

Comparison of Force and Moment Coefficients for the Same Test Article in Multiple Wind Tunnels

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This paper compares the results of force and moment measurements made on the same test article and with the same balance in three transonic wind tunnels. Comparisons are made for the same combination of Reynolds number, Mach number, sideslip angle, control surface configuration, and angle of attack range. Between-tunnel force and moment differences are quantified. An analysis of variance was performed at four unique sites in the design space to assess the statistical significance of between-tunnel variation and any interaction with angle of attack. Tunnel to tunnel differences too large to attribute to random error were detected were observed for all forces and moments. In some cases these differences were independent of angle of attack and in other cases they changed with angle of attack.

I. Introduction

This paper presents an analysis of data acquired in support of the Facility Analysis Verification and Operational Reliability (FAVOR) project, in which data were acquired in four US transonic wind tunnels by executing nominally similar test matrices in each facility on the same test article, balance, and sting. The participating tunnels were the National Transonic Facility at Langley Research Center (LaRC), the 11-Ft Unitary Plan wind tunnel at Ames Research Center (ARC), the 16T wind tunnel at the Arnold Engineering and Development Center (ARDC), and the 8x6-Foot supersonic wind tunnel at Glenn Research Center (GRC). The test article was the AEDC 16T check standard model, a modified 5% scale model of an F-111.

The stated objective of the FAVOR project was to compare test methods, techniques, and procedures, as well as data reduction methods, flow quality, and aerodynamic data acquired across the four facilities in nominally identical wind tunnel tests. In support of these objectives, the NASA Aeronautics Test Program Office requested an independent analysis of the FAVOR data using certain statistical methods that were outlined when this work was presented in 2012¹. The original analysis focused on quantifying unexplained variance in each facility, and partitioning it into random and systematic components. Original plans for statistically-based examinations of between-tunnel differences were impacted by the fact that there was only a single common set of test conditions executed in all four tunnels. A number of identical test conditions were established in three of the four tunnels however (Langley, Ames, and AEDC), and this paper bases between-tunnel comparisons on those results.

Two, three, or four polar replicates were acquired in each facility at four sites within the design space. The angle of sideslip was 0 for all four cases. Two conditions were at Mach 0.60 and two were at Mach 0.85. One of the Mach 0.60 conditions was at a Reynolds number of 3.9 million/ft and the other was at 5.5 million/ft. Both of these samples were acquired at a baseline control surface configuration designated "Configuration 0."

The two Mach 0.85 conditions featured the same Reynolds number of 4.5 million/ft, but one was at Configuration 0 and the other was at a configuration designated "Configuration 1," in which control surfaces in the tail were deflected.

A two-way analysis of variance with replication was performed at all four design-space sites for coefficients of drag, lift, rolling moment, pitching moment, and yawing moment. Within- and between-facility variations are presented graphically.

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II. Test Article and Instrumentation

Figure 1 shows the planform of the test article, the AEDC check standard model consisting of a modified 5% model of the F-111. The wings of the check standard model were modified to provide a 48-inch span at a fixed wing sweep angle of 35 degrees.

Trip dots of the same size were applied at the same location in all facilities. They were located on the nose and upper and lower surfaces on the wing strake, wing, and horizontal and vertical tails.

Two control surface configurations were tested, designated Configuration 0 and Configuration 1. The horizontal tail was not deflected in Configuration 0, but it was deflected 10° in Configuration 1.

The NTF-115 single-piece moment-type balance was designed for use with the F-111 test article. A requirement was for all tests to use the same balance and calibration. Once the sting, balance, model, and instrumentation was built up it remained as one unit for the completion of the four tests. This was to ensure that the bridging of the balance, the installation of the balance to the model and sting, and the build-up of the model did not change from test to test.

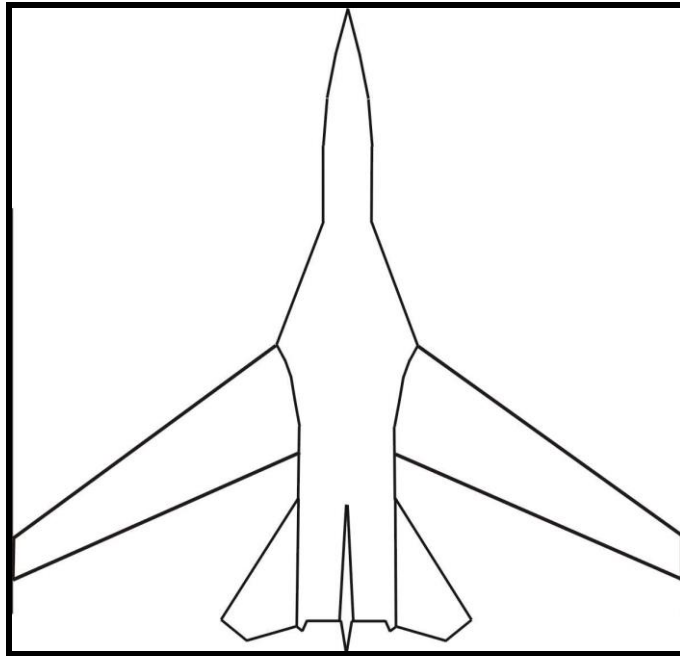


Figure 1. Planform of test article

Figure 2 shows the test article as mounted for testing at AEDC.



Figure 2. F-111 test article. Wing sweep of 35° , 48-inch wing span

III. Analysis Method and Representative Results

This section focuses on the data analysis methodology. The general framework for the analysis is then described, highlighting the role of unexplained variance in the analysis.

A. General Analysis Framework

Eleven combinations of Mach number, Reynolds number, control surface configuration, and sideslip were common to all three tunnels, with angle of attack ranges overlapping sufficiently to facilitate between-facility comparisons. Four of these also featured within-facility polar replicates. These replicates enabled an analysis of variance with replication, in which the total variance in a given sample of data could be partitioned and assigned not only to principal sources of variation, but also to interactions involving those sources.

Specifically, a sample of force or moment data acquired on the same test article under common test conditions in different facilities is expected to display variance attributable to three causes. Most of the variance in the data sample will have been intentionally induced by changes in the independent variables, e.g. angle of attack. After accounting for angle of attack effects, the rest of the variance will be due to facility differences, with some residual unexplained variance commonly attributed to ordinary random experimental error. We say that between-tunnel effects are “significant” (that is, *statistically* significant) if they are too large to be attributed to random error.

Within-facility polar replicates enable a direct estimation of the random error. When this random error variance is subtracted from the residual variance left over after angle of attack changes and tunnel differences have been taken into account, there is often something remaining. This component of variation is attributable to the interaction between changes in angle of attack and changes in facility. It can be statistically significant if facility differences depend on angle of attack. There is no statistically significant interaction when there is the same nominally constant bias of one tunnel with respect to another at all AoA levels.

The interaction between angle of attack and facility is especially interesting. Because an ANOVA with replicated polars also provides the same insight into facility effects as an ANOVA without replicated polars (and actually with higher precision), this paper focuses on analyses performed at that subset of conditions common to all three tunnels for which within-facility polar replicates were available.

B. Analysis of Variance

ANOVA methods have been used extensively in this paper. They are more commonly used outside the experimental aeronautics community than within it, but standard textbooks present the computational details², and Ref. 3 describes specific applications to wind tunnel testing.

