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Analysis of a Split-Plot Experimental Design Applied to a Low-Speed Wind Tunnel Investigation

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June 2013

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Abstract

A procedure to analyze a split-plot wind tunnel experimental design featuring two input factors, two levels of randomization, and two error structures is described in this report. Standard commercially-available statistical software was used to analyze the test results obtained in a randomization-restricted environment often encountered in wind tunnel testing. Experimental data were obtained in a statistically-designed, low-speed wind tunnel test using a small-scale model of a fighter airplane configuration. The input factors were differential horizontal stabilizer incidence and the angle of attack. The response variables were the aerodynamic coefficients of lift, drag, and pitching moment. Using split-plot terminology, the whole plot, or difficult-to-change, factor was the differential horizontal stabilizer incidence, and the subplot, or easy-to-change, factor was the angle of attack. The whole plot factor and subplot factor were each tested at three levels. Degrees of freedom for whole plot error were provided by replication in the form of three blocks, or replicates, which were intended to simulate three consecutive days of wind tunnel facility operation. The analysis was conducted in three stages. The first stage provided an assessment of statistical significance of a combined whole plot term representing the aggregate of linear and quadratic terms at the whole plot level, a combined subplot term representing the aggregate of linear and quadratic terms at the subplot level, and the interaction of the combined whole plot and combined subplot terms. The results of this preliminary assessment justified a second-stage analysis, which was performed in two steps to systematically separate out the individual terms at the whole plot and subplot levels. This analysis yielded the correct error terms at the whole plot and subplot levels, correct mean squares and multiple regression coefficient estimates of all terms at the whole plot and subplot levels, and valid tests of statistical significance at the subplot level. Statistical tests of significance at the whole plot level using the correct whole plot error terms were performed using manual input in a third stage. The combined analyses from the second and third stages yielded the estimated mean squares, multiple regression function coefficients, and corresponding tests of significance for all individual terms at the whole plot and subplot levels for the three aerodynamic response variables. Statistically significant terms included main effects and two-factor interaction terms for the lift coefficient response; main effects, two-factor interaction, and pure quadratic terms for the drag coefficient response; and only main effects for the pitching moment coefficient response.

Introduction

The application of modern design of experiments (MDOE) techniques to wind tunnel testing of aerospace vehicles (references 1-3) improves the data quality and data uncertainty quantification and provides mathematical models describing the complex aerodynamic responses to the selected input factors. Key concepts that are used in designed experiments are randomization, replication, and blocking, which are described in detail in references 4-9. Typical responses in wind tunnel tests are the six-component longitudinal and lateraldirectional aerodynamic coefficients. Common input factors include the model incidence relative to the free stream (for example, angles of attack and sideslip), the model configuration, and the test conditions (for example, Mach number and Reynolds number). Support systems providing movement of the model in the wind tunnel test section are frequently automated, which facilitates the control of the angles of attack and sideslip. However, input factors that are difficult or time-consuming to change are often encountered. For example, scale models of military and civilian aircraft, missiles, manned and unmanned launch vehicles, and atmospheric re-entry vehicles typically require lengthy configuration changes and attendant access to the model in the test section. In addition, changes to the Mach number and Reynolds number might only be achieved through complex tunnel configuration modifications to obtain the desired expansion and compression ratios. Although it is well understood that complete randomization of the experimental run order is desired to average out the effects of all uncontrolled and unknown sources of variability, sometimes it is unreasonable, impractical, or extremely inefficient to conduct experiments in such a fashion. These factors represent a restriction on randomization in statistically-designed experiments as well as a constraint on the operational productivity. The use of a split-plot experimental design (references 10 and 11) is an effective means of experimenting with hard-to-change factors in a randomization-restricted environment.

The objective of this report is to describe a split-plot analysis of a statistically-designed low-speed wind tunnel experiment on a small-scale model of a fighter airplane configuration. The three response variables were the aerodynamic lift, drag, and pitching moment coefficients, and the two input factors were differential horizontal stabilizer incidence and angle of attack. Variation of the horizontal stabilizer incidence was the hard-to-change factor, since the wind tunnel was brought off-line to allow access to the test section for the manual configuration modifications. In contrast, the model angle of attack could be easily and rapidly varied during wind-on operation. Consequently, a simple split-plot design featuring two input factors, two levels of randomization, and two different error structures (references 10 and 11) was used. A three-stage analysis and statistical model-building method derived from reference 12 was used, which was augmented by commercially-available statistical software (reference 13) and spreadsheet software with statistical tools (reference 14).

Nomenclature

a	number of levels of the whole plot factor
ANOVA	analysis of variance
AoA	subplot factor, angle of attack, degrees
b	number of levels of the subplot factor
c_{ref}	reference wing chord, 3.46 inches
Coef	estimated regression coefficient
CD	drag force coefficient, $\dfrac{Drag\ Force}{q\left(S_{ref}\right)}$
CL	lift force coefficient, $\dfrac{Lift\ Force}{q\left(S_{ref}\right)}$
Cm	pitching moment coefficient, $\dfrac{Pitching\ Moment}{q\left(S_{ref}\right)c_{ref}}$
C. Total	calculated total
DF	degrees of freedom
DF Num	degrees of freedom used in the numerator of the *F-test*
DOE	design of experiments
EMS	expected mean squares
F ratio	ratio of the mean squares (*MS*) for the whole model or factor of interest to the error mean square; also *F*

F-test	statistical test of significance in ANOVA to test if the variances of two populations are equal
LaRC	Langley Research Center
MDOE	modern design of experiments
MS	mean squares, sum of squares (*SS*) divided by its associated degrees of freedom (*DF*)
MS Num	mean squares used in the numerator of the *F-test*
MSE	mean square error
NASA	National Aeronautics and Space Administration
ODU	Old Dominion University
Prob > F	probability the test statistic (*F ratio*) will take on a value at least as extreme as the observed value of the statistic, assuming the null hypothesis is true (e.g. no effect due to a factor of interest); also *p-value* or *P*
Prob >|t|	two-tailed probability that the test statistic, *t*, will take on a value at least as extreme as the observed value of the statistic, assuming the null hypothesis is true (e.g. no effect due to a factor of interest); also two-tailed *p-value*
psf	pounds per square foot
q	free stream dynamic pressure, 6.4 pounds per square foot (*psf*)
r	number of replicates
Re	unit Reynolds number, 0.48 million per foot
Re_c	Reynolds number based on reference wing chord, 0.14 million
Rep	replicate
Run	experimental run in wind tunnel test
SE Coef	standard error of regression coefficient
S_{ref}	reference wing area, 36.08 square inches
sptreat	combined subplot factor containing linear and quadratic terms in *AoA*
Source	source of variation in linear statistical model
SP	subplot
SS	sum of squares
t	*t*-ratio or test statistic from Student's *t* distribution, obtained by dividing the regression coefficient *Coef* by the standard error of the coefficient, *SE Coef*
Term	statistical model term at the whole plot or subplot level
Tail_Angle	whole plot factor, horizontal stabilizer incidence, degrees
wptreat	combined whole plot factor containing linear and quadratic terms in *Tail_Angle*
WP	whole plot
Y	response variable
α_s	level of statistical significance, acceptable probability of error (assumed level of significance is 0.05)

Experimental Design Method and Procedures

A split-plot experimental design strategy was used because of the randomization restriction imposed by changes to the differential horizontal stabilizer incidence. Figure 1 illustrates the split-plot experimental design strategy used in the current investigation. Figure 2 presents photographs of the small-scale fighter airplane model used in the low-speed wind tunnel test, including a close-up view of the right-hand horizontal stabilizer. Using standard design of experiments (DOE) terminology (references 10 and 11), the hard-to-change factor, differential horizontal stabilizer incidence (*Tail_Angle*), was designated the whole plot (*WP*) factor, and it was tested at three levels (0, +10, and +20 degrees). In the present application, differential stabilizer incidence was obtained by changing the right-hand stabilizer incidence while maintaining the left-hand stabilizer at 0-degree incidence. An incidence angle with the stabilizer leading edge up is defined as positive.

The easy-to-change factor, angle of attack (*AoA*), was designated the subplot (*SP*) factor, which was run at three levels (0, 4, and 8 degrees). Positive angle of attack is "nose up" relative to the free stream. The simple split-plot design consisting of a single hard-to-change input factor and a single easy-to-change input factor can be viewed as two experiments superimposed on each other with two different randomization schemes and error structures (reference 12). In a split-plot experiment, the subplot error is often less than the whole plot error because the subplots are generally more homogeneous than the whole plots (reference 12). In addition, more degrees of freedom are usually available for the subplot error, which allows the subplot factors to be analyzed with greater precision. A challenge in a split-plot design is to have sufficient degrees of freedom for the whole plot error to perform credible statistical tests of significance on the whole plot terms. In the current experimental design, this was provided by replication in the form of three blocks or replicates (*Rep*) (references 4-9). All three levels of the whole plot factor were tested in random order within each complete block or replicate. The three levels of the subplot factor were tested in random order at each level of the whole plot factor, which comprised an incomplete block. The whole plot and subplot factors and their corresponding levels in run order are shown in the split-plot experimental design strategy previously illustrated in figure 1. The whole plot and subplot factors and their respective levels in actual factor settings and in coded units (where the low, mid, and high levels of each factor are coded -1, 0, and +1, respectively) are also presented in Table I. The use of coded units in the analysis of a statistically-designed experiment can avoid spurious statistical results due to different measurement scales for the input factors, collinearity among the terms in the model, inflated variability in the regression coefficient estimates, and reduced precision in the tests of statistical significance (references 4 and 5).

A listing of the experimental observations in run order is provided in Table II, which was also the data table used as input to the statistical software package in reference 13. A 30-minute time period was allocated between complete blocks or replicates, during which the wind tunnel facility was shut down and facility personnel vacated the area. Consequently, the blocks (replicates) represented an approximation to three consecutive days of facility operation. Blocking defends against unwanted "day-to-day" variability, but also represents a restriction on randomization (references 4-9).

Numerous commercially-available software packages are available for the design and analysis of experimental data. The software cited in reference 13 was the primary analysis tool in the current investigation. The analysis was augmented by spreadsheet software with statistical tools in reference 14.

The lift, drag, and pitching moment coefficient responses to the selected input factors were analyzed in three stages using a split-plot analysis procedure similar to that described in reference 12. The analysis procedure is illustrated in figure 3. An initial, somewhat qualitative, analysis in the first stage assessed the statistical significance of a combined, or aggregate, whole plot factor containing linear and quadratic terms in the differential horizontal stabilizer incidence (*Tail_Angle* and *Tail_Angle*Tail_Angle*), an aggregate subplot factor containing linear and quadratic terms in the angle of attack (*AoA* and *AoA*AoA*), and a two-factor interaction of the aggregate whole plot and subplot factors (*Tail_Angle*AoA* and mixed higher-order terms) for each of the three aerodynamic response variables. The initial analysis was a screening procedure to verify the statistical significance of the aggregate input factors before committing to a more detailed quantitative analysis of individual whole plot and subplot terms that were supported by the assumed linear statistical models. The decomposition of the individual terms at the whole plot and subplot levels was conducted in a two-step, second-stage analysis in which the individual whole plot and subplot terms featuring main effects, two-factor interactions, and quadratic terms were systematically separated out to estimate their respective mean squares and regression function coefficients and the error terms at the whole plot and subplot levels. Valid tests of statistical significance at the subplot level were also performed at this stage of the analysis. Tests of significance of the estimated mean squares and regression function coefficients using the correct error term at the whole plot level were conducted in a third analysis stage. These results were combined with the output from the two-step, second-stage analyses to assemble final listings of the estimated mean squares, multiple regression coefficients, and tests of statistical significance at the whole plot and subplot levels for the three response variables *CL*, *CD*, and *Cm*.

