GPS BASED REDUCED DYNAMIC ORBIT DETERMINATION USING ACCELEROMETER DATA

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Extended Abstract

Currently two gravity field satellite missions, CHAMP and GRACE, are equipped with high sensitivity electrostatic accelerometers, measuring the non-conservative forces acting on the spacecraft in three orthogonal directions. During the gravity field recovery these measurements help to separate gravitational and non-gravitational contributions in the observed orbit perturbations. For precise orbit determination purposes all these missions have a dual-frequency GPS receiver on board. The reduced dynamic technique combines the dense and accurate GPS observations with physical models of the forces acting on the spacecraft, complemented by empirical accelerations, which are stochastic parameters adjusted in the orbit determination process. When the spacecraft carries an accelerometer, these measured accelerations can be used to replace the models of the non-conservative forces, such as air drag and solar radiation pressure. This approach is implemented in a batch least-squares estimator of the GPS High Precision Orbit Determination Software Tools (GHOST), developed at DLR/GSOC and DEOS. It is extensively tested with data of the CHAMP and GRACE satellites. As accelerometer observations typically can be affected by an unknown scale factor and bias in each measurement direction, they require calibration during processing. Therefore the estimated state vector is augmented with six parameters: a scale and bias factor for the three axes. In order to converge efficiently to a good solution, reasonable a priori values for the bias factor are necessary. These are calculated by combining the mean value of the accelerometer observations with the mean value of the non-conservative force models and empirical accelerations, estimated when using these models. When replacing the non-conservative force models with accelerometer observations and still estimating empirical accelerations, a good orbit precision is achieved. 100 days of GRACE B data processing results in a mean orbit fit of a few centimeters with respect to high-quality JPL reference orbits. This shows a slightly better consistency compared to the case when using force models. A purely dynamic orbit, without estimating empirical accelerations thus only adjusting six state parameters and the bias and scale factors, gives an orbit fit for the GRACE B test case below the decimeter level. The in orbit calibrated accelerometer observations can be used to validate the modelled accelerations and estimated empirical accelerations computed with the GHOST tools. In along track direction they show the best resemblance, with a mean correlation coefficient of 93% for the same period. In radial and normal direction the correlation is smaller. During days of high solar activity the benefit of using accelerometer observations is clearly visible. The observations during these days show fluctuations which the modelled and empirical accelerations can not follow.

1 Introduction

Ongoing and future geopotential space missions in a low Earth orbit are equipped with high sensitivity accelerometers, measuring the non-conservative accelerations. Examples are CHAMP (CHAllenging Minisatellite Payload), led by the GeoForschungsZentrum (GFZ) in Potsdam [12] and launched in July 2000 and GRACE (Gravity Recovery And Climate Experiment), a joint US-German mission [13], launched in March 2002. The GRACE mission comprises two identical spacecraft flying at a separation of nominally 220 km at an altitude of roughly 450 km. The ESA GOCE (Gravity field and steady-state Ocean Circulation Explorer) mission [3], to be launched in March 2008 at a very low altitude of about 250 km, will carry a gradiometer, consisting of six accelerometers, orthogonally aligned in three pairs close to the spacecraft's center of mass. The difference of the measurements of one pair gives the gradient of the gravity field plus rotational terms due to inertial forces, while the arithmetic mean results in the non-gravitational acceleration.

All these geopotential missions carry a dual-frequency GPS receiver on board to determine their orbit very precisely. The reduced dynamic precise orbit determination (POD) technique combines the dense and accurate GPS observations with physical models of the forces acting on the spacecraft, complemented by empirical accelerations, which are stochastic parameters adjusted in the orbit determination process [18]. This approach usually results in trajectories of the highest accuracy. When the spacecraft carries an ac-

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celerometer, these measured accelerations can be used to replace the models of the non-conservative forces, such as air drag and solar radiation pressure.

In this paper the implementation of the orbit determination technique using accelerometer observations is discussed and the results from GRACE data analysis presented.

2 The reduced dynamic orbit determination technique

For the analysis of the GRACE data, the GPS High precision Orbit determination Software Tools (GHOST) [17] have been used. The RDOD program of GHOST, described in detail in [10], is a reduced dynamic batch least-squares orbit determination tool. It is extended to accommodate accelerometer observations, which is described in the second subsection. First an overview of the standard RDOD program is given.

