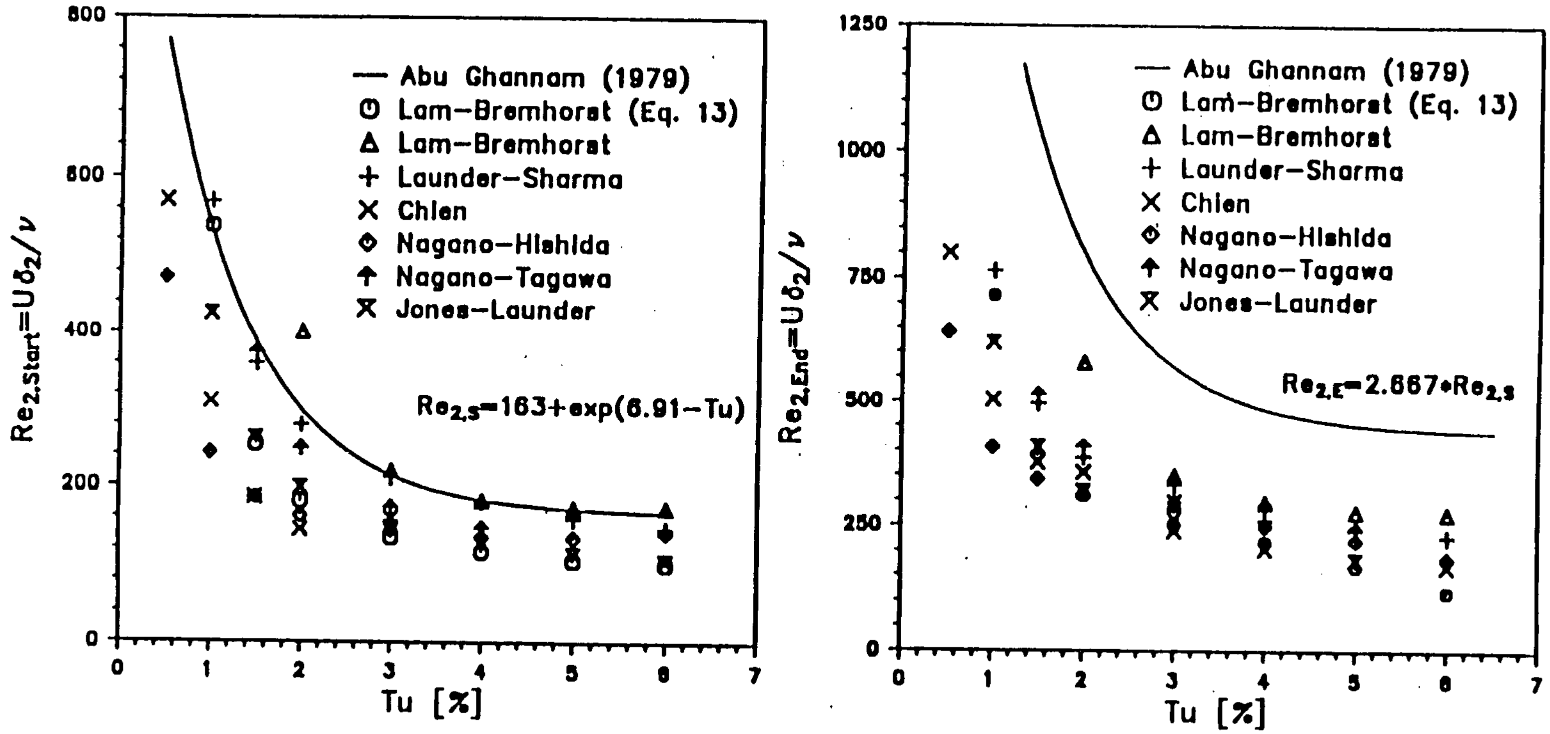
$$\underbrace{\rho\overline{U}\frac{\partial k}{\partial x}+\rho\overline{V}\frac{\partial k}{\partial y}}_{Convection}=\underbrace{\frac{\partial}{\partial y}[(\mu+\frac{\mu_t}{\sigma_k})\frac{\partial k}{\partial y}]}_{Diffusion}+\underbrace{(-\rho\overline{u'v'})\frac{d\overline{U}}{dy}}_{P_k}-\underbrace{\rho\varepsilon}_{D_k}+D$$

$$\underbrace{\rho\overline{U}\frac{\partial \varepsilon}{\partial x}+\rho\overline{V}\frac{\partial \varepsilon}{\partial y}}_{Convection}=\underbrace{\frac{\partial}{\partial y}[(\mu+\frac{\mu_t}{\sigma_\varepsilon})\frac{\partial \varepsilon}{\partial y}]}_{Diffusion}+\underbrace{C_1 f_1\frac{\varepsilon}{k}P_k}_{P_\varepsilon}-\underbrace{\rho C_2 f_2\frac{\varepsilon^2}{k}}_{D_\varepsilon}+E$$

$$\text{where } -\overline{u'v'}=\upsilon_t\frac{d\overline{U}}{dy}=C_\mu f_\mu\frac{k^2}{\varepsilon}\frac{d\overline{U}}{dy}$$



