



# Analysis of active figure control effects on mounting strategy for x-ray optics

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## Outline

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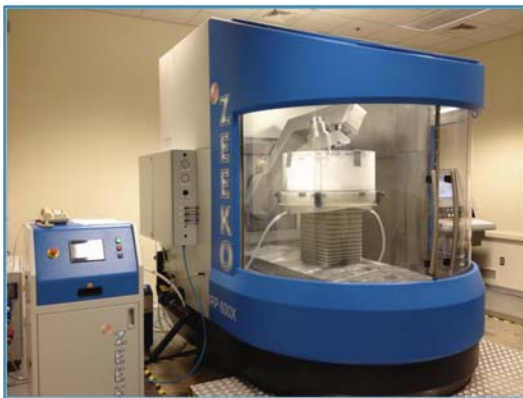
## Overview and Motivation

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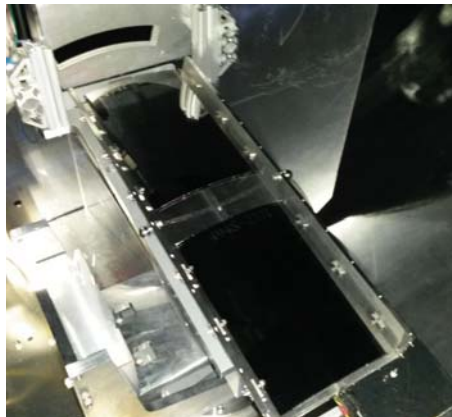
- Manufacturing process development and product design with the goal of performance optimization within specified constraints can often involve a very large trade space.
- Not only the selection of technological methods and design parameters, but the sequence of process steps can significantly influence the resulting performance.
- Such efforts benefit from the early development of high-fidelity simulations, which can illuminate the tall poles and guide the choices toward performance optimization.
- Ideally these simulations would provide an integrated end-to-end picture of the entire process so that one option can be weighed against another, i.e. to investigate strategic alternatives.
- We anticipate that the manufacturing process required for the production of large-area actively-controlled x-ray optics modules falls into this category.

## Overview and Motivation

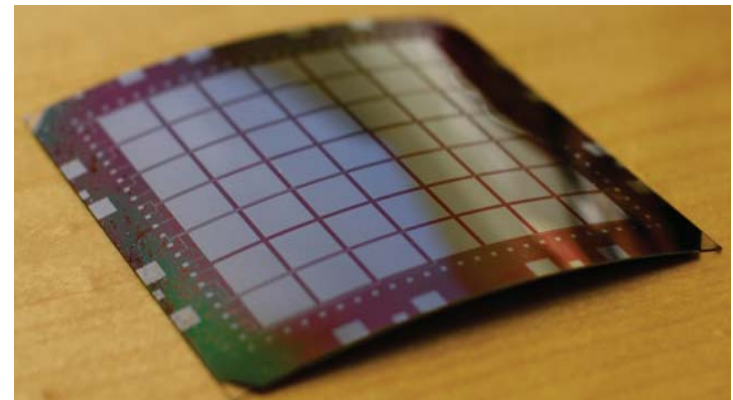
- At MSFC, we are involved in several technology development efforts for higher resolution / larger area / lighter weight x-ray optics.
- All of these technologies share the need to be mounted/assembled/aligned into modular units for testing or for flight.
- We have identified a requirement to develop the capability to assess the impact of mounting options on performance.
- The goal is to develop an optimal mounting strategy or a small set of testable strategic options for any specified x-ray optics technology, which would be applicable to the fully-developed manufacturing process.
- We are in the very early stages of developing this capability for active optics, so results, so far, are relatively simplistic and naive.
- Nevertheless, we feel that including active optics mounting strategies in our repertoire will help to drive our understanding of technical challenges and capabilities to overcome them to a higher level in the future.



