# GPS-BASED PRECISION BASELINE RECONSTRUCTION FOR THE TANDEM-X SAR-FORMATION*

O. Montenbruck(1), P.W.L. van Barneveld(2), Y. Yoon(1), P.N.A.M. Visser(2)

(1) Deutsches Zentrum für Luft- und Raumfahrt (DLR),
German Space Operations Center (GSOC), 82234 Wessling, Germany;
oliver.montenbruck@dlr.de
(2) Delft Institute of Earth Observation and Space Systems,
Delft University of Technology, Delft, The Netherlands

## ABSTRACT

The TanDEM-X formation employs two separate spacecraft to collect interferometric Synthetic Aperature Radar (SAR) measurements over baselines of about 1 km. These will allow the generation ofa global Digital Elevation Model (DEM) with an relative vertical accuracy of 2-4 m and a 10 m ground resolution. As part of the ground processing, the separation of the SAR antennas at the time of each data take must be reconstructed with a 1 mm accuracy using measurements from two geodetic grade GPS receivers. The paper discusses the TanDEM-X mission as well as the methods employed for determining the interferometric baseline with utmost precision. Measurements collected during the close fly-by of the two GRACE satellites serve as a reference case to illustrate the processing concept, expected accuracy and quality control strategies.

## 1. TANDEM-X MISSION OVERVIEW

Germany is presently preparing the first operational formation flying mission for Synthetic Aperture Radar SAR) interferometry in low Earth orbit. It comprises two nearly identical spacecraft, named TSX and TDX, which have a size of 5 m x 2.4 m, a mass of 1200 kg and carry a high-resolution synthetic aperture (SAR) radar operating in the X-band (9.65 GHz). TSX has been launched by a DNEPR rocket on June 15, 2007 and was injected into a 514 km sun-synchronous dusk-dawn orbit with 97° inclination. This orbit has an 11 day repeat cycle and will be maintained troughout the mission with an accuracy of 250 m perpendicular to the flight direction [1]. TDX will join the same orbit in 2009, after which the two spacecraft will fly in a precisely controlled formation to form a radar interferometer with typical baselines of 0.5 km. This allows a much higher resolution than achievable in the X-SAR/SRTM Shuttle Topography mission and thus the generation of digital elevation models (DEMs) with unrivaled accuracy. A global mapping of the Earth will be achieved within 3 years during the guaranteed joint lifetime of TSX and TDX. Secondary objectives of the TanDEM-X mission include the demonstration of

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along-track-SAR interferometry for ocean current measurements as well as polarimetric SAR-interferometry [2].

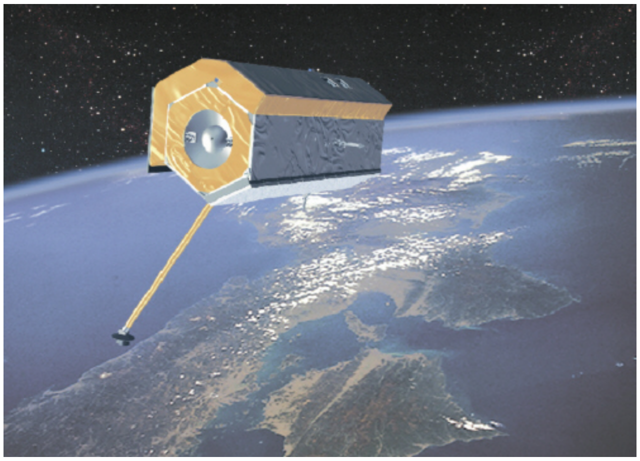

**Fig. 1** The TSX spacecraft (artist's impression, courtesy EADS Astrium)

The relative orbit geometry adopted for the TanDEM-X formation is an ellipse, which combines a periodic radial motion of amplitude 250-400 m with a periodic cross-track motion of 200-4000 m. By proper phasing of the relative eccentricity and inclination vectors [3], the maximum cross-track separation is achieved near the equator crossing, while the largest radial separation occurs in polar regions. In this way a high passive safety against collisions is achieved. The average along-track separation (i.e. the difference of the mean arguments of latitude) is nominally zero except during the initial orbit acquisition and dedicated along-track SAR interferometry data takes. From a global perspective, the motion of the two satellites around the Earth resembles a helix and has accordingly been termed a helix orbit [2] in the SAR community. It allows a flexible baseline selection and is thus optimally suited for interferometric SAR missions [4].

