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Numerical study of transmission loss through a slow gas layer adjacent to a plate

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ABSTRACT

This paper describes a systematic numerical investigation of the sound transmission loss through a multilayer system consisting of a bagged gas and lightweight panel. The goal of the study is to better understand the effect of the gas on transmission loss and determine whether a gas with a slow speed of sound is beneficial for noise control applications. As part of the study, the density and speed of sound of the gas are varied independently to assess the impact of each on transmission loss. Results show that near grazing incidence the plane wave transmission loss through the multilayer system is more sensitive to the speed of sound than the density of the gas. In addition, it was found that a slow wave speed in the bagged gas provides more low-frequency transmission loss benefit than a fast wave speed. At low angles of incidence, close to the plate normal, the benefit is due to the reduction of the characteristic impedance of the gas. At high angles of incidence, the benefit is attributed to the fact that the incident waves at the air/gas interface are bent towards the surface normal. Since transmission loss is angle dependent, refraction in the slow gas layer results in a significant improvement in the transmission loss at high angles of incidence.

1. INTRODUCTION

The goal of this work is to investigate an approach to improve the low-frequency, diffuse-field transmission loss through a structure with little added mass. Low-frequency noise is notoriously difficult to attenuate without using bulky, heavy treatment. In aerospace vehicles, and even ground vehicles, there is a need for lightweight treatment options. Reducing interior noise is desirable for several reasons. In some vehicles, acoustic treatment is necessary to improve passenger and crew comfort. Other types of vehicles require acoustic treatment to attenuate high noise levels that would otherwise present a safety hazard to the crew or damage sensitive equipment.

This work considers the effect of adding a bagged gas layer adjacent to a light-weight plate representative of an aerospace structure. Previous researchers have shown the benefit of using bagged gases to improve the transmission loss through panels[1,2]. These experimental studies considered nitrogen, argon, and helium. The researchers concluded that the benefit was due to the difference in the characteristic impedance between the gas and air. The potential acoustic benefits of gases such as helium, which have a low characteristic impedance, have been considered in a number of other applications as well, ranging from launch vehicle acoustics[3] to damping estimation of lightly damped structures[4].

This paper describes a systematic numerical investigation of the sound transmission loss through a gas layer and plate. Since this is a numerical study, the properties of the gas are not constrained to the finite combination of speeds of sound and densities available to experimentalists. Instead, the density and speed of sound are varied independently to assess the impact of each on transmission loss. The paper begins with the basic theory and equations describing transmission loss through a gas layer and plate separately. A numerical model used to study the problem is then discussed. Results are then presented for several different types of gases. Finally, some concluding remarks are provided.

2. THEORY

This section presents fundamental concepts that help explain the results which will be presented in Section 4. The section is broken into three parts: the first covers sound transmission through a fluid-fluid interface; the second discusses sound transmission through fluid layers; and the final part describes sound transmission through plates. Unless otherwise stated, the discussions in the following subsections pertain to propagating plane waves.

A. Transmission through a fluid-fluid interface

An acoustic wave incident on a boundary between two fluids will generate reflected and transmitted waves as depicted in Figure 1(a). The relative amplitudes of the reflected and transmitted waves depend on the characteristic acoustic impedance and speed of sound in each fluid, as well as the angle of incidence. The transmission coefficient, or fraction of energy transmitted, can be expressed as[5]:

$$\tau = \left(4\frac{\rho_2 c_2}{\rho_1 c_1}\frac{\cos\theta_t}{\cos\theta_i}\right)\Bigg/\left(\frac{\rho_2 c_2}{\rho_1 c_1}+\frac{\cos\theta_t}{\cos\theta_i}\right)^2 \qquad \theta_t \text{ real} \qquad (1)$$

$$\tau = 0 \qquad\qquad\qquad\qquad \theta_t \text{ imaginary} \qquad (2)$$

where ρ is the density, c is the speed of sound, θ_i is the incidence angle, and θ_t is the transmitted angle. When the two fluids have a different speed of sound, the waves are refracted, or bent according to Snell's law,