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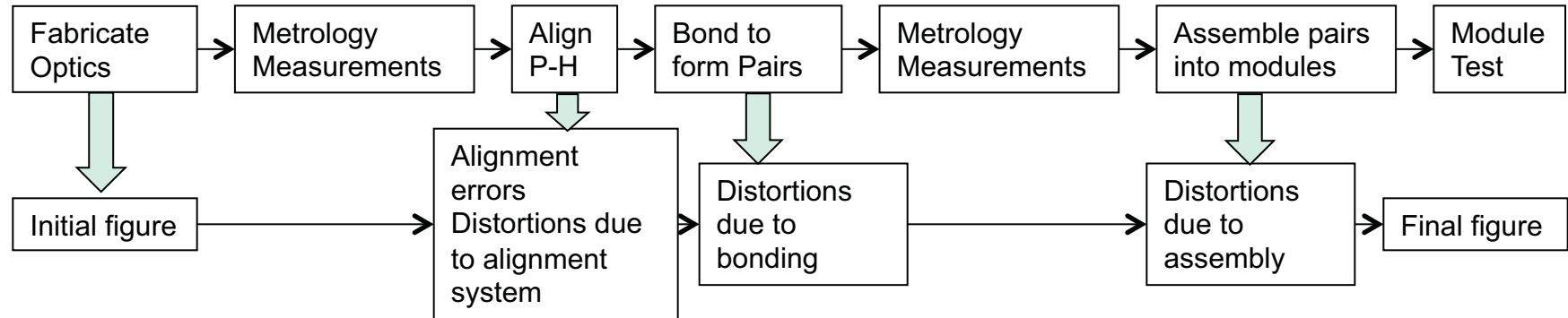


*SPIE: Adaptive X-Ray Optics III*



## Posing Key Questions

- The process involved may be very complex



- Key questions for mounting strategy
  - What kind of distortions are uncorrectable by bimorphs?
  - How are uncorrectable distortions minimized?
    - Which design parameters?
    - Which process techniques?
    - In what sequence are processes performed?



## Approach

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- Since this is virtual development, we use simulations to begin to answer key questions
- Steps:
  - generate large samples of initial figure error maps
  - develop deflection models based on plate/shallow shell theory and validate against FEM
    - Develop and apply boundary conditions as appropriate to address the question in terms of figure distortions
  - compute influence function basis for response to piezo actuation “voltage”
  - fit basis to initial figure + distortions to determine final figure which minimizes RMS axial slope errors or RMS 2-reflection ray divergence
  - Compare results from configuration alternatives



## Modeling Methods

- Monte carlo technique to generate figure error maps
  - Probably should just operate on Fourier components instead.
- Finite difference deflection model based on Kirchhoff's plate bending theory (1850), and Donnell–Mushtari–Vlasov shallow shell theory. (Ventsel & Krauthammer, 2001)
  - Numerically solve “biharmonic” equation with boundary conditions
    - approximate piezo with a force distribution
  - For curved plates plan to solve a pair of 4<sup>th</sup> order equations with boundary conditions ( not yet implemented )
    - For now using influence functions from FEM (Carolyn Atkins) for curved plates
- Rationale
  - Quicker to run, implemented in Mathematica<sup>®</sup>
  - No additional expense and training
  - No need to take time from trained FEM experts

Kirchhoff:

$$\nabla^4 \eta(x, y) = p(x, y)/D,$$

$\eta(x, y)$  = deflection in plate coordinates,

$p(x, y)$  = pressure distribution orthogonal to the plate, and

$$D = \text{flexural rigidity} = \frac{Eh^3}{12(1-\nu^2)},$$

where  $E$  = is Elastic Modulus,

$h$  = thickness, and

$\nu$  = Poisson's Ratio.

DMV:

$$\nabla^4 \eta(x, y) = (p(x, y) + \frac{1}{R \cos^2 \frac{\Theta}{4}} \frac{\partial^2 \Phi(x, y)}{\partial y^2})/D,$$

$$\nabla^4 \Phi(x, y) = -Eh \frac{1}{R \cos^2 \frac{\Theta}{4}} \frac{\partial^2 \eta(x, y)}{\partial y^2},$$

$\Phi(x, y)$  = Airy stress function,

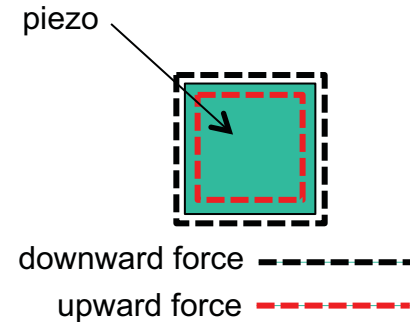
$\Theta$  = shell azimuthal extent, and

$R$  = shell radius.