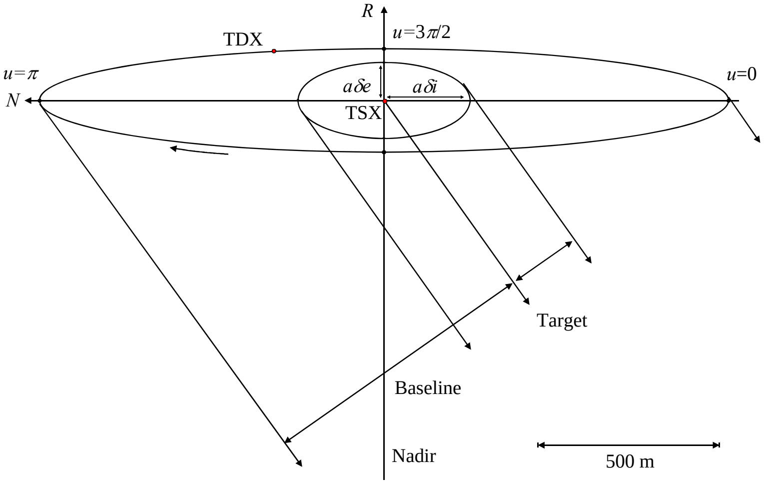

**Fig. 2** Relative motion of TDX with respect to TSX (as seen in flight direction) and projected baseline for different eccentricities (aδe=200…300m) and relative inclinations (aδi=500…2000m).

The interferometric baseline represents the projected distance of the two spacecraft (or more precisely the SAR antenna phase center) perpendicular to the flight direction and the direction of the observed target (Fig. 2). It determines the (latitude dependent) resolution and height of ambiguity in the DEM generation process. Different formation parameters are therefore adopted in a predefined sequence of mission phases.

For proper processing of the SAR measurements, the interferometric baseline during the data take must be known with an accuracy of about 1 mm. To achieve the desired baseline restitution accuracy, both spacecraft are equipped with geodetic grade dual frequency GPS receivers, that have been contributed by the GeoForschungsZentrum (GFZ) Potsdam. The IGOR receivers selected for this purpose represent a commercial version of JPL's BlackJack receiver and supports both navigation and radio occultation measurements with dedicated antennas in the zenith as well as the apex and ant-apex directions. The IGOR receiver has been validated in extensive ground tests using a GPS signal simulator [5] and achieves a representative code and carrier phase noise of 0.1 m and 1 mm, respectively. For backup purposes a redundant pair of MosaicGNSS GPS receivers is, furthermore, available on each spacecraft, which provides single-frequency carrier measurements with an expected accuracy of a few millimeters.

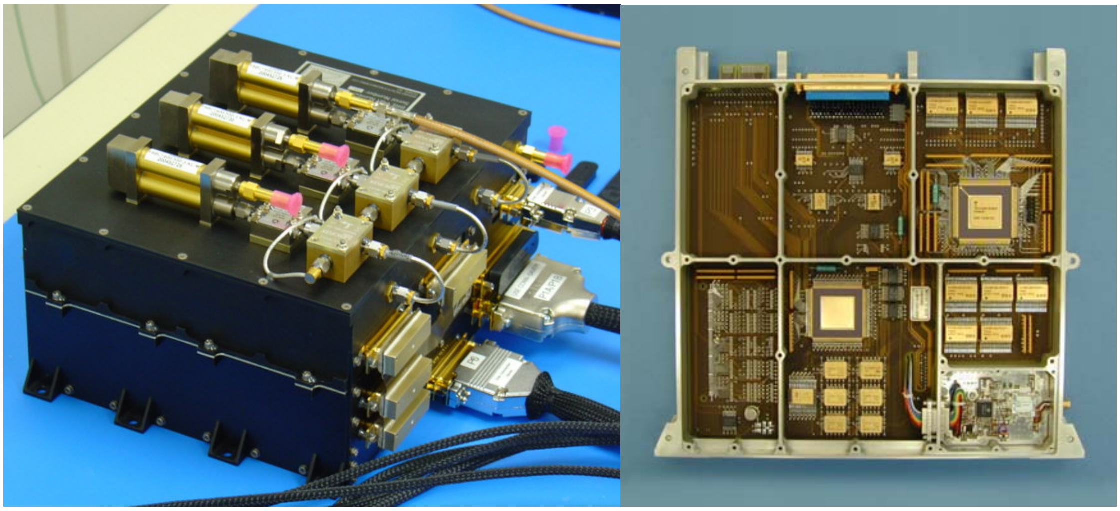

**Fig. 3** The IGOR (*left*) and MosaicGNSS (*right*; courtesy EADS Astrium) GPS receivers onboard the TSX/TDX spacecraft.

Using dual-frequency pseudorange and carrier phase measurements from the IGOR receiver, the absolute motion of each spacecraft can be determined with an expected accuracy of 5-10 cm. In case of TSX, a preliminary validation of the precise orbit determination (POD) solution has been carried out with satellite laser ranging (SLR) measurements, which match the resulting orbits with an rms accuracy of 3 cm [6].

