Transient thermoelectric solution employing Green's functions

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1. Objective:

This study works to formulate convenient solutions to the problem of a thermoelectric couple operating under a time varying condition. Transient operation of a thermoelectric device will become increasingly common as thermoelectric technology permits new applications such as automotive and aerospace energy harvesting. In an effort to generalize the thermoelectric solution, Green's functions are employed. This allows arbitrary time varying boundary conditions to be applied to the system without reformulation. The solution demonstrates that in thermoelectric applications of a transient nature Thermal Diffusivity Factor, Inductance Factor, and leg length ratio must be taken into account.

2. What are Green's Functions?

- The Green's function is an integral kernal which allows for simple expression of the solution for the problem of interest [1].
- Method applies to the linear operator $\mathcal{L}$ and the adjoint operator $\mathcal{L}^*$

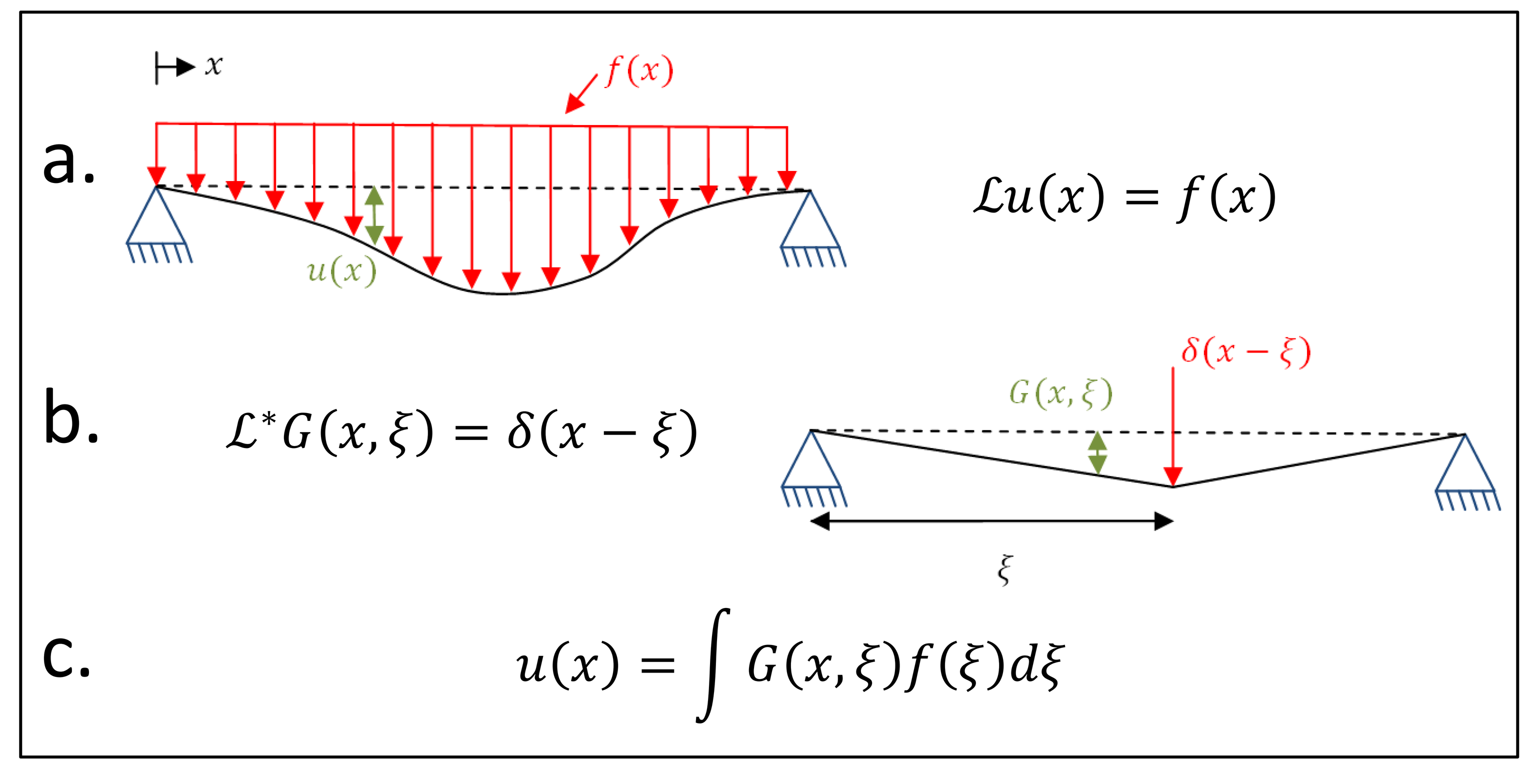

Fig. 1- a. the desired inhomogeneous problem to be solved. b. the corresponding problem to be solved to obtain the Green's function. c. The desired solution in terms of the inhomogeneous function.

