

# **A Patch Density Recommendation based on Convergence Studies for Vehicle Panel Vibration Response resulting from Excitation by a Diffuse Acoustic Field**

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**Producing fluid structural interaction estimates of panel vibration from an applied pressure field excitation are quite dependent on the spatial correlation of the pressure field. There is a danger of either over estimating a low frequency response or under predicting broad band panel response in the more modally dense bands if the pressure field spatial correlation is not accounted for adequately. It is a useful practice to simulate the spatial correlation of the applied pressure field over a 2d surface using a matrix of small patch area regions on a finite element model (FEM). Use of a fitted function for the spatial correlation between patch centers can result in an error if the choice of patch density is not fine enough to represent the more continuous spatial correlation function throughout the intended frequency range of interest. Several patch density assumptions to approximate the fitted spatial correlation function are first evaluated using both qualitative and quantitative illustrations. The actual response of a typical vehicle panel system FEM is then examined in a convergence study where the patch density assumptions are varied over the same model. The convergence study results illustrate the impacts possible from a poor choice of patch density on the analytical response estimate. The fitted correlation function used in this study represents a diffuse acoustic field (DAF) excitation of the panel to produce vibration response.**

## **Nomenclature**

DAF = Diffuse Acoustic Field

eq = Equation

FEM = Finite Element Model

MSFC = Marshall Space Flight Center

NESC = NASA Engineering and Safety Center

SPL = Sound Pressure Level

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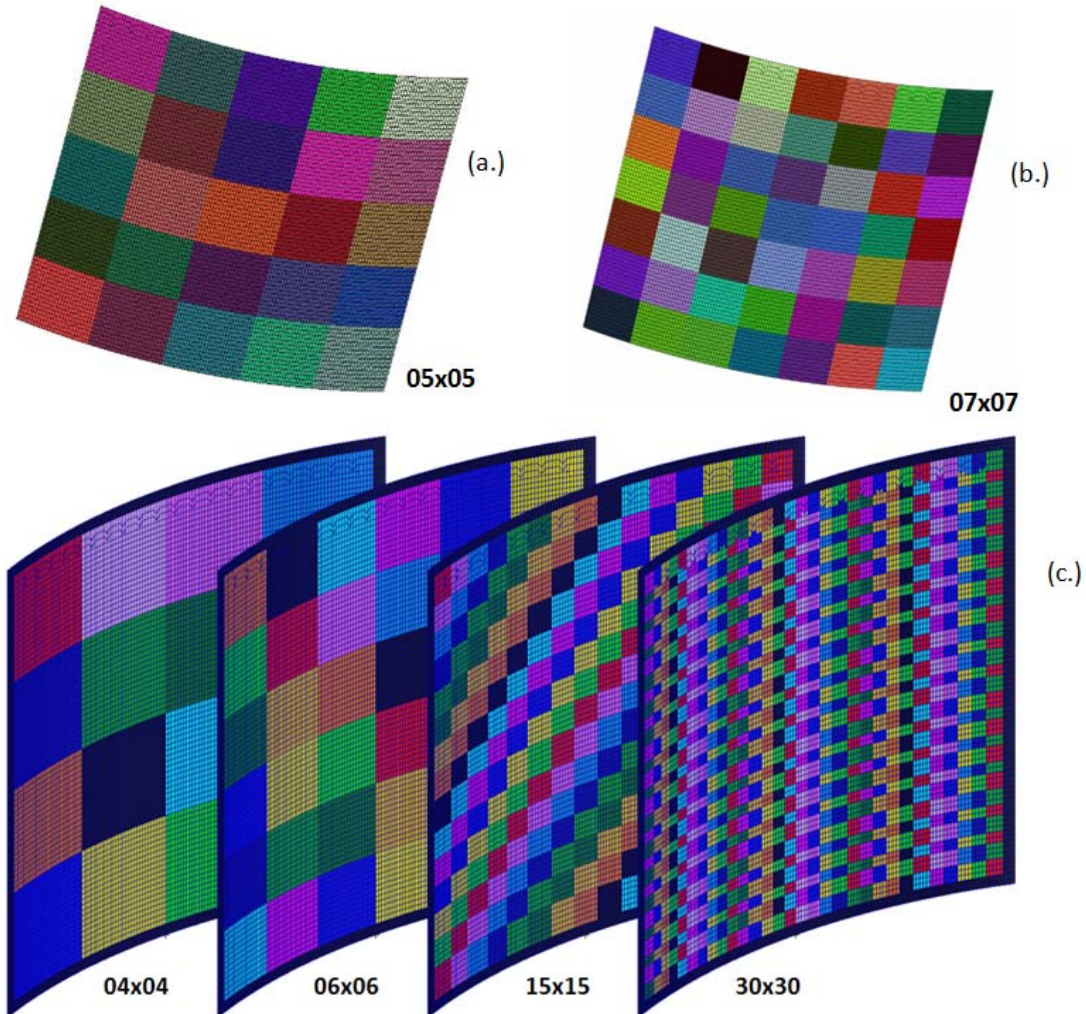
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## I. Introduction

THE importance of adequately representing the pressure field forcing function used in structural response studies of a structural finite element model vehicle panel system is illustrated in the following sections of the paper (Figure 1). The response resulting from acoustic noise excitation calculated using such models is an example of a fluid structural interaction that depends on the spatial correlation of the applied pressures at each frequency of interest. The patch method can be used to approximate a continuously changing spatial correlation function by sampling the values of that function at regular intervals and applying them consistently within each patch. This approximation approaches the continuous function in the limit as the size of each patch gets smaller and the patch



**Figure 1. Patches are defined on the surface of a vehicle panel Finite Element Model in order to specify spatial cross correlation relationships for the applied forcing function. (a.) a 05x05 Patch density with patch center to center distance of 16.2 in. (b.) a 07x07 patch density with adjacent patch center to center distance of 11.6 in. (c.) a range of other possible patch density assumptions depicted over the same panel.**

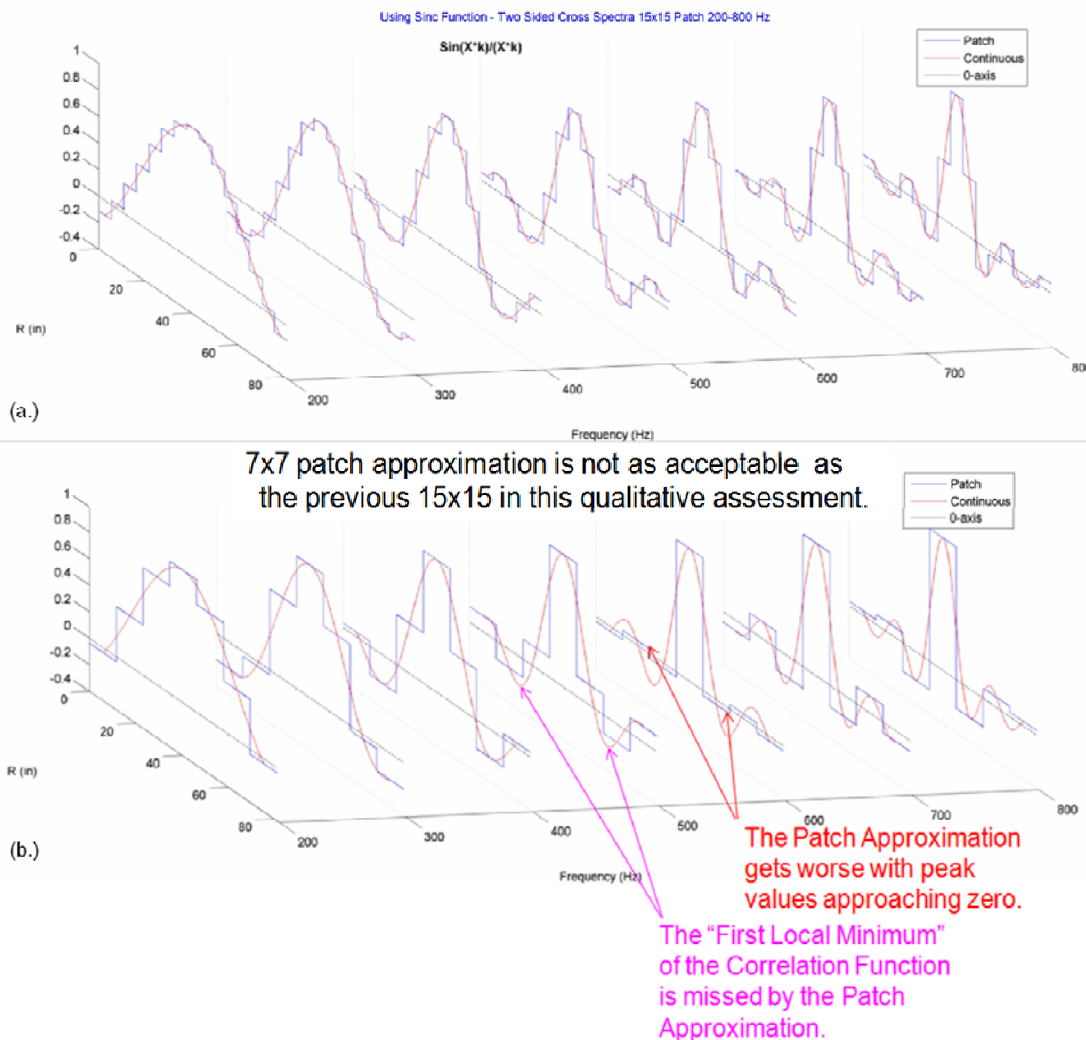
density increases. Since choosing an extremely fine patch density can be computationally expensive, the recommendations of the technical paper are intended to assist the analyst in avoiding errors, and to provide suitably accurate response results for the frequency bands important to their own work.