Test Information

A diagnostic wind tunnel test was performed as part of a split-plot experimental design investigation of the effects of differential horizontal stabilizer incidence and the angle of attack on the lift, drag, and pitching moment responses of a small-scale fighter airplane model previously shown in figure 2. The rolling moment, yawing moment, and side force responses were also measured, since differential stabilizer incidence is a means of augmenting the roll, yaw, and side force control of air combat vehicles. However, the current report focuses on the lift, drag, and pitching moment to demonstrate an analysis procedure of a simple split-plot experimental design implemented in a wind tunnel environment with restrictions on randomization.

A 0.025-scale precision metal model of the selected fighter airplane configuration was tested in an Old Dominion University (ODU) low-speed diagnostic wind tunnel facility (figure 2). The model overall length and span were 16.80 inches and 11.23 inches, respectively. The model incorporated horizontal stabilizers that were manually adjusted to a selected incidence. Scribe lines along the model fuselage provided a visual reference for setting the incidence angle. Differential incidence angles were obtained by changing only the right stabilizer incidence and retaining the incidence of the left stabilizer at 0 degrees. A close-up view of the right stabilizer at +20-degrees incidence (leading-edge up) was previously shown in figure 2.

The ODU facility was an open-circuit, atmospheric wind tunnel and a 1/15th-scale version of the NASA Langley Research Center (NASA LaRC) 30- by 60-Foot Full-Scale Wind Tunnel (references 15 and 16). The wind tunnel was used primarily for diagnostic experiments and as a research laboratory to support joint NASA- and ODU-sponsored graduate-level courses in experimental design for the local technical and academic communities (reference 12). The model was attached to the wind tunnel main support system, which provided remote control of the angle of attack relative to the incident air flow.

The six aerodynamic forces and moments arising from the air flow about the model were measured using a NASA LaRC strain-gage balance designated IR-15 mounted inside the model. The balance was attached to a cylindrical “sting” mounted to the main support system.

The free stream dynamic pressure, q, in the test section was approximately 6.4 pounds per square foot (approximately 50 miles per hour) for all measurements. The corresponding unit Reynolds number, Re, was $0.48(10^6)$. The Reynolds number based on a reference wing chord, c_{ref}, of 3.46 inches was $Re_c = 0.14(10^6)$.

The model attitude was corrected for deflections of the balance, sting, and support system due to aerodynamic loads. Since the testing was exploratory in nature and focused on aerodynamic trends, however, there were no corrections applied to the data to account for tunnel flow angularity, model base pressures, blockage, buoyancy, or wall interference.

Each balance measurement represented the average of 12 five-second samples of data, that is, 60-second data acquisition time for each point. The balance measurements were reduced to coefficient form using an on-line computer, and the data were stored to disk for off-line analysis.

The two quantitative input variables were differential deflection of the horizontal stabilizers (deflection of right tail only), *Tail_Angle*, and the model angle of attack, *AoA*. The three aerodynamic responses, *CL*, *CD*, and *Cm*, were nondimensional (coefficients) and corresponded to the measured lift force and drag force divided by the product of the reference wing area, S_{ref}, and the free-stream dynamic pressure, q, and the measured pitching moment divided by the product of S_{ref}, c_{ref}, and q.

No limitations or problems were encountered during the wind tunnel experiment. The entire experiment was completed in one morning over a 4-hour period of time.

Discussion of Results

First-Stage Analysis

Table II is a listing of the experimental data which shows the input factor settings in coded units and the three aerodynamic coefficient responses in run order. The listing is divided into the three blocks, or "work days." The run order was randomized using the statistical software in reference 13. The whole plot and subplot factors are designated *Tail_Angle* and *AoA*, which correspond to the differential horizontal stabilizer incidence in degrees and the model angle of attack in degrees, respectively. The column labeled *Rep* is identical to *Block* and is assigned values of 1, 2, and 3 corresponding to the three blocks or replicates in the current split-plot experimental design. *Rep* is a term included in the statistical models assumed in this analysis. The three responses are *CL, CD,* and *Cm* which correspond, respectively, to the aerodynamic coefficients of lift, drag, and pitching moment. Two columns labeled *wptreat* and *sptreat* are also included in the data listing in Table II. Since the whole plot factor was tested at three levels, *wptreat* is assigned values of 1, 2, and 3 and represents a combination, or aggregate, of linear and quadratic terms in *Tail_Angle* as previously described in the section "*Experimental Design Method and Procedures*." The subplot factor was also tested at three levels. Consequently, *sptreat* is assigned values of 1, 2, and 3 and represents the aggregate of linear and quadratic terms in *AoA*. Initial statistical models were constructed using the aggregate terms *wptreat*, *sptreat*, and the two-factor interaction *wptreat*sptreat* declared as nominal factors in order to identify statistically significant effects on the aerodynamic responses of the combined whole plot factor, combined subplot factor, and combined two-factor interaction.

The assumed linear statistical model in the first stage of the analysis is

$$Y_{ijk} = \mu + \rho_i + \alpha_j + \delta_{ij} + \tau_k + \left(\alpha\tau\right)_{jk} + \varepsilon_{ijk}$$

where each term in the model is defined as

Y_{ijk}	response in i^{th} block (replicate), j^{th} level of the whole plot factor, k^{th} level of the subplot factor
μ	mean response
ρ_i	effect of the i^{th} block (replicate) on the response (a random effect)
α_j	effect of the j^{th} level of the combined whole plot factor (*wptreat*)
δ_{ij}	whole plot error, calculated from the interaction $(\rho\alpha)_{ij}$
τ_k	effect of the k^{th} level of the combined subplot factor (*sptreat*)
$(\alpha\tau)_{jk}$	interaction between the combined whole plot and subplot factors
ε_{ijk}	subplot error or mean square error (*MSE*)

Figure 4 is a screen capture from the "*Fit Model*" platform in the statistical analysis software package in reference 13, which shows the model specification dialog. The "*Select Columns*" box includes the column names in the split-plot experimental design listing in Table II. The columns labeled *CL*, *CD*, and *Cm* (specified as continuous variables) are input to the "*Pick Role Variables*" box as the "*Y*" response variables. The linear model is specified in the "*Construct Model Effects*" box by adding the appropriate terms from the "*Select Columns*" box. For example, the first term in the model is *Rep*, which is declared as a random effect, *Rep&Random*, using the "*Attributes*" pop-up menu. The software appends *&Random* to any term that is declared a random effect. The inclusion of the term *Rep* in the statistical model acknowledges the blocking scheme used in the current experimental design. This will assign degrees of freedom to *Rep*, which accounts for the restriction on randomization due to the blocking. The second term is the nominal aggregate whole plot factor *wptreat*. The third term corresponds to *Rep* crossed with *wptreat*, which is declared as a random effect, *Rep*wptreat&Random*. The latter term represents the error at the whole plot level, and its inclusion in the model ensures the aggregate whole plot factor, *wptreat*, is tested against the correct whole plot error. The fourth term that is added to the model is the nominal aggregate subplot factor, *sptreat*. The fifth term is *wptreat* crossed with *sptreat*, *wptreat*sptreat*, which represents the two-factor interaction between the aggregate whole plot and subplot factors. The latter two terms are tested against the subplot error, which is the mean square error or *MSE*. The "*Personality*" button indicates that the linear model will be fit using standard least squares. The "*Emphasis*" button specifies a minimal report, which suppresses all plots and arranges the whole model and effect details tables in a vertical format. The "*Method*" button indicates that the expected mean squares (*EMS*) method is used to fit the linear model with random effects.

The first stage of the analysis was intended to test for overall significance of the whole plot and subplot factors before initiating a more detailed quantitative assessment of the individual linear and higher-order terms at both levels. A portion of the "*Fit Model*" platform output from the first-stage analysis is shown in figure 5. The analysis of variance (ANOVA) listing for each aerodynamic response corresponds to the whole model with two sources labeled "*Model*" and "*Error*" under "*Source*". The source "*Error*" corresponds to the residual error, or mean square error, *MSE*. There are 3 levels of *wptreat* ($a = 3$), 3 levels of *sptreat* ($b = 3$), and 3 replicates, *Rep* ($r = 3$). Consequently, the calculated total (*C. Total*) degrees of freedom (*DF*) is $r*a*b - 1 = 26$. The *MSE DF* is $a*(r-1)*(b-1) = 12$. Consequently, there are 14 *DF* for the overall model. The estimated mean squares (*MS*) for a given term is obtained by dividing the sum of squares (*SS*) by the corresponding *DF*. The results from the ANOVA indicate that the overall models for the three aerodynamic responses are statistically significant, since the corresponding probability levels or *p*-values (labeled *Prob > F*) of the test statistic, *F ratio*, are less than the prescribed level of significance of 0.05 ($\alpha_s = 0.05$). The *F ratio* is obtained by dividing the estimated mean squares of the model term of interest by the appropriate error mean squares. A probability value of the *F ratio* that is less than 0.05 is considered unlikely in the presence of random error but, instead, consistent with systematic variation in the data caused by changes in the levels of the input factor. For example, the mean squares for the whole model of the response *CL* (using higher-precision values from the statistical software output) is:

$$MS(model) = SS(Model)/DF(model) = 1.16554482/14 = 0.0832532014$$

The mean square error is:

$$MSE = SS(Error)/DF(error) = 0.0007395953/12 = 0.0000616329$$

Consequently, the *F Ratio* is:

$$MS(model)/MSE = 0.0832532014/.0000616329 = 1350.791$$

Given an F distribution with 14 DF in the numerator and 12 DF in the denominator, the probability value of this F statistic is approximately 0 (references 4-9). Since this value is less than the cutoff value of 0.05, the whole model is declared statistically significant.

A more detailed analysis at the whole plot and subplot levels is provided in the output labeled "*Tests wrt Random Effects*" in figure 5. Table III shows the corresponding breakdown of the degrees of freedom at both levels. Note that the combined whole plot factor, *wptreat*, is tested against the correct whole plot error term, *Rep*wptreat&Random*, and the combined subplot factor, *sptreat*, and the two-factor interaction *wptreat*sptreat* are correctly tested against the subplot error term or mean square error, *MSE*. The combined whole plot factor and the combined subplot factor, *wptreat* and *sptreat*, respectively, have statistically significant effects on the lift and pitching moment coefficients, since their corresponding p-values (*Prob > F*) are less than the cutoff value of 0.05. In the analysis of the *CL* response, for example, the mean squares estimate (listed in the column labeled *MS Num*) for *wptreat* is:

$$MS(wptreat) = SS(wptreat)/DF(wptreat) = 0.0680479905/2 = 0.0340239953$$

The whole plot error mean squares estimate corresponding to *Rep*wptreat&Random* is:

$$MS(WP\ error) = SS(WP\ error)/DF(WP\ error) = 0.000421438/4 = 0.0001053595$$

The F statistic for *wptreat* is:

$$MS(wptreat)/MS(WP\ error) = 0.0340239953/0.0001053595 = 322.9324$$

The corresponding p-value from an F-distribution with 2 *DF* in the numerator and 4 *DF* in the denominator is approximately 0.00003789.

