

Detection of Unexpected High Correlations between Balance Calibration Loads and Load Residuals

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An algorithm was developed for the assessment of strain-gage balance calibration data that makes it possible to systematically investigate potential sources of unexpected high correlations between calibration load residuals and applied calibration loads. The algorithm investigates correlations on a load series by load series basis. The linear correlation coefficient is used to quantify the correlations. It is computed for all possible pairs of calibration load residuals and applied calibration loads that can be constructed for the given balance calibration data set. An unexpected high correlation between a load residual and a load is detected if three conditions are met: (i) the absolute value of the correlation coefficient of a residual/load pair exceeds 0.95; (ii) the maximum of the absolute values of the residuals of a load series exceeds 0.25 % of the load capacity; (iii) the load component of the load series is intentionally applied. Data from a baseline calibration of a six-component force balance is used to illustrate the application of the detection algorithm to a real-world data set. This analysis also showed that the detection algorithm can identify load alignment errors as long as repeat load series are contained in the balance calibration data set that do not suffer from load alignment problems.

Nomenclature

AF	= axial force
AF'	= simulated axial force of load series 1
D_j	= difference of two load values of a balance load component
F_j	= load value of a balance load component with index j
i	= summation index
j	= index of a balance load component where $1 \leq j \leq n$
k	= alternate index of a balance load component where $1 \leq k \leq n$
l	= index of a load series where $1 \leq l \leq m$
n	= total number of balance load components
m	= total number of load series of a balance calibration data set
$N1$	= normal force at the forward normal force gage of the balance
$N2$	= normal force at the aft normal force gage of the balance
p	= total number of points of a data set or of a data subset
rAF	= electrical output of the axial force gage
$rN1$	= electrical output of the forward normal force gage
$rN2$	= electrical output of the aft normal force gage
$rS1$	= electrical output of the forward side force gage
$rS2$	= electrical output of the aft side force gage
rRM	= electrical output of the rolling moment gage
RM	= rolling moment
$S1$	= side force at the forward side force gage of the balance

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$S2$	= side force at the aft side force gage of the balance
x_i	= first quantity
\bar{x}	= arithmetic mean of the first quantity
y_i	= second quantity
\bar{y}	= arithmetic mean of the second quantity
δ	= balance load alignment error in degrees
ΔAF	= axial force residual
ΔD_j	= difference of two residuals of balance load component F_j
ΔF_j	= residual of balance load component F_j (fitted minus applied or tare corrected load)
$\Delta N1$	= forward normal force residual
ρ	= linear correlation coefficient

I. Introduction

Sometimes high correlations between wind tunnel strain-gage balance load residuals and the applied calibration loads are observed after the completion of a calibration data analysis when both the load residuals and loads are plotted versus the load series number. These high correlations are “unexpected” because no correlations between residuals and calibration loads should exist after the completion of the regression analysis of the data.

Currently, it is unknown (1) what the exact sources of these correlations are, and, (2) how the unwanted correlations could systematically be avoided. Preliminary investigations indicate that several factors may influence the magnitude of the correlations. It appears, for example, that (i) the calibration load schedule design, (ii) the regression model term selection, (iii) the complexity of the applied load combinations, (iv) the load alignment, (v) the number of data points of each load series, and (vi) physical limitations of the balance itself are connected in one way or another to the presence of the unexpected high correlations between load residuals and loads.

In general, it is best to avoid unwanted high correlations between load residuals and the applied calibration loads in order (i) to gain confidence in the predictive capability of the regression model of the balance calibration data and (ii) to achieve the best possible performance for a given balance design. Some limited studies indicate that most of the high correlations can be suppressed by increasing the number of the regression model terms that are used to fit the balance calibration data. This approach, however, has its limitations as an increase of the number of regression model terms could also negatively influence the predictive capability of the related regression models of the calibration data. An increase of the number of regression model terms, for example, increases the risk of over-fitting the calibration data. It may also introduce unwanted near-linear dependencies between regression model terms.

It was mentioned above that the exact sources of the unexpected high correlations are still not rigorously understood such that they can systematically be avoided. Therefore, the authors decided to develop an algorithm for the Ames Balance Calibration Laboratory that would be able to objectively detect the presence of unexpected high correlations between load residuals and applied loads in a balance calibration data set. Then, results of this detection algorithm could be used to more efficiently study potential sources of the high correlations.

