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Reduced Order Modeling of Incompressible Flows

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POD-based Reduced Order Modeling

- Develop low-dimensional model that can represent the dynamics of a higher dimensional system
- Uses:
  - Models for real-time control
  - Sub-model generation
- Difficulties:
  - Round-off sensitive
  - Unstable / poorly conditioned
  - Units inconsistency for systems

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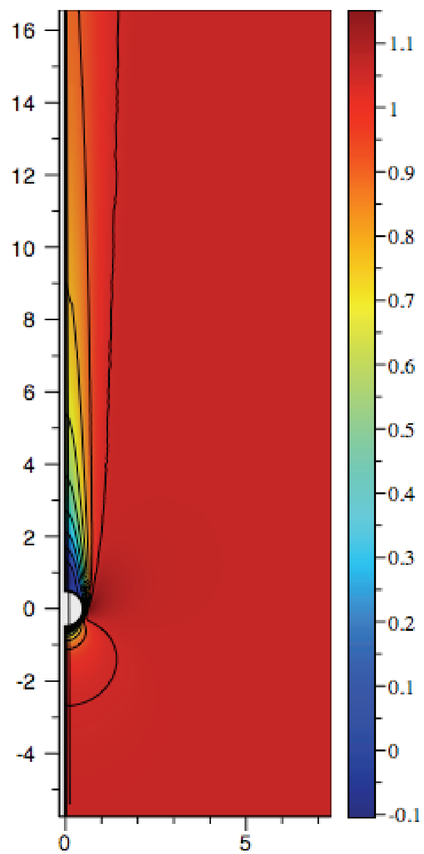
Particle Modeling

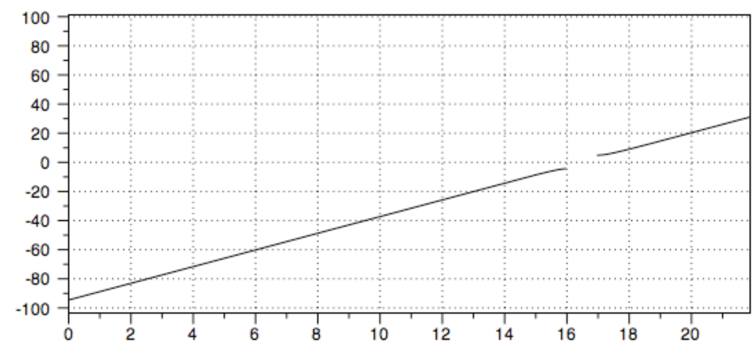
- Incompressible, flow over a sphere with variable inlet velocity

$$u = u_\infty (1 + A \sin(2\pi t / T))$$

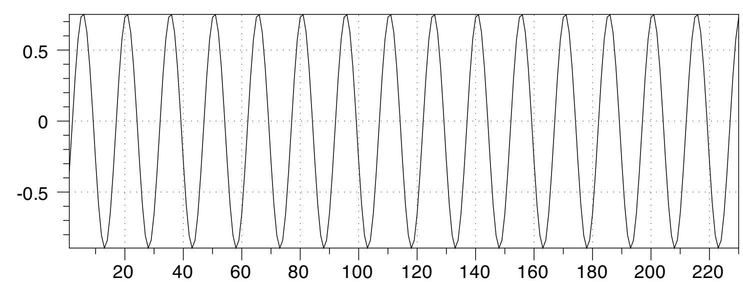
- Axial Velocity



- Centerline pressure



- Force



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Generate Modes

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- Find decomposition $\sum_j a_j(t)\phi_j(x)$ to represent solution $u(x,t)$
- Choose optimal functions by maximizing mean square projection:

$$\Pi(\phi) = \frac{\langle (\phi, u)^2 \rangle}{(\phi, \phi)}$$

- Some trouble:

$$\vec{u} = \begin{bmatrix} u_r \\ u_z \\ p \end{bmatrix}$$

$$\Pi(\phi) = \frac{\langle (\vec{\phi}, \vec{u})^2 \rangle}{(\vec{\phi}, \vec{\phi})}$$

Dimensionally inconsistent?