Without delving deeply into the computational details, every ANOVA entails some prescription for computing various components of variance and comparing each to some irreducible error variance by means of a ratio known as the “F statistic” to honor Ronald Fisher, who first developed the analysis of variance.

The F-statistic is a random variable, and because of ordinary chance variations that can occur in any finite sample of data, the F-statistic takes on a distribution of values as both the numerator and denominator wax and wane with such variations. Just as with any other random variable, while theoretically the F statistic can take on a range of values, some are more likely than others. There is thus a probability distribution associated with the F statistic.

We use the F statistic to assess the “statistical significance” of some effect such as between-tunnel differences by asking whether the associated variance component is too large compared to random error to be due simply to chance variations in the data. In this case we would construct a “null hypothesis” stating that the variance in data acquired across tunnels is just random error, so the ratio of that variance to ordinary random error variance is nearly 1. Under the null hypothesis, we expect F values greater than 1 to be progressively less and less likely the larger F is. This results in a probability density function for F that is skewed to the right. Fig. 3 displays the general behavior of the F distribution.

The vertical red line in Fig. 3 marks the location of what is called the critical F value, F_{crit} . This location is defined by the area under the F distribution to the right of F_{crit} , customarily designated α , which is 0.05 in this figure. We compare computed F values with F_{crit} to infer whether the null hypothesis should be rejected.

If $F > F_{crit}$, we reject the null hypothesis and infer that the effect we are assessing (tunnel to tunnel variations in the data, say) is too great to be attributed entirely to random noise. In that case the probability of an inference error due to ordinary chance variations in the data will be no greater than α , and we will conclude with $100 \times (1 - \alpha)\%$ confidence that the effect is real.

If, on the other hand, the F value lies to the left of F_{crit} , we will be unable to reject the null hypothesis at what is formally described as the α level of significance. We will then conclude that the data do not support with at least $100 \times (1 - \alpha)\%$ confidence an inference that the effect is real.

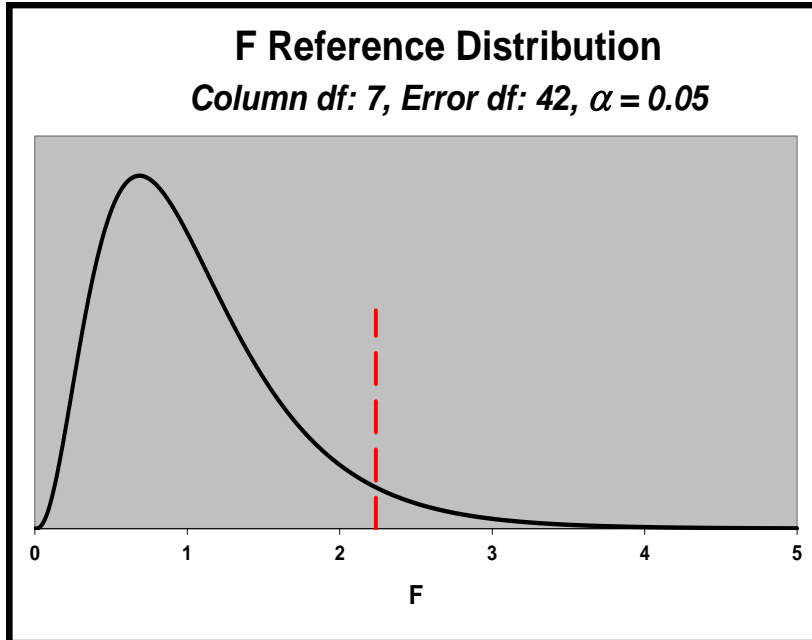


Figure 3: Representative F Distribution. Red line is critical F corresponding to $\alpha = 0.05$.

The analysis of variance entails a certain amount of bookkeeping. We must account for multiple sources of variation. Separate calculations of several variance components must be made for each such source. Critical F values are computed for a specified α based on the number of numerator and denominator degrees of freedom in the F statistic. In practice, the rather tedious calculations of ANOVA are automated in many commercially available software packages.

A standard structure for supporting the bookkeeping of an analysis of variance is the ANOVA table. Table 1 displays a representative ANOVA table for that applies to a sample of data consisting of three ostensibly identical 7-point drag polars acquired in each of three tunnels. Each polar spanned the range of $\pm 3^\circ$ in 1° increments. This table was created by using the “two-way ANOVA with replication” data analysis add-in of Excel.

Table 1: ANOVA table for ostensibly identical drag polars acquired in three tunnels with Configuration 0, Re 3.9E06, Mach 0.60.

<i>Source of Variation</i>	<i>SS</i>	<i>df</i>	<i>MS</i>	<i>F</i>	<i>P-value</i>	<i>F crit</i>
AoA	7.693E-04	6	1.282E-04	1120.6	0.0000	2.3
Tunnel	1.683E-06	2	8.415E-07	7.4	0.0018	3.2
AoA x Tunnel	2.708E-06	12	2.257E-07	1.97	0.0524	1.99
Residual Variance	4.805E-06	42	1.144E-07			
Total	0.000778462	62				

The first column of the ANOVA table identifies the source of variation, and the second and third columns display “sums of squares” and “degrees of freedom,” the ratio of which is called the “Mean Square” (or “variance”) that is tabulated in the “MS” column. Entries in the F column are computed by simply dividing the corresponding Mean Square by the residual variance Mean Square. This calculation reveals that the component of variance attributable to changes in the angle of attack is 1120.6 times larger than the variance attributable to random error, the number in the F column. The P-value simply displays the probability that the corresponding F value would occur by chance because of ordinary random fluctuations in the data, if the null hypothesis is actually true. It represents the area under an F distribution such as Fig. 3 to the right of $F = 1130.6$, a very small number that is “0.0000” to the fourth decimal place. We are entitled therefore to make the rather unremarkable inference that drag coefficients change with angle of attack, and to do so with a vanishingly small probability of an inference error.

The fact that AoA changes are a significant contributor to the total variance in a sample of wind tunnel data may be uninteresting, but the other sources of variation in the ANOVA table are quite important in the current study. For example, the ANOVA table for the drag data of Table 1 displays an F value of 7.4 for the tunnel effect. This indicates that acquiring some data in one tunnel and some in another introduces 7.4 times more variation than acquiring all the data in a single tunnel where random error is the only source of variation beyond changes in angle of attack. This F statistic of 7.4 is larger than the critical F value of 3.2 that is tabulated in the last column of Table 1. We therefore reject the null hypothesis and conclude that between-facility effects are real.

Figure 4: Within- and between-facility variation for drag polar replicates acquired at Configuration 0, Re 3.9E06, Mach 0.60.