2.1 Reduced dynamic orbit determination (RDOD) with piecewise constant empirical accelerations

In the RDOD program undifferenced dual frequency code and carrier phase GPS measurements are processed. The ionosphere-free pseudorange measurement is used, formed by the following linear combination [5]:

$$\begin{aligned} \rho_{P12} &= \frac{f_1^2}{f_1^2 - f_2^2}\rho_{P1} - \frac{f_2^2}{f_1^2 - f_2^2}\rho_{P2} \\ &\approx 2.546\rho_{P1} - 1.546\rho_{P2} \end{aligned} \tag{1}$$

with f_1 and f_2 the L1 and L2 signal frequencies. An equivalent relation holds for the ionosphere-free carrier phase measurement ρ_{L12}. A batch least-squares system is built with all available observations and the parameters of interest are iteratively adjusted by correcting their a priori values with the solution of the system. These parameters are the receiver clock offset at each measurement epoch, a carrier-phase bias for each continuous single GPS satellite pass and dynamical trajectory parameters, comprising the spacecraft state vector with the spacecraft position and velocity at the reference epoch, force model coefficients scaling the solar radiation pressure and atmospheric drag and finally empirical accelerations, to compensate for deficiencies in the employed dynamical force models. The observations are modeled based on the precise GPS satellite ephemeris and 30 s clock products determined by the Center for Orbit Determination (CODE) in Bern, Switzerland. For data editing purposes, an a priori orbit determined from a dynamical smoothing of pseudo-range-based single point position solutions is used.

Each iteration the equation of motion of the satellite is numerically integrated from the reference epoch to the subsequent observation epochs, to obtain the residuals between the observations and the modeled measurements. At the same time the partial derivatives of the state vector with respect to the dynamical trajectory parameters are obtained from the integration of the associated variational equations [9]. The gravitational and non-gravitational forces are modeled following the general discussion in the above reference. The GGM01S gravity field model [15] to degree and order 120 is employed, together with the TOPEX 4.0 ocean tide model [16]. The Jacchia 71 atmospheric density model [6] has been adopted to compute the drag, making use of daily NOAA solar flux and 3 hourly geomagnetic activity values [11]. Direct solar radiation pressure is considered using a constant-area model and a conical Earth shadow function.

Empirical accelerations (a_R, a_T, a_N) are estimated as piecewise constant parameters in radial, transverse (along-track) and normal (cross-track) direction (RTN) in independent intervals of 600 s. This interval length is found to be the most suitable and is a compromise between computational effort and a sufficient sampling of the characteristic time scales of the dynamical model deficiencies. The empirical accelerations are characterized by an expectation value of zero and an a priori variance which constrains them [7].

2.2 Using accelerometer observations

The SuperSTAR accelerometers, manufactured by ONERA (France), on board the GRACE spacecrafts are a successor of the STAR accelerometer flying on CHAMP [14]. The instrument measures the changes in electrostatic forces needed to maintain a proof mass in the center of a cage. The center of mass of the proof mass is placed as close as possible to the center of mass of the spacecraft. Both proof mass and spacecraft are influenced by the same gravitational forces. The non-gravitational forces act only on the spacecraft and consequently on the instrument cage, inducing a movement of the proof mass which is measured by capacitive detectors. The instrument calibration parameters are not known precisely at the time of launch because the voltages required to suspend the proof mass in a laboratory environment are different from those in orbit. Therefore the calibration parameters have to be determined when processing the accelerometer observations for e.g. gravity field determination, atmospheric density retrieval [1] or precise orbit determination [8].

The applied calibration equation is formulated as:

$$\boldsymbol{a}_{cal} = \boldsymbol{S} \cdot \boldsymbol{a}_{obs} + \boldsymbol{B} \tag{2}$$

with $\boldsymbol{a}$ a three-dimensional vector with the accelerometer observations in the Spacecraft Reference Frame (SRF). The X-axis of the SRF is nominally directed to the other GRACE spacecraft, the Z-axis is nadir pointing and the Y-axis completes a right-handed system. Because the GRACE satellites fly in a near-circular orbit, the SRF agrees within a few degrees with the RTN frame in which the empirical accelerations are defined. Furthermore $\boldsymbol{S}$ is a 3×3 diagonal matrix containing a scale factor in each direction and $\boldsymbol{B}$ the bias vector. The bias factor refers the observations to the correct magnitude while the scale factor adjusts the amplitude of the measured differences.

