GUIDANCE ALGORITHMS FOR NON-DRIFTING TRAJECTORY GENERATION AND CONTROL IN RENDEZVOUS MISSIONS INTO ELLIPTICAL ORBITS[1]

Emanuele Di Sotto[†], Juan Carlos Bastante[†], Rémi Drai[‡]

[†] DEIMOS ENGENHARIA, Av. D.João II, Lt. 1.17 10, Lisbon Portugal

[‡] European Space Agency (ESA), ESTEC, Noordwijk, The Netherlands, 2200 AG

emanule.disotto@deimos.com.pt , juan-carlos.bastante@deimos.com.pt ,
Remi.Drai@esa.int

ABSTRACT

Safety requirement represents one of the most critical aspect when defining the operational profile for a RendezVous mission. This requirement specially affects the design of the guidance algorithms that need to be tailored to guarantee what is normally referred to as "Passive Trajectory Protection".

The basic idea of passive trajectory protection is to design all the trajectory elements in an approach sequence such that if, at any point of the trajectory, thrust control ceases, the resulting free drift motion will remain collision free during a certain amount of time.

This paper deals with the design and performances assessment of specific guidance algorithm addressing this issue. Firstly the problem is addressed for circular orbit using the "Traveling Ellipse" formulation for the relative motion. Secondly a solution for the RendezVous into a generic elliptical orbit is presented. This is based on a reduced transition matrix obtained through a description of the relative motion based on the first order variation of the orbital elements.

INTRODUCTION

Safety requirement represents one of the most critical aspects when defining the operational profile for a RendezVous mission. This requirement specially affects the design of the guidance algorithms that need to be tailored to guarantee what is normally referred to as "Passive Trajectory Protection".

The basic idea of passive trajectory protection is to design all the trajectory elements in an approach sequence such that if, at any point of the trajectory, thrust control ceases, the resulting free drift motion will remain collision free during a certain amount of time.

This paper deals with the design and performances assessment of specific guidance algorithm addressing this issue. Firstly the problem is addressed for circular orbit using the "Traveling Ellipse" formulation for the relative motion. Secondly a solution for the RendezVous into a generic elliptical orbit is presented. This is based on a

---

[1] The work presented in this paper was partially performed under the ESA/ESTEC contract for the study of Development of GNC algorithms for Rendezvous and Formation Flying in Non-Circular orbits (GNCO). ESTEC/Contract No. 19495/05/NL/JA/pg

reduced transition matrix obtained through a description of the relative motion based on the first order variation of the orbital elements.

In what regards circular orbit, passive trajectory protection is normally implemented approaching the target vehicle with a series of hops obtained through dedicated maneuvers performed along the local vertical direction, normally referred to as R-BAR axis. This approach guarantees the passive trajectory protection only in the nominal case, when the chaser vehicle is initially located on the target orbit along the axis that is generally referred to as V-BAR. However, non-drifting condition can be achieved also for a generic chaser location into the target orbital plane through the traveling ellipse formulation for the relative motion. It basically consists in describing the analytical solution of the Euler-Hill dynamic system through the motion resulting by a point moving along an ellipse whose centre is also shifting away describing, in this way, a sort of cycloid. This formulation can be effectively used in order to control the approaching path. Correction maneuver is computed to cancel the drifting motion of the ellipse center, (through a dedicated V-Bar maneuver) whereas the maneuver along R-BAR is computed imposing to the resulting ellipse to pass through the initial and final positions.

It is important to remark that this algorithm is not valid when dealing with RendezVous mission into elliptical orbit being the traveling ellipse representation applicable only to circular orbits. On the other side, the interest for performing RendezVous operations in elliptical orbit has recently arisen specially for its application in planetary exploration mission [1]. For Mars Sample Return mission (e.g) this option would allow saving almost 25% in the overall mission mass budget. This aspect mainly drives the effort to derive safety guidance algorithms also for the elliptical orbits scenario.

The algorithm derived for circular orbit using the traveling ellipse formulation can provide a useful suggestion to solve the problem for elliptical orbit if is reformulated in terms of relative orbital elements. Imposing the non-drifting condition means that the chaser orbit semi-major axis is constantly kept equal to the target orbit one for nominal and correction maneuver. Only the chaser orbit eccentricity vector is changed to meet the boundary conditions.

Having this in mind, and using the formulation mapping the Cartesian relative state into linear difference in the orbital target/chaser orbital parameter, a solution has been derived also for a generic elliptical orbit. This solution has been obtained in terms of reduced transition matrix allowing to deriving the trajectory control algorithm for a general elliptical orbit that guarantees the passive trajectory protection.

This algorithm has been prototyped and intensively validated with respect to the expected navigation solution provided through a vision based rendezvous sensor. It has been compared versus more classical trajectory control algorithm based on the inversion of the whole transition matrix (which generally introduces a drift component) in terms of dispersion and total required ΔV. These results show that this non-drifting algorithm allows meeting the passive safety requirements without being adversely affected neither in terms of ΔV nor in terms of dispersion.

NON-DRIFTING PATH FOR CIRCULAR ORBIT

Passive safety concept for RendezVous in circular orbit is normally implemented when designing reference profile (Trajectory Generation). This requirement is generally satisfied imposing that the chaser vehicle approaches the target through a series of nominal maneuvers that are directed along the nadir direction (R-BAR).

This type of maneuvers (V-BAR hops) are used during the Terminal RendezVous Phase (TRP) to close the distance from the chaser vehicle from some kilometers range up to some hundreds of meters. When the target and the chaser vehicle are located on the same orbit, maneuvers directed along the R-Bar perturb only the eccentricity vector of the chaser orbit.