The prediction of transition for flow over flate plate using LS two-equation models



(a.) Start of Transition

(b.) End of Transition

FIGURE 3: Computed momentum thickness Reynolds number at the start and end of transition in comparison to the correlation by Abu Ghannam (1979)

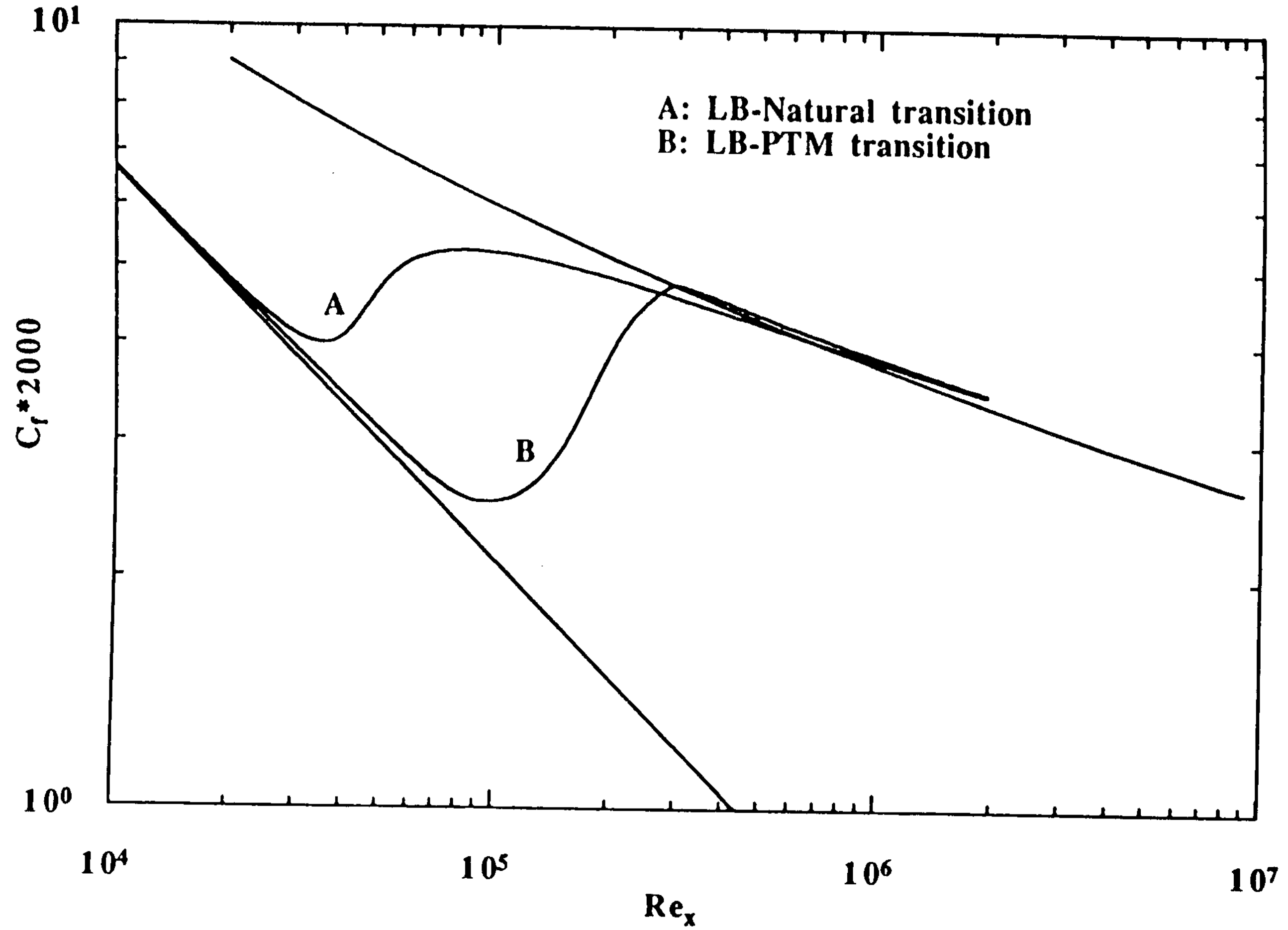


Figure 3: An example of natural transition and PTM transition for free stream turbulence level, Tu, of 3%.