## 2. GPS BASED RELATIVE NAVIGATION

GPS carrier phase measurements collected with the IGOR receivers on both spacecraft will be processed on ground in a dedicated relative orbit determination process. This process builds up on the reconstructed absolute orbits of TerraSAR-X and TanDEM-X, which can be obtained with a position accuracy of 5-10 cm (3D rms) using established reduced dynamics orbit determination techniques. Due to correlation of common-view measurements, the difference of the absolute orbit determination results exhibits a (relative) error of only 1-2 cm, which is still insufficient, however, for the SAR processing. In a separate processing step, the relative positions are therefore determined from differential carrier phase measurements, which offers a high level of common error cancellation. Furthermore, double-differences of the carrier phase ambiguities can be constrained to integer values which effectively converts the phase measurements into high-precision ranges.

Over the past years the feasibility of meeting the TanDEM-X requirements for relative baseline restitution has been verified by various research groups based on data from the GRACE mission. The GRACE satellites orbit the Earth at a separation of nominally 200 km and are equipped with BlackJack GPS receivers that closely match the IGOR receivers on TSX and TDX. Variations in the distance of the two spacecraft can be sensed with a Ka-band microwave link at a resolution of 10 micrometer [7]. Following early efforts with reported baseline accuracies of several millimeters [8], a 1mm single-axis accuracy could later be demonstrated in a joint study of TU Delft and DLR [9] as well as subsequent research of JPL [10] and the University of Bern [11].

The approach adopted for relative orbit determination of the TanDEM-X formation is based on the algorithms and software described in [9] and [12]. Since the absolute orbits are individually determined with high accuracy, it is adequate to consider the orbit of one reference satellite (TSX) fixed and estimate only the relative motion based on differential GPS measurements. Other than the absolute orbit determination, which makes use of a batch least squares approach, the relative orbit determination is performed in a forward-backward Kalman filter-smoother. This architecture has been selected for two reasons. On the one side it avoids a need to propagate the relative spacecraft state over long data arcs, which would be difficult to achieve with the desired 1 mm accuracy. Instead, the orbit propagation is only performed as part of the time-update step in between two measurements over a typical interval of 10-30s.

The second reason for preferring a Kalman filter over a least-squares batch filter is related to the carrier phase ambiguity resolution, which is performed using the LAMBDA (Least Squares Ambiguity Decorrelation) technique. Due to rapid changes in the set of tracked GPS satellites for a receiver in low Earth orbit, a huge number of ambiguities would arise in the batch adjustment of extended data arcs. The Kalman filter approach, in contrast, requires only an “on-the-fly” fixing of new ambiguities as part of an extended measurement update step whenever the constellation of tracked satellites changes.

The measurements provided by the IGOR receivers on TSX and TDX comprise C/A code and carrier phase on the $L_1$ frequency as well as semi-codeless P(Y)-code and carrier phase on the $L_1$ frequency ($f_1$=1575.42 MHz) and the $L_2$ frequency ($f_2$=1227.60 MHz). Only measurements from commonly tracked satellites are utilized and these are differenced between the two receivers (Fig. 4).

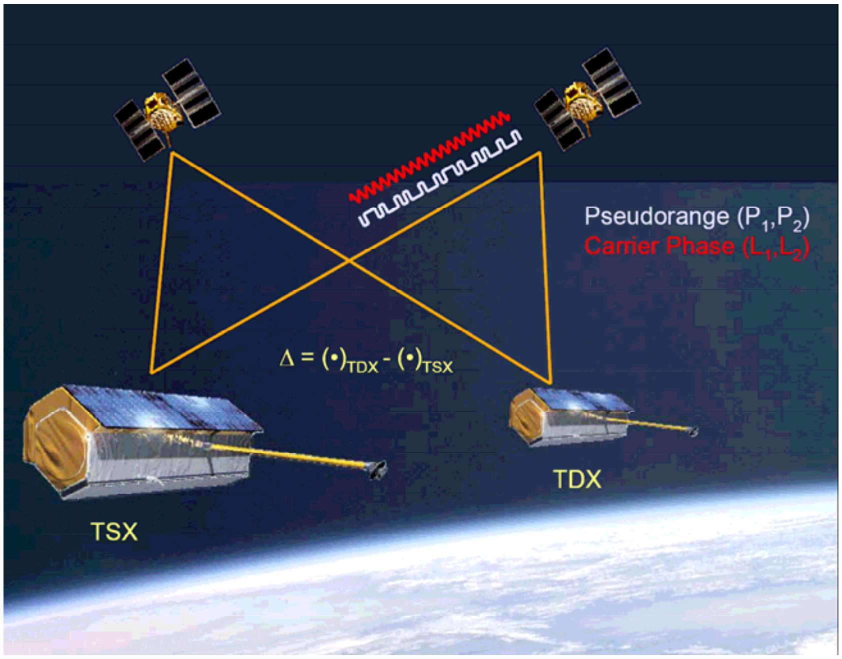

**Fig. 4** Differential GPS observables formed from the measurements collected onboard the TSX and TDX spacecraft.

Denoting by $\Delta$ the difference of quantities between the two spacecraft, by $\boldsymbol{r}$ and $\boldsymbol{r}_{\mathrm{GPS}}$ the positions of the receiving spacecraft and the transmitting GPS satellite, respectively, by $\delta t$ the receiver clock offset and by $I$ the ionospheric path delay, the single-difference observables can be described by the following measurement model:

$$\begin{aligned} \Delta P_1 &= \Delta|\boldsymbol{r}-\boldsymbol{r}_{\mathrm{GPS}}| + c\Delta\delta t + \Delta I \\ \Delta P_2 &= \Delta|\boldsymbol{r}-\boldsymbol{r}_{\mathrm{GPS}}| + c\Delta\delta t + (f_1/f_2)^2\Delta I \\ \Delta L_1 &= \Delta|\boldsymbol{r}-\boldsymbol{r}_{\mathrm{GPS}}| + c\Delta\delta t + \lambda_1\Delta A_1 - \Delta I \\ \Delta L_2 &= \Delta|\boldsymbol{r}-\boldsymbol{r}_{\mathrm{GPS}}| + c\Delta\delta t + \lambda_2\Delta A_2 - (f_1/f_2)^2\Delta I \end{aligned} \tag{1}$$

All measurements depend in the same way on the differential range between transmitter and receiver as well as the differential clock offset. Ionospheric effects, in contrast are frequency dependent and differ in sign for code and phase measurements. Finally, the carrier phase measurements are subject to an ambiguity $A$, which comprises both an unknown integer cycle count as well as fractional-cycle phase biases. An important property of the phase ambiguity relates to the fact that double-differences $\nabla\Delta A$ formed between two receivers and two GPS satellites are always integer valued (at least for properly designed receivers). This provides important constraints for the relative navigation process and effectively converts the ambiguous carrier phases into unbiased range measurements of extreme accuracy.

Besides the relative position and velocity $\Delta\boldsymbol{y} = (\Delta\boldsymbol{r};\Delta\boldsymbol{v})$, various other parameters have to be adjusted as part of the orbit determination process. These include differential force model parameters $\Delta\boldsymbol{p}$ such as the differential drag and radiation pressure coefficients as well as empirical accelerations. Other, non-dynamical estimation parameters comprise the relative clock offset $c\Delta\delta t$, the vector of differential ionospheric path delay $\Delta\boldsymbol{I}$ as well as the vector $\Delta\boldsymbol{A} = \left(\Delta\boldsymbol{A}_1, \Delta\boldsymbol{A}_2\right)$ of $L_1$ and $L_2$ carrier phase ambiguities for all tracked satellites.

Between consecutive measurements epochs the relative motion of the two satellites is propagated through numerical integration of the dynamical equations of motion, while the empirical accelerations are treated as a first-order Gauss-Markov process with exponential autocorrelation. A random walk process with a suitable amount of process noise is assumed for the clock and ionosphere states, while the ambiguity states are treated as constants between epochs.

Following the time-update step a rearrangement of the filter state is performed, if the constellation of commonly tracked satellites has changed. As part of this, the ionosphere and ambiguity states of satellites that are no longer used are reset to zero and constrained to a zero variance. Thereafter, free ionosphere and ambiguity states can be assigned to newly observed satellites with suitably chosen a priori values. The filter can thus work with a fixed state vector dimension but continuously adapts itself to the rapidly changing constellation of visible GPS satellites. In case of a twelve channels GPS receiver, the allocated state vector

$$\boldsymbol{x} = (\Delta\boldsymbol{r}; \Delta\boldsymbol{v}; \Delta C_{\mathrm{R}}; \Delta C_{\mathrm{D}}; \Delta\boldsymbol{a}_{\mathrm{emp}}; \Delta\boldsymbol{I}; \Delta\boldsymbol{A}_1; \Delta\boldsymbol{A}_2) \tag{2}$$

comprises a total 3+3+1+1+3+12+12+12=47 elements:

The subsequent measurement update processes the code and phase measurements on the $L_1$ and $L_2$ frequency to obtain the latest estimate of the individual state parameters. As such, up to 48 measurements per epoch are processed for a set of 12 simultaneously tracked satellites. Due to a noise level of only 20 cm (i.e. about one wavelength), the IGOR code measurements contribute to a rapid estimation of the individual phase ambiguities and a reliable filter performance even in the case of pronounced ionospheric path delays.

As a result of the measurement update step, the filter provides an estimate of the float-valued single-difference carrier phase ambiguities along with their associated covariance. So far, however, no use of the integer property of double difference carrier phases has been made. As such the resulting relative navigation solution is still affected by uncertainties in the estimated ambiguities and achieves a typical accuracy of “only” 1 cm.