The chaser vehicle will close the target onto an orbit having a different eccentricity but the same semi-major axis. In this way no drifting component is added through the maneuver between the chaser and the target vehicle. If any problems should arise when performing the braking maneuver at V-Bar crossing the chaser will naturally drift back to the initially condition. This strategy leaves the opportunity to enter in a contingency phase without dangerously drifting towards the target vehicle.

However the nominal profile cannot be exactly followed due to different sources of perturbation and system characteristics as: orbital perturbations, engines misalignments and noises and, by far the most important, navigation errors.

To compensate for all these errors correction maneuvers are normally executed when the navigation accuracy increases (generally when closing the range). This represents the Trajectory Control process operated through the guidance function that directly introduces this command within the GNC loop as a feed-forward term.

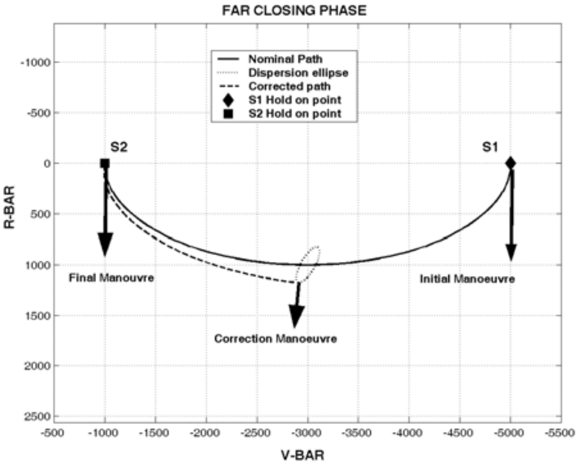


Figure 1 Far-Closing phase bringing the chaser from S1 (5000 km apart from the target) to S2 hold on point. A MCM it is also carried on to counteract for the navigation error at S1

These correction maneuvers are executed through Terminal Point Guidance Algorithm (TPGA) that allows fulfilling the Two Points Boundary Values Problem (TPBVP), which derives by imposing the chaser to fly from the current position up to the final prescribed point.

Two different approaches can be adopted to compute, in a circular RendezVous scenario, this correction maneuvers: the first one based on a typical inversion of the transition matrix and a second one derived by the traveling ellipse formulation. These two approaches will be detailed in the following subsections, however it can be anticipated that only the one based on the traveling ellipse formulation allows to keeping the passive safety requirements.

Classical Approach Based on the inversion of the transition matrix

This method is based on the analytical solution of the Euler-Hill dynamics (well-known as Clohessy–Wiltshire equations). This solution can be usefully expressed in terms of the state transition matrix. This matrix, whose elements are functions of time, establishes a linear relationship between the current state vector (position and velocity) and a future state that has to be attained.

$$\mathbf{X}_f = \Phi(t) \cdot \mathbf{X}_0 \tag{1}$$

This transition matrix is a powerful mean for computing the impulsive maneuvers required by the baseline mission scenario previously described.

Equation (1) can be rearranged in the following way:

$$\begin{aligned} \mathbf{r}(t) &= \Phi_{rr}(t)\mathbf{r}_0 + \Phi_{r\dot{r}}(t)\dot{\mathbf{r}}_0 \\ \dot{\mathbf{r}}(t) &= \Phi_{\dot{r}r}(t)\mathbf{r}_0 + \Phi_{\dot{r}\dot{r}}(t)\dot{\mathbf{r}}_0 \end{aligned} \tag{2}$$

where the transition matrix has been properly split into sub-matrices that explicitly relate initial position/velocity vector $(\mathbf{r}_0, \dot{\mathbf{r}}_0)$ with the current one $(\mathbf{r}, \dot{\mathbf{r}})$.

Once defined a certain time-of-flight equation (2) can be inverted to derive the required initial velocity ($\dot{\mathbf{r}}_0$) to transfer the vehicle from the initial position ($\mathbf{r}_0$) up to the final one ($\mathbf{r}_f$)

$$\dot{\mathbf{r}}_0 = \Phi_{r\dot{r}}^{-1}\left(\mathbf{r}_f - \Phi_{rr}\mathbf{r}_0\right) \tag{3}$$

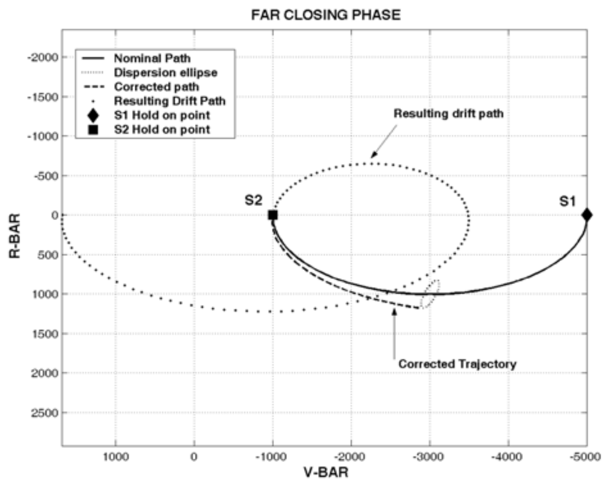


Figure 2 Mid Course Manoeuvre correction and resulting free drift path in case of engine failure at V-bar crossing

Equation (3) represents a classical terminal point guidance that can be always used to attain a final prescribed position in a rendezvous problem. A certain attention must be paid when inverting the sub-matrix $\Phi_{r\dot{r}}$ (that relates the initial velocity with the final position) being singular when time-of-flight is an integer number of half the orbital period.