Modified MTS Model:

$$\rho U\frac{\partial K_p}{\partial x}+\rho V\frac{\partial K_p}{\partial y}=\frac{\partial}{\partial y}[(\mu+\frac{\mu_t}{\sigma_{kp}})\frac{\partial K_p}{\partial y}]+P_k-\rho\varepsilon_p$$

$$\rho U\frac{\partial K_t}{\partial x}+\rho V\frac{\partial K_t}{\partial y}=\frac{\partial}{\partial y}[(\mu+\frac{\mu_t}{\sigma_{kt}})\frac{\partial K_t}{\partial y}]+\rho\varepsilon_p-\rho\varepsilon_t$$

$$\rho U\frac{\partial \varepsilon_p}{\partial x}+\rho V\frac{\partial \varepsilon_p}{\partial y}=\frac{\partial}{\partial y}[(\mu+\frac{\mu_t}{\sigma_{\varepsilon p}})\frac{\partial \varepsilon_p}{\partial y}]+C_{p1}P_k\frac{\varepsilon_p}{k_p}+\rho C'_{p1}k_p(\frac{\partial U}{\partial y})^2-\rho C_{p3}\frac{\varepsilon_p^2}{k_p}$$

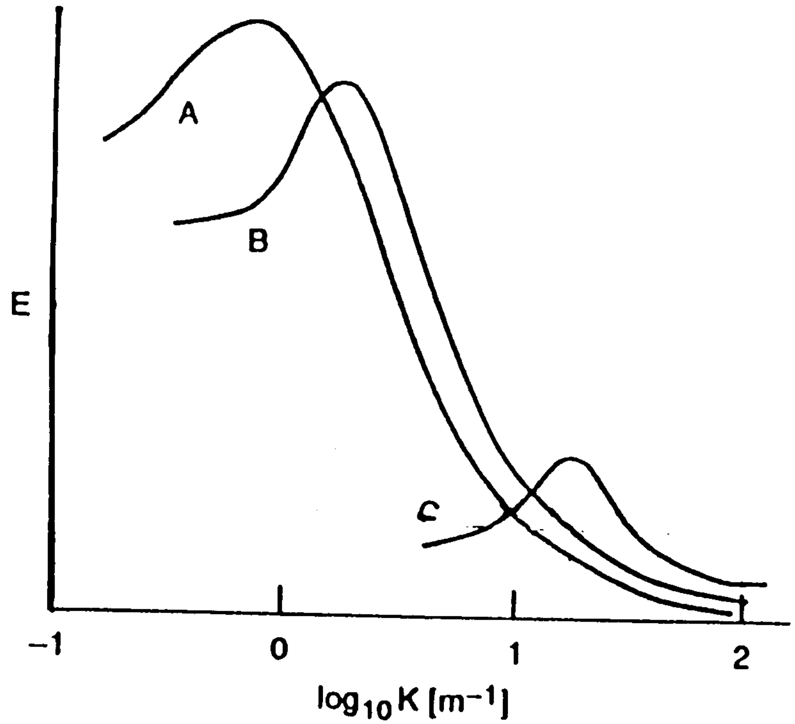
$$\rho U\frac{\partial \varepsilon_t}{\partial x}+\rho V\frac{\partial \varepsilon_t}{\partial y}=\frac{\partial}{\partial y}[(\mu+\frac{\mu_t}{\sigma_{\varepsilon t}})\frac{\partial \varepsilon_t}{\partial y}]+\rho C_{t1}\frac{\varepsilon_t^2}{K_t}+\rho C_{t2}\frac{\varepsilon_p\varepsilon_t}{K_t}-\rho C_{t3}\frac{\varepsilon_t^2}{K_t}$$