$$c_2 \sin\theta_i = c_1 \sin\theta_t. \qquad (3)$$

When $c_2 < c_1$, the transmitted ray is bent towards the surface normal such that $\theta_t < \theta_i$ for all values of θ_i. Conversely, when $c_2 > c_1$, the transmitted ray is bent away from the normal such that $\theta_t > \theta_i$. However, in this case there is a critical angle of incidence, $\theta_c = \sin^{-1}(c_1/c_2)$, which corresponds to $\theta_t = 90°$. Above this angle of incidence, θ_t will be complex and no true propagating plane wave will be generated in the second fluid. Instead an evanescent field will be generated which decays exponentially with distance away from the interface.
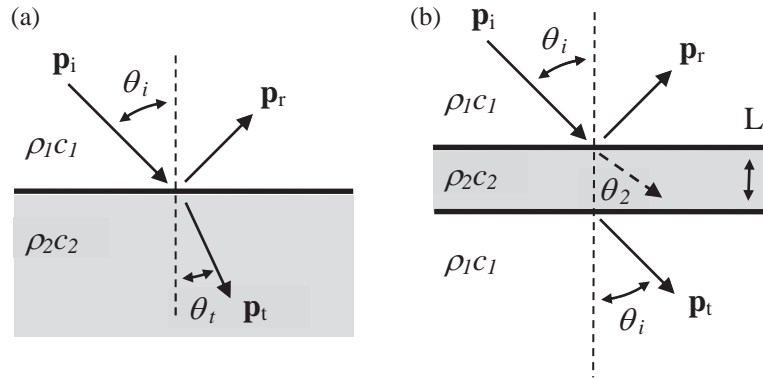
Figure 1: Obliquely incident wave on a (a) fluid-fluid interface, and (b) fluid layer resulting in a reflected and transmitted wave.

B. Transmission through fluid layers

Next consider transmission through a fluid layer with a finite thickness L, as depicted in Figure 1(b). In this case the transmission coefficient can be expressed as[6]:

$$\tau = \frac{1}{1 + \frac{1}{4}\left(\frac{Z_2}{Z_1} - \frac{Z_1}{Z_2}\right)^2 \sin^2 K_2 L} \tag{4}$$

where $Z_1 = \rho_1 c_1 / \cos\theta_i$ and $Z_2 = \rho_2 c_2 / \cos\theta_2$ are the specific acoustic impedances of each fluid, and $K_2 = \omega\cos(\theta_2)/c_2$. Notice that Equation (4) is real for all angles of incidence. This is even true at high angles when $c_2 > c_1$, and θ_2 is complex. Second, notice that the transmission coefficient equals one when $K_2 L$ is a multiple of π. In other words, when $L\cos\theta_2$ is equal to one-half the wavelength of sound in the gas there is complete transmission through the fluid layer. Similarly the expression goes through a minimum when $K_2 L$ is a multiple of $\pi/2$, or when $L\cos\theta_2$ is equal to one-quarter of the wavelength of sound in the gas.

C. Transmission through plates

Finally consider the sound transmission through an unbounded uniform plate with the same gas on both sides. This is discussed in a number of excellent references including Pierce[6] and Fahy *et al.*[7]. The fraction of energy transmitted through the plate can be expressed as[6]:

$$\tau = \frac{1}{\left|1 + \frac{Z_p}{2Z_g}\right|^2} \tag{5}$$

where Z_p is the specific impedance of the plate and $Z_g = \rho c / \cos\theta$ is the specific acoustic impedance of the gas. At low frequencies, well below the critical frequency, an infinite plate can be approximated as a perfectly limp layer. In this case, the specific impedance of the plate simplifies to $Z_p = -j\omega m_s$, where m_s is the mass per unit area of the plate. Therefore the mass-law transmission loss can be expressed as[7]:

$$TL = 10\log_{10}(1/\tau) = 10\log_{10}\left(1 + \left(\frac{\omega m_s \cos\theta}{2\rho c}\right)^2\right). \tag{6}$$

Notice from Equation (5) that reducing the specific acoustic impedance of the gas also reduces the transmission coefficient. Similarly as the specific acoustic impedance of the gas increases, so does the transmission coefficient. In addition, notice that since the specific acoustic impedance of the gas is $Z_g = \rho c / \cos\theta$, the transmission loss through a limp plate can be increased by reducing either the characteristic impedance of the gas or the angle of incidence. Equation (6) can be used to explain the baseline plate transmission loss response, which will be presented in Section 4. It will be shown that transmission loss is largest when the acoustic waves are normal to the panel's surface and decreases as the angle between the surface normal and wave propagation direction approaches 90 degrees.