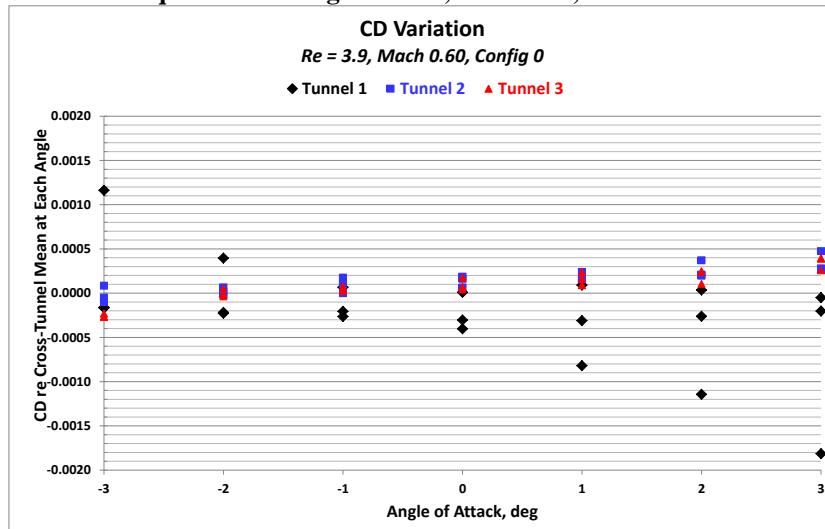


Figure 4 reinforces this conclusion. Nine drag measurements were made at each of the seven angles of attack, for a total of 63 measurements. At each AoA setting, three replicates were acquired in each of the three tunnels. At each angle, the mean of the nine measurements is subtracted from each of them and plotted. To the extent that the average across all tunnels is our most reliable indicator of “truth,” Fig. 4 could be regarded as a display of the deviation from “truth” (that is, the “error”) associated with each measurement.

Tunnels 2 and 3 generally agree with each other, but both disagree significantly with Tunnel 1, which accounts for the significant F statistic for Tunnel in Fig. 4. Incidentally, the within-facility variance is also indicated in Fig. 4. Clearly, at any given angle of attack, results were more reproducible in tunnels 2 and 3 than in Tunnel 1.

The ANOVA table displays an F value of 1.97 for AoA x Tunnel interaction, just under the critical F of 1.99. Formally, this implies that we cannot reject the null hypothesis for interaction and claim that tunnel differences depend on AoA, but because the difference between the F value and the critical F are so small, this is very much a borderline situation. The P-value of 0.0524 says that while we cannot claim to see interactions with 95% confidence, we can with 94.76% confidence, which is obviously a distinction with no practical difference. No doubt the only reason the ANOVA does not declare with greater conviction that an interaction exists in Fig. 4 is that the within-facility variation of Tunnel 1 is too great see a definitive trend.

C. More from the ANOVA Table

Results from an analysis of variance are organized in an ANOVA table that reveals the significance of variance components with respect to ordinary random error variance. This is the primary function of an ANOVA. However, rather more information than this can be gleaned from the ANOVA table, as will be described here.

1. R-squared: The sums of squares column can be used to estimate how much of the total variation in the data is allocated to each of the components, since the sums of squares of all components add up to the total sum of squares. Therefore the ratio of any component SS to the total is an indication of how much variation is associated with that component.

Consider the ratio of the AoA sum of squares to the total sum of squares. Since in this study these variations are attributed to changes we induced intentionally, the ratio of the AoA sum of squares to the total sum of squares, called the R-squared statistic, would be exactly 1.0000 in the absence of any unexplained variance.

The R-squared value for the ANOVA table corresponding to the drag data analyzed above, is 0.9882, which means that by this metric 98.82% of all the variation in this data sample can be explained by the angle of attack changes that were intentionally made. This also means that all of the uncertainty in this sample of drag polars is attributed to the 1.2% percent of the variations that cannot be explained.

2. Standard Random Error: The Mean Square for residual variance from the ANOVA table is just σ^2 for random error, so the square root of this is the ordinary “one sigma” standard random error value. For the drag data analyzed above the error MS is 1.144E-07 and the square root of this is 0.0003, or 3 drag counts. This is a one-sigma estimate of random error based on 42 degrees of freedom.

IV. Results

This analysis consisted of ANOVA calculations resulting in tables such as Table 1 and plots of differential forces and moments such as Fig. 4, designed to reveal within- and between-facility variations. These results are assembled in the Appendix. Of the 20 cases examined in this study (five response variables \times four tunnel states), in not a single one was there found an insignificant tunnel effect. That is, in every instance, differences from tunnel to tunnel were unambiguously larger than ordinary random error.

Significant AoA \times Tunnel interactions were observed in exactly half of the cases examined (10 out of 20). Of the 10 cases in which an AoA \times Tunnel interaction was observed, half of them occurred Configuration 1. In fact, there were no Configuration 1 cases in which this interaction did not occur. Even though Configuration 0 comprised 75% of all the cases examined, only 5 of those cases (25%) displayed significant AoA \times Tunnel interaction.

No particular dependence of these results on Mach number or Reynolds number was observed, although over a wider range of these variables, such effects might have been apparent. See the Appendix for a summary of all 20 cases.

V. Concluding Remarks

The results of this study suggest that wind tunnel results are difficult to reproduce from facility to facility. Bias shifts from one facility to another occurred for every force and moment examined, and for every Mach number and Reynolds number, albeit over limited ranges of both. Both test article configurations displayed significant facility to facility variation.

There was some thought initially that if significant tunnel effects were observed, they might be attributable to differences consistently associated with one tunnel, which would be identified as “the odd man out.” This was not the case. It was common to see all three tunnels yielding unique results, and in cases in which there was a 2-on-1 grouping, there was no consistent pairing of the same two tunnels yielding similar results.

It is interesting to note that deflecting the control surfaces of the test article seemed to increase the frequency of AoA \times Tunnel interactions. That is, with deflected control surfaces, response differences from one tunnel to another were more likely to vary with angle of attack. The sample size was not large enough to support a definitive conclusion, but this result was consistent in the current study.

Appendix

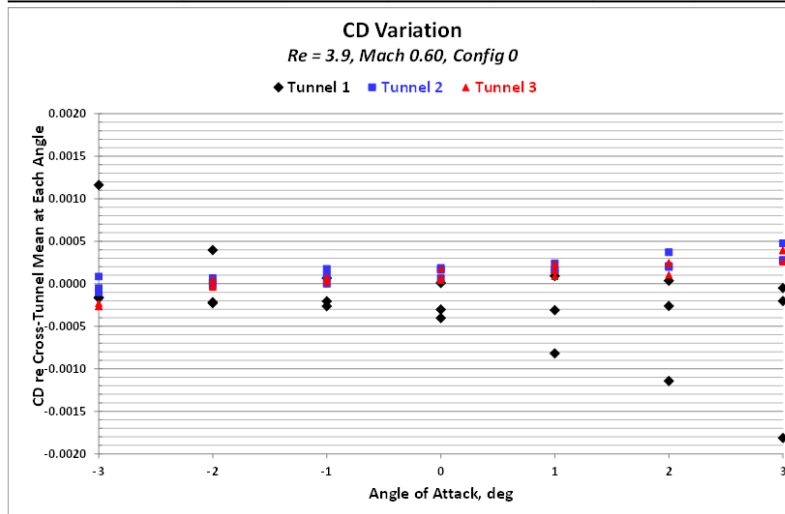
ANOVA tables and graphs of within- and between tunnel variations are collected in this appendix. The ANOVA results reflect a two-way ANOVA with replication, quantifying main effects (AoA and Tunnel) as well as the interaction between them. The F statistic and P-value for effects significant at the 0.05 level (detectable with at least 95% confidence) are bolded and highlighted to call attention to them.