3. The Transient Couple:

- New applications of thermoelectrics demand transient operation.
- A number of numerical efforts have successfully characterized transient couples[2,3] but an analytic approach provides powerful design guidelines.
- Unlike the steady state couple, the transient couple depends additionally on material density and specific heat, which must be captured in Thermal Diffusivity Factor and Inductance Factor.

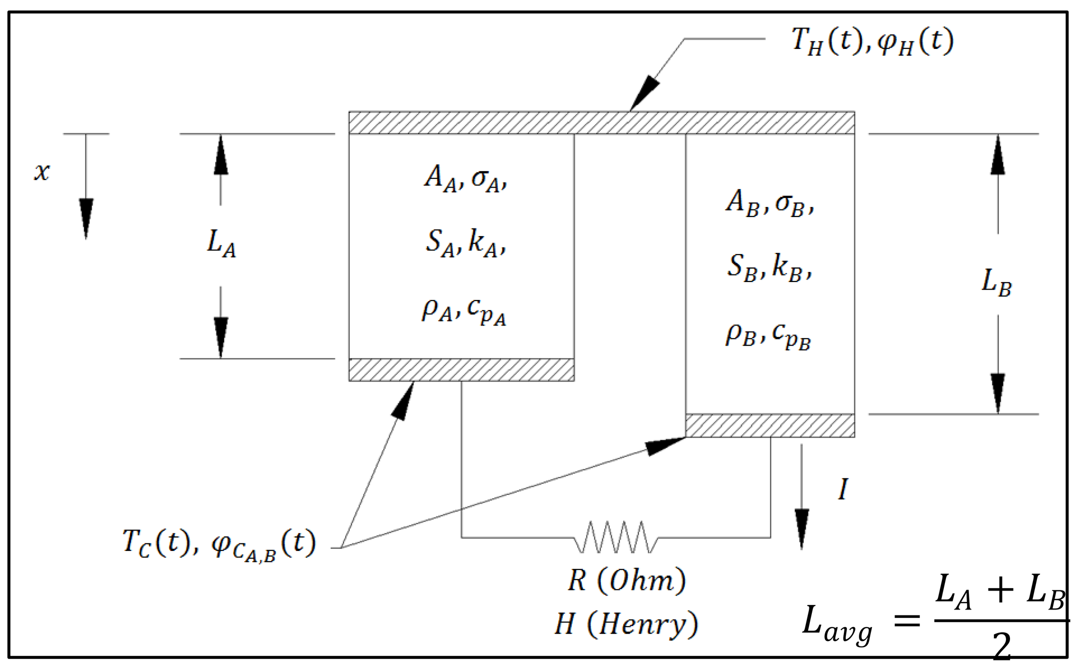

Fig. 2- Sketch of a transient thermoelectric couple, See "Nomenclature" section for clarification. Subscripts: (A & B) are n- and p-type legs, (C & H) are cold and hot shoes, (avg) is leg to leg average.

Thermal
$$\frac{\partial}{\partial x}\left[-k_{A,B}\frac{\partial T_{A,B}}{\partial x}\right] + \frac{I_{A,B}\tau_{A,B}}{A_{A,B}}\frac{\partial T_{A,B}}{\partial x} - \frac{{I_{A,B}}^2}{{A_{A,B}}^2\sigma_{A,B}} = \rho_{A,B}c_{p_{A,B}}\frac{\partial T_{A,B}}{\partial t}$$

Electrical
$$\frac{\partial \varphi_{A,B}}{\partial x} = -S_{A,B}\frac{\partial T_{A,B}}{\partial x} - \frac{I_{A,B}}{A_{A,B}\sigma_{A,B}}$$

Ohm's Law
$$\varphi_B(L_B) - \varphi_A(L_A) = IR + H\frac{dI}{dt}$$

4. Thermal Diffusivity Factor:

- An On/Off cycle is studied, using a unit dimensionless heat flux on the hot shoe and fixed temperature on the cold shoe.
- Solution introduces a new dimensionless parameter, Thermal Diffusivity Factor (Γ).