$$\mu_t=\rho C_\mu f_\mu\frac{k^2}{\varepsilon_p}$$

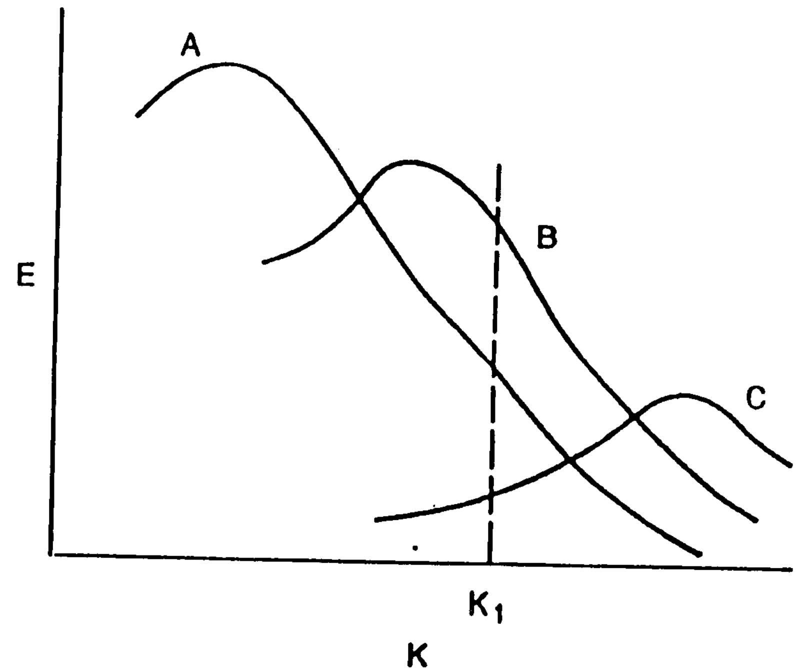
$\sigma_{kp}=0.75$, $\sigma_{kt}=0.75$, $\sigma_{\varepsilon p}=1.15$, $\sigma_{\varepsilon t}=1.15$, $C_{p1}=0.21$, $C'_{p1}=0.11$, $C_{p3}=1.84$, $C_{t1}=0.29$, $C_{t2}=1.28$, and $C_{t3}=1.64$

Boundary Conditions:

$$\varepsilon_p|_w=\varepsilon_t|_w=\nu\partial^2k_p/\partial y^2|_w;\quad k_p|_w=k_p|_t=0$$



(a) Spectral density for inequilibrium turbulent flows, A: maximum shear location in a circular jet [20], B: center of a circular jet [20], C: free stream region of a boundary layer flow in zero pressure gradient [19].



$$k_p = \int_{K=0}^{K_1} E dK, \quad k_t = \int_{K=K_1}^{\infty} E dK$$

(b) k_p/k_t for inequilibrium turbulent flow, A: $Pr/\epsilon_t > 1$, B: $Pr/\epsilon_t = 1$, C: $Pr = 0.0$.

Figure 2.—Spectral density.

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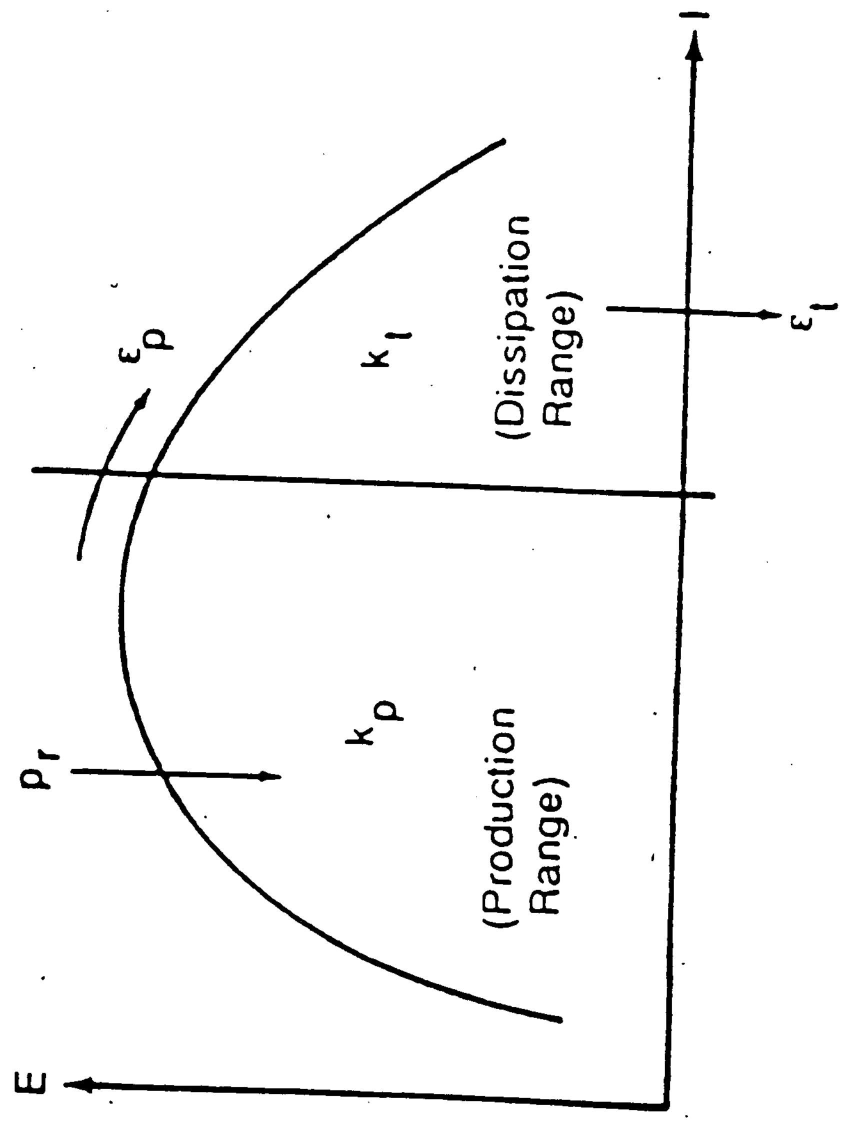