However, this algorithm does not allow keeping the passive safety condition in the resulting trajectory. When it is used for computing (e.g.) a MCM during a closing transfer it introduces, in the subsequent path, a drift component that could represent a serious risk for the target in case of engine failure.

In Figure 2 it is represented the case corresponding to the MCM foreseen for a far-closing phase bringing the chaser from 5km to 1 km range. The correction manoeuvre has been computed through the guidance algorithm reported in equation (3) and the resulting path brings the chaser to the S2 station keeping (SK) point. However, the manoeuvre has excited the drift component and, in case of engine failure at S2 the chaser will drift away of more than 2 km in only one orbital period.

A different approach it is proposed that allows attaining the designed final position without introducing any drift component. This method basically relies on the so-called “travelling ellipse” formulation.

Non-Drifting approach based on the “Travelling Ellipse” formulation

The analytical solution of the Euler-Hill dynamic system shows that, whatever the relative motion in the orbital plane, it can be described by a point moving along an ellipse whose centre is also shifting away (a sort of cycloid).

This formulation is called “travelling ellipse” [2]. The main features of this formulation of the motion are:

- Ellipse semi-major axis is twice the semi-minor axis.
- Ellipse drifting component is directed along the orbital velocity.
- All the ellipse parameters can be related with the initial state vector.
- The point is moving along the ellipse with the orbital frequency

This formulation can be explicitly reported only after introducing a reference system that, as usual in rendezvous literature, is the Local Vertical Local Horizontal reference frame (LVLH). This reference is normally centred in the target CoM and its axes are oriented as follows:

- X-axis is oriented along the velocity vector (only for circular orbit) and is generally referred to as: V-bar
- Y-axis along the orbital momentum but in the opposite direction
- Z-axis towards the centre of the planet

In the LVLH the in-plane component of the equation of motion can be expressed as [3]

$$\begin{aligned} x &= x_c(t) - 2b\cos(\omega t + \varphi) \\ z &= z_c + b\sin(\omega t + \varphi) \end{aligned} \tag{4}$$

where the terms x_c, z_c represent the coordinate of the centre of the ellipse. It should be remarked that the V-bar component has a linear dependence with time that characterizes the drift component in the relative motion. This drift component is related to the initial state vector through the following relationship:

$$\begin{aligned} x_c(t) &= x_0 + 2\frac{\dot{z}_0}{\omega} + \frac{3}{2} z_c \omega t \\ z_c &= 4z_0 - 2\frac{\dot{x}_0}{\omega} \end{aligned} \tag{5}$$

These equations put into evidence that the ellipse must to be centred on V-bar ($z_c = 0$) to cancel out any drift component from the relative motion. It leads to the conclusion that, to leave the chaser onto a pure elliptical path, the initial velocity along V-bar must be equal to

$$\dot{x}_0 = 2z_0\omega \tag{6}$$

The point now is deriving an expression that will provide the R-Bar velocity component that will inject the chaser into a trajectory intersecting the final prescribed point. This can be obtained imposing that the resulting elliptical path has to pass through the initial position and the final one.

$$\dot{z}_0 = \left[\frac{x_f - x_0}{4} - \frac{z_0^2 - z_f^2}{x_f - x_0}\right]\omega \tag{7}$$

Equations (6) and (7) can be used to address a terminal point guidance scheme that will bring the chaser from the initial to the final position without adding any drift component in the resulting trajectory.

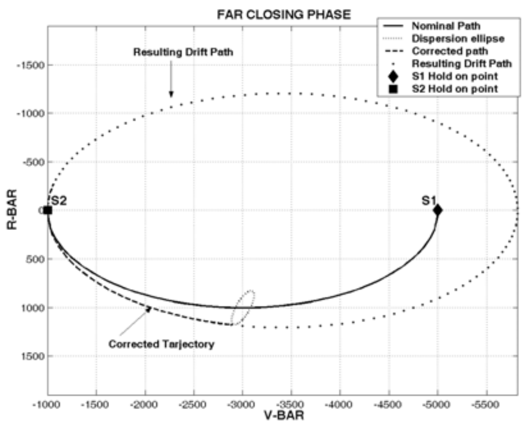


Figure 3 MCM carried out using the elliptical injection algorithm

This is presented in Figure 3 where this algorithm has been used to perform a MCM during the far-closing phase. The corrected path has been further propagated during ¾ of the orbital period after attaining V-Bar simulating in this way a total engine failure at V-Bar crossing. The chaser will come back to the initial position leaving to the onboard system and to the ground specialist the opportunity of recovering the original mission profile.

An important asset for adopting this approach is represented by the required ΔV. At a first glance it could seem that this strategy is more demanding in terms of propellant due to the need of cancelling the drift component. However, an exhaustive analysis, trading the elliptical injection against the classical approach, has shown good performance also in terms of required ΔV. MonteCarlo analysis has been run to assess the performance with respect to the expected dispersion (worst case) at the MCM firing.

Elliptical injection requires almost the same total ΔV used by a classical algorithm with slightly better performance, as reported with deeper details in Table 1. Here the statistics in terms of total ΔV is reported taking into account all the three boosts required: initial maneuver, correction maneuver and final maneuver (see Figure 1). The last two rows represent the increasing of DV with respect to the nominal value (1.7 m/s) due to the correction maneuver for both the analyzed algorithms.