When using the accelerometer observations for orbit determination the calibrated measurements are directly inserted into the equation of motion and the scale and bias factors are estimated in the least-squares adjustment procedure. The non-gravitational force models are no longer considered in the calculation of the accelerations experienced by the spacecraft. Empirical accelerations are still estimated, but as drag and solar radiation pressure models imply a higher amount of uncertainty compared to the direct measurements of them with an accelerometer, the a priori variance of these parameters can be reduced.

The spacecraft equation of motion is defined and integrated in an inertial reference system, within GHOST realized by EME2000 (mean equator and equinox of J2000). Therefore the measured accelerations are transformed from the SRF to the inertial reference frame with attitude information obtained from star camera observations, provided as quaternions. The variational equations are updated accordingly taking into account the proper transformation matrices. The GRACE accelerometer data is preprocessed by JPL and delivered at a 1Hz sampling [2]. To limit computational time and without losing accuracy, 10 second samples are used and linearly interpolated in between.

Tests indicated that the parameters in Y and Z direction are closely correlated with the initial state vector, when estimating the scale and bias factors in the SRF directions. Furthermore the Y direction (out of plane) is the weakest bounded by the dynamics. Therefore the scale and bias factors in Y and Z direction are tightly constrained to their a priori values. This leads to the necessity of good a priori values, especially for the bias factor. As mentioned above, the bias shifts the observations to the correct mean value. The determination of an a priori bias factor is composed of two parts, given by the following equation:

$$\boldsymbol{B}_{a\,priori} = -mean(\boldsymbol{a}_{meas}) + mean(\boldsymbol{a}_{model}) \qquad (3)$$

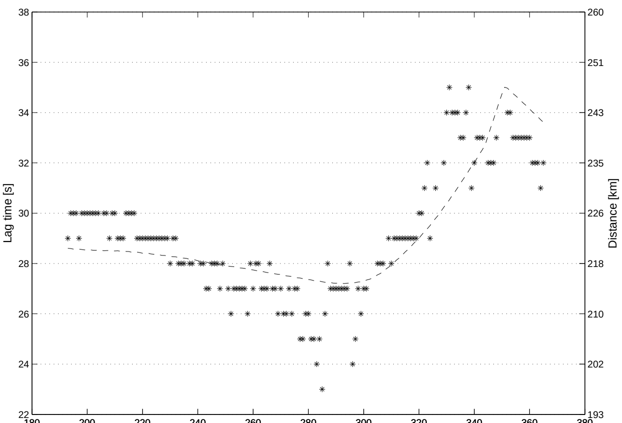

Figure 1: Comparison between time lag of measured accelerations (asterisks) onboard of the GRACE spacecrafts and the inter-satellite distance (dashed line)

First the mean value of the accelerometer measurements $\boldsymbol{a}_{meas}$ is determined, and subtracted from the observations, which shifts them to fluctuate around zero. These corrected measurements are used as accelerometer observations in the POD process. Second, the a priori bias factor is set to the mean value of the modeled non-gravitational and empirical accelerations $\boldsymbol{a}_{model}$, calculated and estimated with the standard reduced dynamic technique described above. This shifts the accelerometer observations, corrected for their mean, to a magnitude which is expected to be close to the correct value.

3 Results and discussion

This section deals with the analysis of GRACE Level 1B data, publicly released by JPL and UT/CSR as part of the Physical Oceanography Distributed Active Archive Center (PODAAC).