Basic elements of the new detection algorithm are discussed in the next section of the paper. Afterwards, data from the baseline calibration of a six-component force balance is used to illustrate the application of the algorithm to a real-world strain-gage balance data set.

II. Detection Algorithm

An algorithm for the detection of unexpected high correlations between strain-gage balance calibration load residuals and the applied calibration loads was developed at the Ames Balance Calibration Laboratory. The algorithm looks at correlations on a load-series-by-load-series basis as both the calibration hardware and the orientation of the balance may change from one load series to the next.

First, the algorithm quantifies correlations between all possible residual/load pairs of the given balance

calibration data set. The corresponding total number of correlation coefficients equals the square of the number of balance load components times the number of load series. Afterwards, three criteria are applied in order to identify those correlation coefficients that are considered “high” and should be studied in more detail.

The correlations between all possible pairs of load residuals and calibration loads are quantified for each load series by using the linear correlation coefficient. In general, the linear correlation coefficient for pairs of data sets x_i and y_i is defined as

$$\rho = \frac{\sum_{i=1}^p (x_i - \bar{x}) \cdot (y_i - \bar{y})}{\sqrt{\sum_{i=1}^p (x_i - \bar{x})^2} \cdot \sqrt{\sum_{i=1}^p (y_i - \bar{y})^2}} \quad \text{where} \quad -1 \leq \rho \leq +1 \quad (1)$$

where \bar{x} and \bar{y} are the arithmetic means of the two given data sets (see Ref. [1] for a definition of the linear correlation coefficient). The correlation coefficient takes on a value of +1 when the two tested data sets are directly proportional and the constant of proportionality is positive. Similarly, the correlation coefficient takes on a value of −1 when the two tested data sets are directly proportional and the constant of proportionality is negative. A value of the correlation coefficient near zero means that the two data sets are uncorrelated.

In our application, the first data set, i.e., x_i , consists of load residuals that are obtained for the data points of a specific load series after analyzing the balance calibration data using either the *Iterative* or the *Non-Iterative Method* (see, e.g., Ref. [2] for a description of the *Iterative Method*). The second data set, i.e., y_i , has the calibration loads of the load series. The variable p in the above equation equals the total number of data points that the selected load series with index l has. Then, all generic parameters used on the left and right hand sides of Eq. (1) can then be described as follows:

Table 1: Application of correlation coefficient to residual/load pairs.

PARAMETER	BALANCE DATA SET	INDEX RANGE
ρ	$\rho(j, k, l)$	$1 \leq j \leq n ; 1 \leq k \leq n ; 1 \leq l \leq m$
p	$p(l)$	$1 \leq l \leq m$
x_i	$\Delta F_j(i, l)$	$1 \leq j \leq n ; 1 \leq i \leq p(l) ; 1 \leq l \leq m$
y_i	$F_k(i, l)$	$1 \leq k \leq n ; 1 \leq i \leq p(l) ; 1 \leq l \leq m$
$\bar{x} = \frac{1}{p} \sum_{i=1}^p x_i$	$\overline{\Delta F_j(l)} = \frac{1}{p(l)} \sum_{i=1}^{p(l)} \Delta F_j(i, l)$	$1 \leq j \leq n ; 1 \leq l \leq m$
$\bar{y} = \frac{1}{p} \sum_{i=1}^p y_i$	$\overline{F_k(l)} = \frac{1}{p(l)} \sum_{i=1}^{p(l)} F_k(i, l)$	$1 \leq k \leq n ; 1 \leq l \leq m$

A more reliable detection of a high correlation between residuals and loads may be achieved if correlations between changes of quantities instead of correlations between the quantities themselves are investigated. Therefore, the authors decided to use the change of the residuals and the change of the calibration loads during a load series for the calculation of the correlation coefficients. The residual and load of the first point of a load series are used as references for the definition of the change of the residuals and loads.

Table 2: Alternate application of correlation coefficient to residual/load pairs.