The graphs each represent force and moment measurements expressed as a departure from the cross-tunnel mean of that measurement. All analyzed polars ranged from -3° to $+3^\circ$ in 1° increments, and at each of the three tunnels 2, 3, or 4 polar replicates were acquired. Therefore, at each AoA value there were $3 \times 2 = 6$, $3 \times 3 = 9$, or $3 \times 4 = 12$ ostensibly identical measurements (differing only by experimental error and cross-tunnel effects). The horizontal axis therefore represents the mean of these 6, 9, or 12 measurements, and each plotted point shows how far it is away from the average of all replicates acquired in the three tunnels for that AoA value, Mach number, Reynolds number, and configuration.

Points for a given tunnel that cluster closely suggest high within-facility precision. To the extent that the grand cross-tunnel mean is a “best estimate” of the true response, points near the horizontal axis in these plots suggest high accuracy. If replicates from a particular facility cluster close to each other but are located far from the mean, it may suggest high precision but low accuracy. Likewise, points from a given facility that exhibit substantial scatter centered about the mean of all tunnel results may be displaying greater accuracy than precision. Obviously, considerable scatter about a within-tunnel mean that is displaced considerably from the grand mean for all tunnels may suggest that both accuracy and precision are lacking.

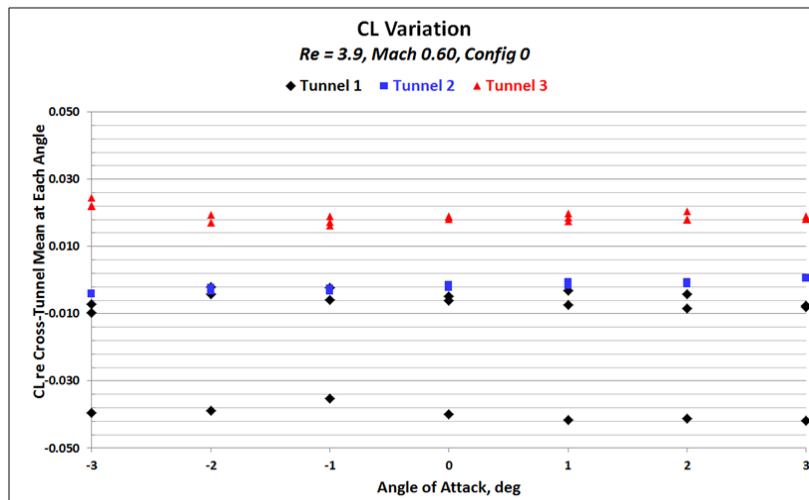
CD for Re# = 3.9, Mach 0.60, Configuration 0

Source of Variation	SS	df	MS	F	P-value	F crit
AoA	7.693E-04	6	1.282E-04	1120.6	0.0000	2.3
Tunnel	1.683E-06	2	8.415E-07	7.4	0.0018	3.2
AoA x Tunnel	2.708E-06	12	2.257E-07	1.97	0.0524	1.99
Residual Variance	4.805E-06	42	1.144E-07			
Total	0.000778462	62				



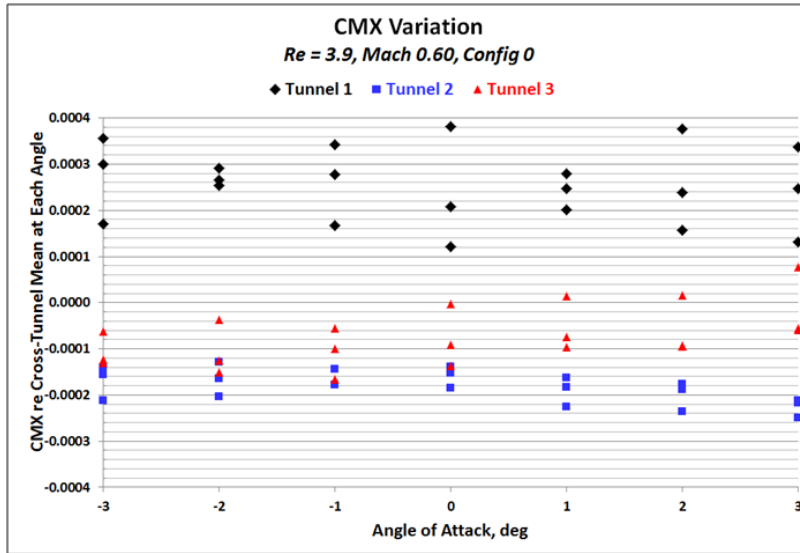
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Source of Variation	SS	df	MS	F	P-value	F crit
AoA	3.85E+00	6	6.41E-01	4938.1	0.0000	2.3
Tunnel	1.38E-02	2	6.88E-03	53.0	0.0000	3.2
AoA x Tunnel	1.57E-04	12	1.31E-05	0.10	0.9999	1.99
Residual Variance	5.45E-03	42	1.30E-04			
Total	3.87E+00	62				



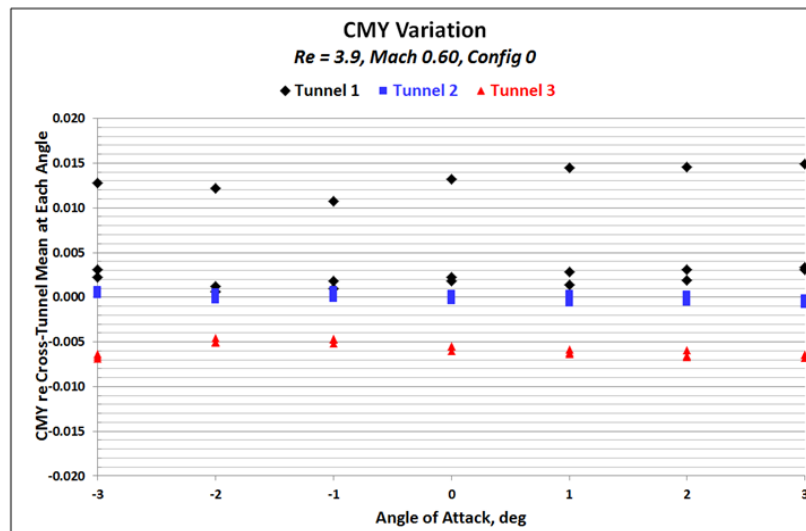
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Source of Variation	SS	df	MS	F	P-value	F crit
AoA	2.82E-06	6	4.7E-07	107.9	0.0000	2.3
Tunnel	2.16E-06	2	1.08E-06	248.4	0.0000	3.2
AoA x Tunnel	3.9E-08	12	3.25E-09	0.75	0.6992	1.99
Residual Variance	1.83E-07	42	4.36E-09			
Total	5.21E-06	62				



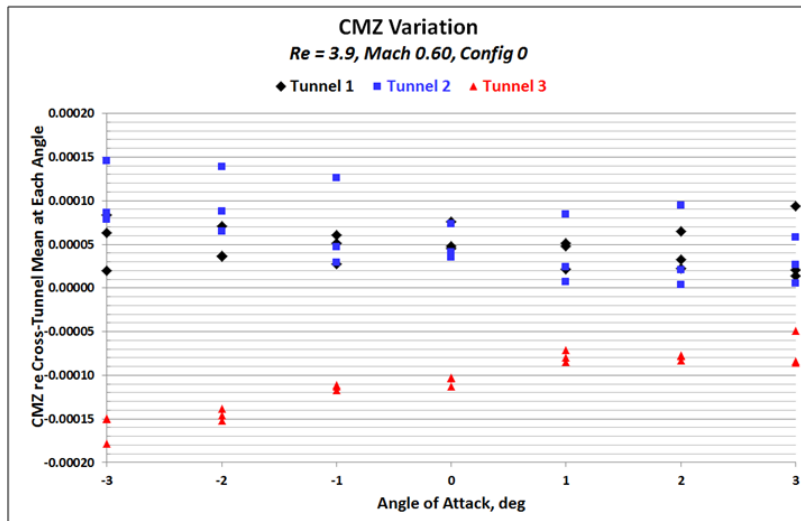
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Source of Variation	SS	df	MS	F	P-value	F crit
AoA	4.46E-01	6	7.43E-02	5275.4	0.0000	2.3
Tunnel	1.44E-03	2	7.19E-04	51.0	0.0000	3.2
AoA x Tunnel	2.86E-05	12	2.38E-06	0.17	0.9990	1.99
Residual Variance	5.92E-04	42	1.41E-05			
Total	4.48E-01	62				