3.1 Along-track acceleration time lag versus distance

The GRACE mission consists of two spacecraft flying in the same orbit at a distance of nominally 220 km. The non-gravitational forces acting on a satellite are predominantly dependent on the Earth-fixed position, therefore GRACE A, the satellite flying first during the considered period, experiences certain variations in these forces earlier than GRACE B, the trailing satellite. As the accelerometer measures the accelerations at a high sampling, the time lag between the sequences of the instruments onboard of both satellites can be determined from the correlation of the GRACE A accelerations with a shifted copy of GRACE B accelerations and compared to the inter-satellite distance. This is done in along-track direction, using the 1s sampled accelerometer data. The results are plotted in Figure 1 for the second half of 2003 and show a strong correlation. This also illustrates that the accelerometer data of the two satellites show a high resemblance. Therefore in the following sections only a GRACE B data set

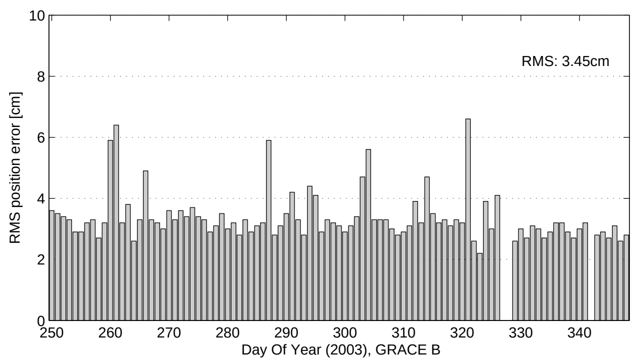

Figure 2: GRACE B orbit determination results using accelerometer observations. Root-mean square position errors (3D) are given with respect to JPL reference trajectories

Table 1: Orbit fit statistics in the RTN-frame (mean, standard deviation and root-mean square of the position differences) of reduced dynamic orbits compared to JPL reference trajectories, for the GRACE B spacecraft and the period DOY 250-350 2003. (1) represents the standard reduced dynamic case, (2) the case using accelerometer observations, (3) the case using accelerometer observations without estimating empirical accelerations

	mean			stand dev			RMS
case	R [cm]	T [cm]	N [cm]	R [cm]	T [cm]	N [cm]	3D [cm]
(1)	-0.40	0.01	-0.33	1.78	2.76	1.65	3.77
(2)	-0.72	-0.04	-0.19	1.31	2.54	1.64	3.45
(3)	-0.75	-0.20	-0.19	2.86	7.59	3.98	9.19

is considered, starting from September 7 until December 16 2003 (100 days).

3.2 External orbit fit and SLR residuals

Reduced dynamic reference orbits produced by JPL, part of the GRACE Level 1B data delivery, serve as reference orbits to compare the orbits determined using accelerometer data with. Figure 2 depicts the root-mean square position differences of the GRACE B orbits. Large outliers, due to the non-availability of auxiliary data or a large number of bad observations, are not included. The total position consistency for the whole period is 3.5 cm (3D RMS), which is slightly better compared to the orbits determined by the standard technique (using non-gravitational force models), which amounts to 3.8 cm.

A detailed overview of the orbit fits of GRACE B with respect to the JPL reference orbits is given in Table 1. These results show that a comparable orbit accuracy can be achieved when replacing the non-gravitational force models with accelerometer observations and that the introduction of scale and bias factors in the adjustment procure is correctly implemented. In that case the standard deviation of the position difference in radial and along-track direction gets smaller, indicating that the scale and bias factor in this direction are well determined.

A disadvantage of external orbit comparisons is that the reference orbits are created using the same observations and with also a reduced dynamic technique. An independent orbit validation test is the computation of the residuals of SLR observations of the spacecraft, fixing the positions with the determined orbits. This validation procedure is implemented in the GHOST software set. The SLR residual statistics for GRACE B are displayed in Table 2 and show a similar trend in both cases. When accelerometer observations are used the mean offset and RMS of the SLR residuals of the whole period are slightly better. For the sake of completeness, the statistics for the JPL reference orbits are also computed and included in the table. On average 76 observations per day are used for this analysis.

Table 2: SLR residual statistics (mean and root-mean square) of reduced dynamic orbits for the GRACE B spacecraft and the period DOY 250-350 2003. (1) represents the standard reduced dynamic case, (2) the case using accelerometer observations, (3) the statistics for the JPL reference orbits

case	mean [cm]	RMS [cm]
(1)	-0.72	2.13
(2)	-0.55	1.96
(3)	-1.11	2.31

3.3 Measured versus modeled accelerations

The scale and bias factors of GRACE B obtained during the orbit determination are presented in Table 3, showing their mean value and standard deviation for the 100 day period. These values show a better consistency compared to the case when no empirical accelerations are estimated. The results of the latter case are presented in the right hand side of the table and are discussed in section 3.4.