Using the patch method to represent the continuous spatial correlation function of a phased pressure field over a structural surface is an approximation. The approximation approaches the continuous function as the patch size becomes smaller.

As a first step, the approximation resulting from several patch assumptions are compared to a typical continuous spatial correlation function realized at different frequencies. These plots may provide insight revealing the answers to the following questions:

1. For what patch size/density does the approximation seem to be representative of the original continuous function?
2. What does the comparison look like when the patch size is too coarse (i.e. too large) for the frequency range and the approximation begins to break down.

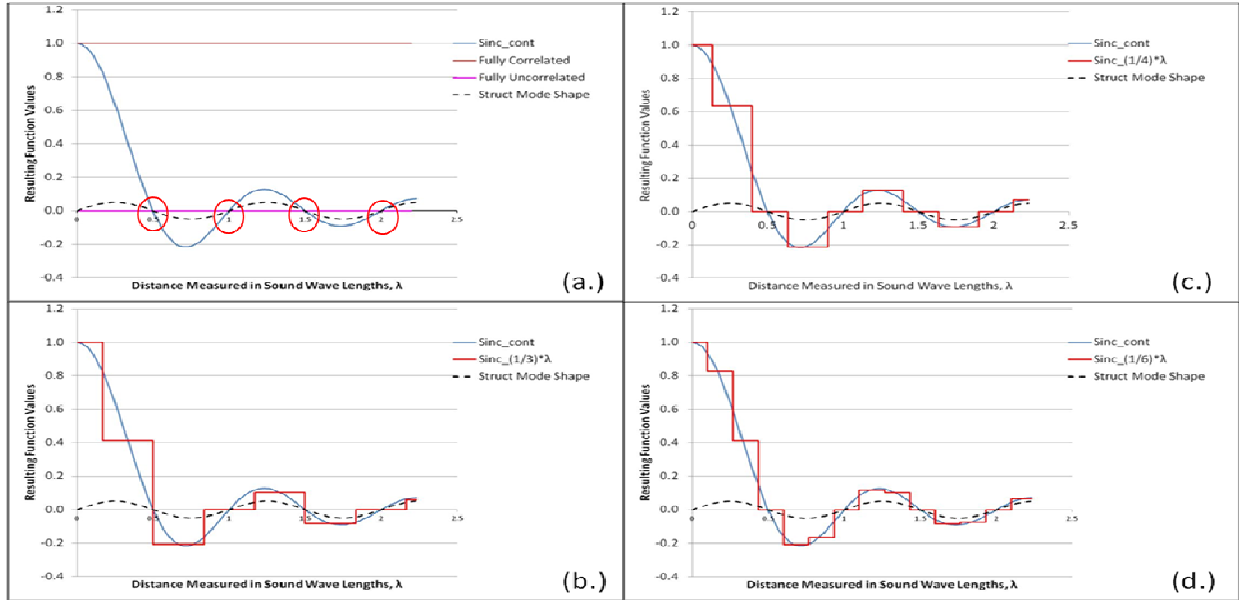
Figure 2 presents this comparison. At the center of each curve is the correlation of the pressure at one location on the surface with itself. The function is an even function with a maximum (the pressure is perfectly correlated) for self correlation. The correlation of the pressure acting at the same point with pressures acting a few inches away is diminished. If sampled far enough away, the function reaches a negative minimum. As distance increases, a series of alternating local positive maximums and negative minimums can be observed. In this qualitative comparison we ask if the patch assumption adequately represent the shape of the continuous correlation function. This becomes



**Figure 2. Waterfall plots depicting the approximation of a continuous spatial function that also depends on frequency. (a.) Consider the 15x15 patch density assumption evaluated in one dimension across frequency. (b.). The 7x7 patch density approximation of the continuous function is inferior to the 15x15 approximation evaluated the same way.**

more challenging with increased frequency. The correlation function varies more rapidly in magnitude as frequency increases because it is a function of both wave number,  $k$  and distance,  $x$  (or  $R$  in equation 4).

These observations can be normalized with respect to frequency by converting the patch dimensions so that they represent a fraction of the fluid wave length using the speed of sound (Figure 3). Imposing the applied pressure field that represents the correlation with local maxima and minima across the surfaces of the panel is desired. Therefore, care must be taken to avoid an approach that flattens out the local maxima and minima to zero. Figure 3a indicates that choosing a patch center to center distance equal to the  $\frac{1}{2}$  the fluid wave length would result in this undesired situation. Therefore, selecting a center to center distance equal to  $\frac{1}{2}$  of the wavelength at the largest frequency of interest would be too coarse an assumption for patch density. The assumed patch density should be finer. Patch densities corresponding to  $\frac{1}{3}$  or  $\frac{1}{4}$  of the fluid wavelength at the highest frequency of interest have a better probability of providing adequate results.



**Figure 3. Spatial Correlation Function Normalized to Fluid Wavelength (a.) DAF Sinc Correlation Function compared to a proposed Structural Panel Bending Mode Shape. Note that zeros occur at half wavelengths. (b.) Approximation Assuming  $(1/3)\lambda$  Patch Size. (c.) Approximation Assuming  $(1/4)\lambda$  Patch Size. (d.) Approximation Assuming  $(1/6)\lambda$  Patch Size.**

An explanation describing how the correlation function is used to apply the pressure field to a finite element model is provided in Section II.

In addition to the qualitative comparisons provided above, the enclosed area of the correlation function is explored visually in Section III. This provides a more quantitative assessment of the spatial distribution of phased pressures across the panel surface.

Afterward, a convergence study is presented using a single launch vehicle panel FEM to demonstrate by comparison the impact of using a poor patch density assumption to complete the analysis. Developing insights that help us to predict sufficient patch density to provide adequate convergence within the intended frequency range of interest for our proposed analyses is a goal of this study.

## II. The Applied Pressure Forcing Function depends on the Spatial Correlation Function

The equations presented below provide the framework for applying a pressure field excitation over the surface of a panel system. A random pressure field on a launch vehicle skin surface may be approximated by dividing the surface into patches, or regions of uniform pressure with no phase offsets within the patch. The size of each patch must be chosen to be small enough to justify the assumption of uniform (though dynamically varying) pressure with zero phase offset across the patch. The selection of the appropriate patch and element size is an important parameter for specifying the forcing function. The pressure may be defined as a stationary Gaussian random field with spatially varying autospectral density. The pressures on any pair of patches may be correlated, exhibiting a non-zero

cross-spectral density between them. The random pressure field is thus a Hermitian matrix of spectral densities of dimension  $N_p$ , the total number of pressure patches. The pressure autospectra occur on the diagonal of the matrix. The cross-spectra appear as off-diagonal terms. The random pressure field on all patches may be written as

$$\mathbf{P}_{N_p}(\omega) = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1N_p} \\ P_{21} & P_{22} & \cdots & P_{2N_p} \\ \vdots & \vdots & \ddots & \vdots \\ P_{N_p 1} & P_{N_p 2} & \cdots & P_{N_p N_p} \end{bmatrix} \quad (1)$$

where  $P_{bc} = P_{cb}^*$ , and the asterisk denotes the conjugate operator. If spatial functions  $\gamma(\omega, R)$  are defined that relate the autospectra to the cross-spectra, eq 1 may be written as

$$\mathbf{P}_{N_p}(\omega) = \begin{bmatrix} \gamma_{11} \hat{P}_{11} & \gamma_{12} \hat{P}_{12} & \cdots & \gamma_{1N_p} \hat{P}_{1N_p} \\ \gamma_{21} \hat{P}_{12} & \gamma_{22} \hat{P}_{22} & \cdots & \gamma_{2N_p} \hat{P}_{2N_p} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{N_p 1} \hat{P}_{1N_p} & \gamma_{N_p 2} \hat{P}_{2N_p} & \cdots & \gamma_{N_p N_p} \hat{P}_{N_p N_p} \end{bmatrix} \quad (2)$$

where  $\hat{P}_{bc} = \sqrt{P_{bb} P_{cc}}$  and  $\gamma_{bb}$  have been added to the diagonals for generalization. The expression for  $\hat{P}_{bc}$  arises from an inequality requirement on the coherence, which states that

$$0 \leq \frac{|P_{bc}(\omega)|^2}{P_{bb}(\omega) P_{cc}(\omega)} \leq 1.0 \quad (3)$$

For a diffuse acoustic field, the spatial correlation functions  $\gamma$  for the pressure field may be expressed as

$$\gamma_{bc}(\omega, R_{bc}) = \frac{\sin(R_{bc} \kappa(\omega))}{R_{bc} \kappa(\omega)} \quad (4)$$

where  $R_{bc}$  is the distance between the centroids of patches  $b$  and  $c$ ,  $\kappa(\omega) = \omega / C_o$ , and  $C_o$  is the speed of sound through the fluid medium adjacent to the panel/patch. The patch centroids are constrained to lie on the curved skin surfaces. When  $b = c$ , the spatial functions coincide with the patch autospectra, the distance  $R$  between patches vanishes and  $\gamma_{bb} \rightarrow 1.0$  in the limit as  $R_{bb} \rightarrow 0$  (L'Hopital's Rule).