Table 1: ΔV performances derived through a MonteCarlo analysis trading the classical MCM algorithm versus an elliptical injection algorithm

Overall ΔV Statistics	Classical MCM	Elliptical MCM
ΔV Mean	2.026 [m/s]	2.008 [m/s]
ΔV STD	0.248 [m/s]	0.274 [m/s]
ΔV increasing Mean	19.8%	18.0%
ΔV increasing STD	15.3%	16.1%

This approach, constraining the trajectory to be a pure elliptical path, reduces the degree of freedom of the system. Time-of-flight is no more an independent variable but it is linearly related to the difference in the phase angle existing between the initial and final position. From equation (4) it can be derived:

$$t_{of} = \frac{1}{\omega}\left[\tan^{-1}\left(\frac{2z_f}{x_c - x_f}\right) - \tan^{-1}\left(\frac{2z_0}{x_c - x_0}\right)\right] \tag{8}$$

This equation allows computing the time-of-flight once known the initial/final positions. It should be noted that the constraint posed on the time-of-flight represents a clear drawback of the proposed approach when compared with a classical terminal point guidance scheme (3). This constraint could in general compromise all the synchronization requirements posed on a typical rendezvous mission.

An exhaustive analysis has been carried out within the mission scenario defined for Mars Sample Return (circular scenario) mission considering the expected relative navigation error. Time-of-flight, as resulting from a MCM, has a standard deviation with respect to the nominal (say a quarter of orbit) of about 200 seconds. This value does not represent any risk with respect to the synchronization needs of the mission profile.

NON DRIFTING PATH FOR ELLIPTICAL ORBIT SCENARIO

Mission benefits in performing RV operation in elliptical Orbit

In the frame of the Mars Sample Return mission a preliminary design has been performed in the context of ESA-Aurora program in which a circular orbit around Mars is proposed for the orbiter vehicle [4]. The orbiter and the Mars Ascent Vehicle (MAV) will carry out autonomous rendezvous operations in this orbit to transfer the collected sample from one vehicle to the other. it appears worth investigate also the elliptical orbit scenario for several advantages provided to the overall mission design.

The elliptic orbit represents a very appealing solution due to the possibility of saving considerably propellant mass at least in two of the most critical maneuvers to be faced with the Orbiter propulsion system: Target Orbit Acquisition (TOA) maneuver and the Mars Escape maneuver (ME).

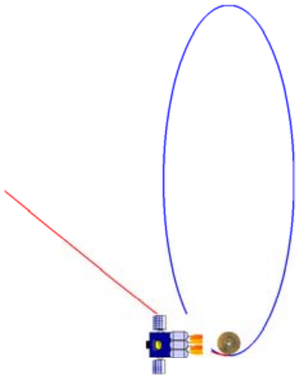


Figure 4 Mars Orbit Insertion (MOI) manoeuvre

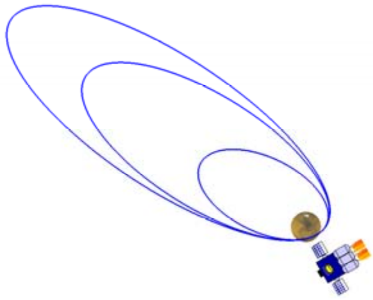


Figure 5 Orbiter Target Orbit Acquisition (TOA) manoeuvres

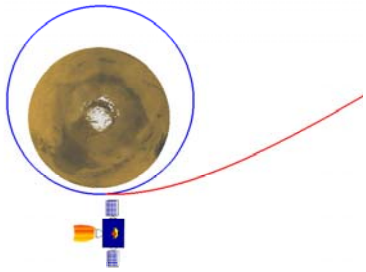


Figure 6 Orbiter Mars Escape Manoeuvre (ME)

Moreover, elliptical orbit would allow saving ΔV also for the rendezvous maneuver foreseen during the approach phase. In this phase, the most demanding maneuver is the one dedicated to bring the two vehicles into coplanar obits, due to eventual difference in orbital inclinations and orbital nodes. An elliptical orbit would permit carrying out these maneuvers with a substantial benefit for the required propellant mass

Taking into account the current design of the Mars Ascent Vehicle (MAV) and its performance, an elliptical orbit has been defined for the Elliptical RV scenario around Mars. This orbit has been obtained optimising the MAV ascent profile in order to maximize the final energy of the attained orbit while constraining several typical parameters (details can be found in [1]). The results show that an elliptical orbits of 2000 km apocenter altitude is attainable through an intermediate ballistic arc. This turns in about 1000 kg propellant mass saved for the orbiter overall mass budget. This amount results from the cascade effect that the saved ΔVs have on the overall mass budget and represents 25% of the current orbiter mass defined for the circular scenario.

Terminal RendezVous Phase and nominal Mission Profile

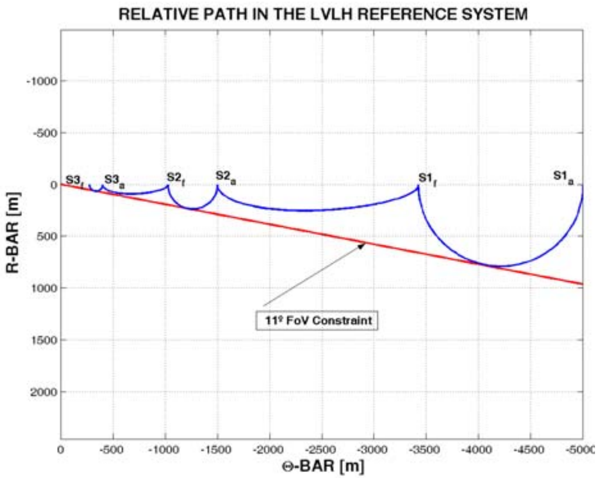
The terminal rendezvous phase, starting at 5 km range and terminating at the docking/capture condition, have been specifically designed for this elliptical scenario. Among the different system and mission requirements and constraints, passive safety requirement has been specifically taken into account when designing the two main hops bringing the chaser form 5 km range up to 275 meters.