To fully exploit the potential of the carrier phase measurements, the integer ambiguities have to be resolved and incorporated into the filter process. To accomplish this, a separate ambiguity resolution step is performed right after the measurement update. It starts with the selection of a comprehensive and non-redundant set of GPS satellite pairs, based on which double-difference ambiguities $\nabla\Delta\boldsymbol{A}$ and their associated covariance can then be formed. Making us of the Least-Squares Ambiguity Decorrelation Adjustment technique (LAMBDA, [13]), the float-valued ambiguities are then “rounded “to the most probable vector $\nabla\Delta\boldsymbol{A} = (\nabla\Delta\boldsymbol{N}_1, \nabla\Delta\boldsymbol{N}_2)$ of integer values, if a sufficient confidence in their estimated values has been obtained. Once know, the integer double-difference ambiguities can be incorporated into the filter state by a pseudo-measurement update which constrains the double difference to the determined integer value with a zero covariance. By maintaining a list of satellites with fixed ambiguities, the LAMBDA method needs only be applied to newly tracked satellites or satellites that have experienced a cycle slip.

## 3. THE GRACE PROXIMITY – A TEST CASE FOR TANDEM-X

As described above, the feasibility of achieving a 1D rms accuracy of 1 mm has been demonstrated for the GRACE mission over a 200 km baseline in reference to Ka-band radar measurements. In case of TanDEM-X, no independent measurement system will be available to validate the accuracy of the relative navigation solution. In order to guarantee the desired baseline accuracy on an operational level, proper validation strategies and self-consistency tests will therefore be required. On the other hand, the TanDEM-X mission benefits from a much smaller separation of the two spacecraft . At a distance of typically 1 km, differential ionospheric effects are notably reduced, which enable different processing strategies

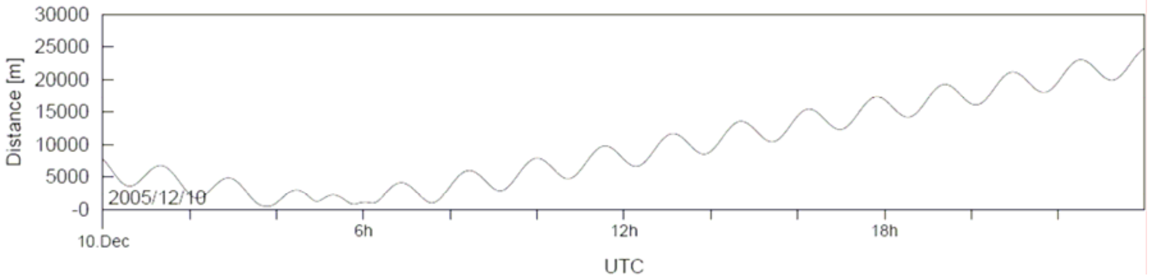

**Fig. 5** Relative distance of the two GRACE satellites during the close encounter in December 2005.

To analyze this condition in more detail, data from the longitude swap maneuver of the GRACE satellites have been considered. When GRACE-B passed by GRACE-A in December 2005 to take the lead of the formation for the second half of the mission [14], the two spacecraft remained in a distance of less than 10 km for roughly 12 hours (Fig. 5). This provides an ideal opportunity to analyze differential ionospheric path delays and the impact on integer ambiguity resolution for close formations in more detail.

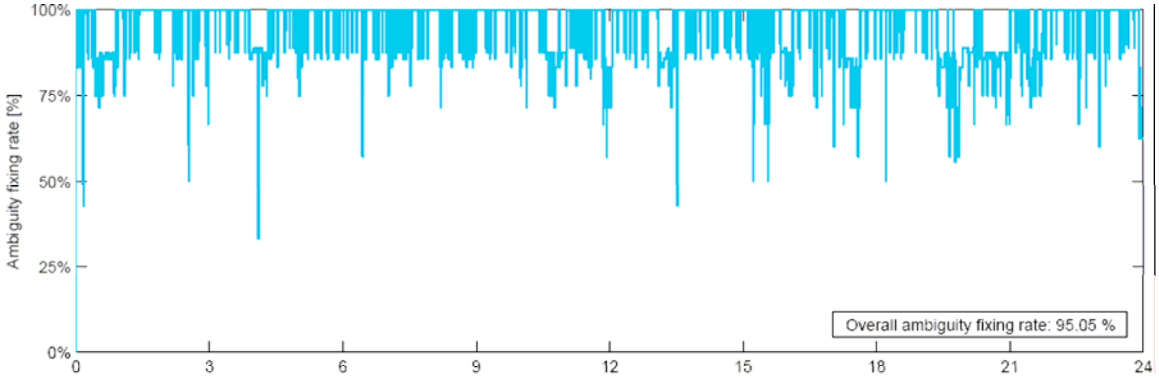

**Fig. 6** Fraction of fixed ambiguities on 10 December 2005.

As shown in Fig. 6, the number of fixed ambiguities during the close encounter is clearly larger than at separations of 10-20 km. With few exceptions, at most one ambiguity remains unfixed at each epoch during the proximity phase. The average ambiguity fixing rate for the entire day amounts to 95% and is roughly 5% higher than observed during the routine mission phase of GRACE with nominal s/c separation of about 200 km.