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Possible Choices for $\vec{u}$

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- Depends on non-dimensionalization:

$$\vec{u} = (u_r(\vec{x}, t), u_z(\vec{x}, t), p(\vec{x}, t)/(\rho u_\infty))$$

- Could be imaginary:

$$\vec{u} = \left(u_r(\vec{x}, t), u_z(\vec{x}, t), \sqrt{2(p(\vec{x}, t))/\rho}\right)$$

- Also has arbitrary constant:

$$\vec{u} = \left(u_r(\vec{x}, t), u_z(\vec{x}, t), \sqrt{2(p(\vec{x}, t) + p_0)/\rho}\right)$$

Eigenvalues of Steady Problem

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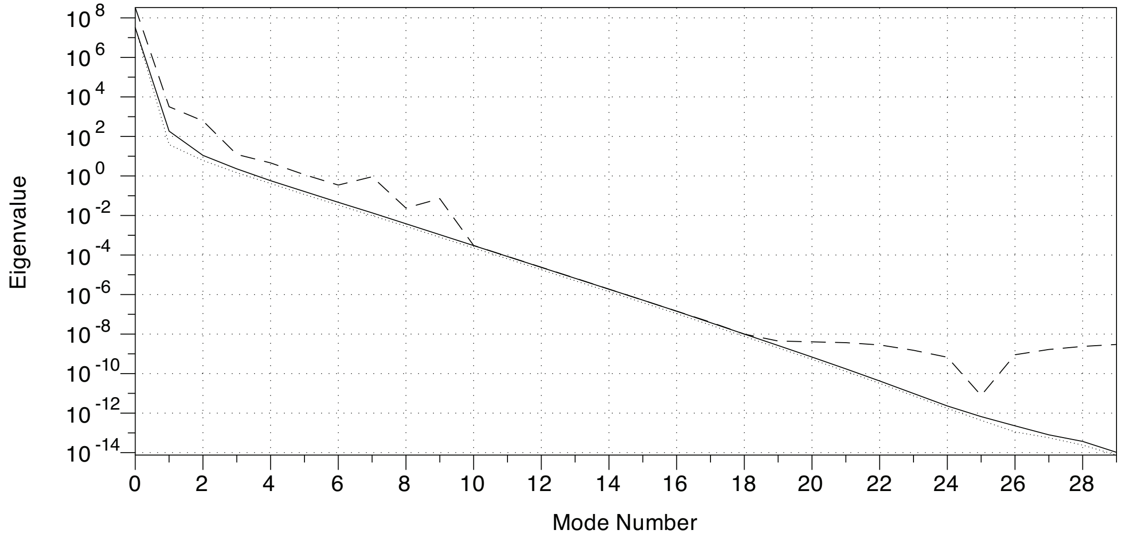

- 31 steady snapshots from $Re = 0.1$ to 100
- Eigenvalues exponentially decay
- Small number of modes can capture most of the energy
- Last five modes of Lapack DGESVD are negative

Generation of Reduced Order Model

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- Failure of Galerkin Projection

$$\int_\Omega [\phi_r, \phi_z, \phi_p] \left[\begin{array}{c} \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla)\vec{V} = -\nabla p + \mu \nabla^2 \vec{v} \\ \nabla \cdot \vec{v} \end{array} \right] d\Omega = 0$$

- Modes are all incompressible
- Continuity is always zero
- ∇p term integrated by parts is zero
- Pressure modes may be undetermined

SUPG Projection

$$\sum_{e=1}^{n_{el}} \left\{ \int\int_{\Omega} \left[-\frac{\partial \vec{\phi}}{\partial \xi}^{T} \vec{\mathrm{e}} - \frac{\partial \vec{\phi}}{\partial \eta}^{T} \vec{\mathrm{f}} \right] d\Omega + \int_{\Gamma} \vec{\phi}^{\,T} \left(\vec{\mathrm{e}}, \vec{\mathrm{f}} \right) \cdot \vec{\mathrm{n}}_{\Gamma} \, d\Gamma \right.$$

$$\left. + \int\int_{\Omega} \left[\frac{\partial \vec{\phi}}{\partial \xi}^{T} \frac{\partial \vec{\mathrm{e}}}{\partial \vec{u}} + \frac{\partial \vec{\phi}}{\partial \eta}^{T} \frac{\partial \vec{\mathrm{f}}}{\partial \vec{u}} \right] \mathrm{T} \left[\frac{\partial}{\partial \xi} \vec{\mathrm{e}} + \frac{\partial}{\partial \eta} \vec{\mathrm{f}} \right] d\Omega \right\} = 0 \quad \forall \vec{\phi}$$