3. NUMERICAL MODEL

A 2D finite element model was used to simulate the transmission loss through the multilayer system. The system considered in this study is shown in Figure 2, which is based on a tutorial provided by COMSOL[8]. The model includes two perfectly matched layer (PML) subdomains approximating infinite air domains. Finite fluid domains are included to model the air on either side of the gas-plate system. A 50.8 mm thick gas layer is represented as a fluid domain and a solid elastic structure is used to model a 1.27 mm thick aluminum plate. The aluminum plate has an elastic modulus of 70×10^9 Pa, a density of 2,700 kg/m³, Poisson's ratio of 0.33, and a structural loss factor of 0.01. Periodic Floquet boundary conditions are used to model infinite periodic fluid and structural domains. A background pressure field is defined in the air domain adjacent to the gas layer. The pressure field approximates an incident plane wave propagating at an angle θ_i with respect to the normal of the air-gas interface. The geometry, which includes everything shown within the dashed lines in Figure 2, is meshed using a mapped structured quadrilateral mesh with an element size of 0.635 mm, which is more than sufficient to resolve the shortest wavelengths in the elastic and acoustic domains throughout the analysis range. A direct frequency solve is performed at 120 frequencies logarithmically distributed between 400 and 6,000 Hz. It is worth noting that the critical frequency, which corresponds to the frequency where the acoustic wavelength in air matches the bending wavelength in the plate, is around 9,500 Hz for this structure.
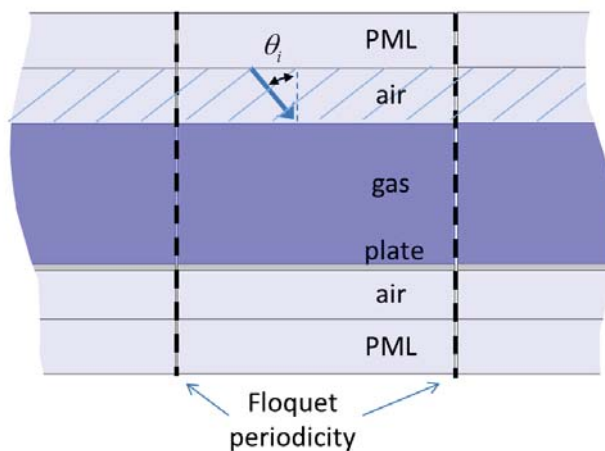

Figure 2: Geometry considered in this study.

All results are presented in terms of transmission loss, which is calculated as:

$$TL(\theta) = 10\log_{10}(1/\tau(\theta)) \tag{7}$$

where the angle-dependent transmission coefficient is defined as $\tau = \Pi_t/\Pi_i$, the incident power per unit area is $\Pi_i = P_i^2 \cos\theta/2\rho c$, and Π_t is the transmitted power per unit area, which is found by taking the spatial average of the time-averaged acoustic intensity normal to the surface of the plate. Since an acoustic diffuse field implies that plane waves are incident from all directions with equal probability and random phase, this type of field can be approximated as the summation of plane waves incident from many different angles. Therefore the diffuse field transmission coefficient is calculated as[7]:

$$\tau_d = \int_0^{\pi/2} \tau(\theta)\sin(2\theta)\,d\theta. \tag{8}$$

For this study the diffuse field transmission loss was estimated based on simulations performed at 21 different angles equally distributed between 0 and 89 degrees.