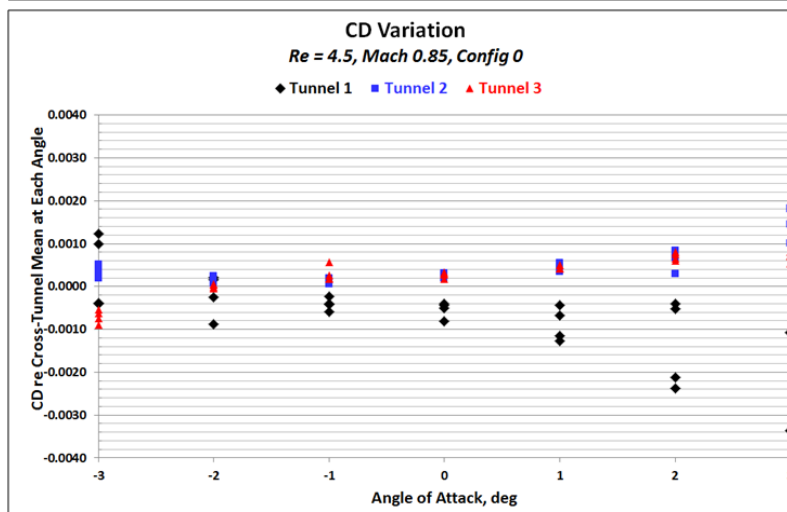
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Source of Variation	SS	df	MS	F	P-value	F crit
AoA	1.01E-08	6	1.68E-09	2.2	0.0635	2.3
Tunnel	3.68E-07	2	1.84E-07	239.1	0.0000	3.2
AoA x Tunnel	3.75E-08	12	3.12E-09	4.05	0.0003	1.99
Residual Variance	3.24E-08	42	7.71E-10			
Total	4.48E-07	62				



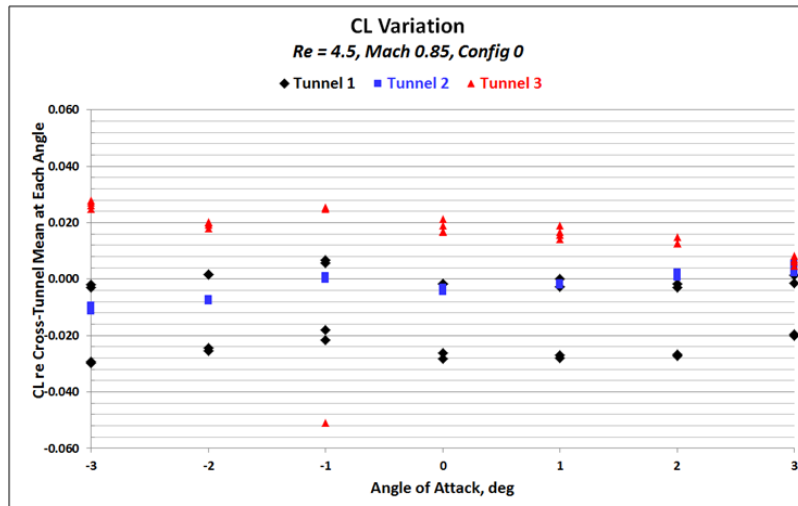
CD for Re# = 4.5, Mach 0.85, Configuration 0

Source of Variation	SS	df	MS	F	P-value	F crit
AoA	4.86E-03	6	8.10E-04	3871.8	0.0000	2.2
Tunnel	2.45E-05	2	1.23E-05	58.6	0.0000	3.1
AoA x Tunnel	2.82E-05	12	2.35E-06	11.25	0.0000	1.91
Residual Variance	1.32E-05	63	2.09E-07			
Total	4.93E-03	83				



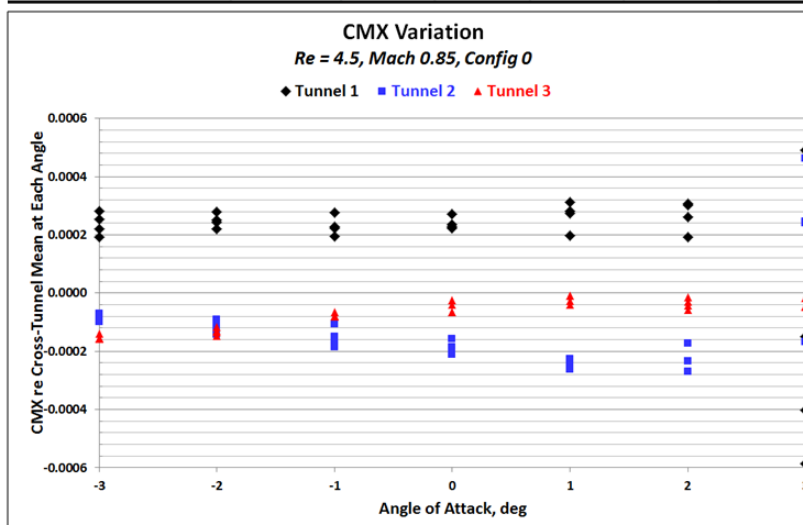
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Source of Variation	SS	df	MS	F	P-value	F crit
AoA	7.41E+00	6	1.23E+00	8756.8	0.0000	2.2
Tunnel	1.11E-02	2	5.54E-03	39.3	0.0000	3.1
AoA x Tunnel	2.14E-03	12	1.78E-04	1.26	0.2624	1.91
Residual Variance	8.88E-03	63	1.41E-04			
Total	7.43E+00	83				



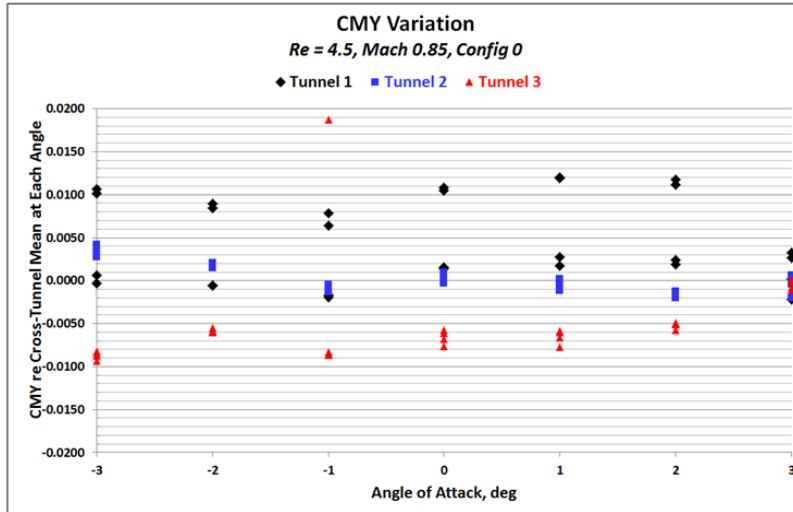
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Source of Variation	SS	df	MS	F	P-value	F crit
AoA	1.35E-05	6	2.25E-06	154.6	0.0000	2.2
Tunnel	1.53E-06	2	7.65E-07	52.6	0.0000	3.1
AoA x Tunnel	1.179E-06	12	9.83E-08	6.75	0.0000	1.91
Residual Variance	9.169E-07	63	1.46E-08			
Total	1.713E-05	83				