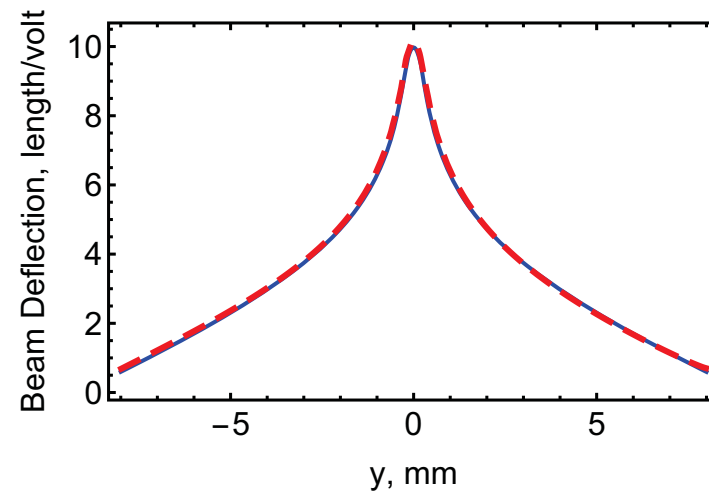
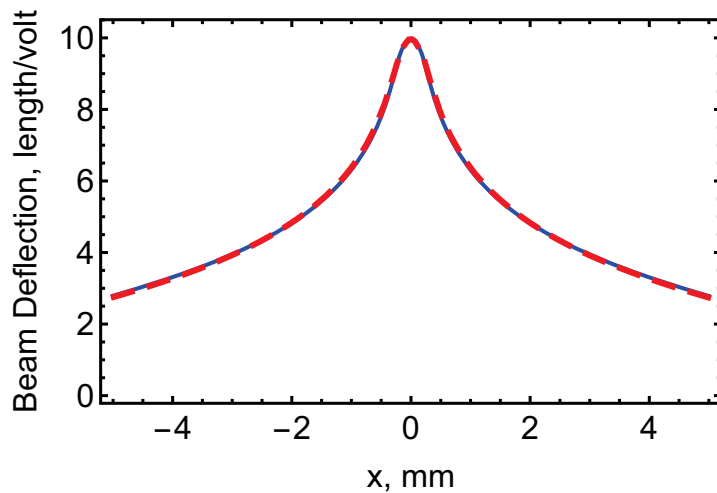


## Model Validation

- Compared finite difference(FD) solutions to FEM solutions with modeled piezos
  - 16 cm x 10 cm simply constrained flat
  - 5 mm x 5 mm piezo with 0.5 mm separation
  - 0.4 mm thick glass
  - 0.2 mm thick piezo
- FEM mesh is automatically generated (Comsol<sup>®</sup>)
- FD is on 0.25 mm grid (x elements)
- FD approximates piezos with a distribution of localized forces
- Agreement is very good.



2-D influence functions for FD (solid blue) and FEM (dashed red)