- Streamwise-upwind-Petrov-Galerkin variational approach
- Allows us to seek u_r, u_z, p with no pressure decoupling
- Upwinds all the terms in the governing equations consistently
- Results in a system of M ODE's (Solved using DIRK & Newton-Rhapson)

Drag Results - Steady

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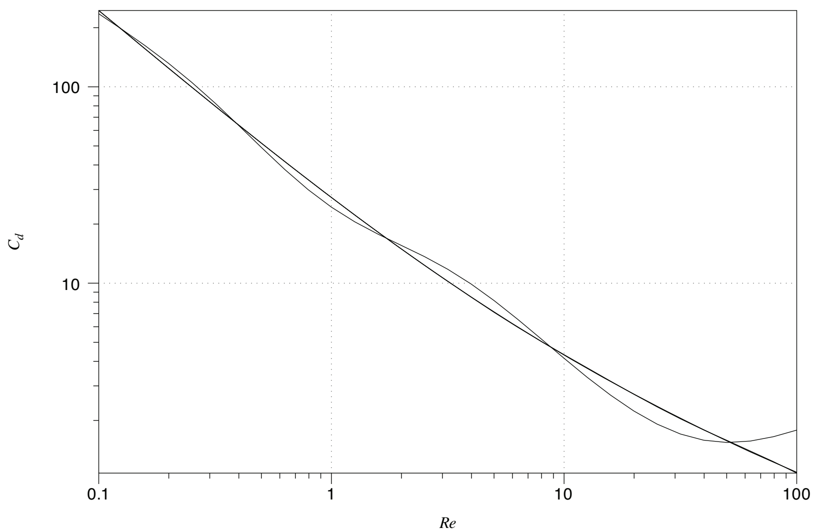

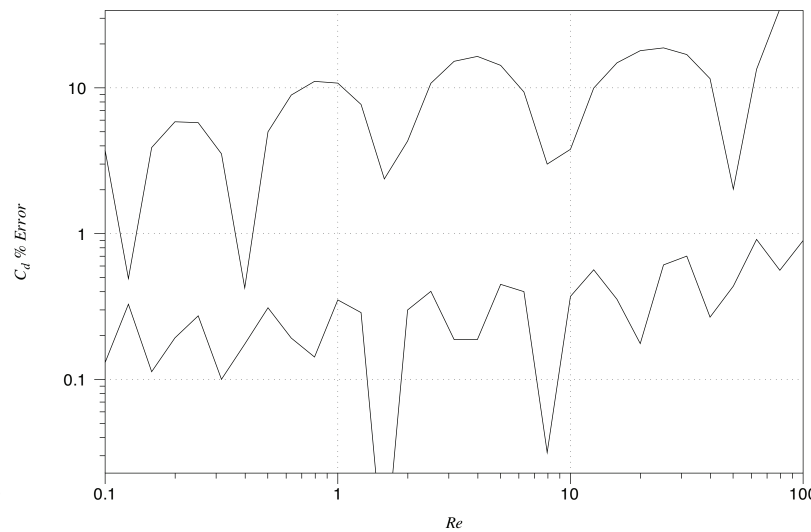

- Accuracy of drag prediction versus *Re* for 5 and 10 modes
- Significant improvement over DNS (10 vs. 21,000 degrees of freedom)
- Close to empirical correlations in performance
- All three sets of modes perform similarly