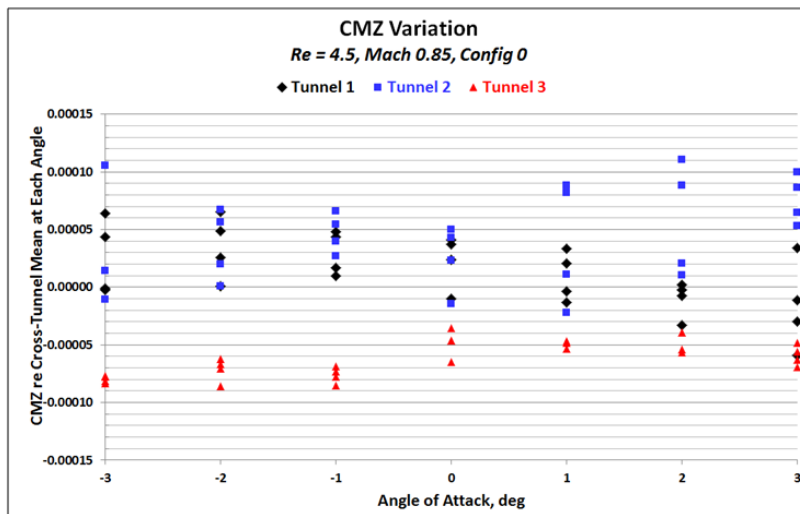
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Source of Variation	SS	df	MS	F	P-value	F crit
AoA	9.44E-01	6	1.57E-01	8779.9	0.0000	2.2
Tunnel	1.32E-03	2	6.62E-04	36.9	0.0000	3.1
AoA x Tunnel	4.00E-04	12	3.33E-05	1.86	0.0573	1.91
Residual Variance	1.13E-03	63	1.79E-05			
Total	9.47E-01	83				



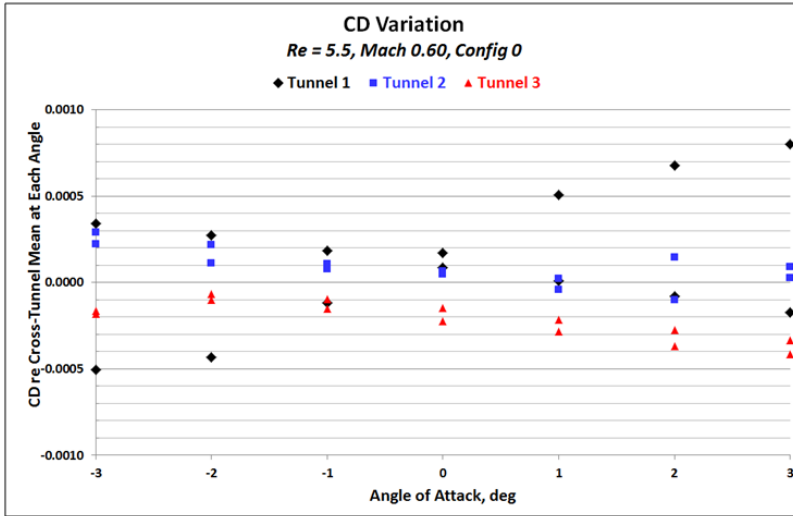
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Source of Variation	SS	df	MS	F	P-value	F crit
AoA	1.613E-07	6	2.69E-08	32.9	0.0000	2.2
Tunnel	1.756E-07	2	8.78E-08	107.6	0.0000	3.1
AoA x Tunnel	2.113E-08	12	1.76E-09	2.16	0.0250	1.91
Residual Variance	5.142E-08	63	8.16E-10			
Total	4.095E-07	83				



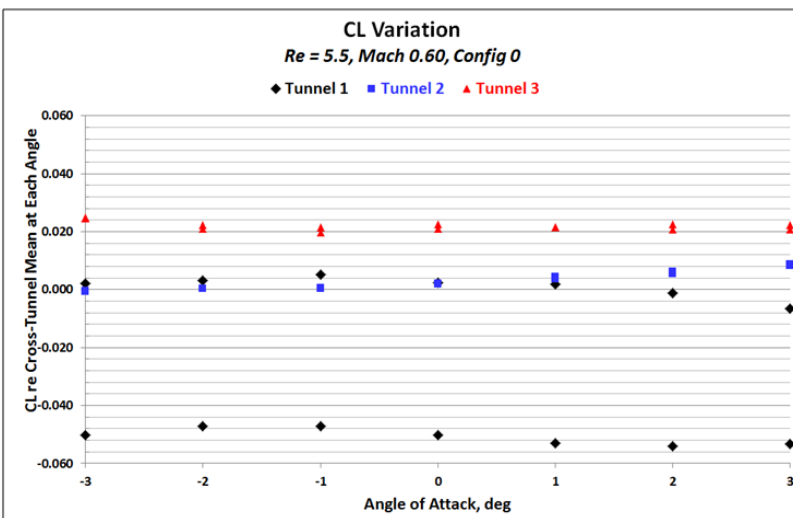
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Source of Variation	SS	df	MS	F	P-value	F crit
AoA	5.38E-04	6	8.97E-05	1179.2	0.0000	2.6
Tunnel	9.90E-07	2	4.95E-07	6.5	0.0063	3.5
AoA x Tunnel	5.86E-07	12	4.88E-08	0.64	0.7847	2.25
Residual Variance	1.60E-06	21	7.61E-08			
Total	0.000542	41				



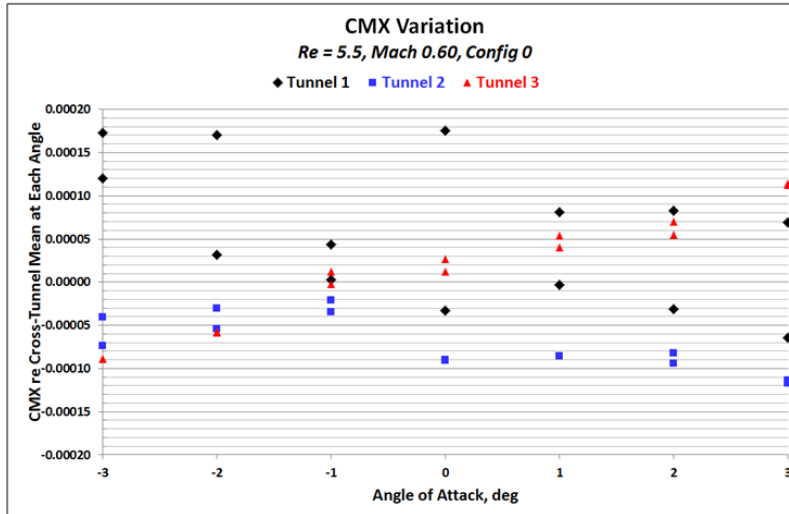
CL for Re# = 5.5, Mach 0.60, Configuration 0