The relationship between patches has therefore been described in equation 4 and applied on the off diagonal terms of the matrix in equation 2. The expression in equation 4 was developed by Rafaely<sup>6</sup>. Approximation of the function represented by equation 4 is explored in the next section.

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<sup>6</sup> (Rafaely, June 2000)



### III. Quick Look Evaluation For Adequacy of the Approximation

Short of performing a convergence study, an analyst might compare the look of the approximation provided by his choice of patch density to the function. They might also quantitatively assess the values of the approximation in regions where the function returns positive and negative values. Three types of function integration areas were calculated in order to assess the values of the function in certain regions on the surface of the panel. These three area types are illustrated in figures 4 through 6.

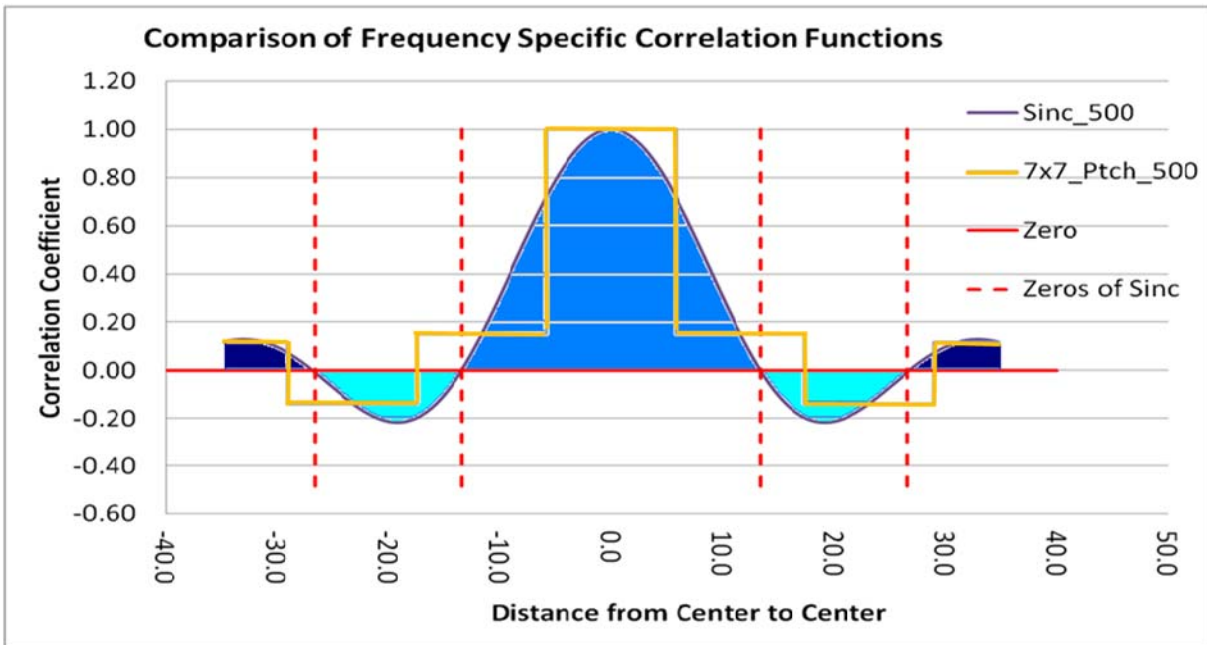


Figure 4. Area Integration Type I. Spatial Correlation Function (i.e. the Sinc Function) Evaluated at 500 Hz. Patch approximation of the same function at 500 Hz. Enclosed area of the continuous Sinc correlation function is presented in shaded lobes above and below zero. This 500 Hz example shows that a 7x7 patch approximation is not ideal or adequate at this frequency.

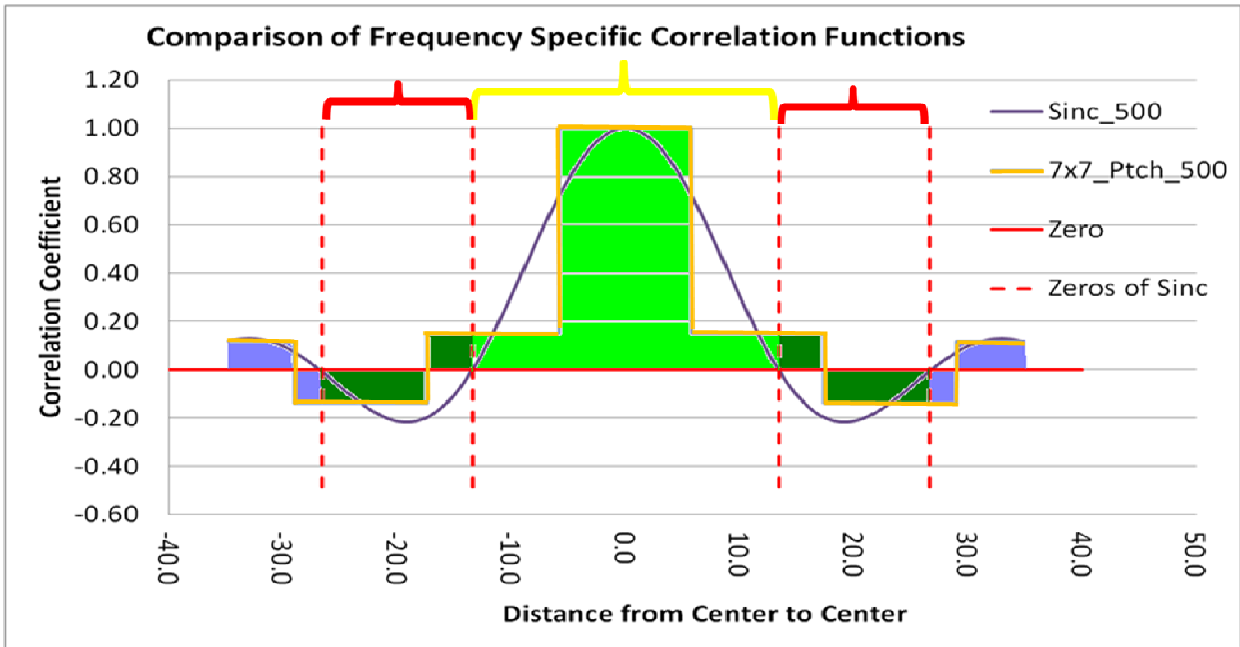


Figure 5. Area Integration Type II. Spatial Correlation Function - Between the yellow bracketed Zeros the Function should be positive with positive enclosed area. Between the red bracketed Zeros the Function should be negative with negative enclosed area. The approximation is not perfect by these evaluation criteria. The net enclosed area in red is negative (but smaller negative).