Figure 3 presents a comparison of the calibrated accelerations with those computed by the non-gravitational force models, together with the estimated empirical accelerations determined with the standard reduced dynamic technique, for October 27 in 2003. Both sequences show a good agreement in the three directions. This also validates the modeled non-gravitational accelerations as implemented in the GHOST software, augmented with the estimated empirical accelerations. The modeled acceleration in radial direction clearly shows the effect of entering the Earth-shadow, which is also visible in the observations,

Table 3: Accelerometer calibration parameters in the SRF obtained during GRACE B orbit determination, DOY 250-350 2003. + or - emp indicates whether empirical accelerations are included in the estimation.

scale/bias	+ emp	- emp
S_X [-]	0.95 ± 0.022	0.94 ± 0.012
S_Y [-]	0.97 ± 0.005	0.96 ± 0.119
S_Z [-]	0.92 ± 0.001	0.92 ± 0.051
B_X [nm/s^2]	-562 ± 11	-558 ± 6
B_Y [nm/s^2]	9226 ± 66	9126 ± 1112
B_Z [nm/s^2]	-778 ± 12	-781 ± 44

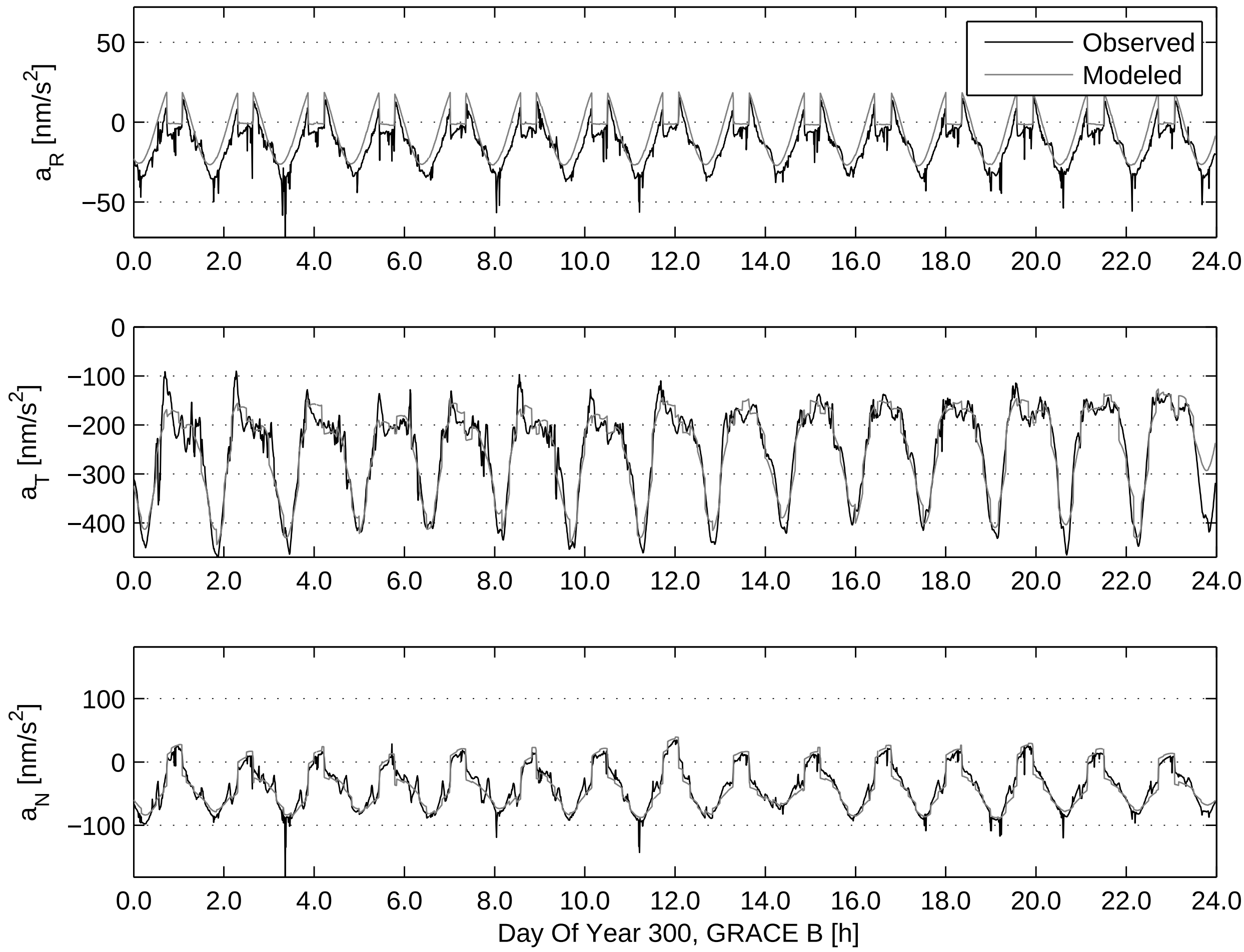

Figure 3: Comparison of observed (1) and modeled (2) non-gravitational accelerations on GRACE B for October 27, 2003, in the RTN-frame ((1) calibrated accelerometer measurements and empirical accelerations, (2) atmospheric drag, solar radiation pressure and empirical accelerations)

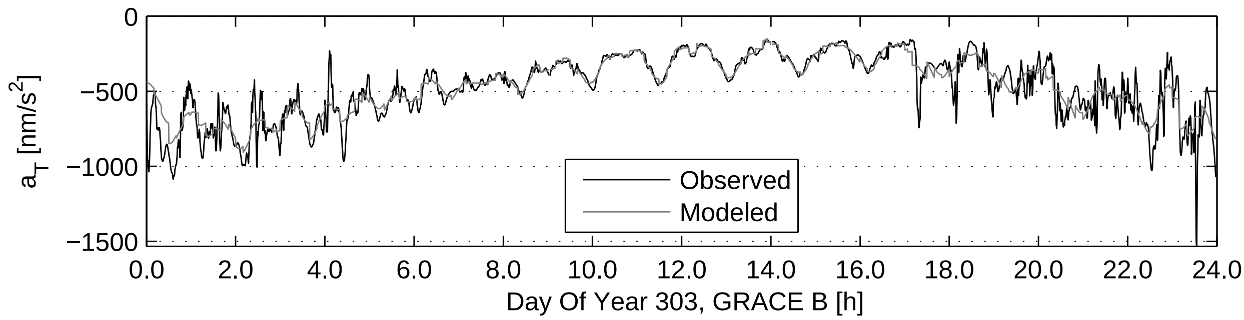