E
l
ε p
k l
(Dissipation Range)
ε l
p r
k p
(Production Range)

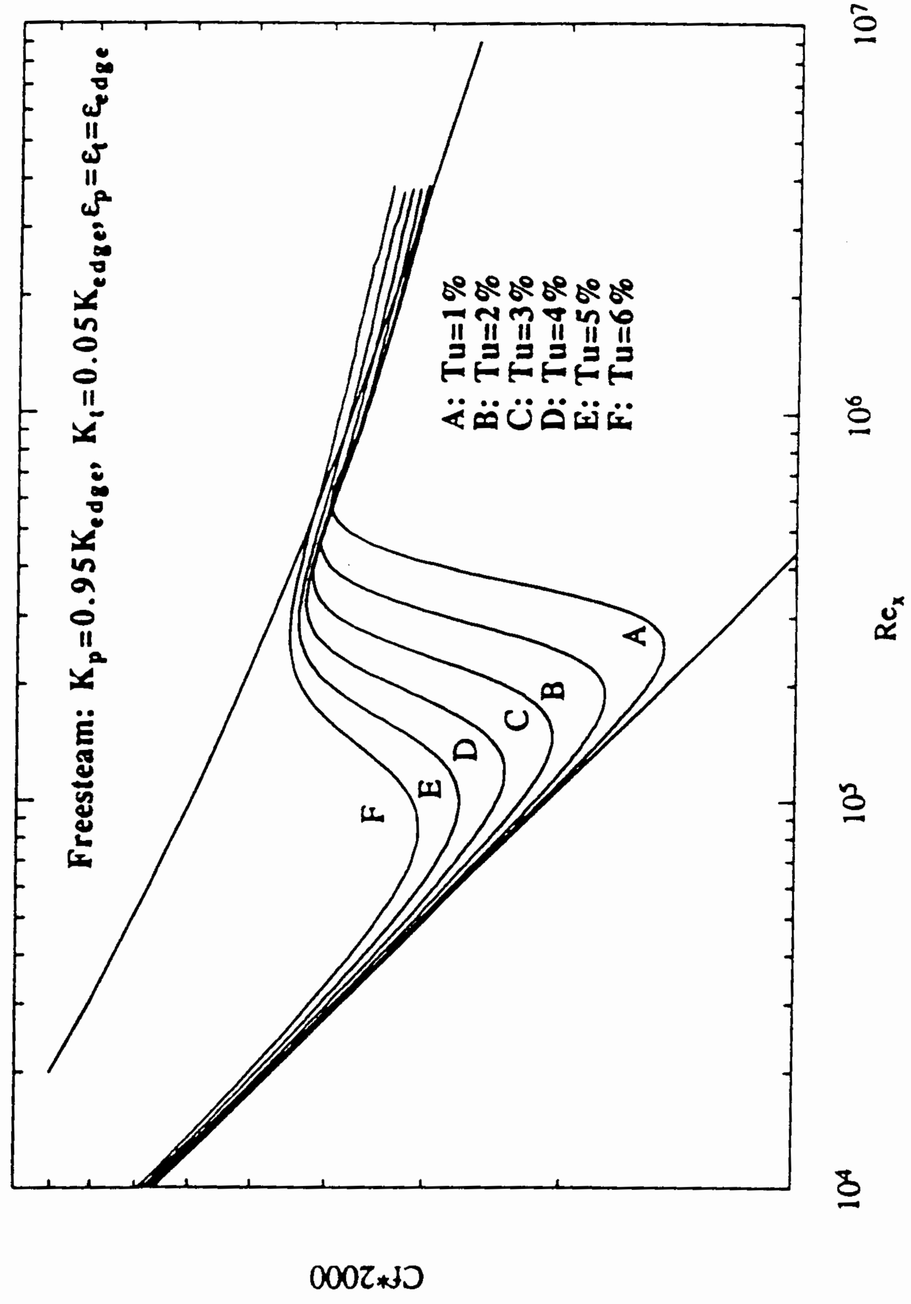
Hanjalic, Launder , Shiestel (1980):