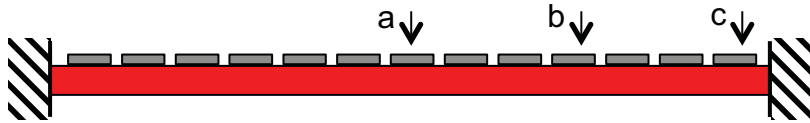




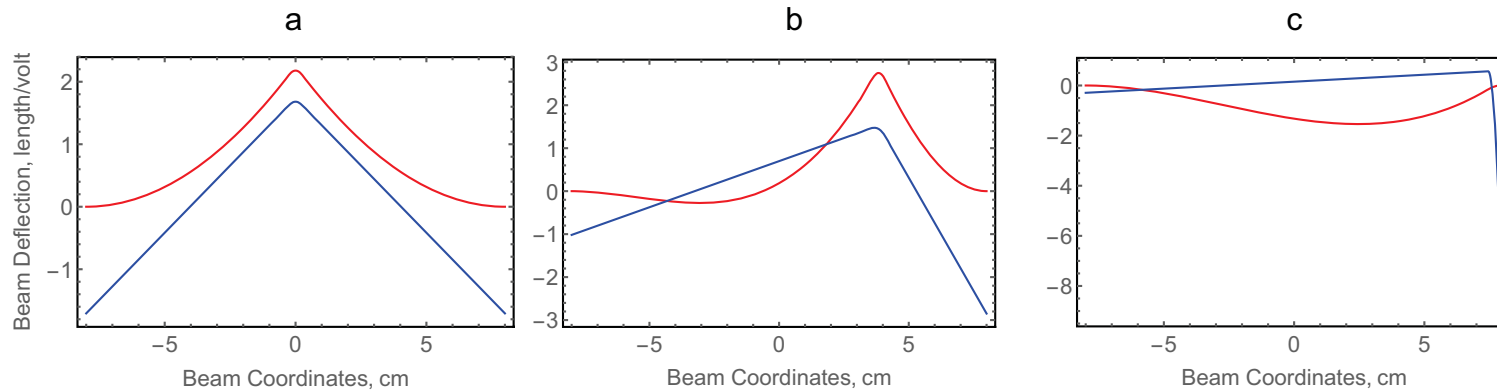



## One-D illustration

- Use simple beam theory to illuminate questions in 1 dimension



- Fixed mount vs. simply supported
  - Influence functions are obviously different for a beam with **fixed** ends, vs. an **unconstrained** beam



- Actuator effectiveness depends on position
- Actuators are relatively ineffective near mounting points
- Completeness of resulting basis composed of the full set of influence functions is affected by mounting  Performance improvement is affected



# One-D illustration II

- Begin to quantify effects using simulated error maps
- Assume a surface error spectrum

$$f(\sigma_s, \omega) = \sigma_s \sqrt{\frac{\sin \frac{3\pi}{k}}{\pi(k-3)\lambda} \frac{k\lambda^{k-1}}{\lambda^k + \omega^k}},$$

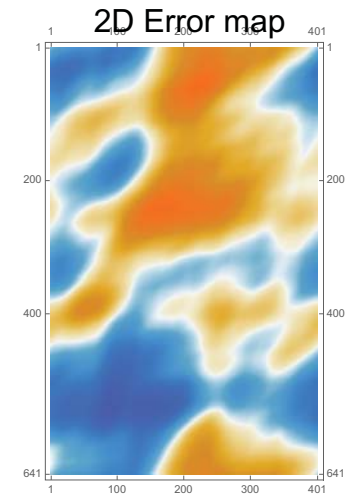
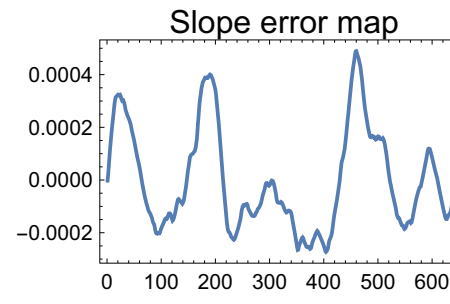
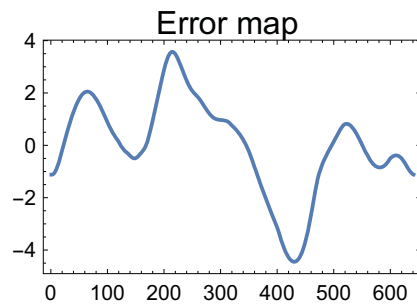
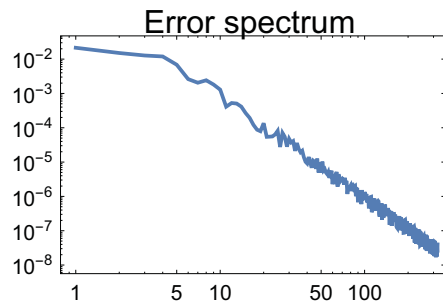
where  $\sigma_s$  is the RMS slope error,

$\omega$  is the spatial frequency,

$k$  is the asymptotic power law index and,

$\lambda$  is a cutoff parameter to tune the low frequencies.