Before giving all the details of the non-drifting algorithm developed for elliptical orbit, appears important to briefly introduce the baseline for the terminal rendezvous phase considered within this study. Further details can be found in [1].

Figure 7 presents the relative path considered for the baseline mission scenario. The reference used to presents the relative path is the typical LVLH system. However, the same trajectory is reported in Figure 8 where a different reference system is used and is here referred to as “Orbital reference System”. The latter has one axis constantly oriented along the target velocity (V-Bar) whereas the other is normal to the previous (N-BAR)

It can be seen in Figure 8 as the chaser is initially located 5 km behind the target and it is positioned on the same elliptical orbit (on V-BAR). This initial position sees the chaser at the pericenter location so that it will naturally drift towards the target during the following half an orbit.

Once reached the apocenter the chaser vehicle will perform a two-boost manoeuvre (Far-Closing phase) to reach a closer range. This sequence will be repeated (through a closing phase) to reach the final position of 275 meters from the target.



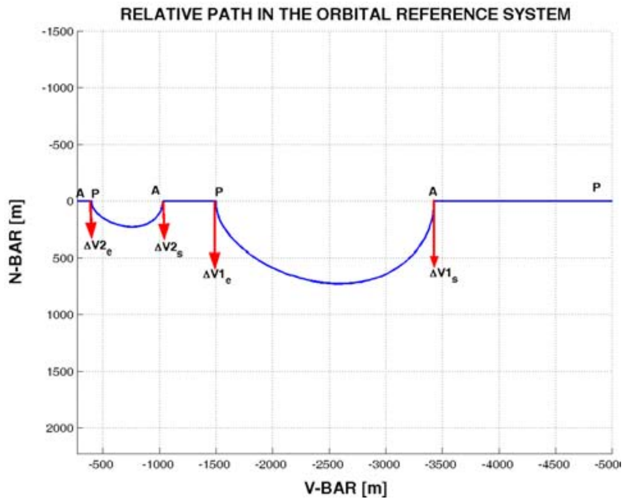
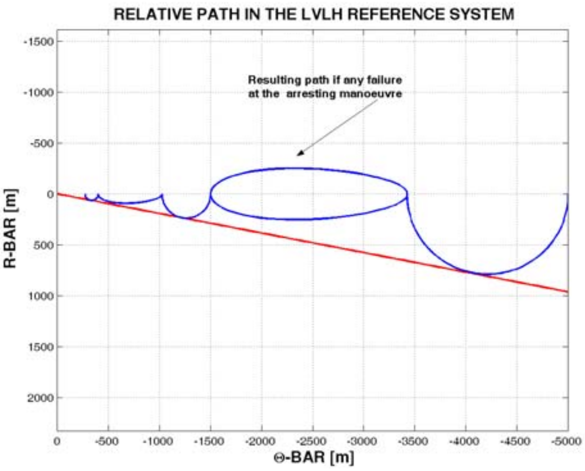


Figure 7 Nominal TRP profile represented in the LVLH reference system

Figure 8 Nominal TRP profile represented in the Orbital reference system (V-BAR, Normal-Bar)

It should be noted that these two boost manoeuvres are not adding any drift component to the resulting path. The orbital semi-major axis of the chaser vehicle is kept equal to the one of the target vehicle. Safety constraints are in this way fully satisfied. If any failure should occur when executing the arresting manoeuvre, the chaser will drift back to initial position as reported in Figure 9 where the resulting path is described in the LVLH and in the orbital reference frame (see Figure 10). This strategy, in close similarity with the circular orbit, allows to satisfying two important requirements: safety and mission recoverability



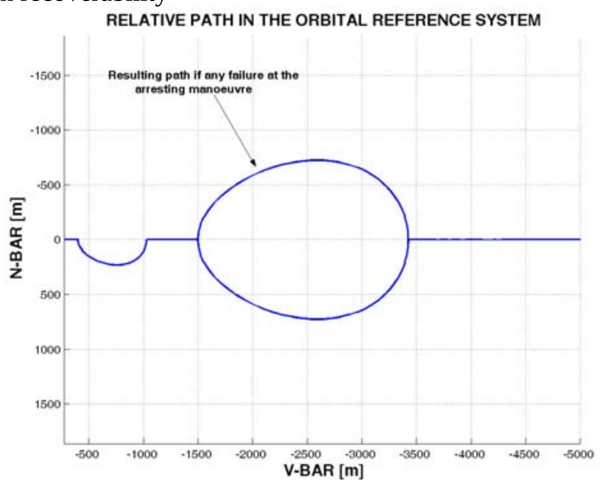


Figure 9 Far Closing manoeuvre where an engine failure is simulated when attaining V-BAR (LVLH representation)

Figure 10 Far Closing manoeuvre where an engine failure is simulated when attaining V-BAR (Orbital frame representation)

Non-Drifting Algorithm for Elliptical Orbit: Reduced transition matrix approach

The passive safety requirement can be fulfilled at guidance level also for the elliptical scenario. However, for the elliptical orbit, the analytical solution available for describing the equation of motion is by far more complex than the one available for the circular scenario.

While a transition matrix formulation is still available through the Yamanaka-Ankersen solution [5] the traveling ellipse (see equation (4)) is no more available for the elliptical scenario. A solution can be still obtained in order to perform nominal and correction maneuvers without altering the chaser semi-major axing and not introducing any drift between the chaser and the target vehicle.