PARAMETER	BALANCE DATA SET
x_i	$\Delta D_j(i, l) = \Delta F_j(i, l) - \Delta F_j(1, l)$
y_i	$D_k(i, l) = F_k(i, l) - F_k(1, l)$
$\bar{x} = \frac{1}{p} \sum_{i=1}^p x_i$	$\overline{\Delta D_j(l)} = \frac{1}{p(l)} \sum_{i=1}^{p(l)} \Delta D_j(i, l)$
$\bar{y} = \frac{1}{p} \sum_{i=1}^p y_i$	$\overline{D_k(l)} = \frac{1}{p(l)} \sum_{i=1}^{p(l)} D_k(i, l)$

Now, basic elements of the detection algorithm can be summarized. First, using the changes of the residuals and loads during a load series as input, the algorithm computes correlation coefficients for all possible residual and load combinations for the given number of load series. Then, each computed correlation coefficient is assessed. Three different conditions have to be fulfilled by a residual/load pair of a load series in order to have an unexpected “high” correlation: \implies **(I) The absolute value of the correlation coefficient of the tested residual/load pair has to exceed the empirical threshold of 0.95.** \implies **(II) The maximum of the absolute values of the residuals of the selected load component and load series has to exceed 0.25 % of the load capacity.** \implies **(III) The load component has to be “intentionally” applied, i.e., the maximum of the absolute values of the loads of the series has to exceed 10 % of the load capacity.**

The detection algorithm was successfully implemented in NASA’s BALFIT regression analysis software package (see Ref. [3] for a detailed description of the software). BALFIT reports correlations between residual/load pairs whenever the mode “Data Reduction Matrix Calculation” is selected in combination with the “Standard” or “Extended Version” of the analysis report. In that case, all residual/load combinations with unexpected high correlations are reported to the user in both table and graphical format for further evaluation. The application of the detection algorithm is illustrated in the next section of the paper by using a baseline calibration data set of a six-component force balance as an example.

III. Discussion of Example

Data from the baseline calibration of the NASA Ames MK40 force balance was selected to illustrate the application of the algorithm that detects unexpected “high” correlations between residuals and calibration loads. The MK40 is a six-component TASK balance that measures five forces ($N1, N2, S1, S2, AF$) and one moment (RM). It has a diameter of 2.5 inches and a total length of 17.31 inches. Table 3 below shows the load capacity of each load component.

Table 3: Load capacities of the NASA Ames 2.5in MK40 balance.

	$N1$, lbs	$N2$, lbs	$S1$, lbs	$S2$, lbs	RM , in-lbs	AF , lbs
CAPACITY	3500	3500	2500	2500	8000	400

The calibration of the balance was performed using the traditional “hand load” method. A total of 164 data points were taken in 16 load series. Figure 1 shows the calibration load schedule of the baseline calibration (the tare corrected calibration load in percent of load capacity is plotted versus the data point index and load series number). It has to be pointed out that the chosen baseline calibration of the MK40 balance is incomplete as calibration hardware limitations did not allow for the simultaneous application of the normal force (side force) and axial force. However, the data set contains enough information about the physical behavior of the balance so that the proposed detection of unexpected correlations between load

residuals and loads can be demonstrated.

All balance loads were corrected for the weight of the balance shell, calibration body, and other hardware before the final regression analysis was performed. Then, the tare corrected calibration data was analyzed using the *Iterative Method* so that residuals of the 164 data points for each load component could be determined (see Ref. [2] for a description of the *Iterative Method*). For simplicity, the following math term group combination was selected for the regression analysis of the balance calibration data: F_j , $|F_j|$, $F_j * F_j$, $F_j * F_k$. The symbols F_j and F_k represent the balance load components as the regression model used by the *Iterative Method* fitted each of the six gage outputs, i.e., $rN1, rN2, \dots, rAF$, as a function of the related six balance load components $N1, N2, \dots, AF$. Figure 2 shows the regression models of the six gage outputs that were used for the analysis of the calibration data. Each column represents the math model selection of a balance gage. The chosen regression model terms are identified by black rectangles. Combined loads were only applied for the normal force and side force components during the calibration of the MK40 balance. Therefore, the chosen calibration data only supported the cross-product terms $N1 * N2$ and $S1 * S2$.