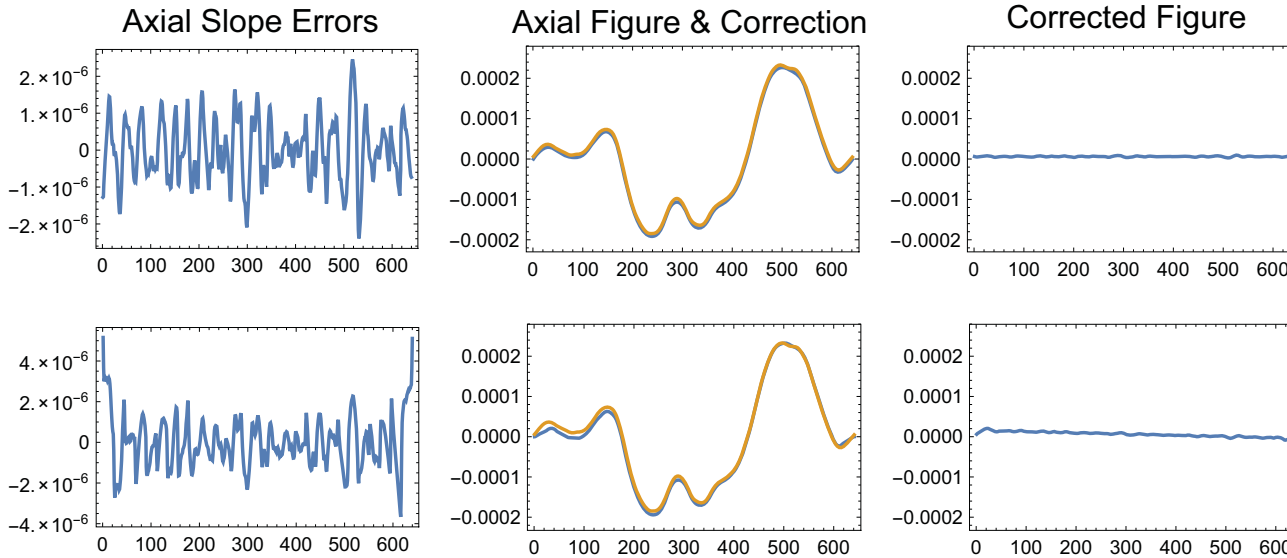
- Surface map is the discrete inverse Fourier transform of the above spectrum
- Monte Carlo phases  $\pm\pi$  and 0.5–2x amplitudes to generate a series of similar surface maps
- Adjust  $\lambda$  to match adjustability response which cuts off at Nyquist of piezo frequency.
- Adjust  $k$  to control high frequency content, performance floor
- For 2D let:  $\omega^2 \rightarrow \omega_{ax}^2 + \omega_{az}^2$  for axial and azimuth frequencies





# One-D illustration III

- Unconstrained (upper) vs. fixed (lower)