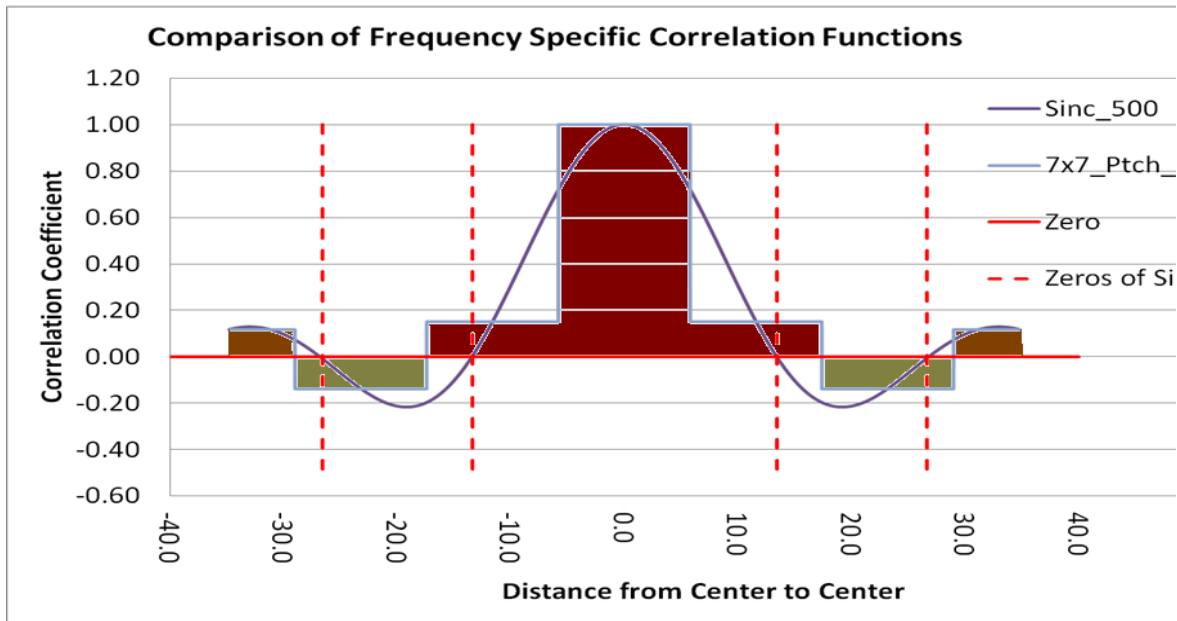
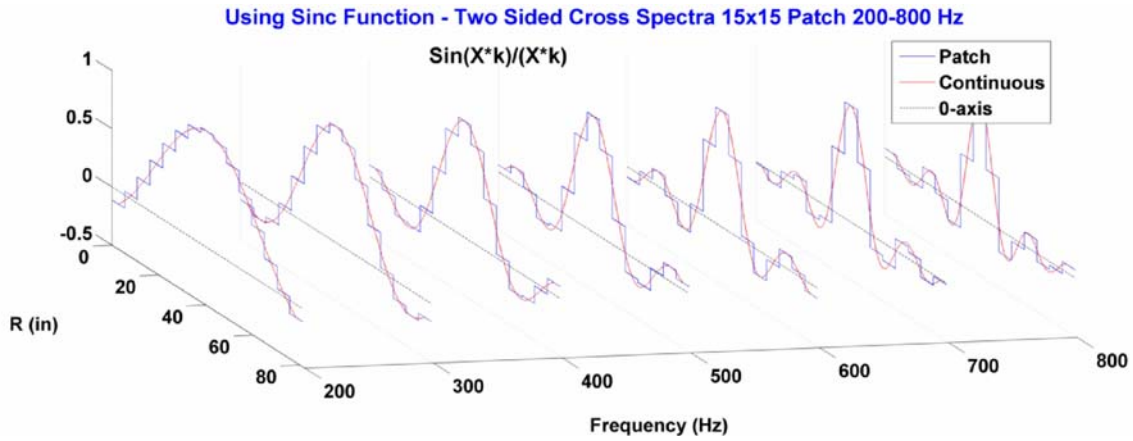


Figure 6. Area Integration Type III. Spatial Correlation Function - The actual enclosed area of the 7x7 approximation is spatially distributed differently than the Sinc. The size of its realized positive and negative lobes was also evaluated without regard to the zeros of the Sinc function.

The 15x15 patch assumption presented in Figure 2 was chosen to present the comparison of the local enclosed area of the approximation to the continuous function. Figures 7 through 12 are provided to illustrate this comparison

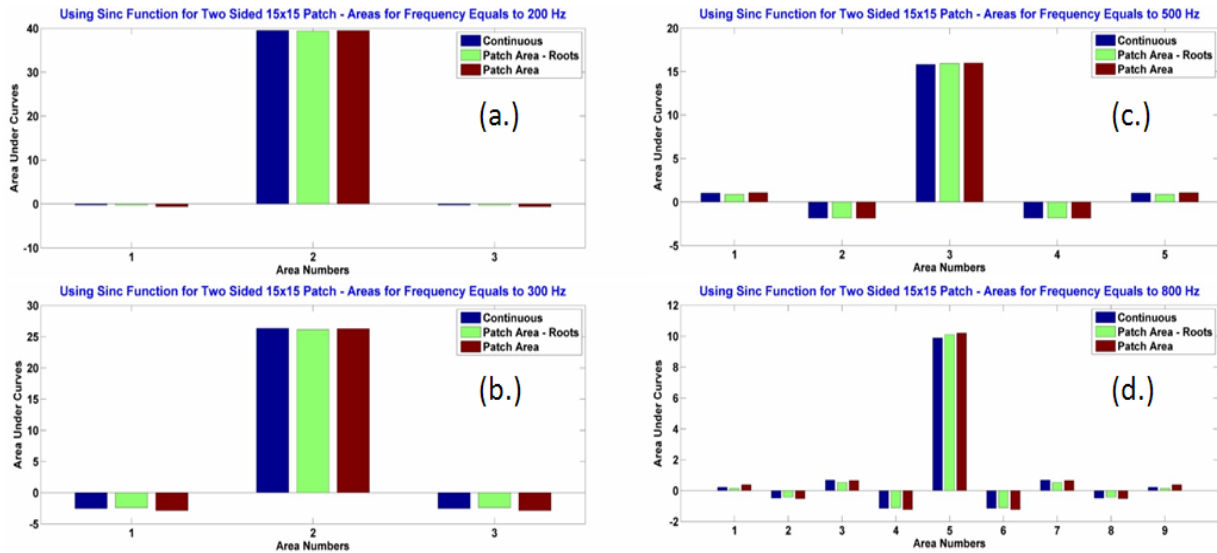
of the differences in these Type I, II, and III area calculations. First the shape (Figure 7) and then the calculated areas are compared in different frequency ranges.

The Type I area calculation is associated with the continuous function and appears blue on the bar graph for each



**Figure 7. Spatial Correlation Function - The plot presents a single row of 15 patches from a 15x15 approximation of the Sinc spatial correlation function providing good approximations in the range from 200 to 800 Hz.**

positive or negative lobe of the function. The zeros of the continuous function make the borders of each integration region lobe. The Type II area calculation is presented as green on the bar graphs and represents the enclosed area for the patch approximation over that same integration region described by the continuous function zeroes. The Type III area calculation is presented as brown on the bar graphs and represents the enclosed area over a different integration area region described by the zeroes of the patch approximation of the continuous function. First the shape and then the calculated areas are compared in different frequency ranges. The integration areas return much the same value in the frequency range up to 500 Hz (Figure 8).

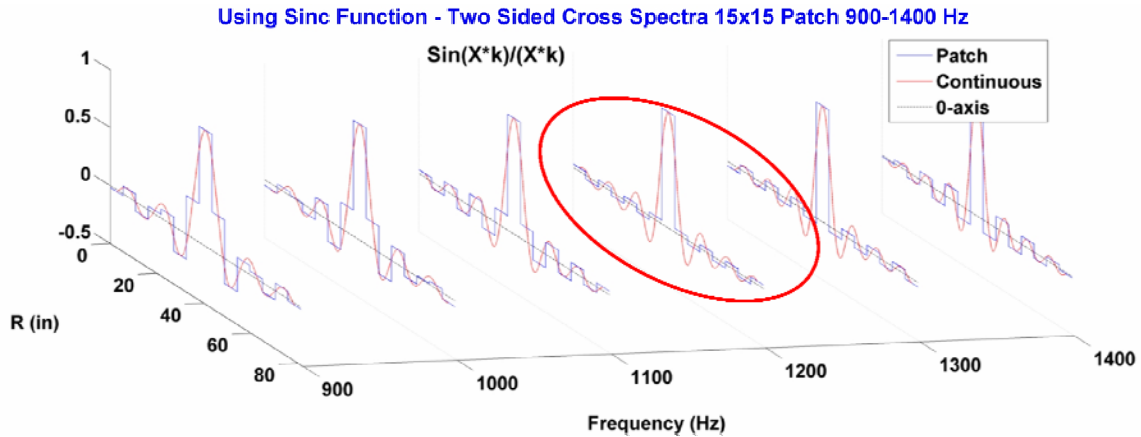


**Figure 8. Spatial Correlation Function Area Comparisons: (a.) Correlation Area Comparison at 200 Hz. (b.) Correlation Area Comparison at 300 Hz. (c.) Correlation Area Comparison at 500 Hz. (d.) Correlation Area Comparison at 800 Hz.**

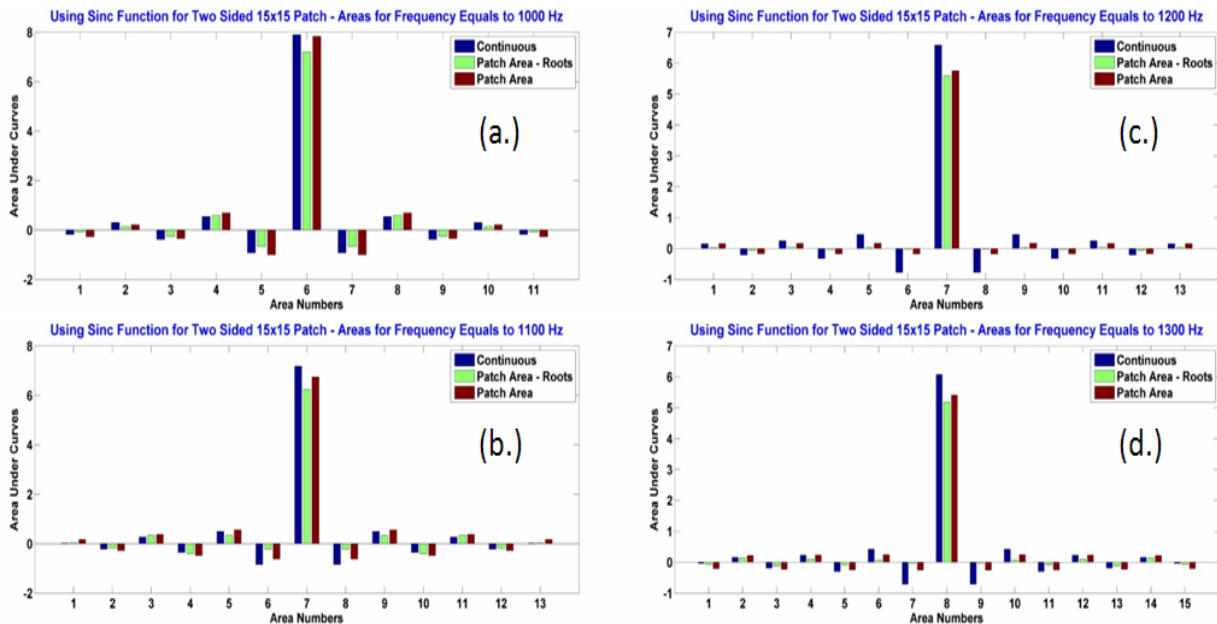
The Sinc function is a function of wave-number and thus changes with frequency. A smaller patch size/higher patch density is required to accurately represent results at higher frequencies. The choice of patch density should depend on the highest frequency of interest for the structural response analysis. Referring to Figure 9, note that the