Figure 4: Comparison of observed and modeled non-gravitational accelerations on GRACE B in tangential direction for October 30, 2003

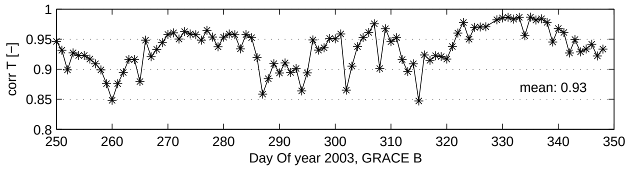

Figure 5: Correlation coefficients between observed and modeled accelerations in along-track direction

though less pronounced. In the observations in normal direction spikes as a result of attitude thruster firings are present. In orbit determination all accelerations experienced by the spacecraft are of importance, while in other disciplines, such as density retrieval, the recordings of such small maneuvers are an unwanted effect.

As mentioned in section 2.2, empirical accelerations are still estimated. The a priori variance attributed to these parameters, especially in along-track direction, is set to a smaller value compared to the case when using non-gravitational force models, as the accelerometer provides direct measurements of these forces. Consequently, the estimated accelerations in along-track direction are considerably smaller, with a standard deviation reduced from 30 nm/s^2 to 5 nm/s^2 when accelerometer observations are used. The estimated empirical accelerations in normal direction show in both cases the same magnitude and pattern, with a standard deviation of around 8 nm/s^2, suggesting that in this direction they mainly account for deficiencies in the applied conservative force models.