The mean squares estimate in the *CL* response analysis for *sptreat* is:

$$MS(sptreat) = SS(sptreat)/DF(sptreat) = 1.0966651483/2 = 0.5483325742$$

The subplot error mean squares estimate is the *MSE* from the ANOVA table and is:

$$MSE = SS(Error)/DF(Error) = 0.0007395953/12 = 0.0000616329$$

Note that the error estimate at the whole plot level is higher than the corresponding error estimate at the subplot level. This is common in split-plot experimental designs, since there are typically more degrees of freedom to estimate the subplot error. Consequently, the statistical tests of significance are conducted with more precision at the subplot level.

The F statistic for *sptreat* is:

$$MS(sptreat)/MSE = 0.5483325742/0.0000616329 = 8896.7457$$

The corresponding p-value from an F- distribution with 2 *DF* in the numerator and 12 *DF* in the denominator is essentially zero. It is noted that the mean squares estimate for the whole plot error term *Rep*wptreat&Random* is listed in the column labeled *MS Num*, since the statistical software also tests this estimate against the *MSE*. However, this test is not relevant to the current analysis. In contrast to the statistical significance of the aggregate whole plot and subplot terms, the interaction term *wptreat*sptreat* is statistically insignificant in the lift and pitching moment coefficient responses, since the p-values exceed the cutoff value of 0.05. The analysis of the drag coefficient response, *CD*, shows that *wptreat*, *sptreat,* and *wptreat*sptreat* are all statistically significant. It is noted, however, that the first-stage analysis does not isolate potentially significant two-factor interactions and quadratic terms, for example, since they are embedded in the terms *wptreat*, *sptreat,* and *wptreat*sptreat*. The decomposition of individual terms at the whole plot and subplot levels is conducted in the second-stage analysis.

Second-Stage Analysis

A second analysis derived from reference 12 featuring a two-step approach was performed to decompose the whole plot and subplot terms into quantitative effects by calculating the individual mean squares associated with linear, two-factor interaction, and quadratic terms supported by the model. For example, Table III previously indicated 2 *DF* for the aggregate whole plot term, *wptreat*; 2 *DF* for the aggregate subplot term, *sptreat*; and 4 *DF* for the aggregate whole plot term crossed with the aggregate subplot term. As a result, statistical models for the *CL*, *CD*, and *Cm* responses will support linear and pure quadratic terms in *Tail_Angle*, linear and pure quadratic terms in *AoA*, and two-factor interactions involving *Tail_Angle* and *AoA* (for example, *Tail_Angle*AoA*, *Tail_Angle*AoA^2*, *Tail_Angle^2*AoA*, and *Tail_Angle^2*AoA^2*). Inclusion of two-factor interaction terms of higher-order than *Tail_Angle*AoA* is problematic, however, since this promotes collinearity of the estimated terms in the statistical models (references 4-9). As a result, only *Tail_Angle*AoA* will be included in the models in the current analyses. It is expected based on previous testing experience (reference 17) that the linear-by-linear term is the "heavy hitter" among the two-factor interaction terms. The estimated sums of squares of the three higher-order terms and their associated degrees of freedom will be pooled into the estimate of the subplot error, or mean square error (*MSE*).

First Step

In the initial step of the second-stage analysis, the whole plot main effect and quadratic terms corresponding to *Tail_Angle* and *Tail_Angle*Tail_Angle* were again combined in the term *wptreat* and specified as a nominal factor as in the first-stage analysis. However, the subplot terms were separated out and declared continuous. Here, *sptreat* from the preceding analysis was broken out into the individual terms *AoA* and *AoA*AoA* corresponding to the linear and quadratic terms in the subplot factor *AoA*. In order to avoid a programmatic error message for a nonhierarchical model (references 4-9), a column was included in the experimental design listing shown previously in Table II to represent the interaction between the whole plot and subplot factors, *Tail_Angle*AoA*. Terms corresponding to the subplot factor main effect, two-factor interaction between the whole plot and subplot factors, and subplot factor pure quadratic effect were supported by the assumed statistical model, since there were sufficient degrees of freedom (as described at the outset of this section) by testing the subplot factor at three levels within each level of the whole plot factor and in three replicates or blocks. The added column for Tail_Angle*AoA in Table II uses 1 of the 4 available *DF* from the *wptreat*sptreat* term shown previously in the *DF* breakdown in Table III. The model specification dialog in the "*Fit Model*" platform is shown in figure 6. Similar to the first-stage analysis in the previous section, the first step in the current analysis stage also estimates the correct whole plot error term, *Rep*wptreat&Random*, without performing statistical tests of the individual terms at the whole plot level. The aggregate term *wptreat* is tested against the correct whole plot error. The key element in the first step of the second analysis is the decomposition of terms at the subplot level, which allows the estimation of the mean squares and multiple regression coefficients of all individual subplot terms, which were tested for statistical significance against the correct subplot error term (*MSE*).

The whole model ANOVA and the tests with respect to random effects from the first-step analyses for each of the three aerodynamic responses are shown in figure 7. The ANOVA for the whole model contains the mean square error (*MSE*) that is used to test all terms at the subplot level. There are 15 *DF* for the subplot error in figure 7 compared to the 12 *DF* shown previously in figure 5 from the first-stage analysis. This reflects the 3 DF for the whole plot and subplot interaction terms that were excluded from the statistical models for the three response variables and, instead, pooled with the *MSE*.

Comparison of the "*Tests wrt Random Effects*" output in figure 7 with the corresponding output in figure 5 from the first-stage analysis indicates the sums of squares, mean squares, error estimates, and statistical tests for *Rep&Random* and *wptreat* at the whole plot level are unchanged. In addition, the sums of squares for *AoA* and *AoA*AoA* in figure 7 add to the corresponding values of the aggregate subplot factor *sptreat* in figure 5. The inclusion of only the *Tail_Angle*AoA* interaction in the statistical models in figure 7 accounts for approximately 75%, 100%, and 50% of the sums of squares attributed to the aggregate *wptreat*sptreat* interaction term in the *CL*, *CD*, and *Cm* responses, respectively, in figure 5. The smaller percentage of the overall interaction *SS* accounted for by *Tail_Angle*AoA* in the *Cm* response is not an issue, however, since the first-stage analysis in figure 5 indicated the aggregate *wptreat*sptreat* term is highly insignificant. These results support the pooling of the sums of squares of the three higher-order interaction terms and their corresponding *DF* with the estimate of the subplot mean square error in the second-stage analysis.

The "*Tests wrt Random Effects*" output shown in figure 7 indicates the main effect of *AoA* has a highly significant effect on the *CL* response, since its *p*-value is much less than the cutoff value of 0.05. The two-factor interaction *Tail_Angle*AoA* is marginally significant with a *p*-value slightly less than the cutoff value. In contrast, the quadratic effect of *AoA* is highly insignificant with a *p*-value nearly an order of magnitude greater than the cutoff value. It is noted that the estimates of the subplot terms are uncorrelated, such that the removal of any term from the statistical model does not affect the estimates of terms that are retained in the model. In addition, the use of coded units allows an assessment of the relative contribution of each model term to the total systematic variation of the aerodynamic response. For example, the sum of squares for *AoA* represents nearly 100% of the variability in the *CL* response attributed to the three subplot terms. This suggests that the exclusion of *Tail_Angle*AoA* and *AoA*AoA* would not have a practical effect on the predictive capability of the assumed statistical model for the *CL* response.

The subplot terms *AoA*, *Tail_Angle*AoA*, and *AoA*AoA* all have statistically significant effects on the *CD* response as shown in the "*Tests wrt Random Effects*" output shown in figure 7. The term *AoA* is again the "heavy hitter" in the statistical model at the subplot level, since it accounts for over 90% of the variability accounted for by the three subplot terms. An interpretation of the statistical significance of the two-factor interaction and quadratic terms is that the effect of *AoA* on the *CD* response depends on the levels of *Tail_Angle* and *AoA*. Drag coefficient is typically the most sensitive component in aerodynamic force and moment wind tunnel testing (reference 18).

Only the main effect of *AoA* has a statistically significant effect on the pitching moment coefficient response in figure 7. The term *AoA* represents approximately 94% of the variability in the *Cm* response accounted for by the three subplot terms. The term *Tail_Angle*AoA* is highly insignificant, since its *p*-value is nearly an order of magnitude greater than the cutoff value. This indicates that the effect of *AoA* on the *Cm* response does not depend on the level of *Tail_Angle*. The quadratic term *AoA*AoA* is marginally insignificant, since its *p*-value is slightly greater than 0.05.

The dominance of the main effect of *AoA* at the subplot level on *CL*, *CD*, and *Cm* is consistent with the current experimental design space featuring angles of attack from 0 degrees to 8 degrees where linear aerodynamics are expected to prevail (reference 19). Significant two-factor interaction and quadratic effects on all three aerodynamic responses would be expected at high angles of attack where the flow field about the current fighter airplane model would be dominated by flow separation, vortex flows, and flow-field interactions between the various airframe components (reference 20).

The output from the analyses of the linear statistical models for the *CL*, *CD*, and *Cm* responses also included valid estimates of the multiple regression model coefficients computed by least squares method (references 4-9) and the corresponding statistical tests of significance for all terms at the subplot level. However, presentation of these results is deferred to the discussion of the third-stage analysis.

Second Step

The next step in the second-stage analysis was to build statistical models for the *CL*, *CD*, and *Cm* responses having only the whole plot main effect and quadratic terms separated out from *wptreat*. This part of the analysis provided the correct estimates of the mean squares and regression coefficients for the linear and quadratic effects of the whole plot factor, which are designated *Tail_Angle* and *Tail_Angle*Tail_Angle*, respectively. The model specification dialog in the "*Fit Model*" platform is shown in figure 8, and partial output from the analyses of the three response variables is presented in figure 9. The valid mean squares estimates at the whole plot level are shown in "*Tests wrt Random Effects*" in figure 9. The *p*-values for all terms at the whole plot level exceed the cutoff value of 0.05 for the *CL* response. Only the main effect of *Tail_Angle* is statistically significant in the *CD* and *Cm* responses. However, the statistical tests are *not* correct since the terms *Tail_Angle* and *Tail_Angle*Tail_Angle* (and *Rep*) are tested against the overall model error shown in the corresponding ANOVA output instead of the whole plot error mean square, *Rep*wptreat&Random*, which was obtained from the first step. Similarly, the estimates of the regression coefficients for the terms at the whole plot level, as presented in the next section, are valid at this stage of the analysis, but the statistical tests of significance are incorrect. The correct tests of statistical significance for the individual terms at the whole plot level are performed in the third and final stage of the analysis.

Third-Stage Analysis

The only missing element in the analysis of the current split-plot experimental design is the application of the correct error terms in the statistical tests of significance of the mean squares estimates of the whole plot terms and the corresponding regression coefficients for the three response variables. The results obtained in the two-step second analysis were combined in a third stage featuring manual input to the spreadsheet software with statistical tools in reference 14 to ensure valid tests of significance at the whole plot level. Results from the third stage of the analysis featuring a modified ANOVA for the lift, drag, and pitching moment coefficient responses are shown in figure 10, figure 11, and figure 12, respectively. Each figure contains three separate listings: (1) results from the first step of the second-stage analysis containing the correct error terms at the whole plot and subplot levels and correct mean squares estimates and statistical tests of all model terms at the subplot level, (2) results from the second step of the second-stage analysis containing valid mean squares estimates of the model terms at the whole plot level but incorrect statistical tests of significance, and (3) modified ANOVA combining the correct mean squares estimates of all terms at the whole plot level (including *Rep*) and at the subplot level and the correct error terms for statistical tests at each level. The decomposition of the aggregate whole plot term *wptreat* is also included in the listings from the third-stage analysis. In the modified third-stage analyses, the subplot error with 15 *DF* is smaller than the whole plot error with 4 *DF* for all three responses. Consequently, the precision of the statistical tests is greater at the subplot level, which is typical of split-plot experimental designs (references 4-9).