Thermal Diffusivity Factor
$$\Gamma_{A,B} = \frac{\alpha_{avg}L_{A,B}^2}{\alpha_{A,B}L_{avg}^2}$$

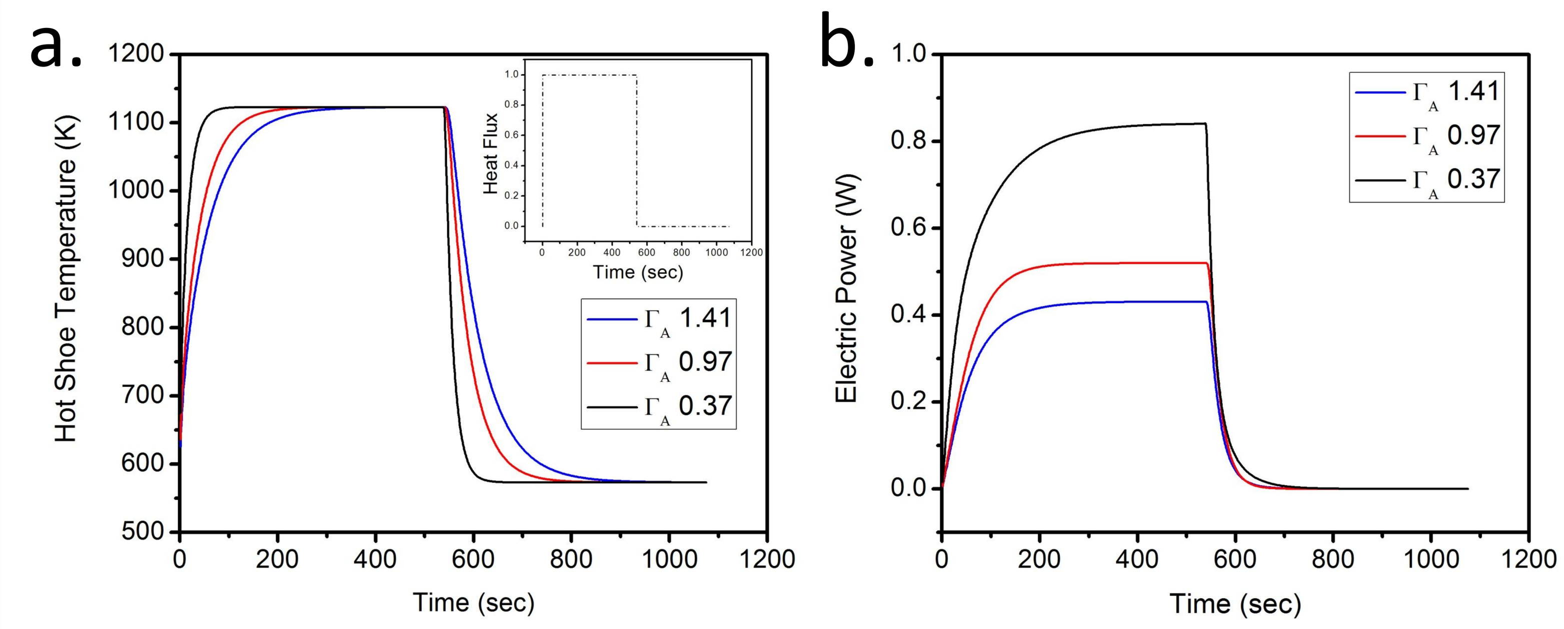

Fig. 3- Study of three geometrically and electrically similar couples under On/Off heat flux cycles; parameter of study is the Thermal Diffusivity Factor (Γ). a. Hot shoe temperature. inset. Applied heat flux. b. Electric power.