      Several types of gases were considered in this study, as listed in Table 1. The properties of the four fictitious gases listed as slow, fast, light, and heavy were chosen such that the transmission coefficient, shown in Equation (1), is the same at normal incidence for each gas. The specific acoustic impedance values given in the table are calculated assuming that the plane wave is incident on the air/gas interface at the designated angle. The slow and fast gases have the same density as air, while the light and heavy gases have the same speed of sound as air. CO_2 is included as an example of a common gas with a slower speed of sound than air.

Table 1: Properties of the gases considered.

Gas type	air	slow	fast	light	heavy	CO_2
Density (kg/m3)	1.20	1.20	1.20	0.90	1.60	1.98
Speed of sound (m/s)	343	257.25	457.33	343	343	259
Characteristic impedance (kg/(m^{2}s))	412	309	549	309	549	513
Specific acoustic impedance (kg/(m^{2}s)) at $0°$	412	309	549	309	549	513
Specific acoustic impedance (kg/(m^{2}s)) at $29°$	471	332	720	353	628	551
Specific acoustic impedance (kg/(m^{2}s)) at $58°$	777	400	-1040i	583	1036	668
Specific acoustic impedance (kg/(m^{2}s)) at $87°$	7872	466	-624i	5904	10490	781

4. RESULTS AND DISCUSSION

This section presents and discusses the results. Although the ultimate goal of this work is to assess the diffuse field transmission loss, it is instructive to consider the plane wave transmission loss at several incident angles before considering the diffuse field response. The following set of figures compares the transmission loss through the bare plate with the transmission loss through the multilayer gas/plate system.

Figure 3(a) shows the benefit of including a slow gas layer adjacent to the plate. The dotted black curves, labeled as baseline, show the transmission loss through the bare plate while the dashed red curves show the transmission loss through the slow-gas/plate system. Notice that while the slow gas layer only provides a small benefit at low angles of incidence, the transmission loss is increased by over 20dB at high angles. Recall from Section 2C that the transmission loss through a plate is sensitive to the specific acoustic impedance of the gas surrounding the plate. Specifically the transmission loss increases as the specific acoustic impedance of the gas decreases. Recall from Table 1 that at 0 degrees the specific acoustic impedance of the slow gas is 25% smaller than air. This causes obliquely incident waves at the air/gas interface to be refracted to smaller angles in the gas. Since the specific acoustic impedance of the gas is a function of the propagation angle, the impedance of the gas does not increase at high angles of incidence as quickly as air. Therefore at an incidence angle of 87 degrees, the specific acoustic impedance of the gas is 94% smaller than air. This explains why the slow gas provides a significant benefit at high angles of incidence and only a modest benefit at low angles.
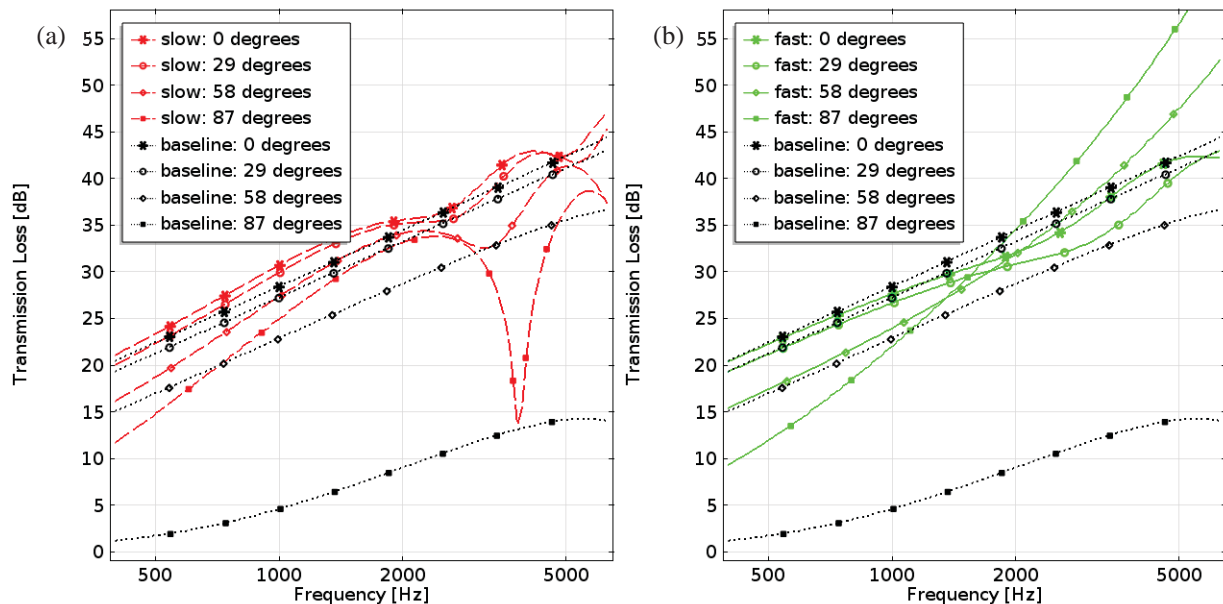