difference between the approximation and the continuous Sinc function at 1250 Hz is quite pronounced. For this case, the 5.4 in patch size is too coarse. The 5.4 inches also is approximately equal to  $\frac{1}{2}$  the fluid wavelength in the range near 1200 Hz. The result is that the continuous function details are not well approximated spatially, in fact the patch approximation magnitude approaches zero for all the side lobes of the function. A condition approximating the field similar to uncorrelated – “rain on the roof”.



**Figure 9. Spatial Correlation Function - The plot presents a single row of 15 patches from a 15x15 approximation of the Sinc spatial correlation function providing good approximations in the range from 900 to 1400 Hz.**



**Figure 10. Spatial Correlation Function Area Comparisons: (a.) Correlation Area Comparison at 1000 Hz. (b.) Correlation Area Comparison at 1100 Hz. (c.) Correlation Area Comparison at 1200 Hz. (d.) Correlation Area Comparison at 1300 Hz.**

The Type II green area calculation grossly under predicts the integration areas in all of the side lobes at 1200 Hz (Figure 10). Both Type II and Type III area calculations provide lower estimate in the central region than the continuous function. The influence of the forcing function in this central region, closest to the location at which the response transfer function is being calculated in the FEM method, may become more important than the side lobes

as frequency increases. Correlation is under predicted for contribution of the fem region closest to the response node.

Both Type II and Type III area shows too much correlation in the central lobe around 1700, 1800, and 1900 Hz (Figure 12). This may be even more important than poorly approximating the side lobes at 1200 Hz, because the more distant pressures do not contribute nearly as efficiently to the response of a node on the panel as the pressures acting more locally in these higher frequencies. When the central lobe becomes narrow enough, an uncorrelated, rain on the roof, approximation can become nearly adequate. But if the patch method is coarse then contribution from the local nodes is over approximated.

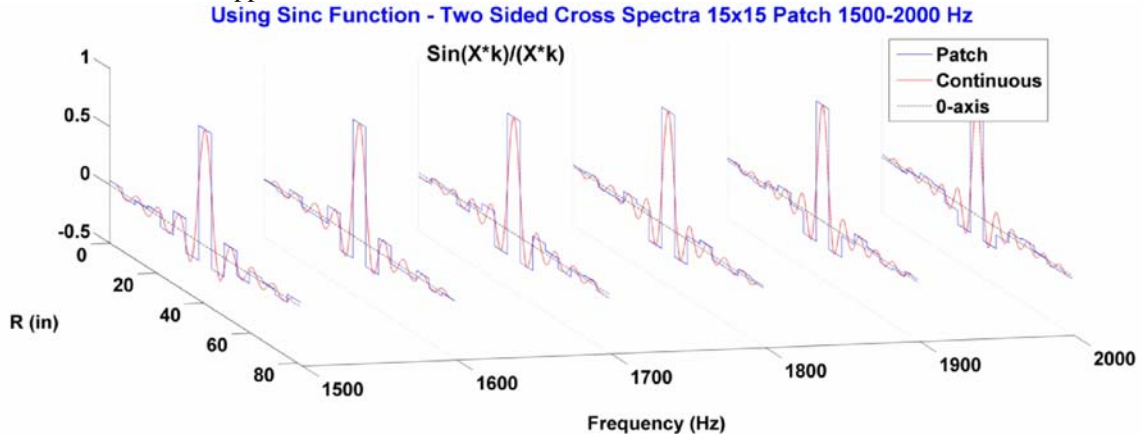


Figure 11. Spatial Correlation Function - The plot presents a single row of 15 patches from a 15x15 approximation of the Sinc spatial correlation function providing poor approximations in the range from 1600 to 2000 Hz.

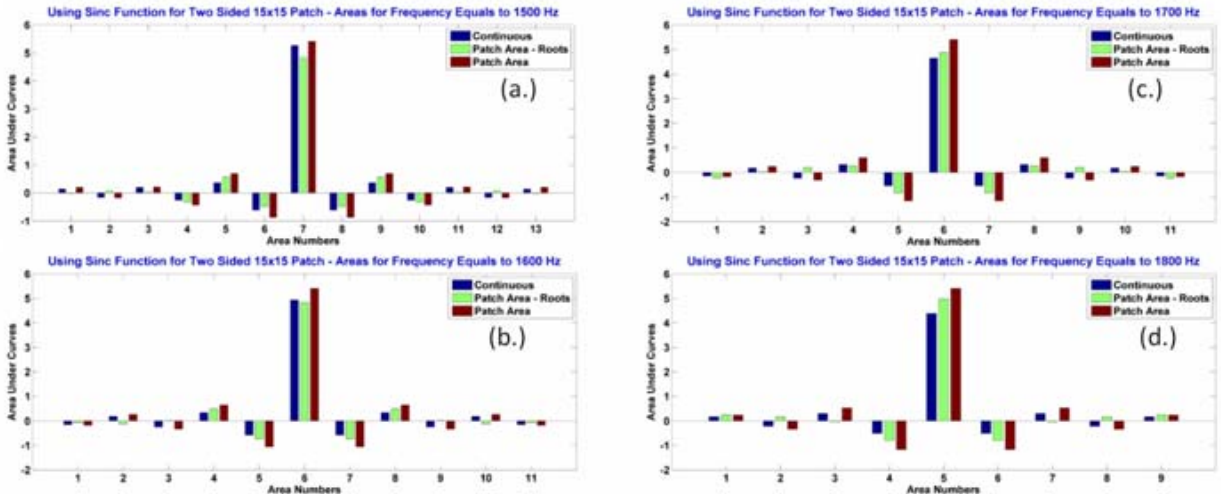
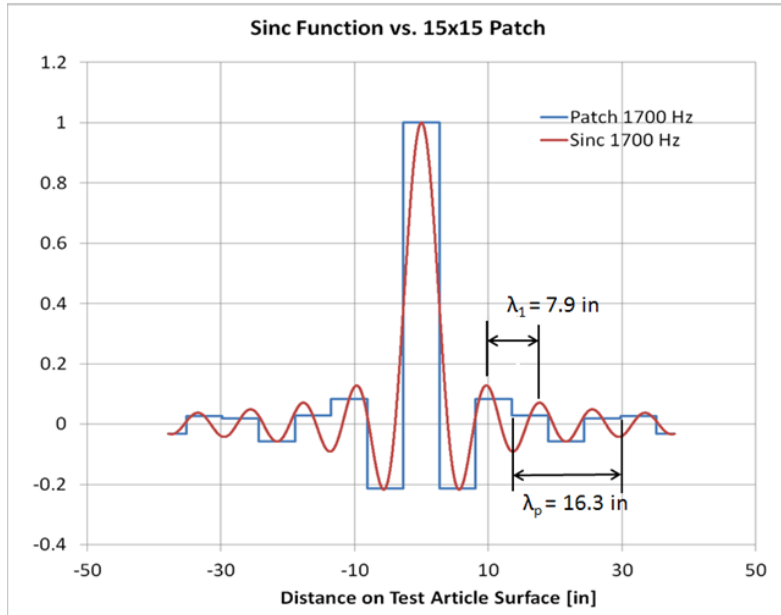


Figure 12. Spatial Correlation Function Area Comparisons: (a.) Correlation Area Comparison at 1500 Hz. (b.) Correlation Area Comparison at 1600 Hz. (c.) Correlation Area Comparison at 1700 Hz. (d.) Correlation Area Comparison at 1800 Hz.

Figure 13 is provided so that the reader can appreciate an effect similar to analog to digital aliasing of time data. When the patch assumption samples the intended correlation function too infrequently, a different wave form is represented by the patch approximation than that which was intended.



The continuous Sinc Function:

$$\gamma = \frac{\sin(kx)}{kx}$$

The apparent but not physical speed of sound implied by the poor patch approximation:

$$c_{apparent} = f * \lambda_p$$

$$c_{apparent} = 1700 * 16.3$$

$$c_{apparent} = 27710 [in / sec]$$

Figure 13. The details of how the continuous function varies spatially are not represented well at 1700 Hz because the patch size is too large to approximate the function well. The poor approximation resembles a frequency aliasing affect. This effect could produce a pseudo coincidence frequency for some designs.