To illustrate the modified calculations in the third-stage analysis, consider the *CL* response in figure 10. The correct *F* statistic for the whole plot term *Tail_Angle* is obtained by dividing the mean squares estimate for *Tail_Angle* (with 1 *DF*) obtained in step 2 of the second-stage analysis (*MS(Tail_Angle)* = 0.06804609) by the whole plot error (with 4 *DF*) obtained in step 1 of the second-stage analysis (*Rep*wptreat&Random* = 0.00010536). The corresponding *F* statistic is 645.8437 as shown in the modified analysis in figure 10. The *p*-value for this *F* statistic from an *F* distribution with 1 *DF* in the numerator and 4 *DF* in the denominator is approximately 0.00001424. Similar calculations for the whole plot term *Tail_Angle*Tail_Angle* yield an *F* statistic of 0.01803341 and a corresponding *p*-value of 0.89966021. Consequently, at the whole plot level, only the main effect of *Tail_Angle* has a statistically significant effect on the lift coefficient response. Modified tests of significance of the pitching moment coefficient response at the whole plot level in figure 12 also reveal the main effect of *Tail_Angle* as the only statistically significant term at this level. Both the main effect and quadratic effect of *Tail_Angle* are statistically significant in the drag coefficient response in figure 11, although the term *Tail_Angle* represents nearly 97% of the variability in the *CD* response accounted for by the two whole plot terms. The valid tests of statistical significance of all terms at the subplot level from the step 1 second-stage analyses of *CL*, *CD*, and *Cm* are retained in the modified ANOVA listings in figure 10, figure 11, and figure 12, respectively. Statistically significant terms at the whole plot and subplot levels are highlighted in figures 10-12. The main effects of *Tail_Angle* and *AoA* and their two-factor interaction *Tail_Angle*AoA* are significant terms in the model for the *CL* response. Whole plot and subplot main effects, their two-factor interaction, and both quadratic effects (*Tail_Angle*Tail_Angle* and *AoA*AoA*) have statistically significant effects on the drag coefficient response. Only the whole plot and subplot main effects are significant in the model for the *Cm* response. For all three responses, the main effects are the dominant terms. The sum of squares due to the subplot main effect, *AoA*, represents approximately 94.1% of the total variability in the *CL* response accounted for by the statistical model. By comparison, the sum of squares due to the whole plot main effect, *Tail_Angle*, represents approximately 5.8% of the total systematic variation accounted for by the model. Although statistically significant, the sum of squares due to the two-factor interaction, *Tail_Angle*AoA*, represents only about 0.02% of the estimated variability of the *CL* response. The subplot main effect *AoA* and the whole plot main effect *Tail_Angle* contribute approximately 60.5% and 32.1%, respectively, to the total variability in the *CD* response accounted for by the model. The remaining terms, *AoA*AoA*, *Tail_Angle*AoA*, and *Tail_Angle*Tail_Angle* contribute approximately 4.7%, 1.6%, and 1.0%, respectively. Of the two significant terms in the model for the *Cm* response, the whole plot main effect *Tail_Angle* contributes about 98.6% of the total variability accounted for by the statistical model, whereas the subplot main effect *AoA* contributes 1.2%.

The valid estimates of the regression coefficients in coded units for the *CL*, *CD*, and *Cm* responses that were obtained by least squares method in the second-stage analysis are listed in figure 13. Modified calculations for the significance tests of the regression coefficients at the whole plot level were conducted in the third-stage analysis, and the results are reflected in figure 13. The column labeled "*Coef*" lists the estimated regression coefficient for the linear model found by least squares. "*SE Coef*" is the standard error of the regression coefficient, or estimate of the standard deviation of the distribution of the coefficient estimate. The column labeled "*t*" is the *t*-ratio obtained by dividing *Coef* by *SE Coef*, and this statistic is assumed to have a Student's *t*-distribution (references 4-9). The column labeled "*Prob* > |*t*|" is the two-tailed *p*-value or the probability the absolute value of the test statistic, *t*, will take on a value at least as extreme as the observed value of the statistic, assuming the null hypothesis is true (e.g. the regression coefficient does not contribute to the prediction capability of the statistical model). In the step 2, second-stage analysis of the *CL* response, for example, the regression coefficient estimate (previously not shown) for *Tail_Angle* was 0.06148446 with a *SE Coef* of 0.05266038. The *t*-ratio was 0.06148466/0.05266038 = 1.16756962 with a corresponding *p*-value of 0. 0.25548085 from a Student's *t* distribution with 22 *DF*. However, this test incorrectly used the overall model error (*MS(Overall Model)* = 0.04991609) with 22 DF instead of the correct whole plot error term *Rep*wptreat&Random* = 0.00010536 with 4 *DF* (see figure 10 for the corresponding error estimates and *DF*). The correct value for *SE Coef* is obtained by using the whole plot error mean squares estimate. For example, multiplying the original value of *SE Coef* by (*MS(WP Error*)/*MS(Overall Model)*)^0.5 yields:

0.05266038*(0.00010536/0.04991609)^0.5 = 0.00241935

The *t*-ratio is then 0.06148466/0.00241935 = 25.41364233. The corresponding two-tailed *p*-value is 0.00001424 based on a Student's *t*-distribution with 4 *DF* (references 4-9). Similar calculations were performed for the significance tests of the terms at the whole plot level for the *CD* and *Cm* responses. Statistically significant estimated regression coefficients are highlighted in figure 13. As discussed in the section on the second-stage analysis, the removal of insignificant terms does not affect the estimates of the regression coefficients that are retained in the statistical model.

Statistically significant coefficients in the estimated regression function for the mean lift coefficient response correspond to the main effects of the whole plot factor (*Tail_Angle*) and the subplot factor (*AoA*) and their two-factor interaction (*Tail_Angle*AoA*). The estimated regression function for the mean drag coefficient response includes statistically significant terms corresponding to the whole plot and subplot main effects (*Tail_Angle, AoA*), whole plot and subplot quadratic effects (*Tail_Angle*Tail_Angle*, *AoA*AoA*), and whole plot and subplot interaction (*Tail_Angle*AoA*). The least complex regression function corresponds to the pitching moment coefficient response, which features only the main effects of *Tail_Angle* and *AoA*. The terms that are highlighted in the estimated regression functions in figure 13 are statistically significant based on the assumed level of significance of 0.05. However, the practical significance of the individual terms would require engineering judgment of the relative magnitudes of the regression coefficients and the corresponding test statistics. For example, the largest coefficient in the regression function for the lift coefficient response corresponds to the main effect of the subplot factor *AoA*. The magnitude of the next most significant coefficient corresponding to the main effect of the whole plot factor *Tail_Angle* is smaller by a factor of four. The interaction of the whole plot and subplot factors *Tail_Angle*AoA* marginally satisfies the criteria of statistical significance. However, the magnitude of the regression coefficient is smaller than the coefficient for the main effect of the subplot factor *AoA* by over a factor of 50.

Concluding Remarks

A procedure to analyze a split-plot experimental design implemented in a low-speed diagnostic wind tunnel was described in this report. A statistically-designed experiment was conducted in an ODU facility to obtain the lift, drag, and pitching moment coefficient responses of a small-scale precision metal model of a fighter airplane configuration. The two input factors were differential horizontal stabilizer incidence and the angle of attack relative to the incident air flow. The differential horizontal stabilizer incidence was a difficult-to-change factor and, therefore, a restriction on randomization, since each level of this input factor required physical access to the test section to perform the required model change. The angle of attack was an easy-to-change input factor, since its levels were remotely controlled during wind-on operation. This investigation featured a simple split-plot experimental design consisting of two input factors, where the hard-to-change factor was tested in incomplete blocks called whole plots, and all levels of the easy-to-change factor were run in subplots within each incomplete block. The experimental design and analysis methods accounted for testing in a randomization-restricted environment with two different error structures. The key design-of-experiment concepts of replication, randomization, and blocking were used to defend against known and unknown sources of variation and to provide sufficient degrees of freedom to perform valid tests of statistical significance at both the whole plot and subplot levels. A three-stage analysis procedure was used to estimate the mean squares of all terms supported by the statistical models, to estimate the error terms at the whole plot and subplot levels, to perform valid tests of statistical significance at both levels, and to construct multiple regression models of the three response variables. Statistically significant terms in the estimated regression function for the lift coefficient response included the main effects of the whole plot factor (differential horizontal stabilizer incidence) and the subplot factor (angle of attack) and their two-factor interaction. The estimated regression function for the mean drag coefficient response included statistically significant terms corresponding to the whole plot and subplot main effects, whole plot and subplot quadratic effects, and whole plot and subplot two-factor interaction. The only significant terms in the estimated regression function for the mean pitching moment coefficient response corresponded to the main effects of the differential horizontal stabilizer incidence and the angle of attack. The more complex regression function for the drag coefficient response was consistent with aerodynamic testing experience, where the drag component typically exhibits a higher-order dependency on input factors related to the vehicle configuration and the flight conditions. More complex regression functions would be expected if the experimental design space was expanded to include high angles of attack where the vehicle aerodynamics are dominated by separated flows and flow-field interactions.

References

1. DeLoach, R., "Applications of Modern Experiment Design to Wind Tunnel Testing at NASA Langley Research Center," AIAA-1998-0713, January 1998.
2. Luner, J. J. and Healey, M. D., "Modern Design of Experiments Techniques to Optimize a Leading Edge Extension," AIAA-2003-0655, January 2003.
3. Deloach, R., "Improved Quality in Aerospace Testing Through the Modern Design of Experiments (Invited)," AIAA-2000-0825, January 2000.
4. Montgomery, D. C., *Design and Analysis of Experiments*, 7th edition, John Wiley & Sons, Inc., New York, 2009.
5. Myers, R. H. and Montgomery, D. C., *Response Surface Methodology: Process and Product Optimization Using Designed Experiments*, 2nd edition, John Wiley & Sons, Inc., New York, 2002.
6. Kuehl, R. O., *Design of Experiments: Statistical Principles of Research Design and Analysis*, 2nd edition, Duxbury Press, 1999.
7. Wu, C. F. J. and Hamada, M., *Experiments: Planning, Analysis, and Parameter Design Optimization*, John Wiley & Sons, Inc., 2000.
8. Box, G. E. P., Hunter, W. G., and Hunter, J. S., *Statistics for Experimenters*, John Wiley & Sons, Inc., 1978.
9. Lentner, M. and Bishop. T., *Experimental Design and Analysis*, 2nd edition, Valley Book Company, 1993.
10. Kowalski, S. M. and Potcner, K. J., "How to Recognize a Split-Plot Experiment," *Quality Progress*, Vol. 36, Issue: 11, pp. 60-66, November 2003.
11. Potcner, K. J. and Kowalski, S. M., "How to Analyze a Split-Splot Experiment," *Quality Progress*, Vol. 37, Issue: 12, pp. 67-74, December 2004.
12. Simpson, J. R., Kowalski, S. M., and Landman, D., "Experimentation with Randomization Restrictions: Targeting Practical Implementation," *Quality and Reliability Engineering International* 2004; 20: 481-495.
13. Sall, J., Creighton, L., and Lehman, A., *JMP® Start Statistics: A Guide to Statistics and Data Analysis Using JMP®*, 4th edition, SAS Institute Inc., 2007.
14. Microsoft, *Microsoft Excel*, Redmond, Washington, Microsoft, 2010.
15. Landman, D., Simpson, J. R., Vicroy, D., and Parker, P. A., "Efficient Methods for Complex Aircraft Configuration Aerodynamic Characterization using Response Surface Methodologies," AIAA-2006-0922, January 2006.
16. Landman, D., Simpson, J. R., Mariani, R., Ortiz, F., and Britcher, C., "Hybrid Design