$$\frac{DK_p}{Dt} = -\varepsilon_p$$

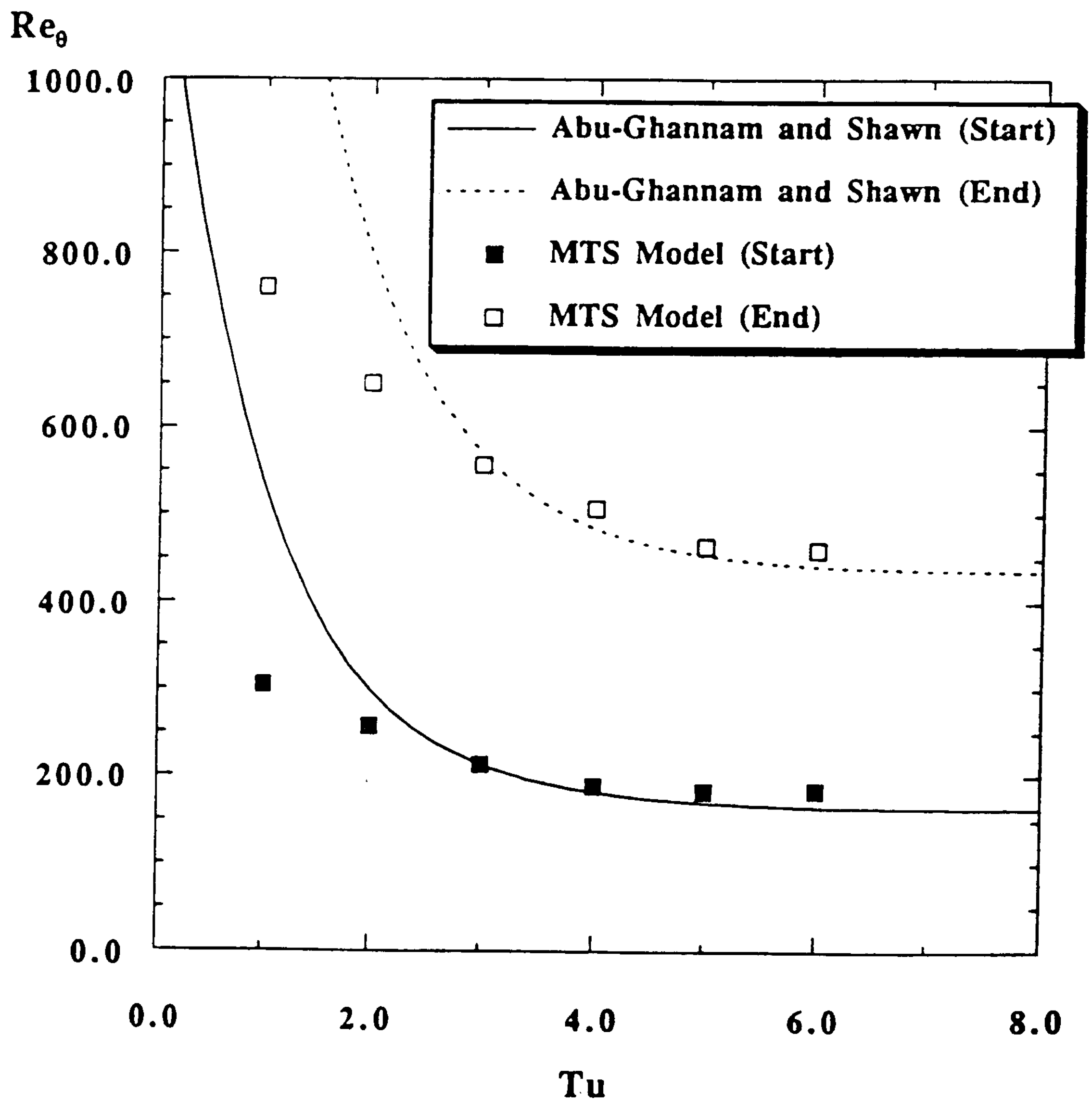
$$\frac{DK_t}{Dt} = \varepsilon_p - \varepsilon_t$$

$$\frac{D\varepsilon_p}{Dt} = -C_{p2}\frac{\varepsilon_p^2}{K_p}$$

$$\frac{D\varepsilon_t}{Dt} = C_{t1}\frac{\varepsilon_p \varepsilon_t}{K_t} - C_{t2}\frac{\varepsilon_t^2}{K_t}$$