Source of Variation	SS	df	MS	F	P-value	F crit
AoA	2.56E+00	6	4.27E-01	953.3	0.0000	2.6
Tunnel	1.55E-02	2	7.76E-03	17.3	0.0000	3.5
AoA x Tunnel	2.67E-04	12	2.23E-05	0.05	1.0000	2.25
Residual Variance	9.40E-03	21	4.48E-04			
Total	2.584737	41				



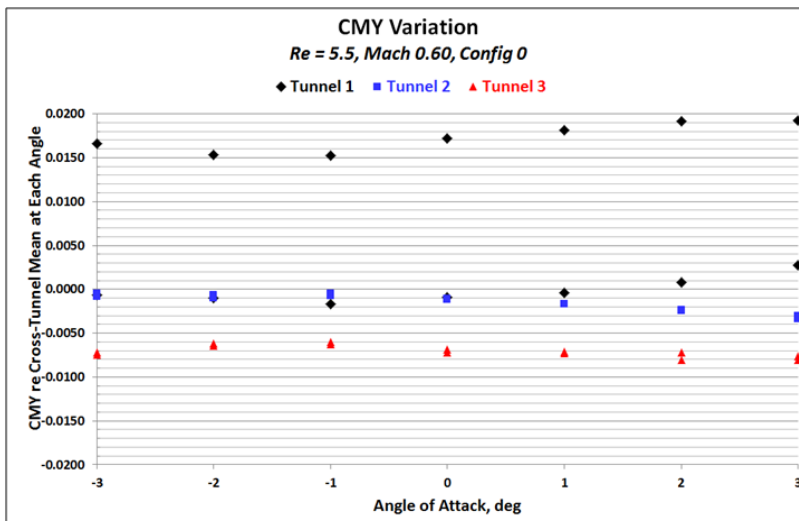
CMX for Re# = 5.5, Mach 0.60, Configuration 0

Source of Variation	SS	df	MS	F	P-value	F crit
AoA	1.94E-06	6	3.23E-07	126.0	0.0000	2.6
Tunnel	1.24E-07	2	6.22E-08	24.2	0.0000	3.5
AoA x Tunnel	1.01E-07	12	8.42E-09	3.28	0.0084	2.25
Residual Variance	5.39E-08	21	2.57E-09			
Total	2.22E-06	41				



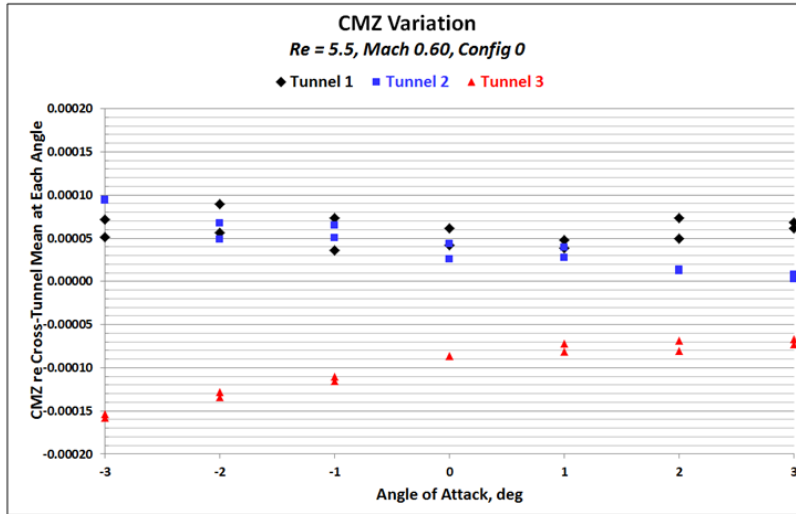
CMY for Re# = 5.5, Mach 0.60, Configuration 0

Source of Variation	SS	df	MS	F	P-value	F crit
AoA	2.94E-01	6	4.89E-02	964.9	0.0000	2.6
Tunnel	1.76E-03	2	8.78E-04	17.3	0.0000	3.5
AoA x Tunnel	4.41E-05	12	3.68E-06	0.07	1.0000	2.25
Residual Variance	1.06E-03	21	5.07E-05			
Total	2.96E-01	41				



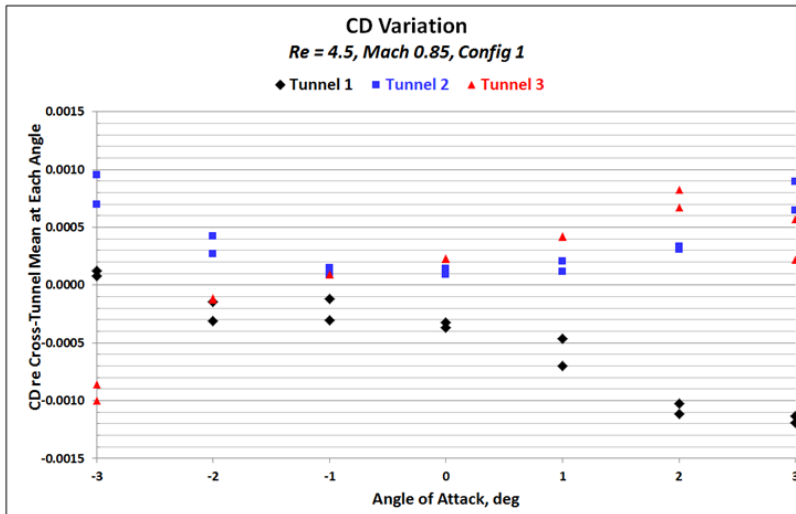
CMZ for Re# = 5.5, Mach 0.60, Configuration 0

Source of Variation	SS	df	MS	F	P-value	F crit
AoA	5.74E-09	6	9.57E-10	7.4	0.0002	2.6
Tunnel	2.16E-07	2	1.08E-07	841.3	0.0000	3.5
AoA x Tunnel	2.53E-08	12	2.11E-09	16.42	0.0000	2.25
Residual Variance	2.7E-09	21	1.29E-10			
Total	2.5E-07	41				



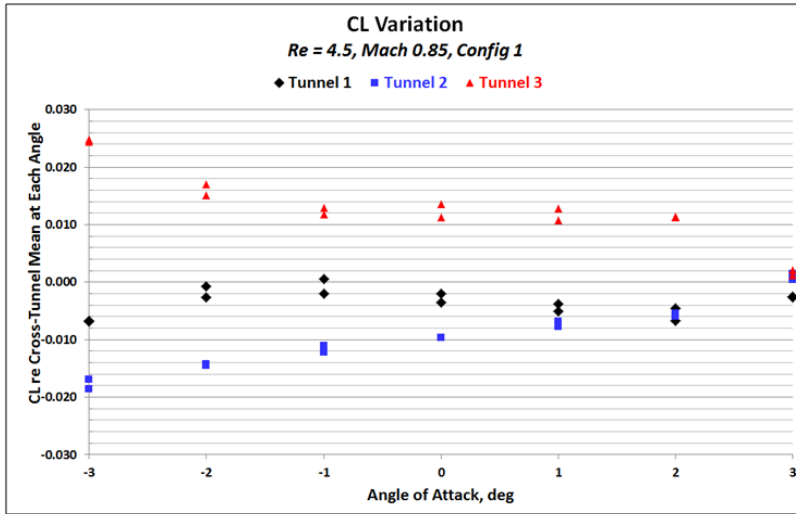
CD for Re# = 4.5, Mach 0.85, Configuration 1