This approach has been derived also for elliptical orbits using the formulation relating the state vector in the LVLH reference system with the orbital elements variation. This formulation is available in literature [6] and can be also used to derive the system transition matrix that is perfectly equivalent to the Yamanka-Ankersen formulation [5]

The formulation mapping the δElements (meaning the difference between the target and chaser orbital elements) is schematically reported in equation (9)

$$\begin{bmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial a} & \frac{\partial x}{\partial e} & 0 & 0 & \frac{\partial x}{\partial \psi} & \frac{\partial x}{\partial M_0} \\ 0 & 0 & \frac{\partial y}{\partial i} & \frac{\partial y}{\partial \Omega} & 0 & 0 \\ \frac{\partial z}{\partial a} & \frac{\partial z}{\partial e} & 0 & 0 & 0 & \frac{\partial x}{\partial M_0} \\ \frac{\partial \dot{x}}{\partial a} & \frac{\partial \dot{x}}{\partial e} & 0 & 0 & \frac{\partial \dot{x}}{\partial \psi} & \frac{\partial \dot{x}}{\partial M_0} \\ 0 & 0 & \frac{\partial \dot{y}}{\partial i} & \frac{\partial \dot{y}}{\partial \Omega} & 0 & 0 \\ \frac{\partial \dot{z}}{\partial a} & \frac{\partial \dot{z}}{\partial e} & 0 & 0 & 0 & \frac{\partial \dot{z}}{\partial M_0} \end{bmatrix} \cdot \begin{bmatrix} \delta a \\ \delta e \\ \delta i \\ \delta \Omega \\ \delta \psi \\ \delta M_0 \end{bmatrix} \tag{9}$$

where the δElements are defined as follows

- δa: semi-major axis difference
- δe: eccentricity difference
- δi: inclination difference
- δΩ: right ascension of ascending node difference
- δψ: it is a linear combination of δΩ and δω (argument of pericenter) given in equation (10)where (*i*) represents the inclination of the target orbit.
- δM_0: Initial Mean Anomaly difference

$$\delta\psi = \cos(i)\delta\Omega + \delta\omega \tag{10}$$

A rapid analysis of equation (9) shows that:

- The out-of-plane motion (y-coordinate) is affected only by those δΩ and δi, say by those δElements related with the difference in the orbital plane orientation.
- The evolution of the nadir component of the motion (z-coordinate) is driven only by those δElements related with the orbit dimension (δa and δe) and with the difference in time between the target and chaser passage through a specific landmark (δM_0).
- The Local Horizontal component of the motion (x-coordinate), apart from the orbital dimension (δa and δe) and chaser location ($\delta\psi$,δM_0), is also affected by the orbital plane orientation δΩ.

Formulation reported in equation (9) can be properly used to derive algorithms that allow computing the required initial velocity to bring the chaser vehicle from the current initial location to the prescribed final location without introducing any change in the orbital semi-major axis (δa).

Algorithm derivation follows the main steps identified through the following subsections

<u>Decoupling of the in-plane motion</u>

Only the in-plane motion is represented as reported in equation (11), being the out-of-plane motion completed decoupled. In equation (11) the matrix *R* is composed by the corresponding terms already presented in equation (9)

$$\begin{bmatrix} x \\ z \\ \dot{x} \\ \dot{z} \end{bmatrix} = R \cdot \begin{bmatrix} \delta a \\ \delta e \\ \delta\psi \\ \delta M_0 \end{bmatrix} \tag{11}$$

Variation of semi-major axis (δa) is imposed to be zero

Variation of semi-major axis δa is imposed to be zero. This condition establishes a linear relationship between the in-plane state components $\{x, z, \dot{x}, \dot{z}\}$ that can be obtained inverting the system reported in equation (11).

In-Plane mapping reduction

Imposing the condition δa=0 allows to reducing the system reported in (11). It is here chosen to assume as dependent variable the velocity along the Local Horizontal $(\dot{x})$, so that the system reported in equation (11) reduces of one degree of freedom and can be expressed in the following form:

$$\begin{bmatrix} x \\ z \\ \dot{z} \end{bmatrix} = D_{XE} \begin{bmatrix} \delta e \\ \delta\psi \\ \delta M_0 \end{bmatrix} \tag{12}$$

In equation (12) the matrix D_{XE} can be derived by the corresponding terms reported in equation (9). They represent the linear coefficient relating the three states in-plane components and the corresponding δElements.

Reduced Transition Matrix Formulation

The following consideration represents the core of the algorithm. Once assigned an initial state vector the δElements vector remains fully identified, as reported in equation (13) where the suffix (0) indicates that the matrix D_{XE} is computed at the initial time and at the initial true anomaly.

$$\vec{X}_0 = D_{EX0}\delta\vec{E} \tag{13}$$

The identified set of δElements will remain unchanged till a new manoeuvre is executed, so that it can be stated that at a final prescribed time the relation between the final state and δElements will be given by equation (14)

$$\vec{X}_F = D_{EXF}\delta\vec{E} \tag{14}$$

It is clear at this stage that the initial and final state can be related by imposing that the same δElements are involved in equation (13) and equation (14) if no manoeuvre is executed in between.

$$\vec{X}_F = D_{EXF}\left(D_{EX0}\right)^{-1}\vec{X}_0 \tag{15}$$

Equation (15) can be useful rewritten introducing a *Reduced Transition Matrix* Φ_r. Here the term “reduced” referred to as a 3x3 transition matrix imposing the condition δa=0.

$$\begin{bmatrix} x_F \\ z_F \\ \dot{z}_F \end{bmatrix} = \Phi_r \begin{bmatrix} x_0 \\ z_0 \\ \dot{z}_0 \end{bmatrix} \tag{16}$$