Figure 3 shows the load residuals of the baseline calibration data after (i) the regression models described in Fig. 2 were determined, (ii) the resulting data reduction matrix coefficients were obtained, and (iii) the load iteration was performed. The load residuals, expressed as a percentage of the load capacity of each load component, are plotted versus the data point index and load series number. The dot-dashed lines mark the 0.25 % threshold that is traditionally used to assess the magnitude of balance load residuals. In addition, alternating gray regions mark the 16 load series of the calibration data set. It can be seen that the load residuals of only a few data points are outside of the region that is bound by the ± 0.25 % threshold.

In the next step, correlation coefficients for all residual/load pairs of each load series were computed. The MK40 balance has six load components and the calibration data set consisted of sixteen load series. Therefore, the total number of possible correlation coefficients is 576 ($6 \times 6 \times 16$). The tables depicted in Figs. 4a and 4b list all 576 computed correlation coefficients including the maximum of the absolute values of the residuals for each load residual and load series. No “high” correlation coefficients between the residuals and loads were detected as no residual/load pair met the three conditions that are highlighted in boldface in the previous section of the paper. However, several residual/load pairs had correlation coefficients near ± 0.95 . As an example, four of these correlation coefficients are highlighted by the blue and green boxes in Fig. 4a. The blue box shows that the forward normal force $N1$ of load series 1 and 2 was correlated to its residual $\Delta N1$. In addition, the green box shows that the aft normal force $N2$ of load series 3 and 4 was also correlated to the residual $\Delta N1$. The reader could have also “qualitatively” come to the same conclusion by visually inspecting the corresponding residual/load pairs that are highlighted by blue and green boxes in Fig. 5 (Fig. 5 shows the forward normal force residuals $\Delta N1$ of the calibration data set plotted versus the six tare corrected calibration $N1, N2, \dots, AF$, the data point index and the load series). The MK40 calibration data example illustrates that the proposed detection algorithm is working as intended.

IV. Detection of Load Alignment Errors

In the introduction it was asserted that the detection algorithm could potentially be used to automatically identify a load alignment error in a balance calibration data set. This characteristic of the algorithm can be demonstrated, for example, by simulating effects of an assumed misalignment of the forward normal force component ($N1$) on the axial force gage outputs (rAF) of a single load series of the MK40 balance calibration data set. In that case, the absolute value of the correlation coefficient of the data pair defined by the axial force load residual (ΔAF) and the forward normal force ($N1$) is expected to be “high” for the selected load series if the modified calibration data set is processed.

The simulation of the axial force gage outputs associated with the assumed misaligned forward normal force was done in several steps. Data of load series 1 was chosen to be modified. Figure 1 shows the original tare corrected calibration loads of load series 1. First, the maximum axial force AF_{max} of load series 1 was computed that is assumed to be caused by a misalignment of $\delta = 0.05^\circ$ of the applied forward normal force. It is the largest axial force error of load series 1 that is caused by the misalignment of the forward normal force. The following relationship was used to obtain AF_{max} where $N1_{max}$ is the largest value of $N1$ that is applied during load series 1:

$$AF_{max} = N1_{max} \cdot \sin \delta = 2200 [lbs] \cdot \sin 0.05^\circ = 1.92 [lbs] \quad (2)$$

The computed axial force error AF_{max} of 1.92 [lbs] for the alignment error of $\delta = 0.05^\circ$ is large. It corresponds to approximately 0.48 % of the axial force capacity of the MK40 balance.

In the next step, the tare corrected loads of the original calibration data set were modified. Therefore, the tare corrected axial forces of load series 1 were changed by adding a small perturbation that is associated with the assumed misalignment of the forward normal force. Consequently, the modified axial force was computed by using the following linear relationship between the axial force perturbation and the applied forward normal force component:

$$\underbrace{AF'}_{\text{simulated load}} = \underbrace{AF}_{\text{original load}} + \underbrace{AF_{max} \cdot \frac{N1}{N1_{max}}}_{\text{perturbation}} \quad (3)$$

Other load components of load series 1, i.e., $N1$, $N2$, $S1$, $S2$, and RM , were not modified. At this point, all loads of load series 1 are known that are caused by the misaligned forward normal force. It only remains to predict the gage outputs of the modified load set of series 1 so that the desired data simulation can be completed. The regression models of the six gage outputs of the original calibration data set are needed for this purpose because they make it possible to predict gage outputs of the modified load set of load series 1. These regression models are easy to obtain because they are an intermediate analysis result if the *Iterative Method* is used for the processing of balance calibration data (see Ref. [2] for a detailed description of the *Iterative Method*).