Differential ionospheric effects in the vicinity of the encounter vary both due to the varying geographical location and the changing separation of the two satellites. On Dec. 10, 2005, the GRACE satellites were flying in a dusk-dawn orbit, which passes the equator in a fairly large distance from the ionospheric density maximum. At the same time the low solar activity resulted in moderate vertical electron content (VTEC) of up to 50 TECU for terrestrial users. The VTEC values experienced by the GRACE satellites were likewise limited and reached a peak values of 5-15 TECU twice per orbit near the equator crossing (Fig 7). This corresponds to a representative ionospheric path delay of 1-2 m (at L1) near zenith and roughly 4 times higher values at low elevations.

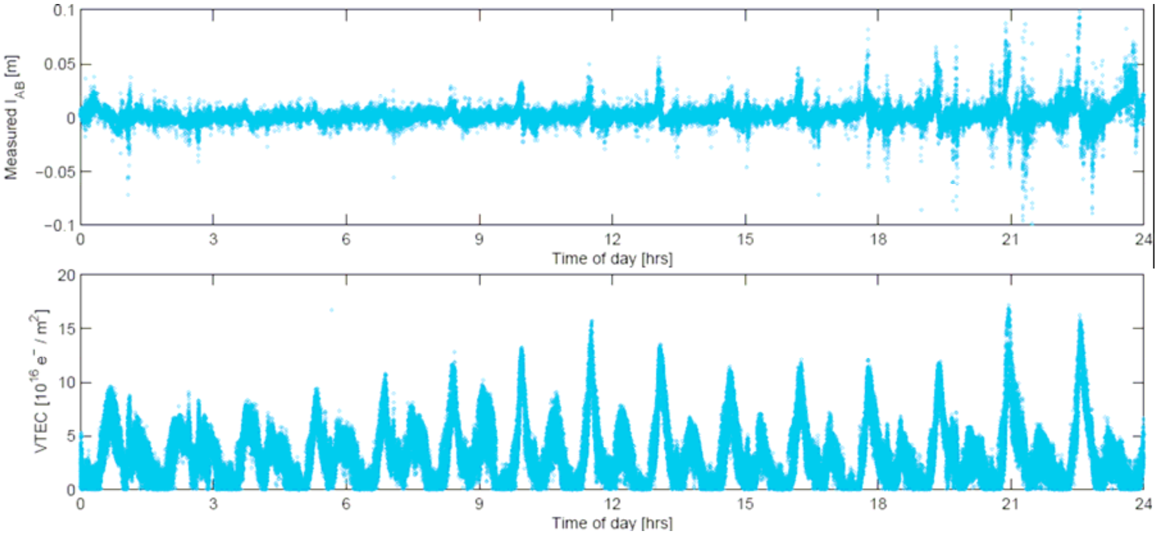

**Fig. 7** Vertical electron content above the GRACE satellites (*bottom*) and differential ionospheric path delays (*top*) on Dec. 10, 2005.

Due to the high regional correlation of the ionospheric electron density and the nearly identical elevations under which a given GPS satellite is observed from the two spacecraft, the ionospheric path delays are nearly identical for GRACE-A and GRACE-B during the proximity phase. As such, the differential path delays are almost negligible for short baselines. This is illustrated in Fig. 7, which shows the ionospheric path delays derived from the dual-frequency carrier phase measurements after estimation of the individual ambiguities. For separations of less than 5 km (3h-8h), the resulting differential path delays $\Delta I$ are dominated by the carrier phase measurements noise, which results in a scatter of roughly 1 cm. At distances of 10-20 km, however, path delay differences of up to 10 cm may be observed for measurements taken on GRACE-A and GRACE-B, respectively. These extreme values are closely correlated with the VTEC maxima along the orbit and grow in proportion to the baseline between the two satellites.

For the short baselines foreseen for most of the TanDEM-X mission, the differential path delays are expected to remain less than a few centimeters or, equivalently, a tenth of a cycle. Thus, even a single-frequency differential GPS solution can be expected to deliver relative positions with a centimeter-level accuracy. As shown in Fig. This in fact the case for the GRACE proximity as illustrated in Fig. 8. Here, single- and dual-frequency kinematic relative navigation solutions are against the dynamically filtered relative navigation solution. Similar to the long-baseline cases discussed in [9], the kinematic dual-frequency exhibits an rms uncertainty of roughly 8 mm, which reflects the average position dilution of position (roughly 3) and the noise of the ionosphere-free L1/L2 carrier phase combination of about 3 mm. Due to the elimination of ionospheric errors, the rms error is essentially constant over the entire day and does not exhibit a dependence on the relative distance of the two satellites. A rather different behavior may, be observed for the kinematic single-frequency solution. While a degradation due the impact of differential path delays is clearly obvious at separations above 10 km, the error at small baselines is roughly 3 times smaller than that of the dual-frequency solution.