#### IV. Vehicle Panel Response Convergence Study

Actual vibration response was then calculated using a range of different patch density assumptions with the same vehicle panel FEM. The applied pressure field was identical in each case except for the number of patches and therefore the fidelity of realized cross correlation relationships represented by the cross terms. The 1x1 case

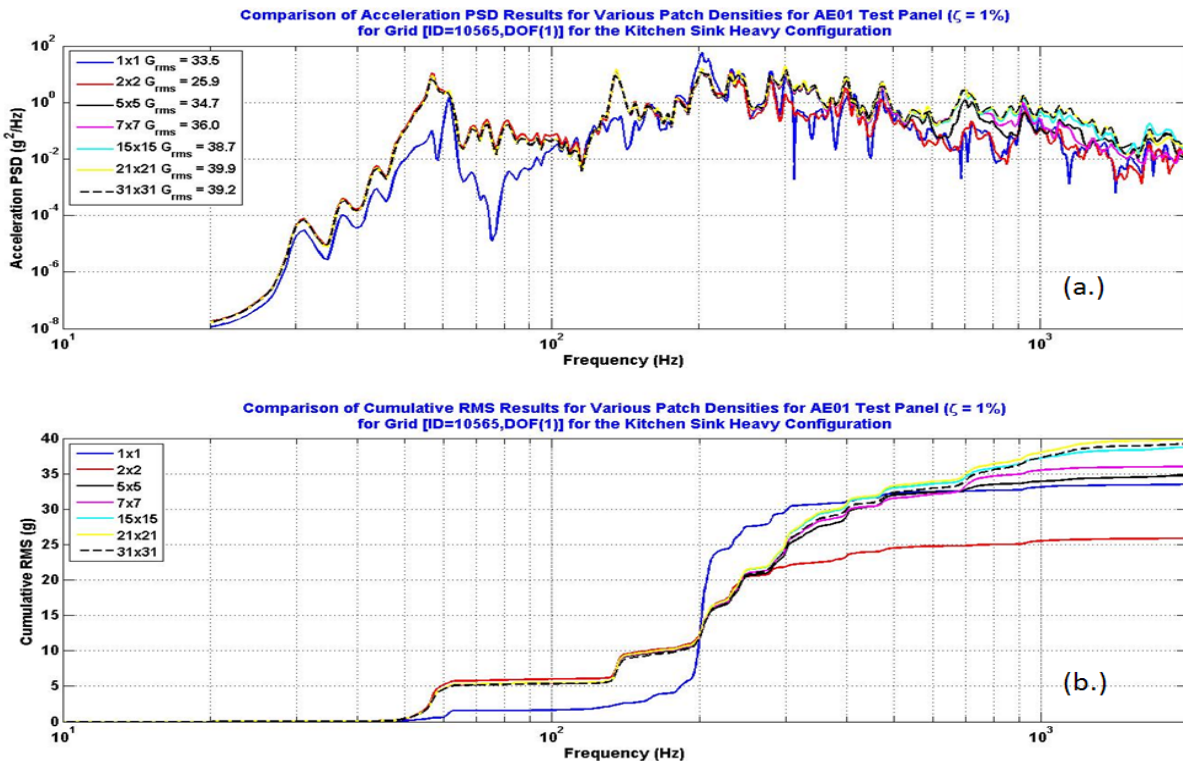
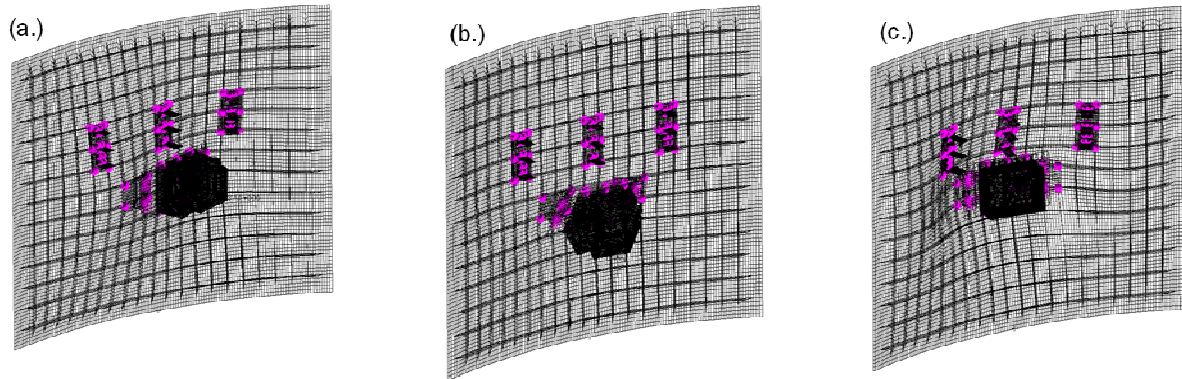


Figure 14. Convergence Study Results: (a.) Overlaid Response from 7 different patch density assumptions. (b.) Overlay of Cumulative RMS plots from the same 7 patch density assumptions.

provides just one forcing function patch which covers the entire vehicle panel. Treating the pressure field as “perfectly correlated and in phase” over the entire panel surface results in a large error. If figure 14a is examined from left to right, the response represented by the blue line is incorrect even before the major response peaks in the 50-60 Hz region. Clearly the 1x1 patch assumption goes wrong right away.

The 2x2 assumption goes awry before we reach 300 Hz, as is most easily seen in the cumulative RMS response plot 14b where the red curve peels off from the other curves, underestimating the cumulative rms response for the



**Figure 15. The first three system mode shapes of the test article (a.) 57.0 Hz an even mode with nodal line extending vertically near center of panel dividing two equal areas of panel which respond 180 degrees out of phase from each other (b.) 59.5 Hz another even mode with nodal line extending horizontally near center of panel dividing two nearly equal areas of panel which respond 180 degrees out of phase from each other (c.) 61.5 Hz an odd mode with the largest area of panel displacing in the same direction.**

rest of the frequency range. The 5x5 and 7x7 cumulative rms curves also peel off at somewhat higher frequency. From the convergence example, the reader should begin to see the need to choose a patch density assumption that is adequate for the frequency range over which he desires to successfully calculate response estimates.

Of the three important structural modes in the range from 50-65 Hz (Figure 15), the totally correlated response case (the 1x1 blue response trace of Figure 14) is only effective at exciting the odd mode. It therefore provided a very poor approximation. The cases are summarized in Table 1. Figures 14 and 15 both correspond to the response calculated for the same configuration of hardware integrated onto the vehicle panel, (an electronics box assembly).

The results for a second configuration are provided in figures 16 through 19 in order to confirm the trends which require choosing an adequate patch density for the frequency range of concern in analysis. This second configuration consisted of a single shelf and strut assembly. The shelf was configured with two small footprint stacks of increment plates which were simulating electronics boxes on the shelf (Figure 20). Figure 20 also illustrates that the fundamental mode of the vehicle panel shelf assembly system is an even function.

The results for the shelf configuration were examined at different locations and in different directions on the assembly. The response plots from this shelf configuration all demonstrate the inadequacy of the fully correlated approach. Figures 16, 17 and 19 all show that the fully correlated solution does poorly beginning with the first resonant mode of response (an even mode shape). The results from Figure 17 show that the fully correlated assumption can produce extremely poor estimates of the response.

The 2x2 patch assumption provides four regions on the panel which are just larger than 40 inches on a side. This patch assumption and the other finer assumptions tend to produce excellent response estimates at low frequency. As the number of patches used to approximate the spatial correlation of the forcing function increased, so did the frequency range over which the assumption could provide adequate response results.

The finer patch densities are necessary as the frequency range of interest expands to higher frequencies. However there is a tradeoff between patch density and computational expense. For cases where the analyst only needs accuracy over a smaller frequency range, a perfectly adequate set of results may be obtained using a smaller number of patches.

The choice of the number of patches will vary from problem to problem depending on the size of the surfaces which are receivers for acoustic energy. Therefore, the size of the individual patch or the smallest distance between patch centers is the general parameter which is portable to different scenarios in this class of problem.



## V. Conclusion

Spatial correlation function approximation using the patch method was illustrated across the frequency range for several different patch assumptions. Ranges where the approximation is not ideal were identified by qualitative/quantitative comparison. Two different convergence studies illustrating the sensitivity to patch density were presented.

The recommend patch size should be based on the highest frequency range of interest. Maintain “patch center to center distance” less than 1/2 the fluid wavelength at highest frequency of interest. Note 1/3 or 1/4 of fluid wavelength is preferred.

A summary relating rectangular “Patch Size” to “Fluid Acoustic Wavelength” is provided in Table 1 below.