For the whole 100 days period analyzed, the correlation between the modeled and observed accelerations is determined. The mean value of the correlation coefficients is 0.85 in radial direction, 0.93 in along-track and 0.91 in cross-track direction. These values reflect the quality of the estimated scale and bias factors. Figure 5 shows the daily correlation value in along-track direction. The higher value in this direction supports that the estimation of scale and bias factors and empirical accelerations, using the respective techniques, is the strongest in along-track direction. Days when the coefficient is smaller than average show attitude maneuvers in the measured accelerations or have experienced a high solar activity, as illustrated below.

An interesting case is the period of late October and early November 2003, when a series of violent solar eruptions took place with a broad impact on space weather [4]. Because the solar activity has a strong effect on the atmospheric drag and a smaller effect on the radiation pressure acting on a spacecraft, these events are visible in the accelerometer data. An example of the observed and modeled accelerations in along-track direction during these days are presented in Figure 4 for October 30 in 2003. The modeled and estimated accelerations follow the same trend but cannot account for the high frequency fluctuations, visible at the start and the end of the day. This is illustrated in Figure 6, where the frequency versus amplitude of

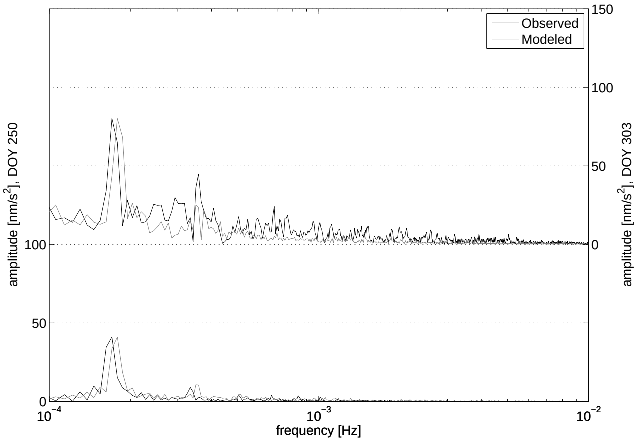

Figure 6: Spectral analysis of the observed and modeled along-track accelerations, for September 7 (DOY 250, bottom) and October 30 (DOY 303, top) 2003

the observed and modeled along-track accelerations is presented. This shows clearly that the accelerometer picks up the higher frequency part of the experienced accelerations in case of the strong solar events. In both cases a one revolution and half a revolution period is visible. For reference purposes, the same analysis is shown for a more quiet day. For both days the accelerometer signal is stronger for the half a revolution period, indicating that also for the low frequency part of the spectrum the use of an accelerometer is beneficial.

3.4 Purely dynamic orbit determination

The accelerometer data provides highly accurate observations of the non-gravitational forces acting on the spacecraft. This allows for a purely dynamic orbit determination, where only twelve parameters are estimated for the whole 1-day data arc, three initial position and velocity components and three scale and bias factors. In this case no pseudo-stochastic parameters enter the estimation. A position precision under the decimeter level is obtained, with a root-mean square position error (3D RMS) for the whole period of 9.2 cm. The orbit fit statistics compared to the JPL reference orbits are presented in Table 1. The mean value and deviation of the estimated scale and bias factors are listed in Table 3. Comparing these with the values obtained when also empirical accelerations are estimated, indicates that the estimation in X direction is not affected, while the factors in the two other directions show a bigger spread in this case.

4 Conclusions

A GPS based reduced dynamic orbit determination tool using accelerometer data is developed and used in analyzing 100 days of GRACE B data of 2003. Six calibration factors, a scale and bias factor for each instrument axis, are estimated during the orbit adjustment. Orbits determined using the accelerometer data show a good fit with respect to the reference trajectories. The total root-mean square position error amounts to 3.5 cm (3D RMS), which is comparable to the standard reduced dynamic technique, where force models are applied to calculate the non-gravitational forces acting on the spacecraft. This is also verified by an independent SLR validation.

The calibrated accelerations show a good agreement with the modeled non-gravitational forces augmented with estimated empirical accelerations. This is especially the case in along-track direction, with an average correlation factor of 0.93. When empirical accelerations are included, the scale and bias factors are more stable compared to the case without. Finally, during days of high solar activity, the application of accelerometer measurements is more beneficial.