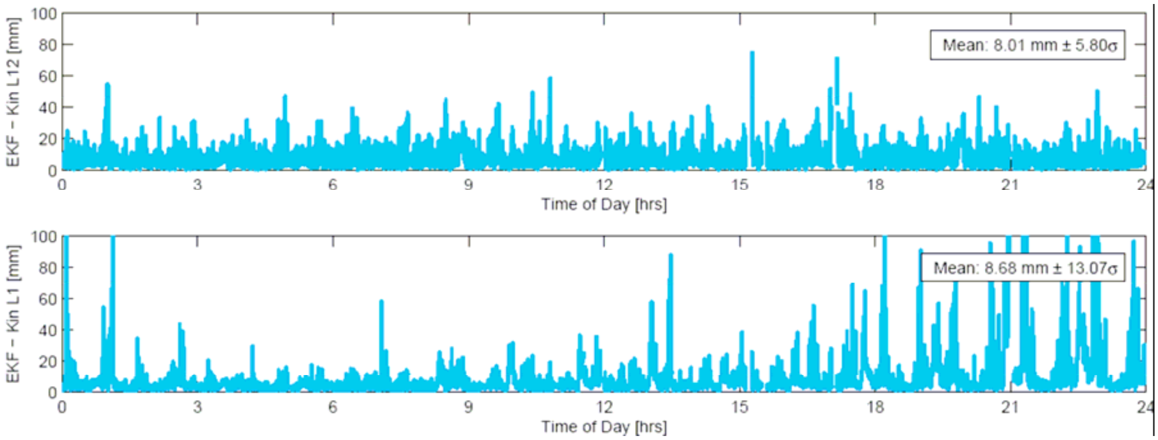

**Fig. 8** Differences between kinematic relative navigation solutions and the dynamically filtered dual-frequency solution for GRACE-A/B on Dec. 10, 2005 (*top*: kinematic dual-frequency solution; *bottom*: L1 single-frequency solution).

Even though the kinematic single-frequency solution is not considered to be more accurate than the dynamically filtered dual-frequency solution, it provides important information for the validation of the ambiguity fixing. While dual-frequency measurements are required to eliminate or compute ionospheric path delays at large baselines, they are partly redundant if the ionospheric path delays are sufficiently small. This redundancy can be used to independently verify the correctness of the estimated ambiguities. In general, the so-called widelane combination, i.e. the difference $\nabla\Delta \boldsymbol{N}_1 - \nabla\Delta \boldsymbol{N}_2$ of the L1 and L2 ambiguities can be obtained with high confidence, whereas the resolution of the narrow-lane ambiguity $\nabla\Delta \boldsymbol{N}_1 + \nabla\Delta \boldsymbol{N}_2$ is more error prone due to its smaller effective wavelength. A good consistency of single-and dual-frequency solution necessarily implies that both the L1 and L2 ambiguities have independently been fixed to their correct values. Since even a single incorrectly fixed ambiguity might notably degrade the relative orbit determination result, the independent single-frequency ambiguity test provides an important diagnostic method in the high-precision baseline reconstruction.

## 4. DIRTY DETAILS

Similar to CHAMP and GRACE, the TSX and TDX spacecraft employ an aeronautical dual-frequency GPS antenna (Sensor-System S67-1575-14) that has been selected because of its robustness and survivability under space conditions (Fig. 9). To suppress multipath and to achieve high-grade code and phase measurements the antenna is equipped with a light-weight choke ring developed by the GeoForschungsZentrum (GFZ), Postdam.

**Fig. 9** GPS antenna and choke ring for the TSX/TDX satellites (engineering model).

On the other hand, the antenna has not been designed for geodetic applications, which demand a very high phase center stability. A dedicated measurement campaign has therefore been conducted to assess the overall phase center variation (PCV) and its repeatability between different antenna units. These calibrations have been conducted with a robotic measurement system developed by the Institute of Geodesy at the University of Hannover [15], which does not require a reference antenna. This approach provides both the absolute 3-dimensional phase center offset relative to a mechanical reference point as well as the phase center variation as a function of the viewing direction.