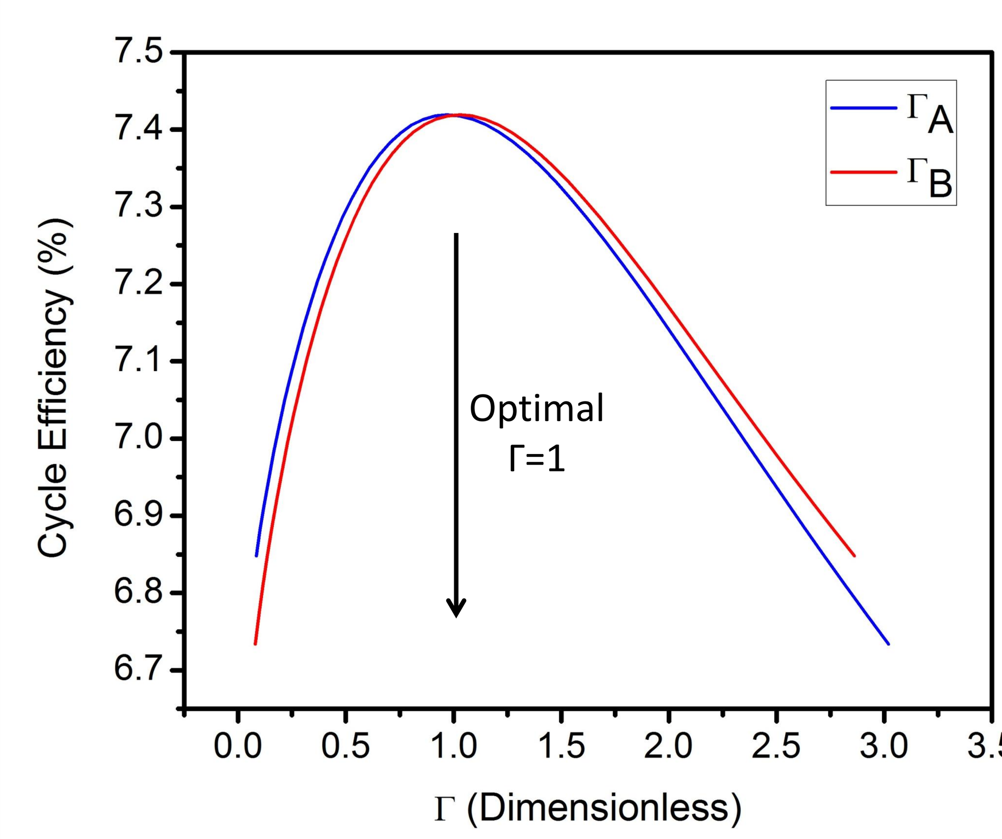

Fig. 4- Cycle conversion efficiency as a function of Thermal Diffusivity Factor (Γ). For geometrically and electrically similar couples

- Cycle conversion efficiency is optimized for Γ=1.
- For selected materials the leg length ratio can be optimally designed as follows:

Optimal Length Ratio
$$\frac{L_A}{L_B} = \frac{\sqrt{2a}+1}{2a-1}$$
$$a = 1 + \frac{\alpha_B}{\alpha_A}$$

5. The A/C Couple:

- The behavior of a couple under a sinusoidal heat flux is interesting for applications such as energy harvesting on a pulse detonation engine.
- Solution introduces the Inductance Factor (β) a dimensionless parameter with strong effect on amplitude and phase angle between thermal and electrical response.
- β and frequency are not found to alter periodic steady average values.

$$Heat\ Flux = C_0\sin(w\cdot t) + C_1$$

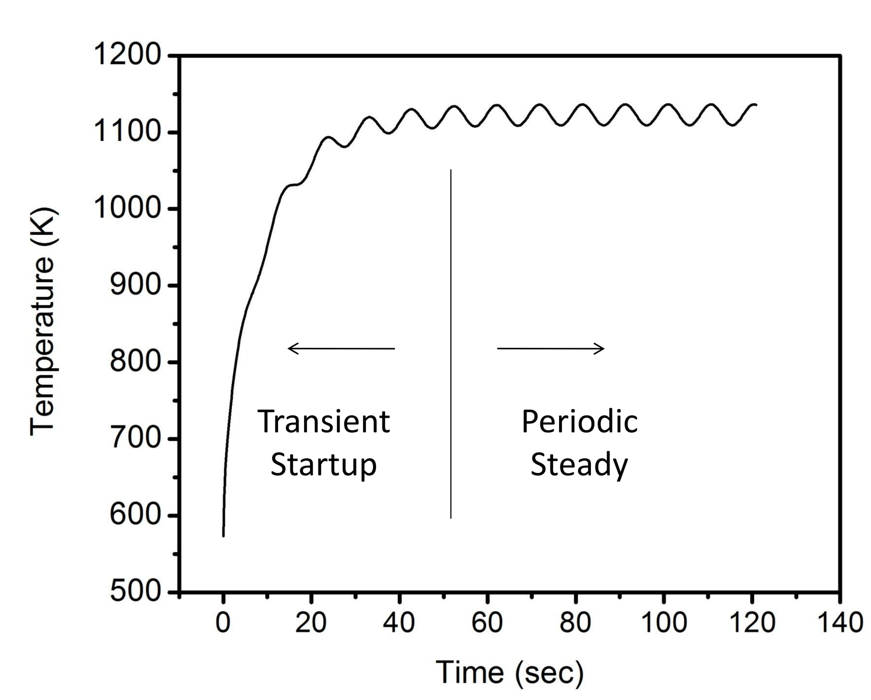

Fig. 5- Hot shoe temperature due to sinusoidal heat flux input.