Unsteady Data

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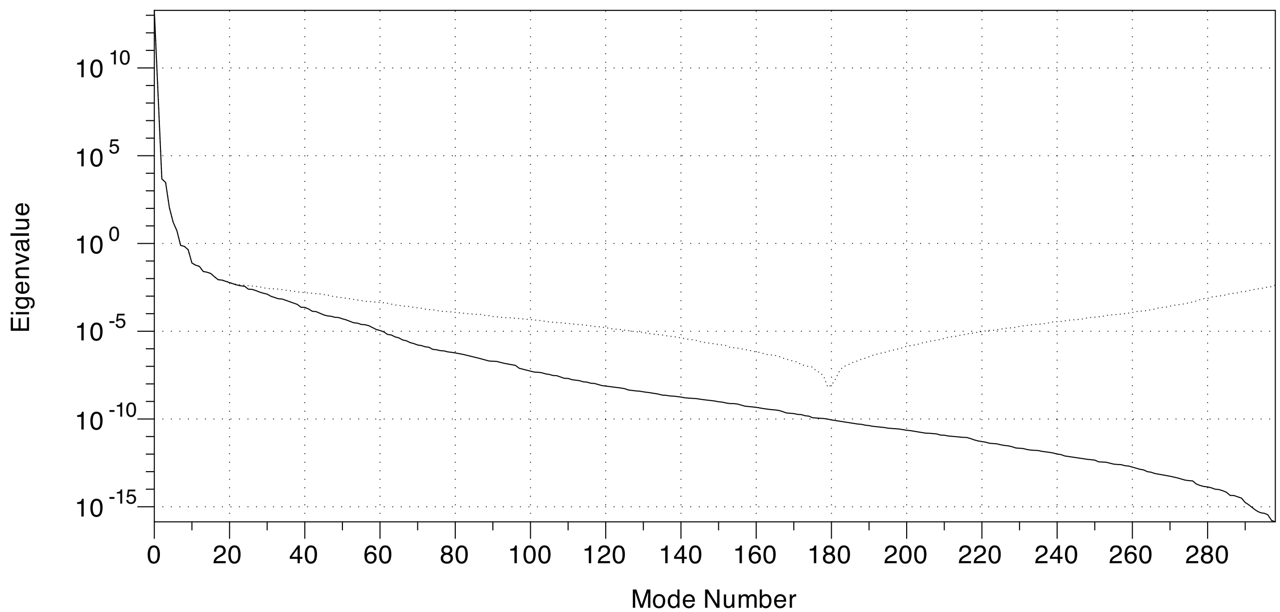
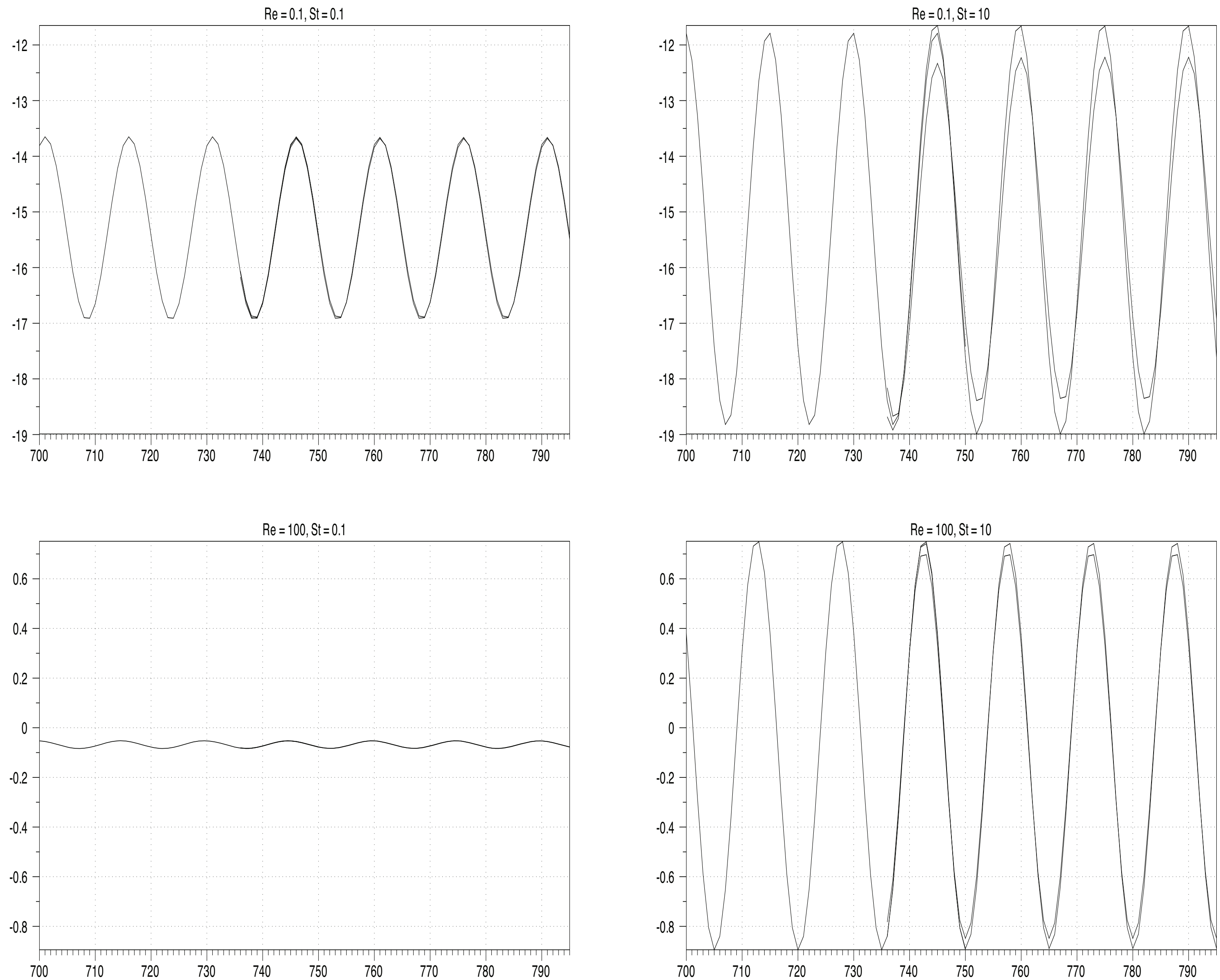
- $Re = 0.1, 1, 10, 100 \times St = 0.1, 0.5, 1, 2, 10$
- 15 time-steps/period, 20x15 snapshots