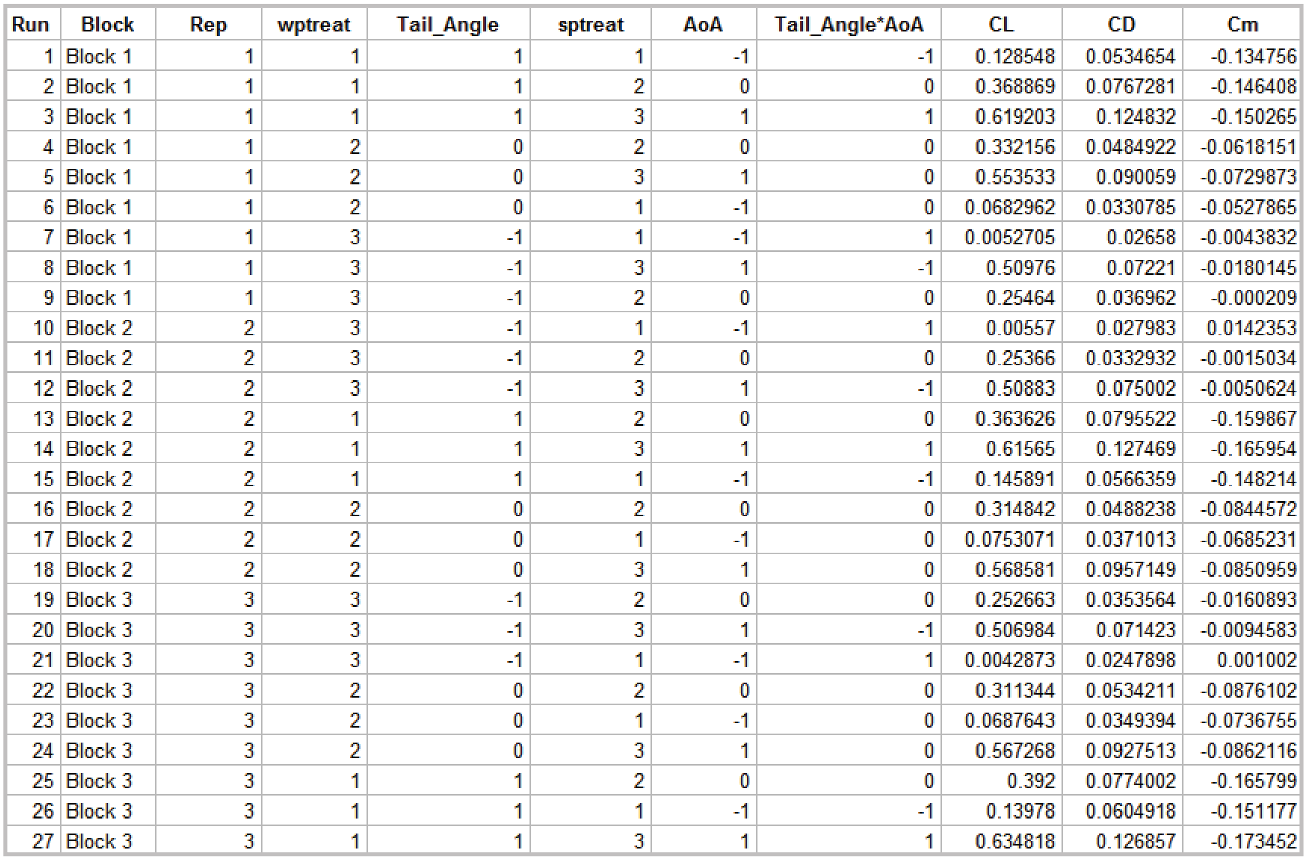
for Aircraft Wind-Tunnel Testing Using Response Surface Methodologies," *Journal of Aircraft*, Vol. 44, No. 4, pp. 1214-1221, 2007.

17. Moore, W. A., Erickson, G. E., Lorincz, D. L., and Skow, A. M., "Effects of Forebody, Wing, and Wing-Body-LEX Flowfields on High Angle of Attack Aerodynamics," SAE Technical Paper 791082, February 1979.
18. Küchemann, D., *The Aerodynamic Design of Aircraft*, Pergamon Press, June 1978.
19. Hoerner, S. F. and Borst, H. V., *Fluid-Dynamic Lift*, Hoerner Fluid Dynamics, 1975.
20. Erickson, G. E., "Wind Tunnel Investigation of Vortex Flows on F/A-18 Configuration at Subsonic through Transonic Speeds," NASA Technical Paper 3111, December 1991.

Table I. Whole plot and subplot factors and levels in actual factor settings and coded units.

	Factor	***Levels (Actual Factor Settings)***	***Levels (Coded Units)***
Whole plot factor	Differential horizontal stabilizer incidence	0, 10, 20 degrees	-1, 0 1
Subplot factor	Angle of attack	0, 4, 8 degrees	-1, 0, 1

Table II. Data table showing the experimental observations in run order and the input factors and factor levels in coded units for the split-plot design.



Run	Block	Rep	wptreat	Tail_Angle	sptreat	AoA	Tail_Angle*AoA	CL	CD	Cm
1	Block 1	1	1	1	1	-1	-1	0.128548	0.0534654	-0.134756
2	Block 1	1	1	1	2	0	0	0.368869	0.0767281	-0.146408
3	Block 1	1	1	1	3	1	1	0.619203	0.124832	-0.150265
4	Block 1	1	2	0	2	0	0	0.332156	0.0484922	-0.0618151
5	Block 1	1	2	0	3	1	0	0.553533	0.090059	-0.0729873
6	Block 1	1	2	0	1	-1	0	0.0682962	0.0330785	-0.0527865
7	Block 1	1	3	-1	1	-1	1	0.0052705	0.02658	-0.0043832
8	Block 1	1	3	-1	3	1	-1	0.50976	0.07221	-0.0180145
9	Block 1	1	3	-1	2	0	0	0.25464	0.036962	-0.000209
10	Block 2	2	3	-1	1	-1	1	0.00557	0.027983	0.0142353
11	Block 2	2	3	-1	2	0	0	0.25366	0.0332932	-0.0015034
12	Block 2	2	3	-1	3	1	-1	0.50883	0.075002	-0.0050624
13	Block 2	2	1	1	2	0	0	0.363626	0.0795522	-0.159867
14	Block 2	2	1	1	3	1	1	0.61565	0.127469	-0.165954
15	Block 2	2	1	1	1	-1	-1	0.145891	0.0566359	-0.148214
16	Block 2	2	2	0	2	0	0	0.314842	0.0488238	-0.0844572
17	Block 2	2	2	0	1	-1	0	0.0753071	0.0371013	-0.0685231
18	Block 2	2	2	0	3	1	0	0.568581	0.0957149	-0.0850959
19	Block 3	3	3	-1	2	0	0	0.252663	0.0353564	-0.0160893
20	Block 3	3	3	-1	3	1	-1	0.506984	0.071423	-0.0094583
21	Block 3	3	3	-1	1	-1	1	0.0042873	0.0247898	0.001002
22	Block 3	3	2	0	2	0	0	0.311344	0.0534211	-0.0876102
23	Block 3	3	2	0	1	-1	0	0.0687643	0.0349394	-0.0736755
24	Block 3	3	2	0	3	1	0	0.567268	0.0927513	-0.0862116
25	Block 3	3	1	1	2	0	0	0.392	0.0774002	-0.165799
26	Block 3	3	1	1	1	-1	-1	0.13978	0.0604918	-0.151177
27	Block 3	3	1	1	3	1	1	0.634818	0.126857	-0.173452

Table III. Degrees of freedom breakdown for the first-stage analysis.

Source	***Degrees of Freedom (DF)***
Replicate (*Rep&Random*)	*r*-1 = 2
Aggregate whole plot factor (*wptreat*)	*a*-1 = 2
Whole plot error (*Rep*wptreat&Random*)	(*r*-1)*(*a*-1) = 4
Aggregate subplot factor (*sptreat*)	*b*-1 = 2
Aggregate whole plot factor crossed with aggregate subplot factor (*wptreat*sptreat*)	(*a*-1)*(*b*-1) = 4
Subplot error (*MSE*)	*a**(*r*-1)*(*b*-1) = 12
Total	*r***a***b* – 1 = 26

Number of replicates (*Rep*), $r = 3$
Levels of aggregate whole plot factor (*wptreat*), $a = 3$
Levels of aggregate subplot factor (*sptreat*), $b = 3$