The tool described here will be further developed to accommodate common-mode accelerations measured by the GOCE spacecraft, which will fly at a lower altitude than the GRACE satellites and is equipped with a drag compensation system.

References

[1] S. Bruinsma, D. Tamagnan, and R. Biancale. Atmospheric densities from CHAMP/STAR accelerometer observations. *Planetary and Space Sciences*, 52:297–312, 2003.

[2] K. Case, G. Kruizinga, and S. Wu. *GRACE level 1B data product user handbook*, 2002. JPL Publication D-22027, Jet Propulsion Laboratory, Pasadena.

[3] ESA. *Gravity Field and Steady-State Ocean Circulation Explorer, Reports for Assessment: The Four Candidate Earth Explorer Core Missions*. ESA SP-1233(1), ESA Publications Division, ESTEC, Noordwijk, 1999.

[4] N. Gopalswamy, L. Barbieri, E.W. Cliver, G. Lu, S.P. Plunkett, and R.M. Skoug. Introduction to violent Sun-Earth connection events of October-November 2003. *Journal of Geophysical Research*, 110(A09S00), 2005.

[5] B. Hofmann-Wellenhof, H. Lichtenegger, and J. Collins. *GPS Theory and Practice*. Springer, Berlin Wien New York, 1997.

[6] L.G. Jacchia. Revised Static Models of the Thermosphere and Exosphere with Empirical Temperature Profiles. *SAO Special Report 332, Cambridge*, 1971.

[7] A. Jäggi, U. Hugentobler, and G. Beutler. Pseudo-stochastic orbit modeling techniques for low-Earth orbiters. *Journal of Geodesy*, 80(1):47–60, 2006.

[8] Z. Kang, B. Tapley, S. Bettadpur, J. Ries, and P. Nagel. Precise orbit determination for GRACE using accelerometer data. *Advances in Space Research*, 38:2131–2136, 2006.

[9] O. Montenbruck and E. Gill. *Satellite Orbits: Models, Methods and Applications*. Springer Verlag, Heidelberg, 2000.

[10] O. Montenbruck, T. Van Helleputte, R. Kroes, and E. Gill. Reduced dynamic orbit determination using GPS code and carrier measurements. *Aerospace Science and Technology*, (9):261–271, 2005.

[11] NOAA/SEC. Daily Space Weather Indices. ftp://ftp.sec.noaa.gov/pub/indices/dayind/, 2006. US Dept. of Commerce, NOAA, Space Environment Center, 2006, Last accessed: September 2006.

[12] Ch. Reigber, H. Lühr, and P. Schwintzer. CHAMP mission status. *Advances in Space Research*, 30(2):129–134, 2002.

[13] B.D. Tapley, S. Bettadpur, J.C. Ries, P.F. Thompson, and M. Watkins. GRACE measurements of mass variability in the Earth system. *Science*, 305(5683):503–505, 2004.

[14] P. Touboul, B. Foulon, M. Rodrigues, and J.P. Marque. In orbit nano-g measurements, lessons for future space missions. *Aerospace Science and Technology*, (8):431–441, 2004.

[15] UT/CSR. GRACE Gravity Model GGM01, 2003. http://www.csr.utexas.edu/grace/gravity/ggm01/GGM01_Notes.pdf, University of Texas, Center for Space Research, Last accessed: September 2005.

[16] UT/CSR. UT/CSR Ocean Tide Models. ftp://ftp.csr.utexas.edu/pub/grav/OTIDES.TOPEX 4.0, 2006. University of Texas, Center for Space Research, November 2006, Last accessed: November 2006.

[17] T. Van Helleputte. *GPS High Precision Orbit Determination Software Tools: User Manual*, 2004. Doc. No. FDS-SUM-3110, Deutsches Zentrum für Luft- und Raumfahrt, Oberpfaffenhofen.

[18] S.C. Wu, T.P. Yunck, and C.L. Thornton. Reduced-dynamic technique for precise orbit determination of low Earth satellites. *Journal of Guidance Control and Dynamics*, 14(1):24–30, 1991.