The regression models of the six gage outputs were obtained from the original calibration data of the MK40 data set. Then, the perturbed axial force and the remaining five original load components of load series 1 were used as input for these regression models in order to generate the simulated outputs of load series 1. They are the final modified gage outputs that the calibration points of load series 1 have as a result of the misalignment of the forward normal force.

Finally, the original gage outputs of load series 1 in the calibration data set of the MK40 balance were replaced by the corresponding modified outputs. Afterwards, the 16 load series of the modified calibration data set were processed as before and correlation coefficients of all possible load and residual pairs were computed.

Figure 6 shows the axial force residual (ΔAF) and the forward normal force ($N1$) of the modified calibration data set plotted versus the data point index and load series number. Initially, it was a surprise to the authors that no “high” correlation between the axial force residual and the forward normal force of load series 1 is visible in Fig. 6 even though effects of the misalignment are “hidden” in the modified gage outputs. A detailed investigation of this observation revealed that the unexpected result is caused by the fact that no alternate load series with a positive forward normal force component is contained in the modified calibration data set for comparison. Therefore, the least squares fit implicitly treated the modified gage outputs of load series 1 like “perfect” error-free balance data. The least squares analysis simply did not have access to a “repeat” of series 1 that could be used to examine the validity of the loads and outputs of series 1. Consequently, the authors concluded that the addition of a “repeat” of series 1 to the modified calibration data set could potentially reveal the “hidden” alignment problem in load series 1 as long as this “repeat” series would have no alignment errors. The authors tested their hypothesis by simply including the original loads and outputs of load series 1 as load series 17 in the modified calibration data set. Afterwards, this extended data set was processed as before.

Figure 7a shows the axial force residual (ΔAF) and the forward normal force ($N1$) of the extended calibration data set plotted versus the data point index and the load series number. The red box in Fig. 7b lists the computed correlation coefficients of the data pair defined by the axial force residual (ΔAF) and the forward normal force ($N1$) for all 17 load series. Several observations can be made after reviewing results that are shown in Fig. 7a and Fig. 7b. First, a “high” correlation between the axial force residual and the forward normal force is visible for load series 1. The corresponding correlation coefficient was computed to be “-1” (see Fig. 7b). This result indicates that the detection algorithm correctly identified the “hidden” connection between the axial force residual and the forward normal force of load series 1. It is also interesting to observe that the largest axial force residuals of series 1 and series 17 are of similar magnitude (0.25 % for series 1; 0.33 % for series 17). It seems that the maximum alignment error computed in Eq. (2), i.e., 0.48 %, was approximately split in half and afterwards assigned to the two load series. Consequently, the

least squares fit recognized that some kind of “contradiction” between the axial force gage outputs of load series 1 and load series 17 exists. However, it is also important to point out that the axial force residuals of load series 17 are large (up to 0.33 % of capacity) but not correlated with the forward normal force (the corresponding correlation coefficient is listed in Fig. 7b as “+0.81”). In other words, the detection algorithm correctly concluded that the data of load series 1 (and not the data of load series 17) is causing the elevated levels of the axial force residual.

Two important conclusions can be drawn from results that are shown in Fig. 7a and Fig. 7b. First, “repeat” load series are important in order to mathematically detect “hidden” misalignments in balance calibration data. In addition, the correlation coefficient definition of the proposed detection algorithm has the ability to positively identify a load series that has load alignment errors.

V. Summary and Conclusions

Basic elements of a new detection algorithm were presented that tries to identify unexpected “high” correlations between residual/load pairs of a strain–gage balance calibration data set. The detection algorithm was successfully implemented in NASA’s BALFIT software package. Data from a baseline calibration of the NASA’s MK40 six–component force balance was used to illustrate the application of the new detection algorithm to a real–world data set. It was also shown that the detection algorithm can positively identify load alignment errors in a load series as long as the given balance calibration data set has a “repeat” of the questionable load series that does not suffer from a load alignment error.