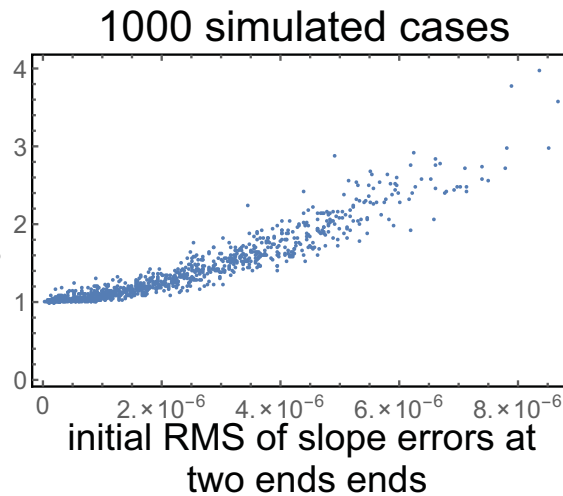


Initial RMS slope errors:  
4.4 arcsec

After unconstrained  
adjustment:  
0.48 arcsec

After adjustment in fixed  
configuration:  
0.71 arcsec

ratio fixed-to-  
unconstrained  
RMS slope errors

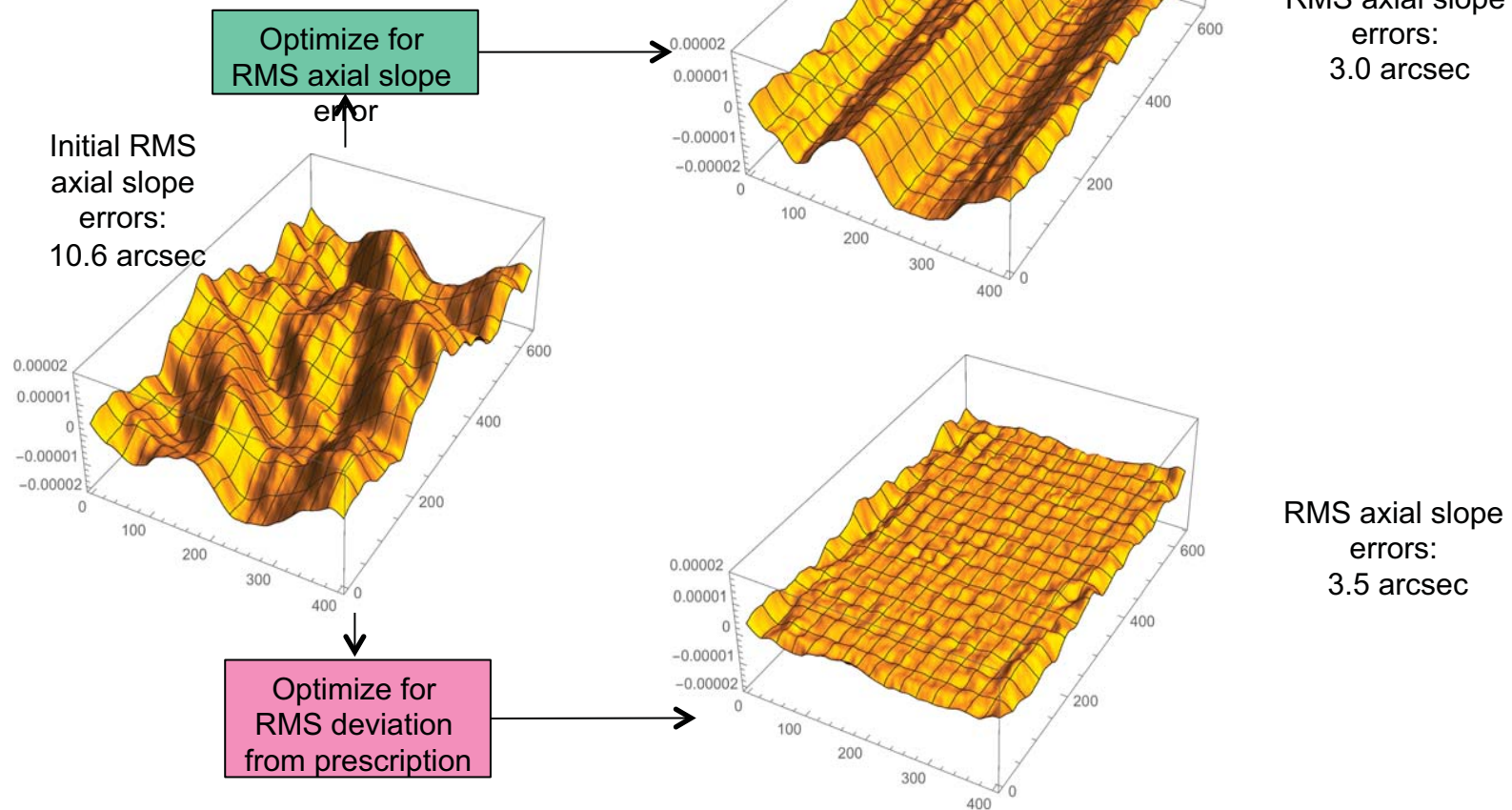


**Mounting with adjusters  
activated would  
generally yield improved  
performance**



## 2-D Flat illustration

- Similar analysis for a 2-D flat plate

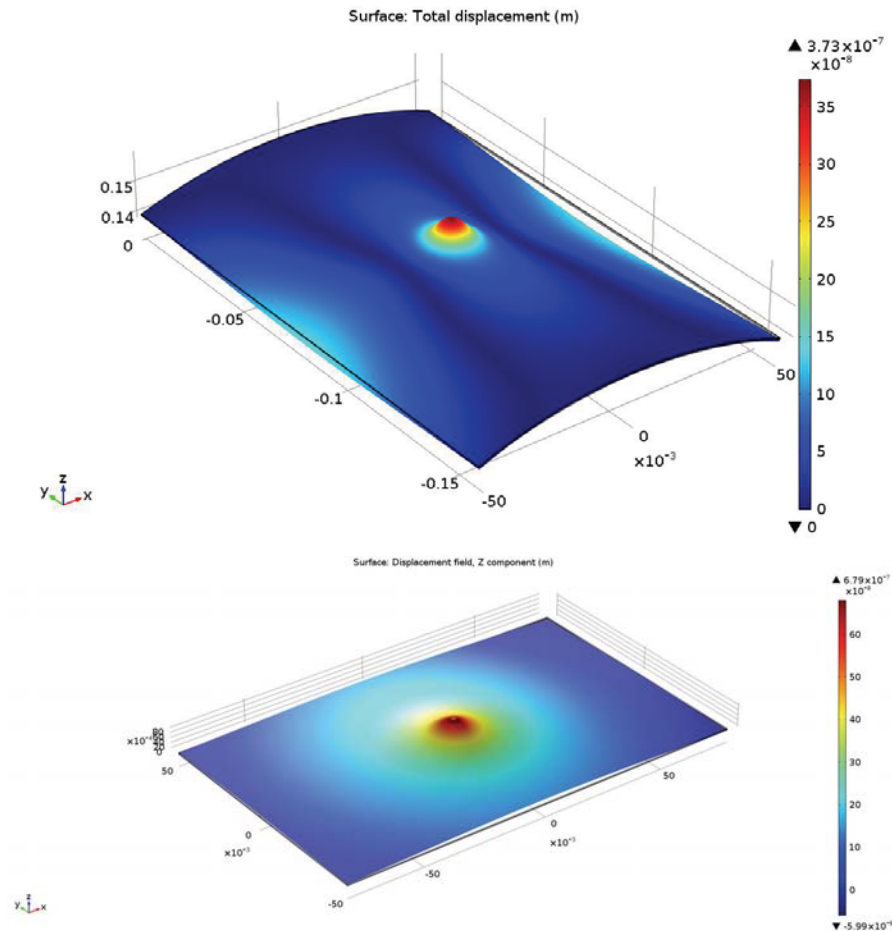


- Which results in better performance after mounting?



## 2-D shell segment

- Differences in influence functions



- Influence functions on curved optics can differ significantly from flat.
  - due to the effective rigidity caused by the curvature
  - The central displacement/volt is smaller.
  - The edge displacement/volt is about the same.
  - The surface deforms more along the azimuthal direction than the axial direction
- So edge displacement for a given central displacement is larger.
  - This must be included in the analysis of mounting distortions



- Some sample end-to-end trade spaces

### Adjusted performance of initially 10 arcsec optics (arcsec)

		k: asymptotic power law index:		
		3	3.5	4
Dimensions	Case			
1	unconstrained beam	0.9	0.7	0.4
1	fixed ends beam	1.4	1.1	0.9
2	unconstrained mirror surface	0.9	0.7	0.4
2	3 fixed points mirror surface	1.2	0.9	0.7
2	4 fixed points mirror surface	1.4	1.1	0.9