Derivation of the required initial velocity

It is now possible to derive the initial velocity along R-BAR that solves the TPBVP, say assigned the initial in-plane position (x_0, z_0) the $\dot{z}_0$ velocity which guarantees the final position (x_F, z_F) is reached in a time that is a dependent variable. It is important to remark that the time-of-flight cannot be chosen a priori but results from solving the following non-linear system:

$$\begin{aligned} x_F &= \phi_{xx0}(t)x_0 + \phi_{xz0}(t)z_0 + \phi_{x\dot{z}0}(t)\dot{z}_0 \\ z_F &= \phi_{zx0}(t)x_0 + \phi_{xz0}(t)z_0 + \phi_{z\dot{z}0}(t)\dot{z}_0 \end{aligned} \tag{17}$$

The formulation reported in equation (17) allows to computing the non-drifting path for whatever set of initial and final positions. However it should be noted that once established an initial value for the initial true anomaly of the target orbit, the final true anomaly (say the time-of-flight) is part of the non-linear system expressed in equation (17).

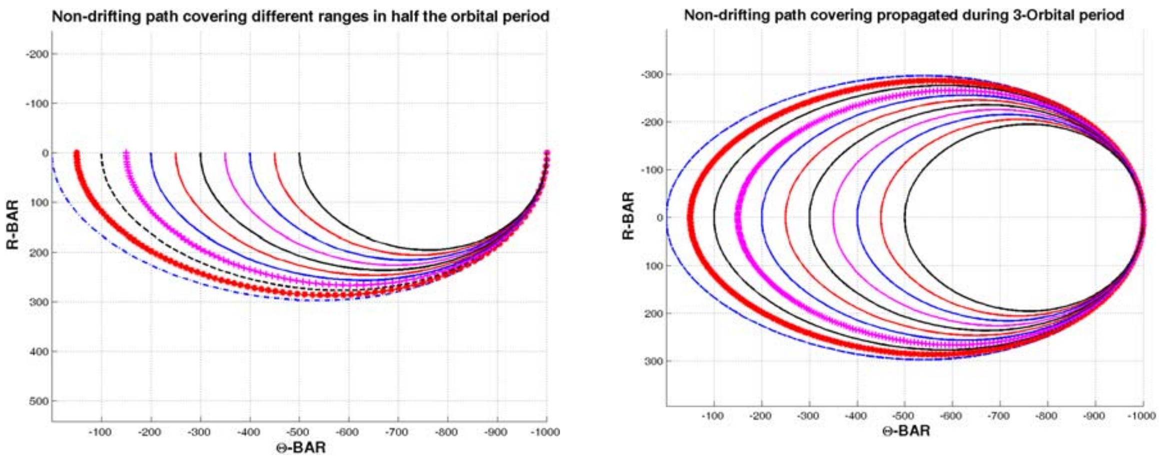


Figure 11 Non-drifting path covering different ranges on Local Horizontal (Θ-Bar) axis

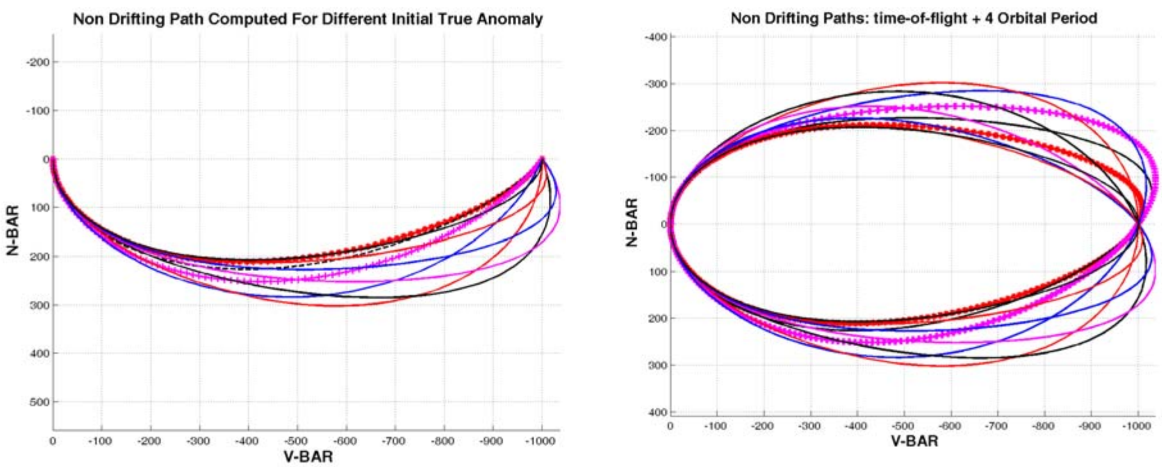


Figure 12 Non-drifting paths computed for different initial true anomaly at 1000 meters range

Figure 11 presents some non drifting paths obtained through the application of the algorithm described in (17) to transfer the chaser vehicle from 1000 meters range to different range in half the orbital period. On the right side the same trajectories are reported propagating during 3 orbital period. It can be seen that no drifting component is added and the paths come back exactly to the initial position in case the final stopping maneuver should be missed.

Figure 12 presents the same characteristic but imposing the covered range along the V-Bar axis and considering different initial true anomalies. It should be said that the relative paths reported in Figure 11 Figure 12 have been obtained for a small eccentricity (ecc = 0.18) defined for the mission scenario considered within this study.

In order to validate the algorithms sensitivity with the eccentricity a higher apocenter orbit has been assumed with an apocenter altitude of 20000 km. This leads (assuming 300 km pericentre) to an orbit eccentricity of 0.72. The results of this analysis have been presented in Figure 13 where it is seen that all the paths computed at different initial true anomalies (so implying the solution of the non-linear system reported in equation (17) reaches the desired target point without introducing any drift component in the relative motion.