Figure 3: Plane wave transmission loss versus frequency for 4 different incident angles. The dotted black curves are for the bare plate, the dashed red curves in (a) are for the slow-gas/plate system, and the solid green curves in (b) are for the fast-gas/plate system.

The frequency dependent peaks and troughs are due to the finite layer thickness, as described in Section 2B. Recall that when $L\cos\theta_2$ is equal to one-half the wavelength of sound in the gas, there is complete transmission through the gas layer. At those frequencies the slow gas provides no benefit. On the other hand, when $L\cos\theta_2$ is equal to one-quarter of the wavelength of sound in the gas, the benefit of the gas layer is maximized. This occurs between 1250 and 2000 Hz, depending on the angle of incidence.

Figure 3(b) shows the effect of adding a fast gas layer adjacent to the plate. Notice that the addition of the fast gas layer reduces the transmission loss at low angles while increasing the transmission loss at high angles. Once again, this can be explained by comparing the specific acoustic impedance of the gas to air, using Table 1. At low angles, the specific acoustic impedance of the gas is larger than air, which results in less transmission loss. However the

story changes above the critical incidence angle, which corresponds to 48.6 degrees for this gas. Above this angle no true propagating plane wave is generated in the gas layer. Instead an evanescent field is generated in the fluid layer which couples the air to the plate. The evanescent field decays exponentially with distance and the decay rate is frequency dependent. The field decays more quickly at high frequencies, resulting in higher transmission loss. To summarize, the fast gas layer is not as beneficial as the slow gas layer at small angles of incidence; however there is a benefit at high angles of incidence, particularly as frequency increases.