Results of the detection algorithm will be used in the future to systematically study potential sources for “high” correlations between residuals and applied loads of a balance calibration data set. In particular, the following three questions will guide these investigations: How exactly does the math term selection influence correlations between residuals and loads? How can a calibration load schedule be improved such that “high” correlations between residual and loads are avoided? How do load alignment errors influence the magnitude of the correlations? These investigations may lead to recommendations that could help minimize or avoid the unwanted correlations between load residuals and loads of a balance calibration data set.

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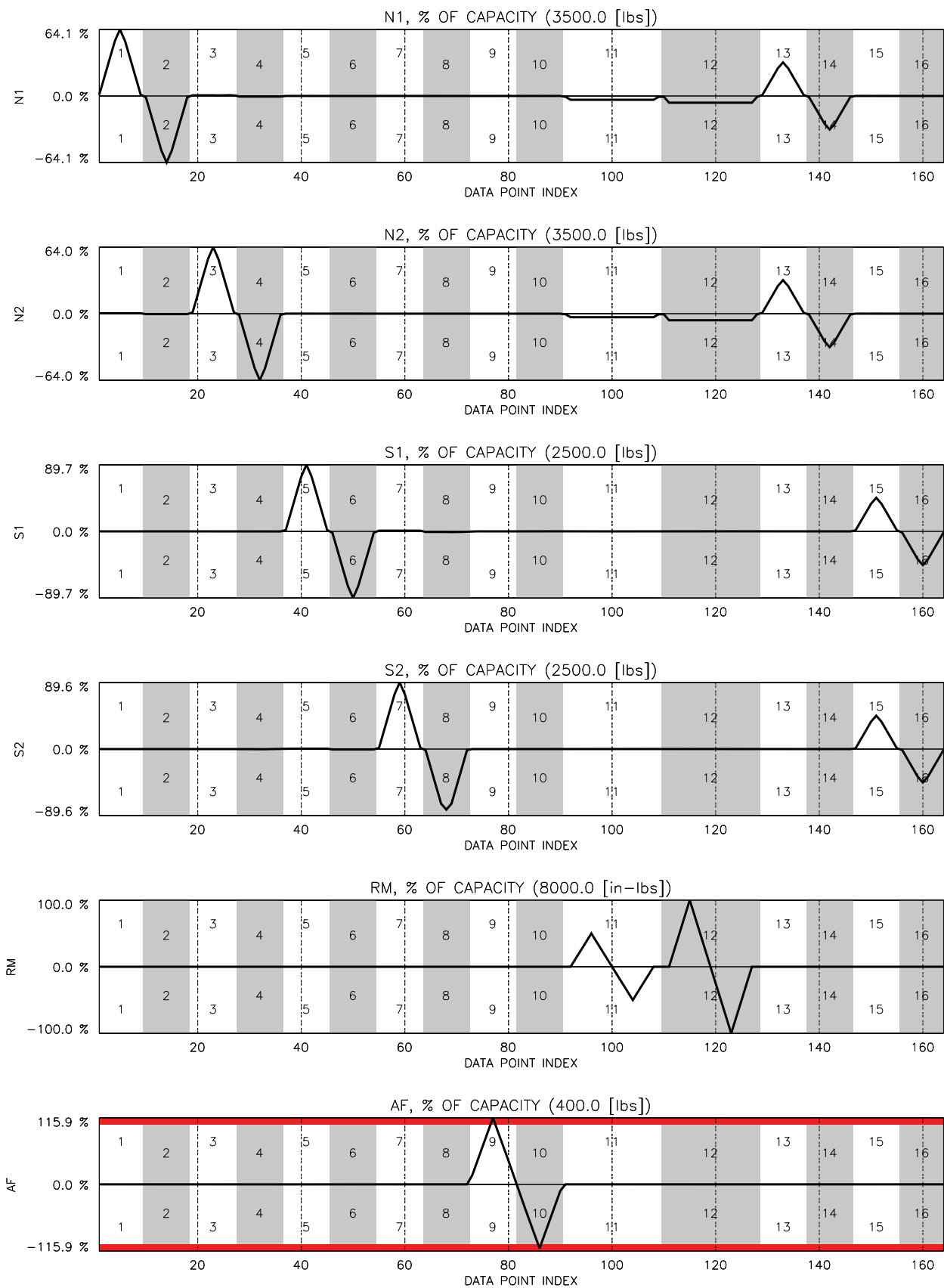


Fig. 1 Tare corrected calibration loads of the MK40 strain-gage balance.