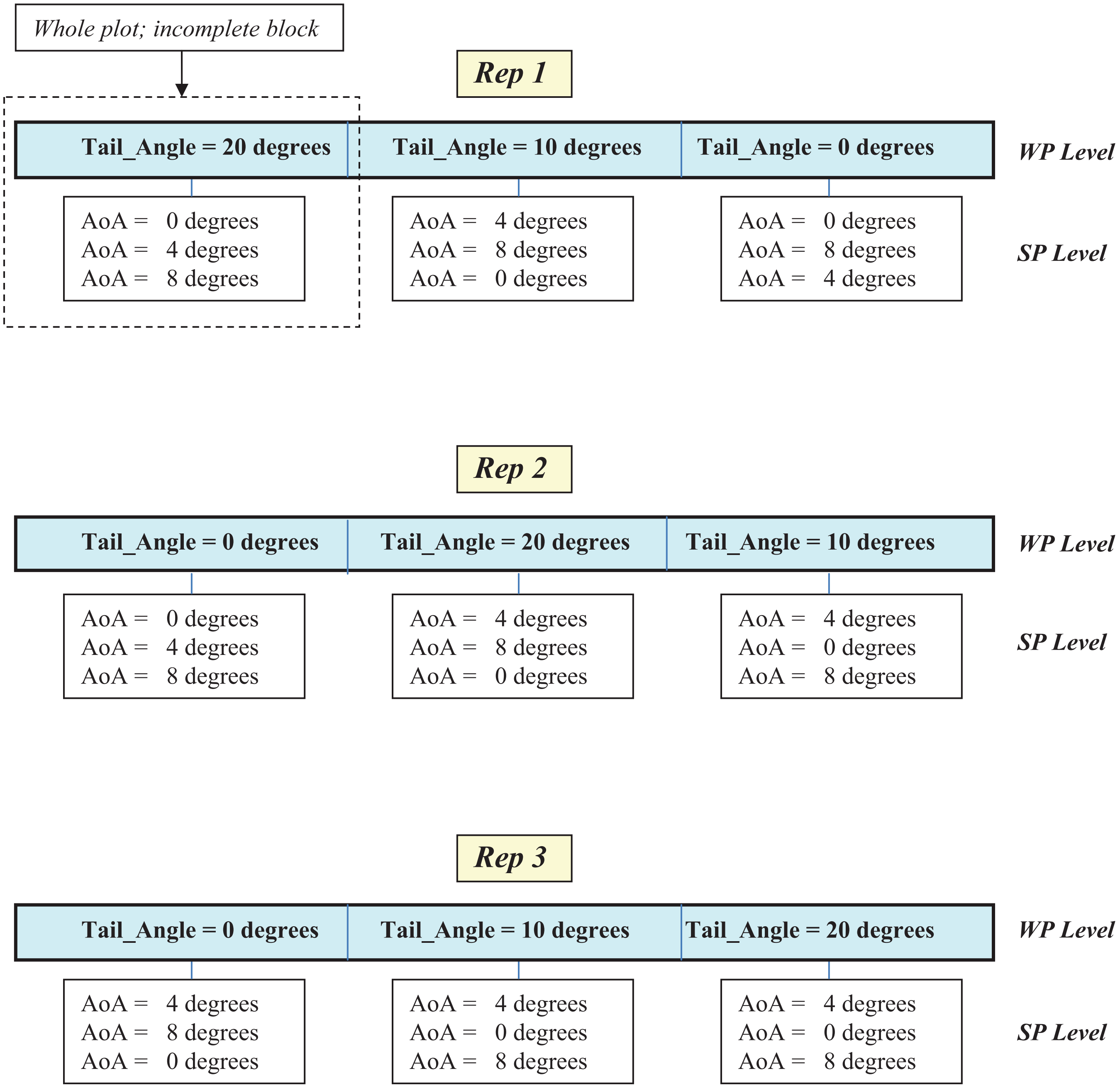


Figure 1. Split-plot design in run order for the low-speed wind tunnel experiment. The design contains nine whole plots (three whole plots within each block or replicate (*Rep*)), three subplot runs within each whole plot, and a total of 27 runs.

(a) overall view

(b) close-up view of the right horizontal stabilizer at +20-degrees incidence

Figure 2. Photographs of the 0.025-scale fighter airplane model installed in the ODU low-speed diagnostic wind tunnel facility.

Stage 1 Analysis

Objective: Verify the overall statistical significance of linear models of *CL*, *CD*, and *Cm* responses at the whole plot and subplot levels

Approach: Construct and analyze linear statistical models containing aggregate whole plot and subplot terms and their two-factor interaction (all terms declared nominal) with correct error terms and statistical tests at the whole plot and subplot levels. Verification of statistical significance warrants follow-on analyses.

Model Specification

WP Level

wptreat (nominal)
WP error

SP Level

sptreat (nominal)
*wptreat*sptreat* (nominal)
SP error

Stage 2 Analysis – Step 1

Objective: Decompose all terms, estimate the mean squares and multiple regression coefficients, and perform correct statistical tests **at the subplot level**.

Model Specification

Approach: Construct and perform correct analysis of linear statistical models and estimated multiple regression functions with continuous individual terms at the subplot level; retain aggregate, nominal whole plot factor *wptreat* in the analysis.

WP Level

wptreat (nominal)
WP error

SP Level

AoA
Tail_Angle*AoA
AoA*AoA
SP error
(all terms continuous)

(a) stage 1 analysis and stage 2, step 1 analysis

Figure 3. Three-stage analysis approach for the split-plot experimental design.

Stage 2 Analysis – Step 2

Objective: Decompose all terms and estimate the mean squares and multiple regression coefficients **at the whole plot level**

Approach: Construct linear statistical models and estimated multiple regression functions with continuous individual terms at the whole plot level; tests of significance are *not* valid at this level because of incorrect error term

Model Specification

WP Level

Tail_Angle
Tail_Angle*Tail_Angle
Overall model error
(all terms continuous)

Stage 3 Final Analysis

Objective: Obtain correct tests of significance of the individual model terms at the whole plot level and assemble final analyses with estimated regression coefficients for the three response variables.

Approach: Use the correct whole plot error term from stage 2, step 1 to perform correct tests of significance of the estimated mean squares and regression coefficients at the whole plot level obtained in stage 2, step 2. Assemble final listings of estimated mean squares and regression coefficients with corresponding tests of significance at the whole plot and subplot levels.

Model Specification

WP Level

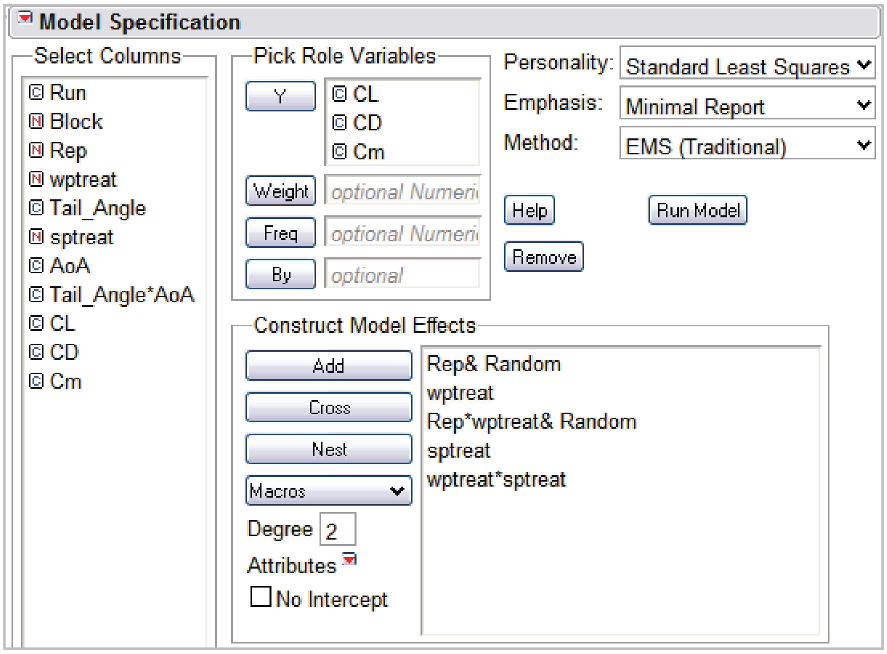
Tail_Angle
Tail_Angle*Tail_Angle
WP error
(all terms continuous)

SP Level

AoA
Tail_Angle*AoA
AoA*AoA
SP error
(all terms continuous)

(b) stage 2, step 2 analysis and stage 3 final analysis

Figure 3. Concluded.



Model Specification

Select Columns

Run
Block
Rep
wptreat
Tail_Angle
sptreat
AoA
Tail_Angle*AoA
CL
CD
Cm

Pick Role Variables

Y: CL, CD, Cm

Weight: optional Numeric

Freq: optional Numeric

By: optional

Personality: Standard Least Squares

Emphasis: Minimal Report

Method: EMS (Traditional)

Construct Model Effects

Rep& Random
wptreat
Rep*wptreat& Random
sptreat
wptreat*sptreat

Degree: 2

☐ No Intercept

Figure 4. Model specification with the combined whole plot term (*wptreat*) and the combined subplot term (*sptreat*) for the first-stage analysis.

```
                         CL Response
                      Analysis of Variance
 Source     DF   Sum of Squares   Mean Square      F Ratio       Prob > F
 Model      14     1.16554482     0.08325320    1350.79074000   5.9895E-17
 Error      12     0.00073960     0.00006163
 C. Total   26     1.16628442

                         CD Response
                      Analysis of Variance
 Source     DF   Sum of Squares   Mean Square      F Ratio       Prob > F
 Model      14     0.02498387     0.00178456     458.83201900   3.8463E-14
 Error      12     0.00004667     0.00000389
 C. Total   26     0.02503055

                         Cm Response
                      Analysis of Variance
 Source     DF   Sum of Squares   Mean Square      F Ratio       Prob > F
 Model      14     0.10519991     0.00751428     334.97972900   2.5207E-13
 Error      12     0.00026918     0.00002243
 C. Total   26     0.10546910
```

(a) analysis of variance

Figure 5. Partial output from the first-stage analysis of the split-plot design with the combined whole plot and combined subplot terms.

CL Response

Tests wrt Random Effects

Source	SS	MS Num	DF Num	F Ratio	Prob > F
Rep&Random	0.00008245	0.00004123	2	0.39128300	0.69951662
wptreat	0.06804799	0.03402400	2	322.93238300	0.00003789
*Rep*wptreat&Random*	0.00042144	0.00010536	4	1.70946748	0.21234330
sptreat	1.09666515	0.54833257	2	8896.74571000	9.3705E-20
*wptreat*sptreat*	0.00032779	0.00008195	4	1.32961523	0.31459038

CD Response

Tests wrt Random Effects

Source	SS	MS Num	DF Num	F Ratio	Prob > F
Rep&Random	0.00002260	0.00001130	2	2.06360525	0.24223504
wptreat	0.00827416	0.00413708	2	755.40947900	0.00000697
*Rep*wptreat&Random*	0.00002191	0.00000548	4	1.40810040	0.28979894
sptreat	0.01626442	0.00813221	2	2090.88669000	5.4886E-16
*wptreat*sptreat*	0.00040078	0.00010020	4	25.76154780	0.00000821

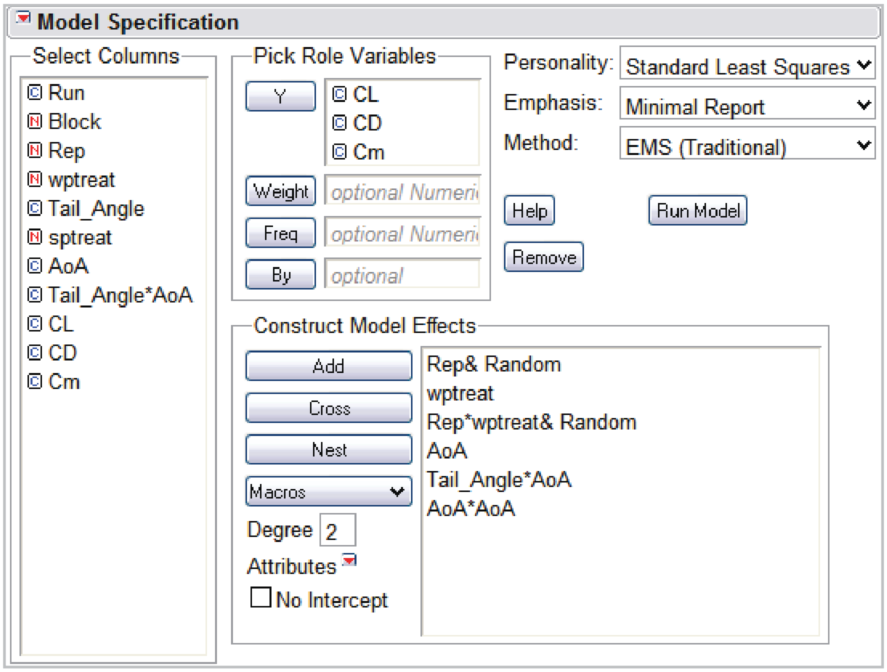
Cm Response

Tests wrt Random Effects

Source	SS	MS Num	DF Num	F Ratio	Prob > F
Rep&Random	0.00081175	0.00040587	2	2.26741461	0.21964955
wptreat	0.10236051	0.05118026	2	285.91833000	0.00004825
*Rep*wptreat&Random*	0.00071601	0.00017900	4	7.97979326	0.00223038
sptreat	0.00129345	0.00064672	2	28.83035090	0.00002613
*wptreat*sptreat*	0.00001819	0.00000455	4	0.20275193	0.93199657

(b) tests with respect to random effects

Figure 5. Concluded.