Figure 4(a) shows the effect of adding a light gas layer adjacent to the plate. Since the speed of sound in the gas layer is the same as air, there is no refraction and the benefit at low angles can be attributed to the fact that the specific acoustic impedance of the light gas is 25% smaller than air at all angles. However, the effect is once again frequency dependent showing a benefit at some frequencies and not at others. This is due to the finite layer thickness as described previously. Notice that when $\cos(\theta_2)/c_2$ is small, the $\sin^2$ term in Equation (4) will also be small and therefore the inclusion of the gas layer will have little effect. This is the case when the incidence angle equals 87 degrees.
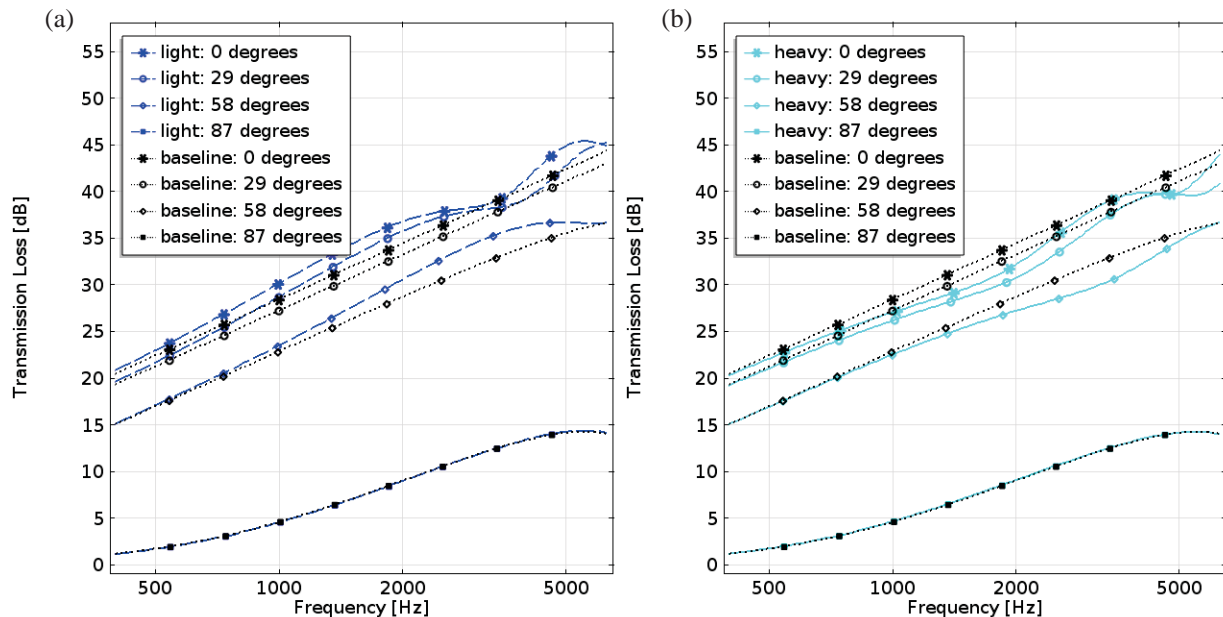

Figure 4: Plane wave transmission loss versus frequency for 4 different incident angles. The dotted black curves are for the bare plate, the dashed blue curves in (a) are for the light-gas/plate system, and the solid cyan curves in (b) are for the heavy-gas/plate system.

Finally consider Figure 4(b), which shows the effect of adding a heavy gas layer to the plate. In this case the addition of the gas layer reduces the transmission loss. This is due to the fact that the characteristic impedance of the heavy gas is 33% larger than air. Since the speed of sound is equal to air, there is no refraction and therefore the specific acoustic impedance at every angle will also be 33% larger than the specific acoustic impedance of air. Once again the effect is frequency dependent and is only noticeable at the lower three angles of incidence for the same reasons described above.

Based on the previous results and discussion, a slow, light gas is desirable to increase the transmission loss in the lower frequency range. Since a common, inert gas with both properties could not be found, CO_2 was considered. As shown in Table 1, the speed of sound in CO_2 is approximately equal to the slow gas considered in this study; however the density and also