Since neither the flight units of the TSX antennas nor those of the TDX satellite could be made available for the test, the campaign has been conducted with reference tests units from different manufacturing lots. The results can then be used to establish an upper bound on the uncertainty of the phase center knowledge. For illustration, results for two different units are shown in Fig. 10. Depending on the elevation angle, individual PCV measurements achieve an accuracy of 0.3 – 0.6 mm. Due to the use of a choke ring, the phase pattern exhibits a high rotational symmetry and depends primarily on the elevation. For elevations above 15°, variations of less than ±1mm have been obtained on both the L1 and L2 frequency. Extreme variations of up to 4 mm are only encountered at very low elevations for the L2 frequency.

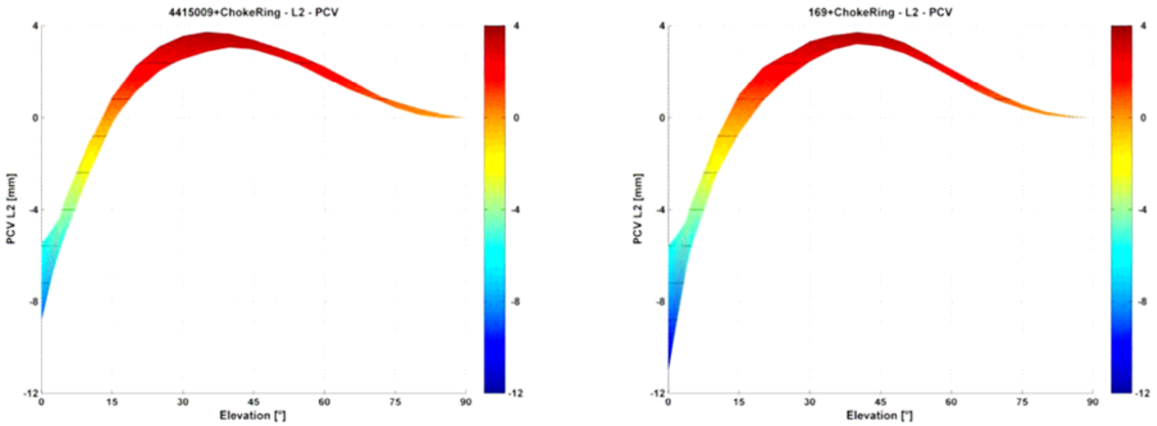

**Fig. 10** L2 phase center variations for two test units of a Sensor Systems S67-1575-14 (*left*) and -141 antenna (*right*). The width of the both graphs represents the variation of the phase center for different azimuth angles.

The results imply that a high level of common error cancellation will indeed exist when differencing measurements between the TSX and TDX satellites, even if the phase pattern were not explicitly considered in the relative navigation. Residual uncertainties of up to 1 mm rms are still to be expected in the modeling of individual differential carrier phase measurements due to the scatter of different antenna samples. However, these can partly be compensated by the dynamic trajectory modeling and are not expected to impact the baseline determination in the same magnitude.

Besides the antenna induced phase center variations proper attention will also have to be paid to receiver internal effects that may result in systematic carrier phase measurement errors. As has first been noted on the CHAMP satellite, the BlackJack receiver is susceptible to cross-talk between the different antenna inputs, which causes multipath-like errors of the code (and presumably) carrier phase measurements. The existence of this problem has likewise been confirmed for GRACE as well as the IGOR receiver onboard the TSX spacecraft [6]. It is expected to impact the relative navigation accuracy when both the zenith antenna and the occulation antenna of either TSX or TDX are operated simultaneously. Preliminary tests conducted for the GRACE formation have indeed revealed a degradation of the kinematic relative navigation solution during occultation data takes but shown a good robustness of the dynamically filtered solution. Further investigations will address this problem in more detail and a decision to disable occultation measurements may be taken if deemed necessary to guarantee the desired relative navigation accuracy during the TanDEM-X mission phase.

## SUMMARY AND CONCLUSIONS

The TanDEM-X mission constitutes the first formation flying mission demanding a post-facto knowledge of the relative spacecraft position with an accuracy of 1mm. A prototype software for the relative navigation using dual-frequency GPS measurements has been developed by Delft University and DLR as part of their GPS High precision Orbit determination Software Tools (GHOST). Using GPS data from the GRACE mission, the feasibility of achieving a 1D rms accuracy of 1 mm has been demonstrated over a 200 km baseline in reference to Ka-band radar measurements. Further effort will, however, be required to increase the reliability of the ambiguity fixing and thus the availability of high precision TSX/TDX relative orbit determination results on an operational basis. On the other hand, this mission benefits from a much smaller separation of the two spacecraft than GRACE. Over a distance of typically 1 km, differential ionospheric effects are notably reduced, which even enables a single-frequency ambiguity resolution. Tests conducted during the close fly-by of the two GRACE satellites in December 2005 provide supporting evidence that single-frequency measurements can indeed be used to independently validate the L1 and L2 ambiguity resolution and thus improve the robustness and reliability of the relative navigation process. Further attention will have to be paid to systematic measurement errors caused by antenna phase center variations and receiver internal cross-talk between antenna strings.

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