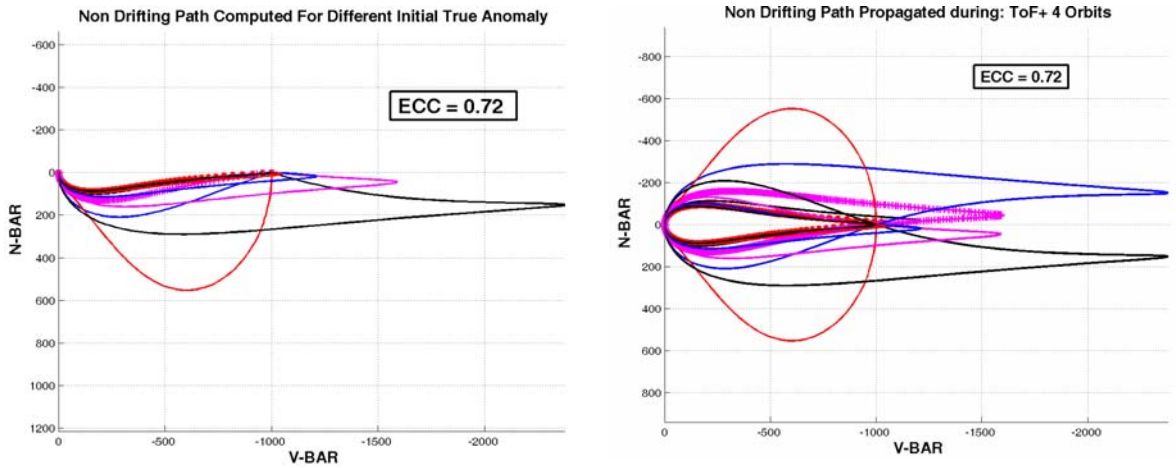


Figure 13 Non-Drifting path bringing the chaser from 1000 meters range up to the target location in an high elliptical orbit

Some specific analytical solution

A systematic validation of the non drifting algorithm as shown that a particular case of non-drifting path, linking two different locations on the Local Horizontal axis (Θ-BAR), can be obtained under the following conditions

- ϑ_0 = 0|pi: Initial true anomaly corresponding to the apo/pericenter locations
- ToF = Period/2. Time-of-flight is equal to half the orbital period

Under these conditions the required initial velocity is directed along the R-BAR (z-component) and can be provided directly using the first one of equation (17) as reported in equation (18)

$$\dot{z}_0 = \frac{(x_F - \phi_{xx0} x_0)}{\phi_{x\dot{z}0}} \qquad (18)$$

The reduced transition matrix components $(\phi_{xx0}, \phi_{x\dot{z}0})$, which appear in equation (18), have been computed for the two different cases:

<u>Pericenter-Apocenter Transfer</u>

- $\phi_{xx0} = -\frac{1+e}{e-1}$
- $\phi_{x\dot{z}_0} = -\frac{4\eta}{n(1+e)}$

<u>Apocenter-Pericenter Transfer</u>

- $\phi_{xx0} = -\frac{e-1}{e+1}$

- $\phi_{x\dot{z}_0} = -\dfrac{4(1+e)}{n\eta}$

In the previous equations the term $\eta = \sqrt{1-e^2}$, whereas *n* represent the orbit mean motion $n = \sqrt{\mu/a^3}$

It is important to remark that, for the circular orbit (e = 0), equation (18) provides the well-known result valid for circular orbit

$$\Delta V = \frac{n}{4}\Delta r \qquad (19)$$

This represents an effective validation of the non-drifting algorithm derived within this work.

CONCLUSIONS

The relevance of the non-drifting algorithm when designed nominal and correction maneuver in RendezVous scenario has been presented. Algorithms dealing with this concept have been derived from both circular and elliptical orbit using different approaches:

- Travelling Ellipse formulation for circular orbit
- Inversion of the reduced transition matrix for the elliptical scenario.

These algorithms allows to design and correct the closing phase without introducing any drift between the chaser and target orbit ensuring in this way the passive safety condition and recoverability requirement.

It has been shown that these algorithms do not include any side effect having the same characteristic of more classical algorithms (Inversion of transition matrix) in terms of both

- ΔV consumption
- Dispersion with respect to the navigation error.

REFERENCES

[1] E. Di Sotto et Al., *System and GNC concept for RendezVous into elliptical Orbit for Mars Sample Return mission*, paper presented at AIAA Guidance, Navigation, and Control Conference and Exhibit 2007, Hilton Head, South Carolina, August 2007

[2] E. Di Sotto et Al., *Design and Performance Assessment of GNC Algorithms for Vision-Based RendezVous in Mars Orbit*, paper (IAC-06-C1.7.06) presented at 57th International Astronautical Congress, Valencia, Spain, October 2006.

[3] Fehse W., "*Automated Rendezvous and Docking of Spacecraft*", Cambridge Univ. Press, Cambridge, 2003.

[4] Mars Sample Return, CDF Final Presentation, ESA-ESTEC, February 2003.

[5] K. Yamanaka, F. Ankersen, "*New State Transition Matrix for Relative Motion on an Arbitrary Elliptic Orbit*", Journal of Guidance, Control, and Dynamics, Vol. 25, No 1, Jan-Feb 2002, pag 60–66

[6] R. Broucke, "*Solution of the Elliptic Rendezvous Problem with Time as Independent Variable*", Journal of Guidance, Control, and Dynamics, Vol. 26, No 4, Jul-Aug 2003, pag 615–621