Freesteam: $K_p=0.95K_{edge}$, $K_t=0.05K_{edge}$, $\varepsilon_p=\varepsilon_t=\varepsilon_{edge}$
A: Tu=1%
B: Tu=2%
C: Tu=3%
D: Tu=4%
E: Tu=5%
F: Tu=6%
A
B
C
D
E
F
10^4
10^5
10^6
10^7
Re_x
Cf*2000

MTS Model


Re_θ
1000.0
800.0
600.0
400.0
200.0
0.0
Abu-Ghannam and Shawn (Start)
Abu-Ghannam and Shawn (End)
MTS Model (Start)
MTS Model (End)
0.0
2.0
4.0
6.0
8.0
Tu

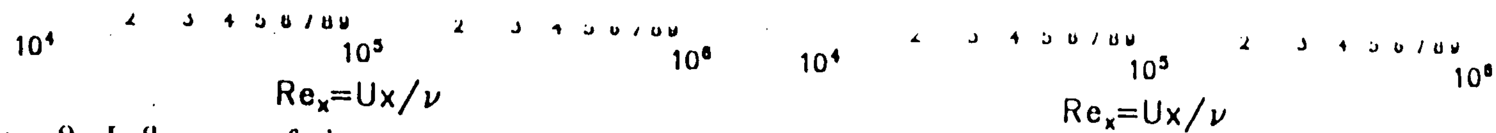


Figure 2. Influence of the starting location on the transition prediction

4.2. Effect of free-stream dissipation length scale

The laminar-turbulent transition depends strongly on the turbulence field in the free-stream, which is characterized by the turbulence level and the dissipation rate (or dissipation length scale) in the k, ϵ typ of model. The importance of a correct specification of the length scale is demonstrated in Fig. 3. Shown are calculations of the skin friction coefficient c_f for the test case T3B performed with the LS model. The corresponding turbulence decay is shown on the right hand side. The experimentally determined turbulence decay is given by the included correlation. The upper bound of the chosen length scales represents a frozen turbulence, i.e. k_∞ is constant. It becomes obvious that severe errors occure in the reproduction of the decay rate if the length scale specified at the starting location is incorrect. Due to this the predicted