For a Vehicle Panel of Length 81.125 in				Number of Patches	Patch Size [in]	Fraction of Wavelength									
						67 in 200 Hz	26.8 in 300 Hz	19.1 in 400 Hz	14.9 in 500 Hz	12.2 in 600 Hz	10.3 in 700 Hz	8.9 in 800 Hz	6.4 in 900 Hz	4.3 in 1000 Hz	
1	X	1	=	1	81.1	1.21	3.03	4.24	5.45	6.66	7.87	9.08	12.71	18.77	
2	X	2	=	4	40.6	0.61	1.51	2.12	2.72	3.33	3.94	4.54	6.36	9.38	
5	X	5	=	25	16.2	0.24	0.61	0.85	1.09	1.33	1.57	1.82	2.54	3.75	
7	X	7	=	49	11.6	0.17	0.43	0.61	0.78	0.99	1.12	1.30	1.82	2.68	
9	X	9	=	81	9.0	0.13	0.34	0.47	0.61	0.74	0.87	1.01	1.41	2.09	
11	X	11	=	121	7.4	0.11	0.28	0.39	0.50	0.61	0.72	0.83	1.16	1.71	
13	X	13	=	169	6.2	0.09	0.23	0.33	0.42	0.51	0.61	0.70	0.98	1.44	
15	X	15	=	225	5.4	0.08	0.20	0.28	0.36	0.44	0.52	0.61	0.85	1.25	
21	X	21	=	441	3.9	0.06	0.14	0.20	0.26	0.32	0.37	0.43	0.61	0.89	
31	X	31	=	961	2.9	0.04	0.10	0.14	0.18	0.21	0.25	0.29	0.41	0.61	

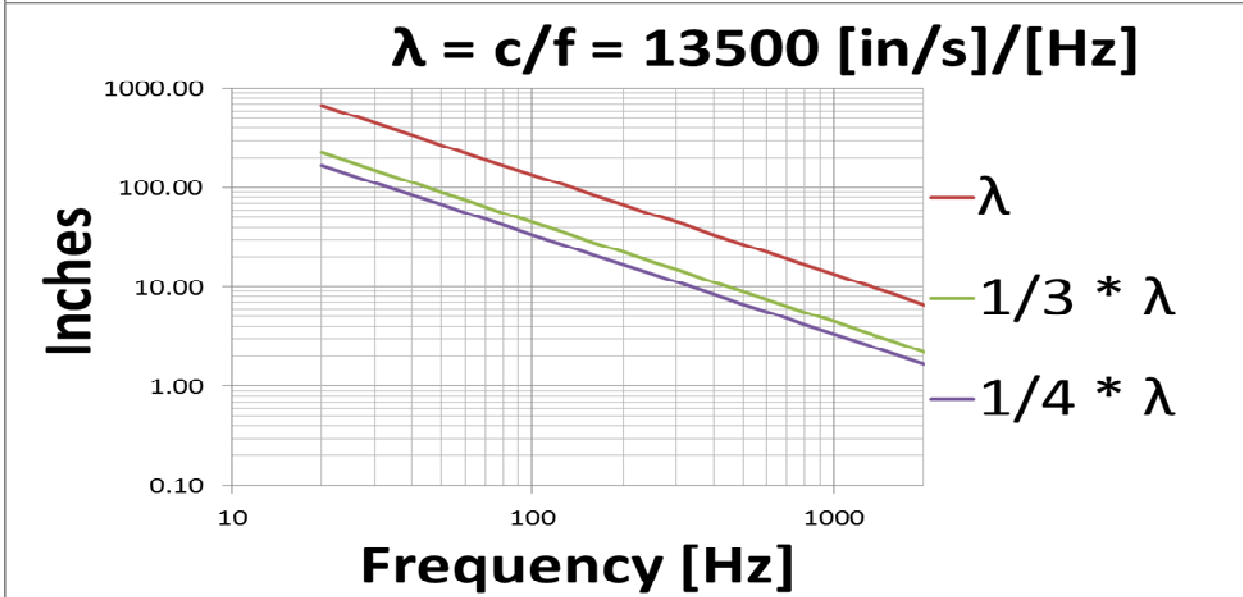


Table 1. Relating the patch density of example to the patch center to center distance. The yellow highlights correspond to patch assumption rows that are approximately 1/4 of the fluid wavelength at each frequency listed in red above. The orange highlights correspond to patch assumption rows that are approximately 1/3 of the fluid wavelength. The plot below table is a helpful graphic for picking minimum patch to patch center distance relative to frequency range of interest.

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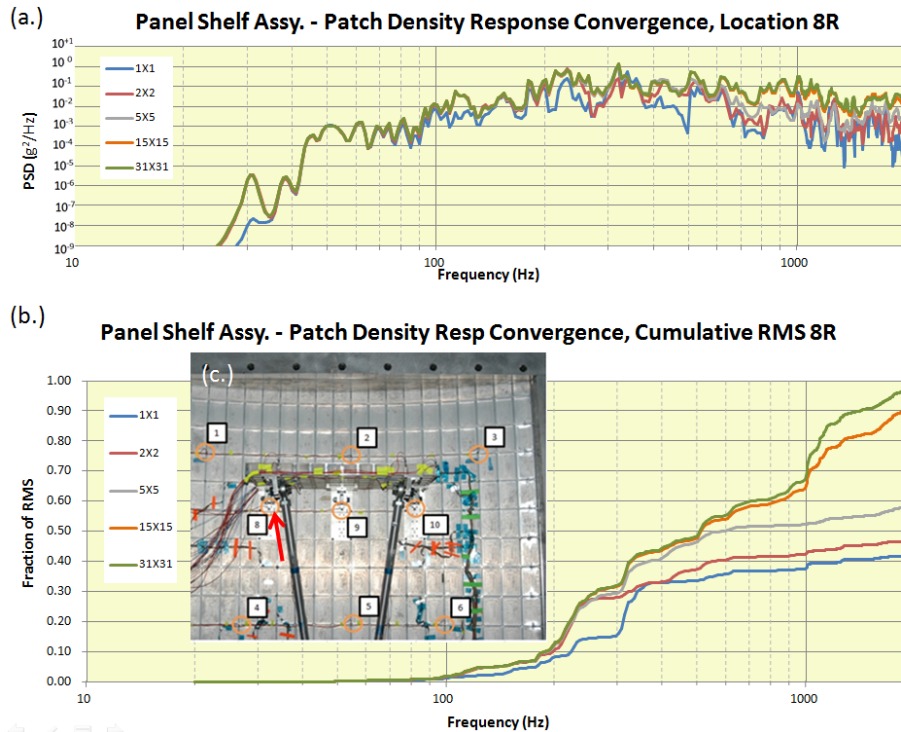
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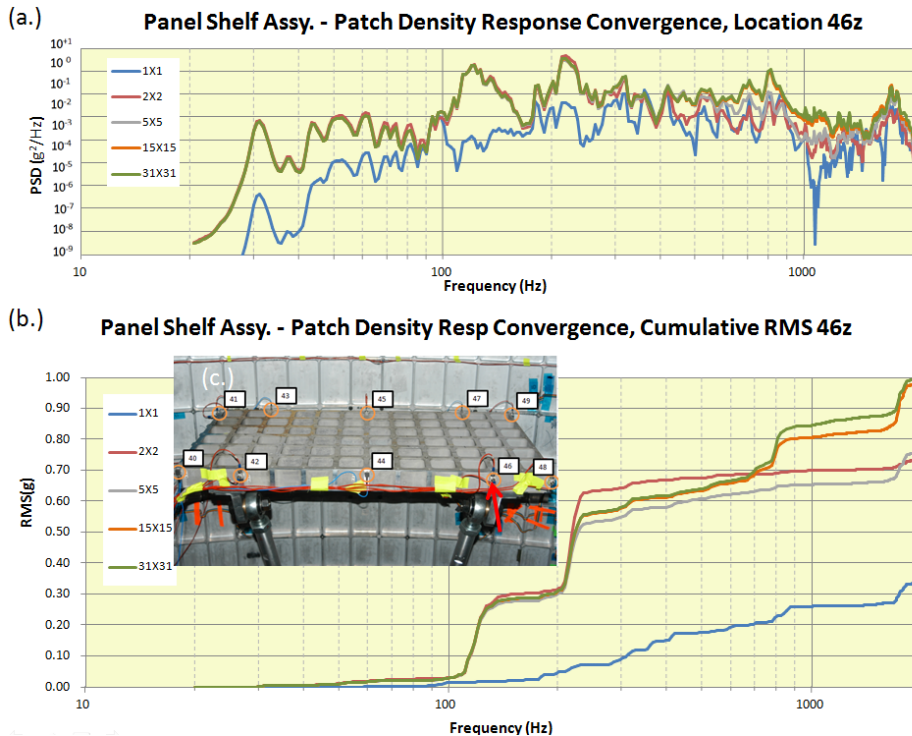
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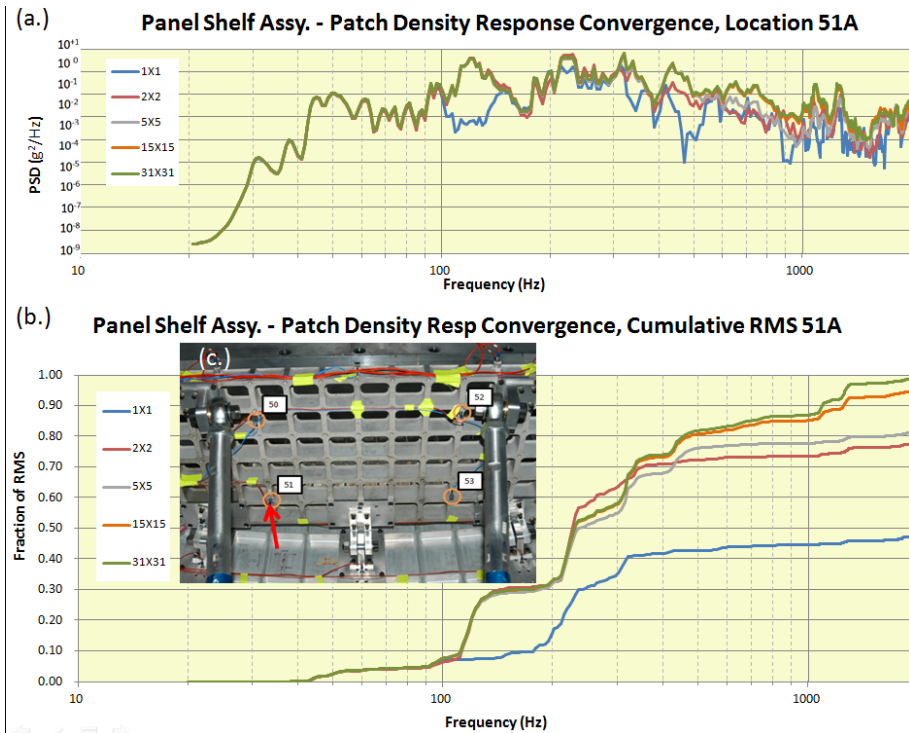