2	6 fixed points mirror surface	1.8	1.3	1
2	aligned unmatched pairs	1.3	1.0	0.6
2	aligned adjust 1 mirror only	1.4	1.0	0.6
2	aligned matched pairs	1.2	0.9	0.5
2(R = 15 cm)	unconstrained mirror surface	0.9	0.7	0.4
2(R = 15 cm)	3 fixed points mirror surface	1.2	0.9	0.7
2(R = 15 cm)	4 fixed points mirror surface	1.4	1.1	0.9
2(R = 15 cm)	6 fixed points mirror surface	1.8	1.3	1



## Conclusions

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- Expect that considering mounting strategy early, and allowing some flexibility in options could lead to better overall result
  - NOT: given mounting, what's the best optical configuration
  - NOT: given optics, what's the best mounting
  - INSTEAD: given nothing, what's the best combination of design parameters, technologies and processing sequence.
- Considering adjustable x-ray optics in a non-specific context, while something of an exercise, helps us to quickly come in-tune with significant mounting issues, in addition to producing some interesting strategic principles.
- We plan to:
  - Expand the finite difference approach to curved segments (shallow shells) and full shells
  - Develop an interface to efficiently investigate larger parameter spaces
  - Develop and test prototypes of promising concepts
- Attention to mounting details will obviously not improve optical system performance by orders magnitude but may lead to factors in the 1.4-2 range.