Select Columns: Run, Block, Rep, wptreat, Tail_Angle, sptreat, AoA, Tail_Angle*AoA, CL, CD, Cm

Y: CL, CD, Cm

Personality: Standard Least Squares

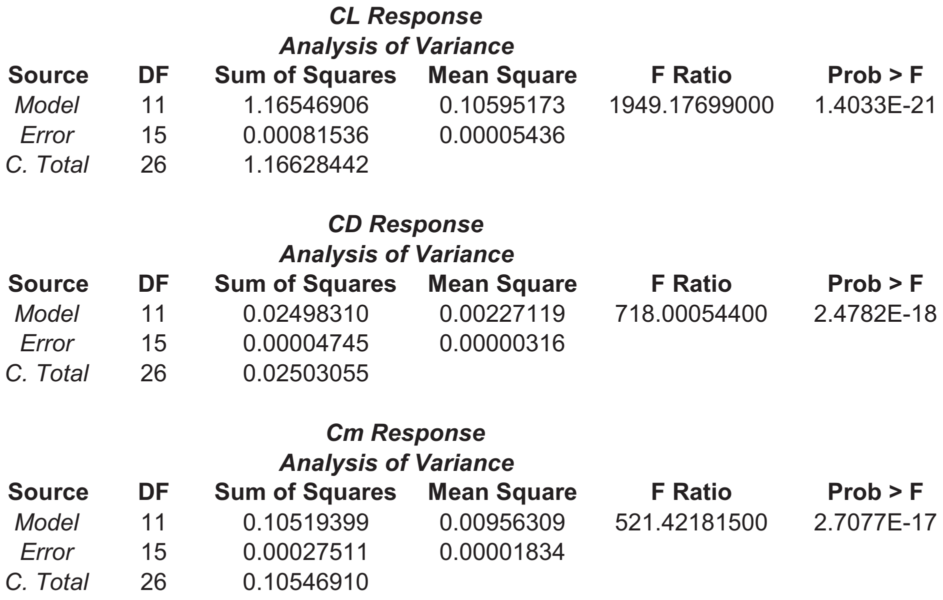
Emphasis: Minimal Report

Method: EMS (Traditional)

Degree: 2

Model effects:
Rep& Random
wptreat
Rep*wptreat& Random
AoA
Tail_Angle*AoA
AoA*AoA

Figure 6. Model specification for the first step of the second-stage analysis of the split-plot design.



```
                          CL Response
                      Analysis of Variance
 Source    DF  Sum of Squares  Mean Square       F Ratio    Prob > F
 Model     11      1.16546906   0.10595173  1949.17699000  1.4033E-21
 Error     15      0.00081536   0.00005436
 C. Total  26      1.16628442

                          CD Response
                      Analysis of Variance
 Source    DF  Sum of Squares  Mean Square       F Ratio    Prob > F
 Model     11      0.02498310   0.00227119   718.00054400  2.4782E-18
 Error     15      0.00004745   0.00000316
 C. Total  26      0.02503055

                          Cm Response
                      Analysis of Variance
 Source    DF  Sum of Squares  Mean Square       F Ratio    Prob > F
 Model     11      0.10519399   0.00956309   521.42181500  2.7077E-17
 Error     15      0.00027511   0.00001834
 C. Total  26      0.10546910
```

(a) analysis of variance

Figure 7. Partial output from the first step of the second-stage analysis of the split-plot design.

CL Response

Tests wrt Random Effects

Source	**SS**	**MS Num**	**DF Num**	**F Ratio**	**Prob > F**
Rep&Random	0.00008245	0.00004123	2	0.39128300	0.69951662
wptreat	0.06804799	0.03402400	2	322.93238300	0.00003789
*Rep*wptreat&Random*	0.00042144	0.00010536	4	1.93828183	0.15623996
AoA	1.09663735	1.09663735	1	20174.66110000	6.9027E-25
*Tail_Angle*AoA*	0.00025203	0.00025203	1	4.63655851	0.04797679
AoA*AoA	0.00002779	0.00002779	1	0.51132859	0.48554603

CD Response

Tests wrt Random Effects

Source	**SS**	**MS Num**	**DF Num**	**F Ratio**	**Prob > F**
Rep&Random	0.00002260	0.00001130	2	2.06360525	0.24223504
wptreat	0.00827416	0.00413708	2	755.40947900	0.00000697
*Rep*wptreat&Random*	0.00002191	0.00000548	4	1.73134181	0.19546921
AoA	0.01509471	0.01509471	1	4771.95070000	3.3690E-20
*Tail_Angle*AoA*	0.00040001	0.00040001	1	126.45602100	0.00000001
AoA*AoA	0.00116971	0.00116971	1	369.784266	5.5695E-12

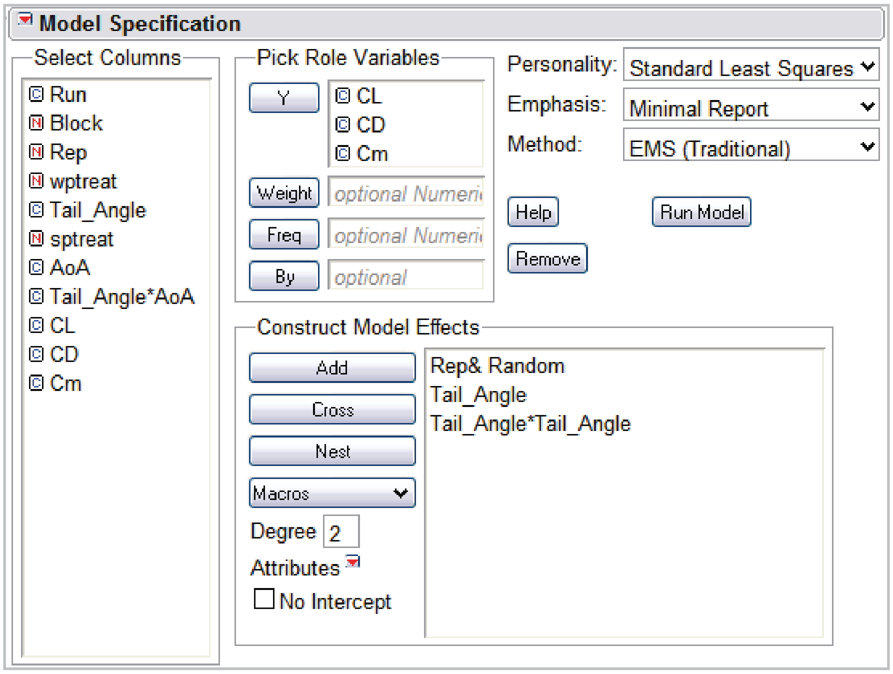
Cm Response

Tests wrt Random Effects

Source	**SS**	**MS Num**	**DF Num**	**F Ratio**	**Prob > F**
Rep&Random	0.00081175	0.00040587	2	2.26741461	0.21964955
wptreat	0.10236051	0.05118026	2	285.91833000	0.00004825
*Rep*wptreat&Random*	0.00071601	0.00017900	4	9.76003571	0.00042926
AoA	0.00122056	0.00122056	1	66.55024270	0.00000068
*Tail_Angle*AoA*	0.00001227	0.00001227	1	0.66906420	0.42618129
AoA*AoA	0.00007289	0.00007289	1	3.97420462	0.06471406

(b) tests with respect to random effects

Figure 7. Concluded.



Model Specification

Select Columns: Run, Block, Rep, wptreat, Tail_Angle, sptreat, AoA, Tail_Angle*AoA, CL, CD, Cm

Pick Role Variables

Y: CL, CD, Cm

Weight: optional Numeric

Freq: optional Numeric

By: optional

Personality: Standard Least Squares

Emphasis: Minimal Report

Method: EMS (Traditional)

Construct Model Effects

Rep& Random

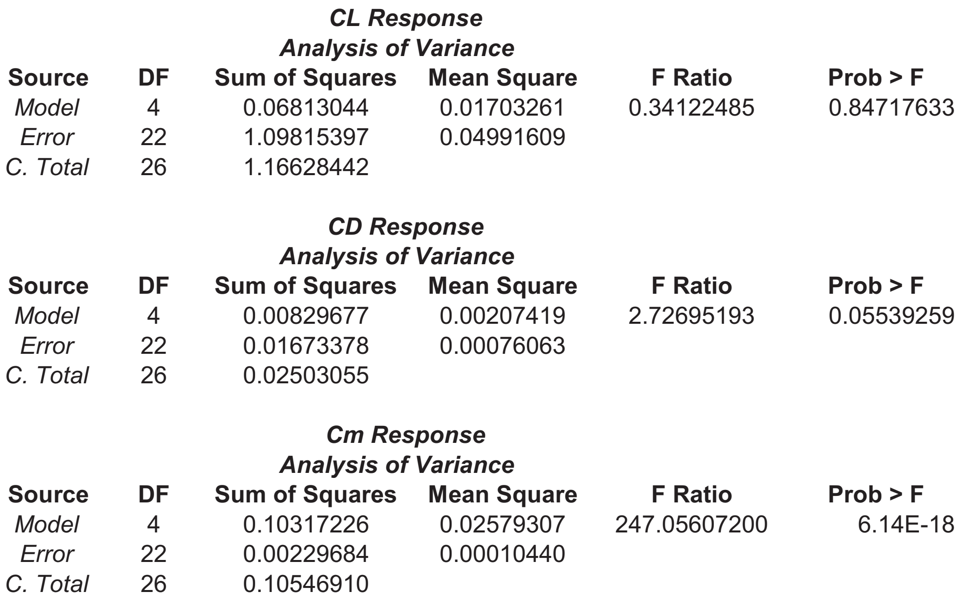
Tail_Angle

Tail_Angle*Tail_Angle

Degree: 2

☐ No Intercept

Figure 8. Model specification for the second step of the second-stage analysis of the split-plot design.



```
                         CL Response
                     Analysis of Variance
Source    DF   Sum of Squares   Mean Square     F Ratio        Prob > F
Model      4     0.06813044     0.01703261     0.34122485     0.84717633
Error     22     1.09815397     0.04991609
C. Total  26     1.16628442

                         CD Response
                     Analysis of Variance
Source    DF   Sum of Squares   Mean Square     F Ratio        Prob > F
Model      4     0.00829677     0.00207419     2.72695193     0.05539259
Error     22     0.01673378     0.00076063
C. Total  26     0.02503055

                         Cm Response
                     Analysis of Variance
Source    DF   Sum of Squares   Mean Square     F Ratio        Prob > F
Model      4     0.10317226     0.02579307   247.05607200       6.14E-18
Error     22     0.00229684     0.00010440
C. Total  26     0.10546910
```

(a) analysis of variance

Figure 9. Partial output from the second step of the second-stage analysis of the split-plot design.

CL Response

Tests wrt Random Effects

Source	SS	MS Num	DF Num	F Ratio	Prob > F
Rep&Random	0.00008245	0.00004123	2	0.00082589	0.99917448
Tail_Angle	0.06804609	0.06804609	1	1.36320952	0.25548085
*Tail_Angle*Tail_Angle*	0.00000190	0.00000190	1	0.00003810	0.99513098

CD Response

Tests wrt Random Effects

Source	SS	MS Num	DF Num	F Ratio	Prob > F
Rep&Random	0.00002260	0.00001130	2	0.01485822	0.98526149
Tail_Angle	0.00801514	0.00801514	1	10.53755030	0.00370563
*Tail_Angle*Tail_Angle*	0.00025902	0.00025902	1	0.34054096	0.56545251

Cm Response

Tests wrt Random Effects

Source	SS	MS Num	DF Num	F Ratio	Prob > F
Rep&Random	0.00081175	0.00040587	2	3.88762149	0.03582952
Tail_Angle	0.10221366	0.10221366	1	979.04245800	0.00000000
*Tail_Angle*Tail_Angle*	0.00014685	0.00014685	1	1.40658721	0.24827524

(b) tests with respect to random effects

Figure 9. Concluded.