- All eigenvalues after ≈ 60 are probably round-off dominated.
- Using Lapack DGESVD, all modes beyond 180 have negative eigenvalues

Unsteady Results



Comparison of Modes

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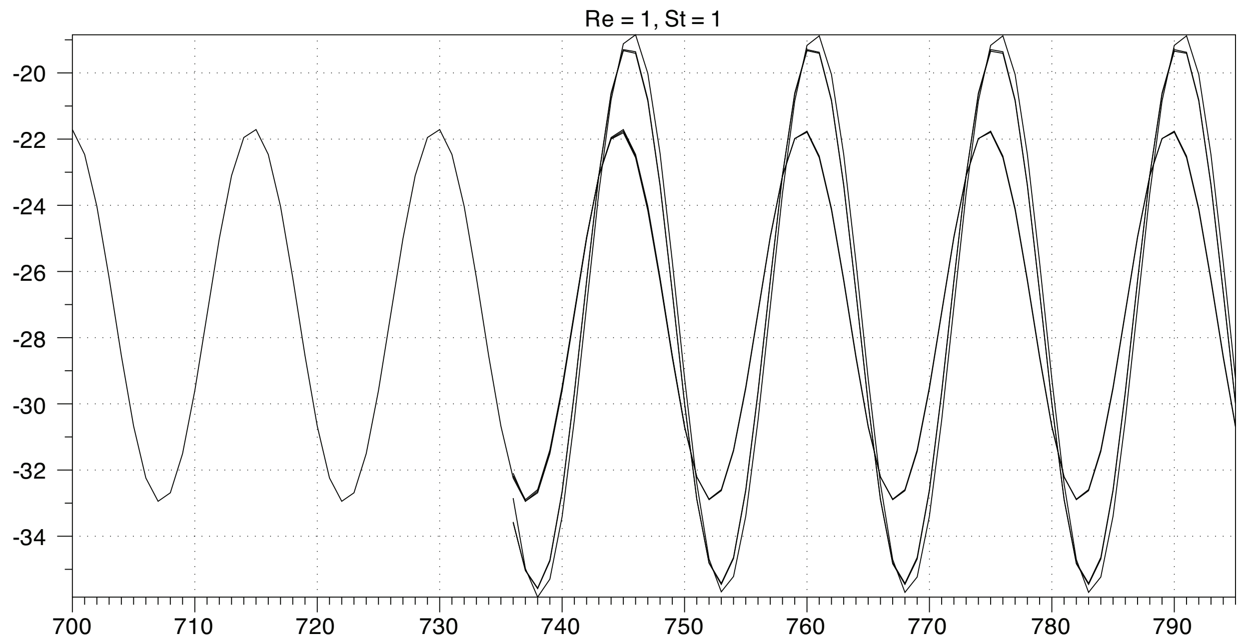

- $Re = 1$ and $St = 1$.
- SVD modes seem ok in spite of eigenvalue distribution.
- Low Mach # modes give similar results with 20 modes, but for 60 modes Newton diverges.

Conclusions

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- The devil is in the details!!!
  - Need stable numerical methods
  - Round off error can be considerable
  - Not convinced modes are correct for incompressible flow
- Nonetheless, can derive compact and accurate reduced-order models.
- Can be used to generate actuator models or full flow-field models