Source of Variation	SS	df	MS	F	P-value	F crit
AoA	1.65E-03	6	2.76E-04	25103.9	0.0000	2.6
Tunnel	5.74E-06	2	2.87E-06	261.2	0.0000	3.5
AoA x Tunnel	7.17E-06	12	5.97E-07	54.38	0.0000	2.25
Residual Variance	2.31E-07	21	1.10E-08			
Total	1.67E-03	41				



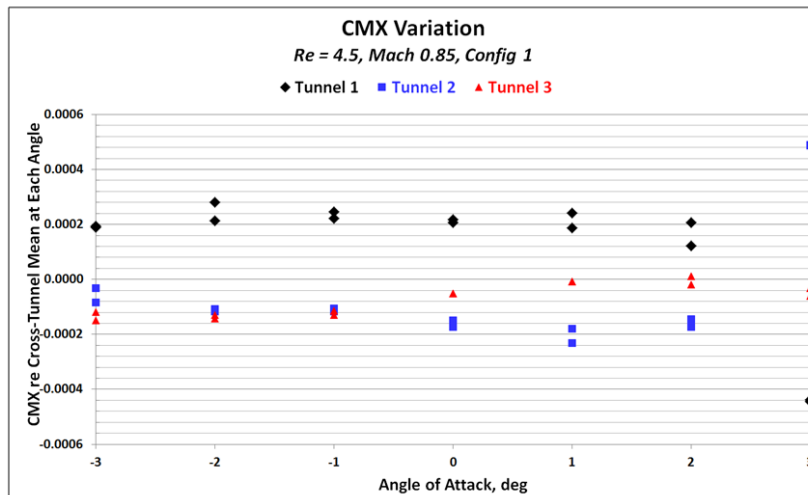
CL for Re# = 4.5, Mach 0.85, Configuration 1

Source of Variation	SS	df	MS	F	P-value	F crit
AoA	3.74E+00	6	6.24E-01	657872.6	0.0000	2.6
Tunnel	3.72E-03	2	1.86E-03	1961.8	0.0000	3.5
AoAx Tunnel	1.06E-03	12	8.85E-05	93.27	0.0000	2.25
Residual Variance	1.99E-05	21	9.48E-07			
Total	3.75E+00	41				



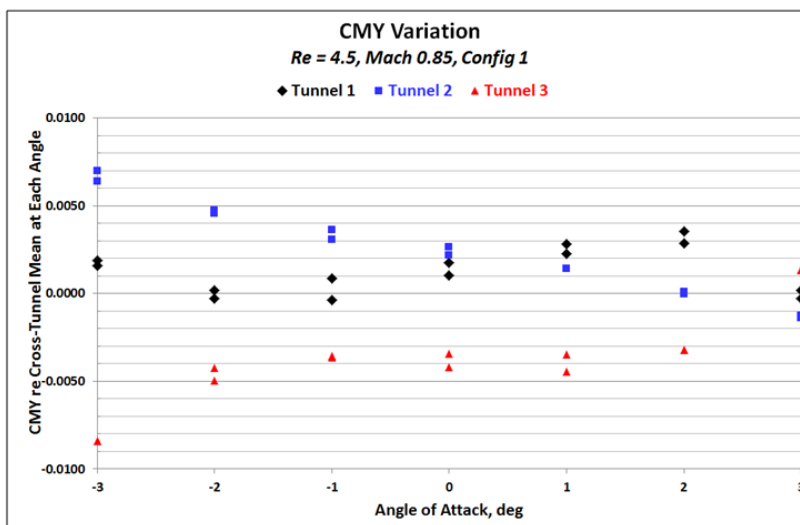
CMX for Re# = 4.5, Mach 0.85, Configuration 1

Source of Variation	SS	df	MS	F	P-value	F crit
AoA	1.01E-05	6	1.68E-06	2771.6	0.0000	2.6
Tunnel	2.94E-07	2	1.47E-07	242.1	0.0000	3.5
AoA x Tunnel	1.47E-06	12	1.22E-07	201.54	0.0000	2.25
Residual Variance	1.28E-08	21	6.08E-10			
Total	1.19E-05	41				



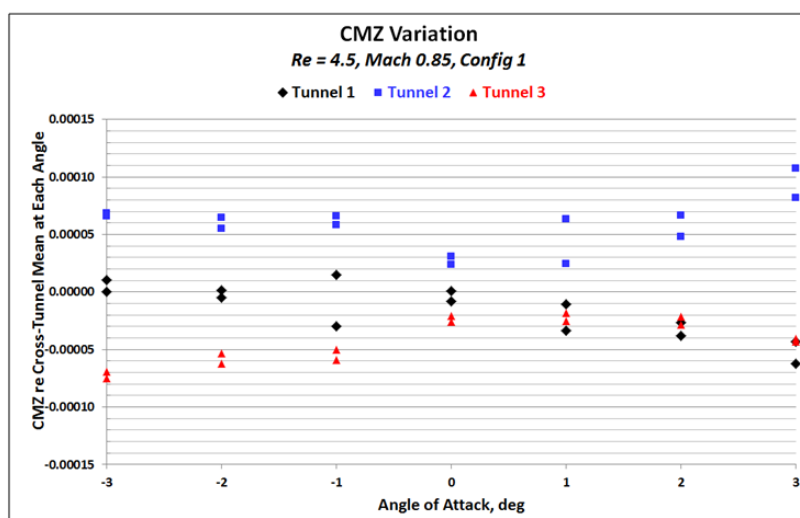
CMY for Re# = 4.5, Mach 0.85, Configuration 1

Source of Variation	SS	df	MS	F	P-value	F crit
AoA	4.98E-01	6	8.30E-02	540669.3	0.0000	2.6
Tunnel	3.06E-04	2	1.53E-04	996.2	0.0000	3.5
AoA x Tunnel	2.07E-04	12	1.73E-05	112.44	0.0000	2.25
Residual Variance	3.22E-06	21	1.54E-07			
Total	4.99E-01	41				



CMZ for Re# = 4.5, Mach 0.85, Configuration 1

Source of Variation	SS	df	MS	F	P-value	F crit
AoA	5.18E-08	6	8.64E-09	57.2	0.0000	2.6
Tunnel	7.75E-08	2	3.87E-08	256.7	0.0000	3.5
AoA x Tunnel	1.49E-08	12	1.24E-09	8.21	0.0000	2.25
Residual Variance	3.17E-09	21	1.51E-10			
Total	1.47E-07	41				



Acknowledgements

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References

¹DeLoach, R., “Check-Standard Testing Across Multiple Transonic Wind Tunnels with the Modern Design of Experiments,” AIAA 2012-3174, 28th Ground Testing Conference, New Orleans, LA, June 2012.

²Scheffe, H., *The Analysis of Variance*, John Wiley and Sons, New York, 1959.

³DeLoach, R., “Analysis of Variance in the Modern Design of Experiments,” AIAA 2010-1111, 48th AIAA Aerospace Sciences Meeting and Exhibit, Orlando, Florida, January 4-7, 2010.