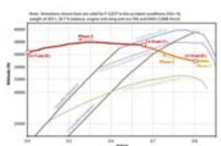
Bayesian Statistics and Uncertainty Quantification for Safet[y B]oundary Analysis in Complex Systems

Dr. Yuning He
UARC, Robust Software Engineering Group, NASA Ames Research Center

Abstract

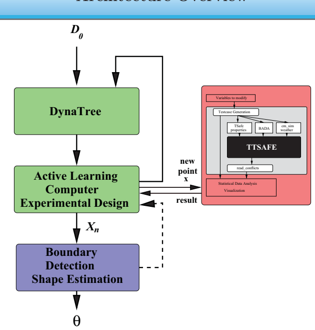
The analysis of a safety-critical system often requires detailed knowledge of safe regions and their high-dimensional non-linear boundaries. We present a statistical approach to iteratively detect and characterize the boundaries, which are provided as parameterized shape candidates. Using methods from uncertainty quantification and active learning, we incrementally construct a statistical model from only few simulation runs and obtain statistically sound estimates of the shape parameters for safety boundaries.

Introduction



- All spacecraft, aircraft, and other complex systems can only work safely within a given operational envelope (Figure shows the flight path (red) of the ill-fated flight AF447 as altitude over mach number; important boundaries are shown in gray colors)
- Multiple, non-linear boundaries in a high-dimensional parameter space and slow/expensive simulation runs limit the use of current analysis techniques like single-variable and linear techniques.
- We use statistical emulation and hierarchical Bayesian modeling to quantify the uncertainites in models and make reliable predictions of complex phenomena like number, location, and shapes of boundaries.

Architecture Overview

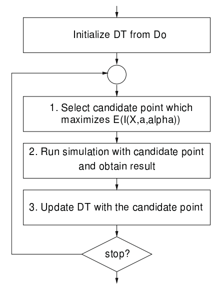


- We use DynaTrees: dynamic regression trees and sequential tree model for online applications [Taddy,Gramacy,Polson 2011]
  - Recursive partition of input space
  - Particle learning for posterior simulation

$$\begin{aligned} p([T,S]_t|[x,y]^t) &= \int p([T,S]_t|[T,S]_{t-1})dP([T,S]_{t-1}|[x,y]^t) \\ &\propto \int p([T,S]_t|[T,S]_{t-1},[x,y]_t) \int p([x,y]_t|[T,S]_{t-1})dP([T,S]_{t-1}|[x,y]^{t-1}) \end{aligned}$$

  solved with resampling and propagation
- High efficiency through tree-based partitioning in higher dimensions
- Particle mechanism suitable for active learning and experimental design

Active Learning Architecture



- General goal: candidate points should be near boundaries
- Maximum entropy $Y = -\sum_{c \in c_1,\ldots,c_n} p_c \log p_c$ is too greedy
- Active Learning McKay (ALM): select maximum variance
- Active Learning Cohn (ALC): maximize reduction in predictive variance
- Expected Improvement (EI): maximize posterior expectation of improvement statistic

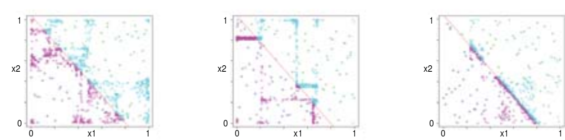
Limitation: ALM, ALC, EI do not take boundaries into account.

Our Extension: Boundary-EI

- Focus on x with $0.5 - \epsilon \leq \hat{y}(x) \leq 0.5 + \epsilon$ for $0 < \epsilon$
- Improvement (Jones 1998, Ranjan 2008): $I(x) = \epsilon^2(x) - \min\{(y(x) - 0.5)^2, \epsilon^2(x)\}$
- Expectation of $I(x)$: ($\alpha > 0$, $\epsilon(x) = \alpha s(x)$, std deviation $s(x)$, $y(x) \sim N(\hat{y}(x), s^2(x))$)

$$\begin{aligned} E[I(x)] = &-\int_{0.5-\alpha s(x)}^{0.5+\alpha s(x)} (y - \hat{y}(x))^2 \phi\left(\frac{y-\hat{y}(x)}{\sigma(x)}\right) dy \\ &+2(\hat{y} - 0.5)\sigma^2(x)\left[\phi\left(\frac{0.5-\hat{y}(x)}{\sigma(x)} + \alpha\right) - \phi\left(\frac{0.5-\hat{y}(x)}{\sigma(x)} - \alpha\right)\right] \\ &+(\alpha^2\sigma^2(x) - (\hat{y}(x) - 0.5)^2)\left[\Phi\left(\frac{0.5-\hat{y}(x)}{\sigma(x)} + \alpha\right) - \Phi\left(\frac{0.5-\hat{y}(x)}{\sigma(x)} - \alpha\right)\right] \end{aligned}$$

- Term 1 variability of response in ϵ neighborhood
  Term 2 farther away and in areas with high variance
  Term 3 is active close to estimated boundary



ALC EI Bounday-EI

Modeling Boundary Shapes

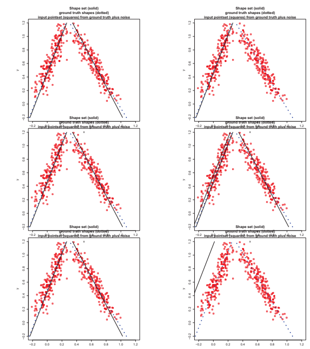
- Task: estimate sha[...]laries given points X_n near boundaries
- Boundary shapes c[...]porate physics and domain knowledge
- Shape dictionary ca[...]ed by domain expert

Metrics for shape estimation

Completeness $\overline{D}^2_{X,[...]}$ [...] $_{X,\mathcal{S}}(x)$

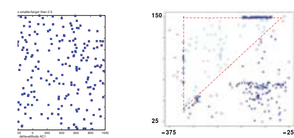
Minimality $\overline{D}^2_{\mathcal{S}} = $ [...] $d^2_{S_i,\mathcal{S}_{-i}}(s_i)$ [...] $S_i|$

Summary $\overline{D}^2_{\mathcal{S},X_n} = $ [...] $d^2_{S_a,X_n}(s)$ [...] $S_a|$



Experimental Result

Uncertainty in TTSA[...]al Tactical Separation Assured Flight Environment) track data. We analyzed TTSAFE be[...] respect to bias in the measured Radar data.



Summary

- We developed a sta[...]ework to support analysis and uncertainty quantification of non-linear complex systems.
- We used Bayesian [...]ethology in combination with active learning techniques for efficient detection and chara[...]safety regions and their boundaries.
- Case studies includ[...]lligent Flight Control System (IFCS) and Terminal Tactical Separation Assured Flight Env[...]TSAFE) for Next Generation Air Traffic Control.
- Future work will fo[...]er uncertainty quantification study, optimization of the active learning for high dimensiona[...]d application of the framework to other domains.

References

Y. He, M. Davies. Valida[...]affic Management Concept of Operation using Statistical Modeling. AIAA Modeling and Simulation, 2013.

Y. He, HK. Lee, M. .Da[...] Validation of an Adaptive FLight Control Simulation Using Statistical Emulation. AIAA Infotech@Aerosp[...]