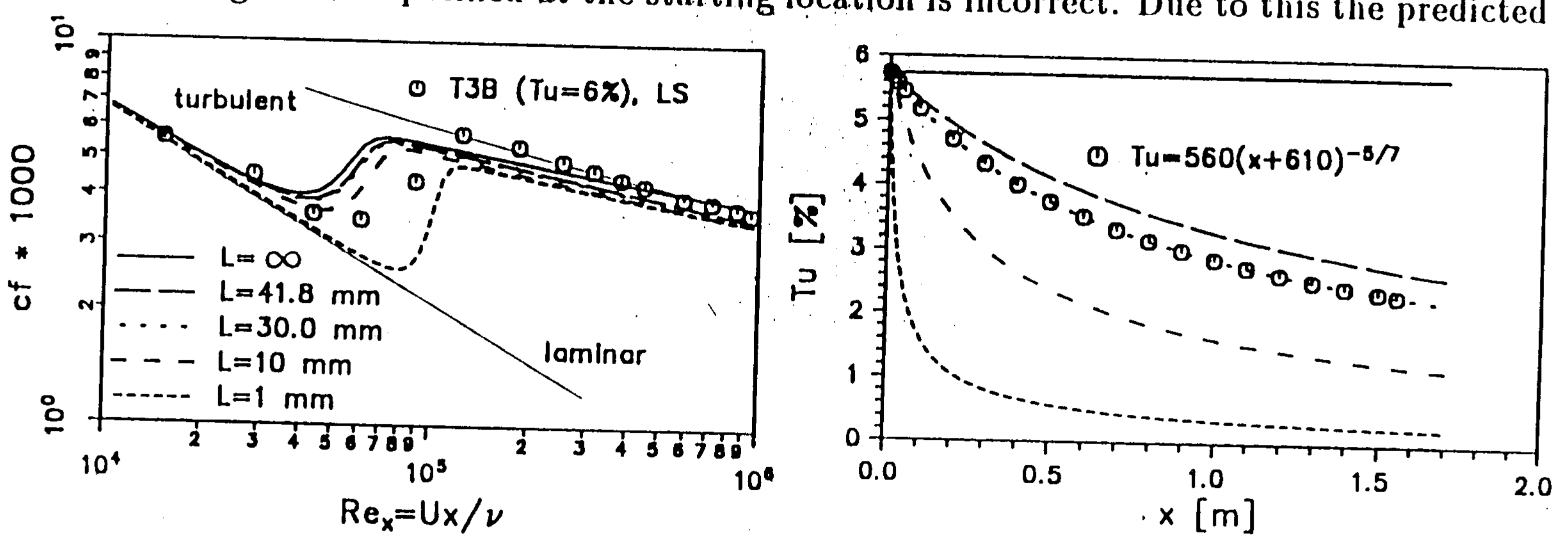
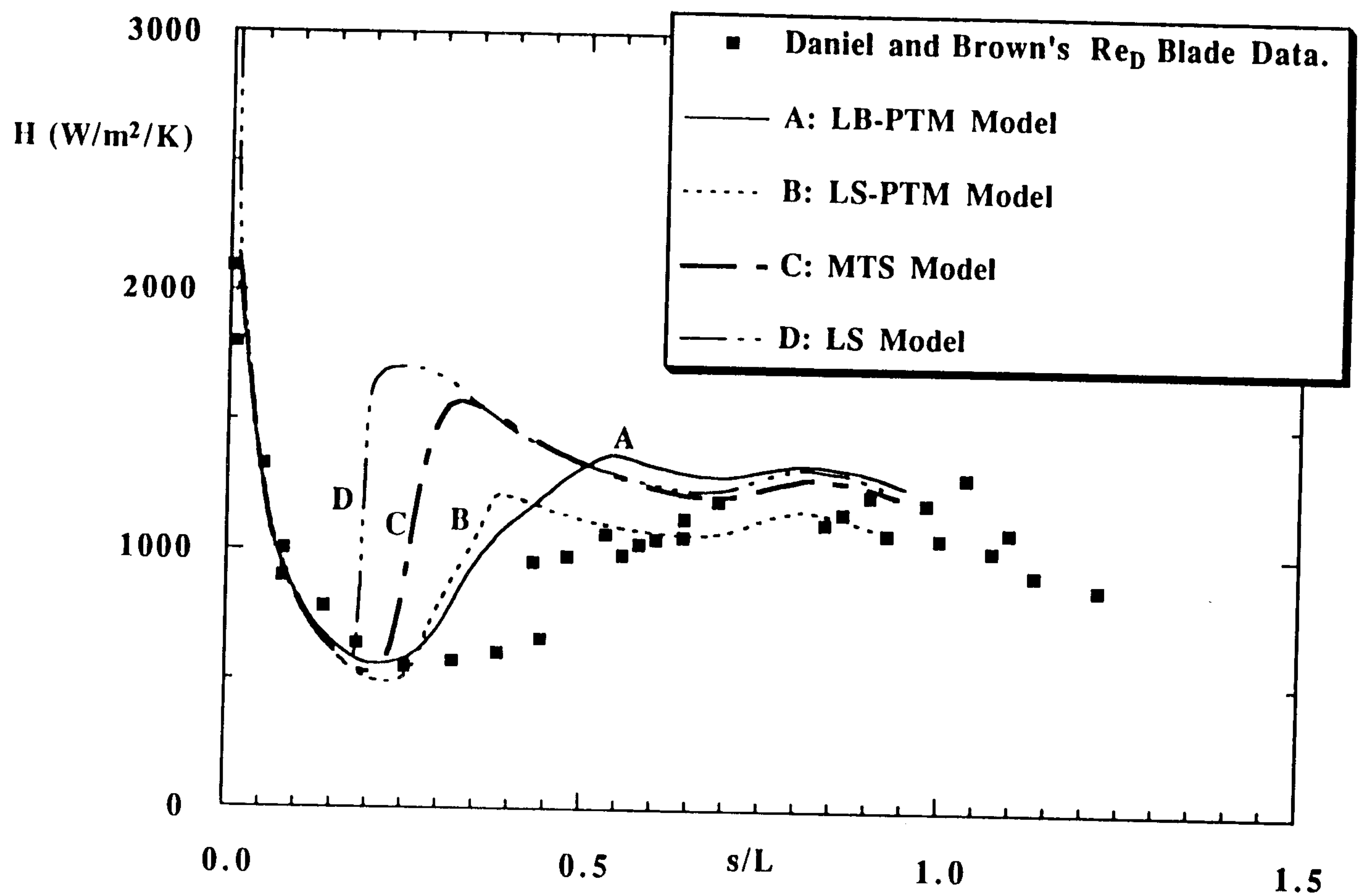


Figure 3. Influence of the free-stream dissipation length scale



The comparison of the predicted heat transfer coefficient with the suction side of Daniel & Brown's blade data.

Conclusions & Recommendations

- Multi-time-scale model shows promise
- Free-Stream partition ?
- Fmu ? Time scales?
- Pressure gradient effect?

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