Inductance Factor
$$\beta = \frac{H\alpha_{avg}}{RL_{avg}^2}$$

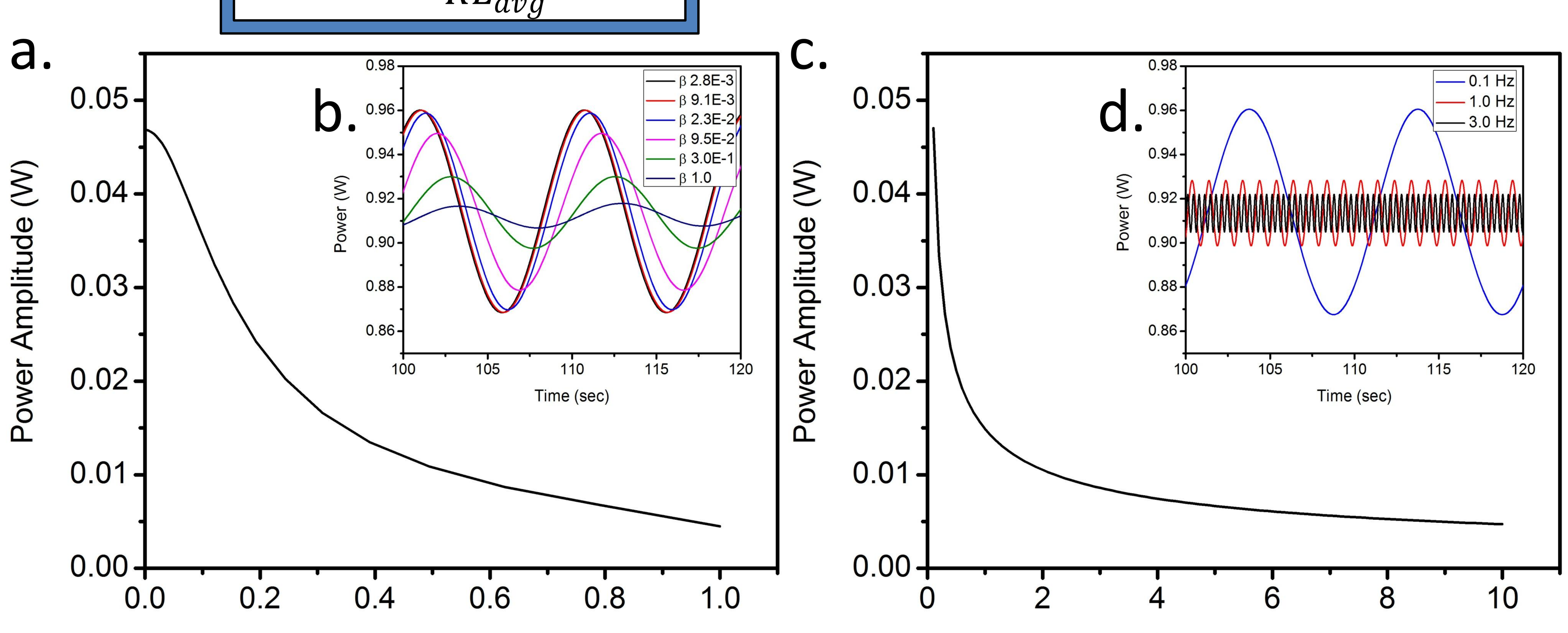

Fig. 6- a. Power amplitude dependent on Inductance Factor (β) for 0.1 Hz. b. Set of power curves for a range of Inductance Factors (β). c. Power amplitude dependent on frequency for β 0.001. d. Set of power curves for a range of frequencies.

6. Conclusion:

The analytic solution of a transient couple leads to the introduction of a Thermal Diffusivity Factor (Γ) and an Inductance Factor (β). The behavior of couples as a function of these parameters has been investigated and for the case of Γ an optimal design point exists. This optimal Γ leads to the design guideline for the selection of optimal leg length ratio.

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References:

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Nomenclature:

Independent Variables	Dependant Variables	Parameters	Material Properties	
x – Space Coordinate	T – Temperature	L – Leg Length	S – Seebeck Coefficient	ρ – Density
t – Time	I – Current	A – Leg Area	σ – Electrical Conductivity	c_p – Specific Heat
	φ – Voltage	R – Resistance	τ – Thomson Coefficient	$\alpha = \frac{k}{\rho c_p}$ – Thermal Diffusivity
		H – Inductance	k – Thermal Conductivity	