Analysis of Lift Coefficient

First Step, Second-Stage Analysis

Decomposition of Subplot Terms

Source	***SS***	***DF***	***MS***	***F Ratio***	***Prob > F***
Rep&Random	0.00008245	2	0.00004123	0.39128300	0.69951662
wptreat	0.06804799	2	0.03402400	322.93238300	0.00003789
Rep*wptreat&Random	0.00042144	4	0.00010536	1.93828183	0.15623996
AoA	1.09663735	1	1.09663735	20174.66110000	6.9027E-25
Tail_Angle*AoA	0.00025203	1	0.00025203	4.63655851	0.04797679
AoA*AoA	0.00002779	1	0.00002779	0.51132859	0.48554603
Error	0.00081536	15	0.00005436		
Total	1.16628442	26			

Second Step, Second-Stage Analysis

Decomposition of Whole Plot Terms

Source	***SS***	***DF***	***MS***	***F Ratio***	***Prob > F***
Rep&Random	0.00008245	2	0.00004123	0.00082589	0.99917448
Tail_Angle	0.06804609	1	0.06804609	1.36320952	0.25548085
Tail_Angle*Tail_Angle	0.00000190	1	0.00000190	0.00003810	0.99513098
Error	1.09815397	22	0.04991609		
Total	1.16628442	26			

Modified ANOVA - Third-Stage Analysis

Combined Analyses from Second-Stage, Steps 1 and 2 with Correct Tests of Signficance at Whole Plot and Subplot Levels

Source	***SS***	***DF***	***MS***	***F Ratio***	***Prob > F***
Whole Plot Level					
Rep&Random	0.00008245	2	0.00004123	0.39128300	0.69951662
wptreat	0.06804799	2	0.03402400	322.93238300	0.00003789
Decomposition of wptreat					
Tail_Angle	0.06804609	1	0.06804609	645.84367882	0.00001424
Tail_Angle*Tail_Angle	0.00000190	1	0.00000190	0.01803341	0.89966021
Rep*wptreat&Random	0.00042144	4	0.00010536	1.93828183	0.15623996
Subplot Level					
AoA	1.09663735	1	1.09663735	20174.66110000	6.9027E-25
Tail_Angle*AoA	0.00025203	1	0.00025203	4.63655851	0.04797679
AoA*AoA	0.00002779	1	0.00002779	0.51132859	0.48554603
Error	0.00081536	15	0.00005436		
Total	1.16628442	26			

Note: Statistically significant terms are highlighted

Figure 10. Combined second- and third-stage analyses for the lift coefficient response.

Analysis of Drag Coefficient

First Step, Second-Stage Analysis

Decomposition of Subplot Terms

Source	*SS*	*DF*	*MS*	*F Ratio*	*Prob > F*
Rep&Random	0.00002260	2	0.00001130	2.06360525	0.24223504
wptreat	0.00827416	2	0.00413708	755.40947900	0.00000697
Rep*wptreat&Random	0.00002191	4	0.00000548	1.73134181	0.19546921
AoA	0.01509471	1	0.01509471	4771.95070000	3.3690E-20
Tail_Angle*AoA	0.00040001	1	0.00040001	126.45602100	0.00000001
AoA*AoA	0.00116971	1	0.00116971	369.78426600	5.5695E-12
Error	0.00004745	15	0.00000316		
Total	0.02503055	26			

Second Step, Second-Stage Analysis

Decomposition of Whole Plot Terms

Source	*SS*	*DF*	*MS*	*F Ratio*	*Prob > F*
Rep&Random	0.00002260	2	0.00001130	0.01485822	0.98526149
Tail_Angle	0.00801514	1	0.00801514	10.53755030	0.00370563
Tail_Angle*Tail_Angle	0.00025902	1	0.00025902	0.34054096	0.56545251
Error	0.01673378	22	0.00076063		
Total	0.02503055	26			

Modified ANOVA - Third-Stage Analysis

Combined Analyses from Second-Stage, Steps 1 and 2 with Correct Tests of Significance at Whole Plot and Subplot Levels

Source	*SS*	*DF*	*MS*	*F Ratio*	*Prob > F*
Whole Plot Level					
Rep&Random	0.00002260	2	0.00001130	2.06360525	0.24223504
wptreat	0.00827416	2	0.00413708	755.40947900	0.00000697
Decomposition of wptreat					
Tail_Angle	0.00801514	1	0.00801514	1462.61678832	0.00000279
Tail_Angle*Tail_Angle	0.00025902	1	0.00025902	47.26642336	0.00234501
Rep*wptreat&Random	0.00002191	4	0.00000548	1.73134181	0.19546921
Subplot Level					
AoA	0.01509471	1	0.01509471	4771.95070000	3.37E-20
Tail_Angle*AoA	0.00040001	1	0.00040001	126.45602100	0.00000001
AoA*AoA	0.00116971	1	0.00116971	369.78426600	5.5695E-12
Error	0.00004745	15	0.00000316		
Total	0.02503055	26			

Note: Statistically significant terms are highlighted

Figure 11. Combined second- and third-stage analyses for the drag coefficient response.

Analysis of Pitching Moment Coefficient

First Step, Second-Stage Analysis

Decomposition of Subplot Terms

Source	SS	DF	MS	F Ratio	Prob > F
Rep&Random	0.00081175	2	0.00040587	2.26741461	0.21964955
wptreat	0.10236051	2	0.05118026	285.91833000	0.00004825
Rep*wptreat&Random	0.00071601	4	0.00017900	9.76003571	0.00042926
AoA	0.00122056	1	0.00122056	66.55024270	0.00000068
Tail_Angle*AoA	0.00001227	1	0.00001227	0.66906420	0.42618129
AoA*AoA	0.00007289	1	0.00007289	3.97420462	0.06471406
Error	0.00027511	15	0.00001834		
Total	0.10546910	26			

Second Step, Second-Stage Analysis

Decomposition of Whole Plot Terms

Source	SS	DF	MS	F Ratio	Prob > F
Rep&Random	0.00081175	2	0.00040587	3.88762149	0.03582952
Tail_Angle	0.10221366	1	0.10221366	979.04245800	9.8149E-20
Tail_Angle*Tail_Angle	0.00014685	1	0.00014685	1.40658721	0.24827524
Error	0.00229684	22	0.00010440		
Total	0.10546910	26			

Modified ANOVA - Third-Stage Analysis

Combined Analyses from Second-Stage, Steps 1 and 2 with Correct Tests of Significance at Whole Plot and Subplot Levels

Source	SS	DF	MS	F Ratio	Prob > F
Whole Plot Level					
Rep&Random	0.00081175	2	0.00040587	2.26741461	0.21964955
wptreat	0.10236051	2	0.05118026	285.91833000	0.00004825
Decomposition of wptreat					
Tail_Angle	0.10221366	1	0.10221366	571.02603352	0.00001819
Tail_Angle*Tail_Angle	0.00014685	1	0.00014685	0.82039106	0.41629108
Rep*wptreat&Random	0.00071601	4	0.00017900	9.76003571	0.00042926
Subplot Level					
AoA	0.00122056	1	0.00122056	66.55024270	0.00000068
Tail_Angle*AoA	0.00001227	1	0.00001227	0.66906420	0.42618129
AoA*AoA	0.00007289	1	0.00007289	3.97420462	0.06471406
Error	0.00027511	15	0.00001834		
Total	0.10546910	26			

Note: Statistically significant terms are highlighted

Figure 12. Combined second and third-stage analyses for the pitching moment coefficient response.

CL Regression Coefficients

Term	*Coef*	*SE Coef*	*t*	*Prob > |t|*
Intercept	0.31597778	0.00245758	128.57288900	3.0713E-24
Tail_Angle	0.06148446	0.00241935	25.41364233	0.00001424
Tail_Angle*Tail_Angle	-0.000563	0.00419044	-0.13435337	0.89961212
AoA	0.24682848	0.00173777	142.03753400	6.9027E-25
Tail_Angle*AoA	-0.0045828	0.00212832	-2.15326690	0.04797679
AoA*AoA	0.0021523	0.00300990	0.71507244	0.48554603

CD Regression Coefficients

Term	*Coef*	*SE Coef*	*t*	*Prob > |t|*
Intercept	0.05444769	0.00059285	91.84093160	4.7463E-22
Tail_Angle	0.02110179	0.00055180	38.24155595	0.00000279
Tail_Angle*Tail_Angle	0.00657044	0.00095566	6.87529109	0.00234472
AoA	0.02895851	0.00041921	69.07930730	3.3690E-20
Tail_Angle*AoA	0.00577356	0.00051342	11.24526660	0.00000001
AoA*AoA	0.01396249	0.00072609	19.22977550	5.5695E-12

Cm Regression Coefficients

Term	*Coef*	*SE Coef*	*t*	*Prob > |t|*
Intercept	-0.08041760	0.00142752	-56.33362700	7.1019E-19
Tail_Angle	-0.07535610	0.00315306	-23.89932687	0.00001818
Tail_Angle*Tail_Angle	-0.00494720	0.00546156	-0.90582231	0.41625914
AoA	-0.00823460	0.00100941	-8.15783320	0.00000068
Tail_Angle*AoA	-0.00101120	0.00123627	-0.81796340	0.42618129
AoA*AoA	0.00348541	0.00174835	1.99354072	0.06471406

Note: Statistically significant terms are highlighted

Figure 13. Final regression model coefficients (in coded units) and statistical tests of significance for the lift, drag, and pitching moment coefficient responses.

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13. SUPPLEMENTARY NOTES

14. ABSTRACT

A procedure to analyze a split-plot experimental design featuring two input factors, two levels of randomization, and two error structures in a low-speed wind tunnel investigation of a small-scale model of a fighter airplane configuration is described in this report. Standard commercially-available statistical software was used to analyze the test results obtained in a randomization-restricted environment often encountered in wind tunnel testing. The input factors were differential horizontal stabilizer incidence and the angle of attack. The response variables were the aerodynamic coefficients of lift, drag, and pitching moment. Using split-plot terminology, the whole plot, or difficult-to-change, factor was the differential horizontal stabilizer incidence, and the subplot, or easy-to-change, factor was the angle of attack. The whole plot and subplot factors were both tested at three levels. Degrees of freedom for the whole plot error were provided by replication in the form of three blocks, or replicates, which were intended to simulate three consecutive days of wind tunnel facility operation. The analysis was conducted in three stages, which yielded the estimated mean squares, multiple regression function coefficients, and corresponding tests of significance for all individual terms at the whole plot and subplot levels for the three aerodynamic response variables. The estimated regression functions included main effects and two-factor interaction for the lift coefficient, main effects, two-factor interaction, and quadratic effects for the drag coefficient, and only main effects for the pitching moment coefficient.

15. SUBJECT TERMS

Aerodynamics; Experimental design; Modern design of experiments; Randomization restriction; Regression analysis; Statistical tests; Wind tunnels

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