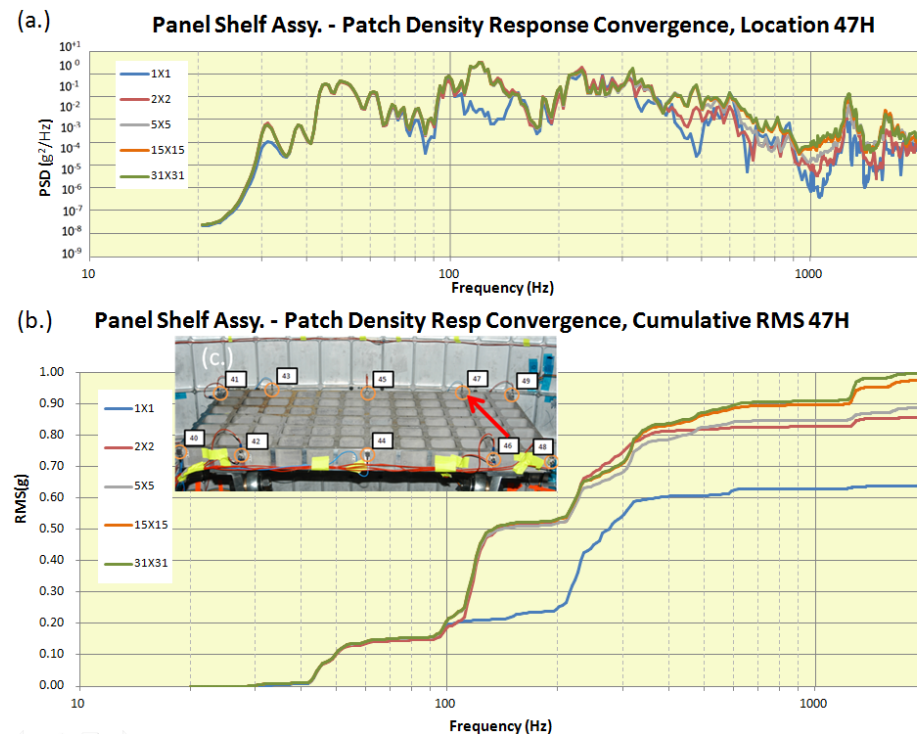
**Figure 16. Convergence Study Results for shelf assembly integrated to vehicle panel (Case Single Shelf Heavy): (a.) Overlaid Response from 5 different patch density assumptions. (b.) Overlay of Fraction of Cumulative RMS plots from the same 5 patch density assumptions. (c.) Depiction of the location, 8R, corresponding to the acceleration response prediction in the radial direction for the cylindrical panel to shelf interface.**



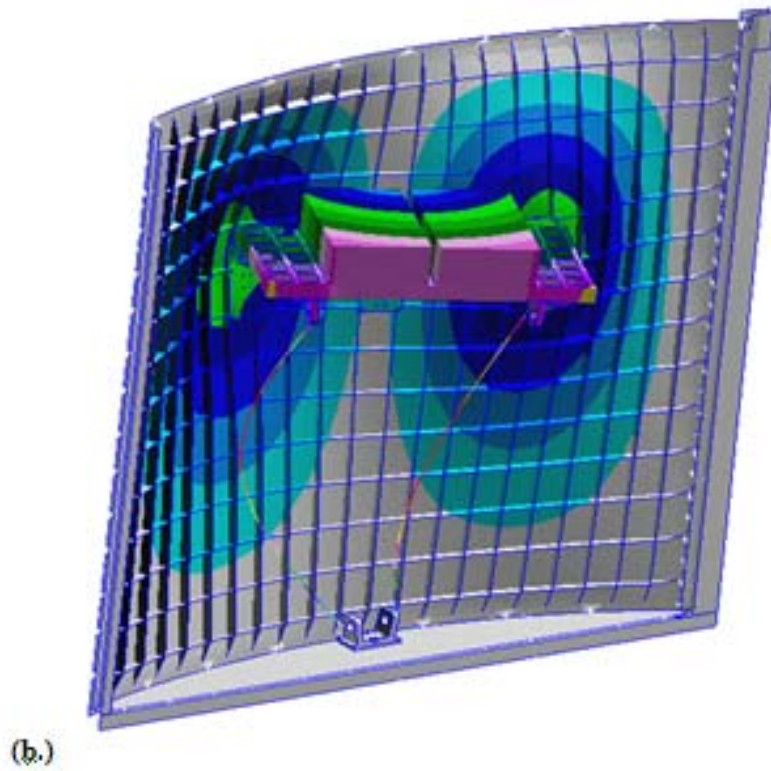
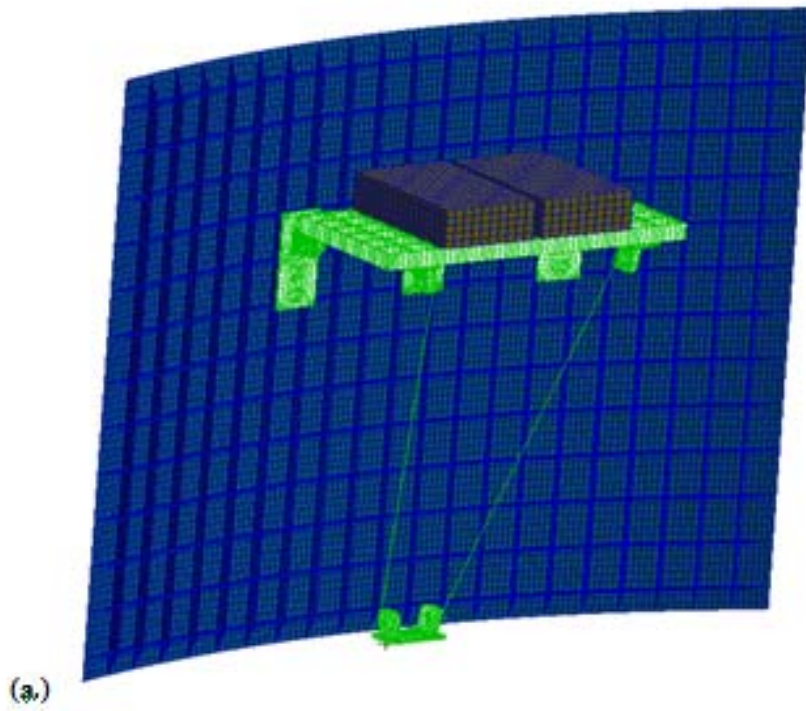
**Figure 17. Convergence Study Results for shelf assembly integrated to vehicle panel (Case Single Shelf Heavy): (a.) Overlaid Response from 5 different patch density assumptions. (b.) Overlay of Fraction of Cumulative RMS plots from the same 5 patch density assumptions. (c.) Depiction of the location, 46z, corresponding to the acceleration response prediction in the radial direction for the strut to shelf interface.**



**Figure 18. Convergence Study Results for shelf assembly integrated to vehicle panel (Case Single Shelf Heavy): (a.) Overlaid Response from 5 different patch density assumptions. (b.) Overlay of Fraction of Cumulative RMS plots from the same 5 patch density assumptions. (c.) Depiction of the location, 51A, corresponding to the acceleration response prediction in the axial direction for the Equipment to shelf interface.**



**Figure 19. Convergence Study Results for shelf assembly integrated to vehicle panel (Case Single Shelf Heavy): (a.) Overlaid Response from 5 different patch density assumptions. (b.) Overlay of Fraction of Cumulative RMS plots from the same 5 patch density assumptions. (c.) Depiction of the location, 47H, corresponding to the acceleration response prediction in the hoop direction for the shelf edge.**



**Figure 20. Vehicle Panel integrated with Single Shelf Heavy: (a.) General undeformed view of the finite element model. (b.) Deformed view of the fundamental mode shape (an even function for the panel surface).**