characteristic impedance of CO_2 are larger than air. This means that the specific acoustic impedance of the gas will also be larger than air at small angles of incidence. Therefore adding a layer of CO_2 will decrease the transmission loss at these angles. However, since the speed of sound in CO_2 is 24% slower than air, the propagation angle in the gas will be smaller than the angle of incidence at the air/gas interface. Therefore at high angles of incidence the specific acoustic impedance of CO_2 is less than air. At these angles, adding the layer of gas increases the transmission loss. Figure 5(a) shows the effect of adding the layer of CO_2 above the plate.
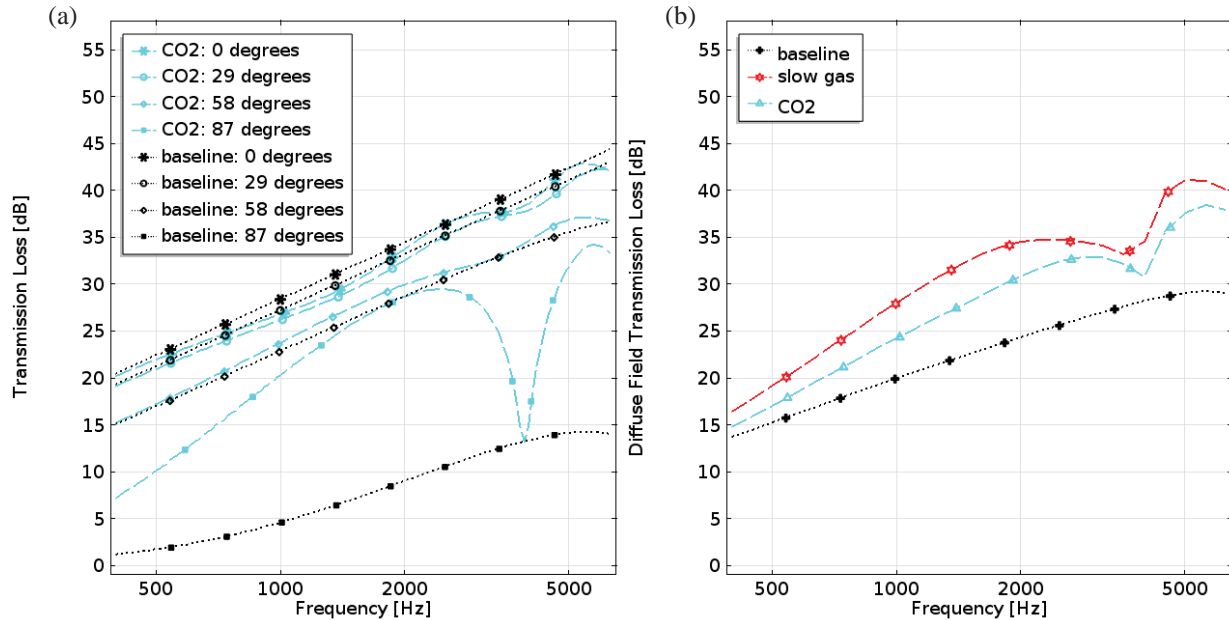

Figure 5: (a) Plane wave transmission loss versus frequency for the bare plate (dotted black curves) and for the CO2/plate system (dashed cyan curves). (b) Diffuse field TL for the bare plate (dotted black curve), plate plus layer of slow gas (dashed red curve), and plate plus layer of CO_2 (dashed cyan curve).

Finally consider Figure 5(b), which shows the diffuse field transmission loss for the baseline plate, slow gas, and CO_2. The slow gas provides a 3.5 dB benefit at 500 Hz, an 8 dB benefit at 1000 Hz and a 10 dB benefit at 2000 Hz. The layer of CO_2 provides less benefit due to the performance penalty at low angles, but still increases the transmission loss of the plate by 1.8 dB at 500 Hz, 4.2 dB at 1000 Hz and 6.5 dB at 2000 Hz. If an ideal slow gas were available, the transmission loss benefit could be realized with essentially no weight penalty. However adding a 50.8 mm thick layer of CO_2 adds approximately 1% to the plate mass.

5. CONCLUDING REMARKS

This paper describes a numerical study of the sound transmission loss through a multilayer system consisting of a gas layer adjacent to a plate. As part of the study, both the speed of sound and density of the gas were varied independently. Results show that the oblique incidence transmission loss through the multilayer system is more sensitive to the speed of sound of the gas than density. In addition, it was found that a slow wave speed provides more low-frequency transmission loss benefit than a fast gas. A slow gas is beneficial for multiple reasons. Assuming all else is equal, reducing the speed of sound will reduce the characteristic impedance, which improves the transmission loss at low angles of incidence. In addition, reducing the speed of sound results in refraction at the air/gas interface, which causes the incident wave to be bent towards the surface normal. This results in a significant improvement in the transmission loss at high angles of incidence.

To quantify the benefit, a 50.8 mm thick layer of gas with the same density as air, but with a 25% slower speed of sound provides an 8 dB increase in the diffuse field transmission loss of a thin aluminum plate at 1000 Hz. Although a real gas with the same properties is not readily available, simulations show that that a denser gas, such as CO_2, is still able to increase the diffuse field transmission loss of the aluminum plate at 1000 Hz by 4.2 dB. Finally, it is worth noting that this benefit could be achieved in practice using bags of gas that add minimal weight to the structure.

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