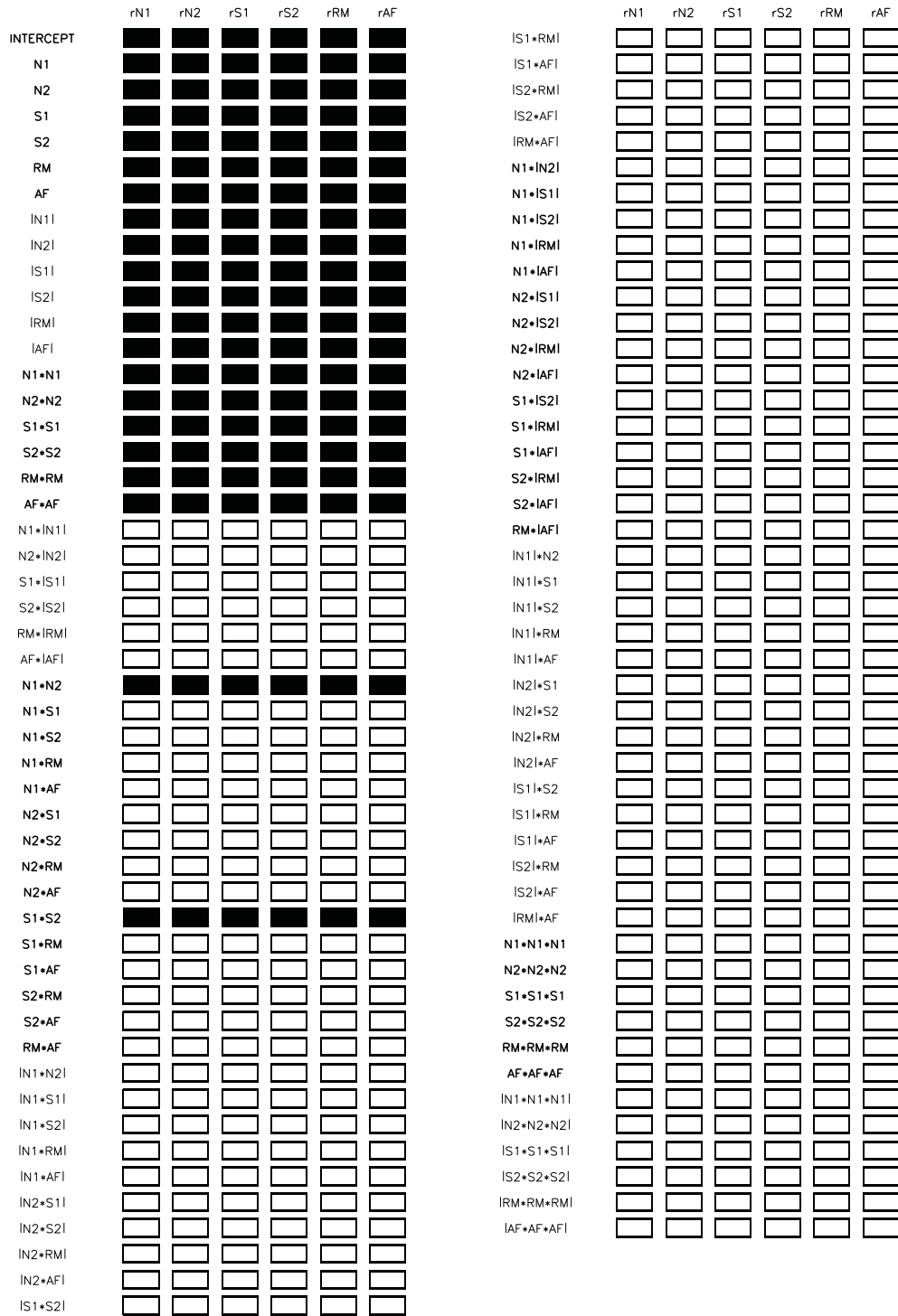


Fig. 2 Math models of the six gage outputs of the baseline calibration data set.

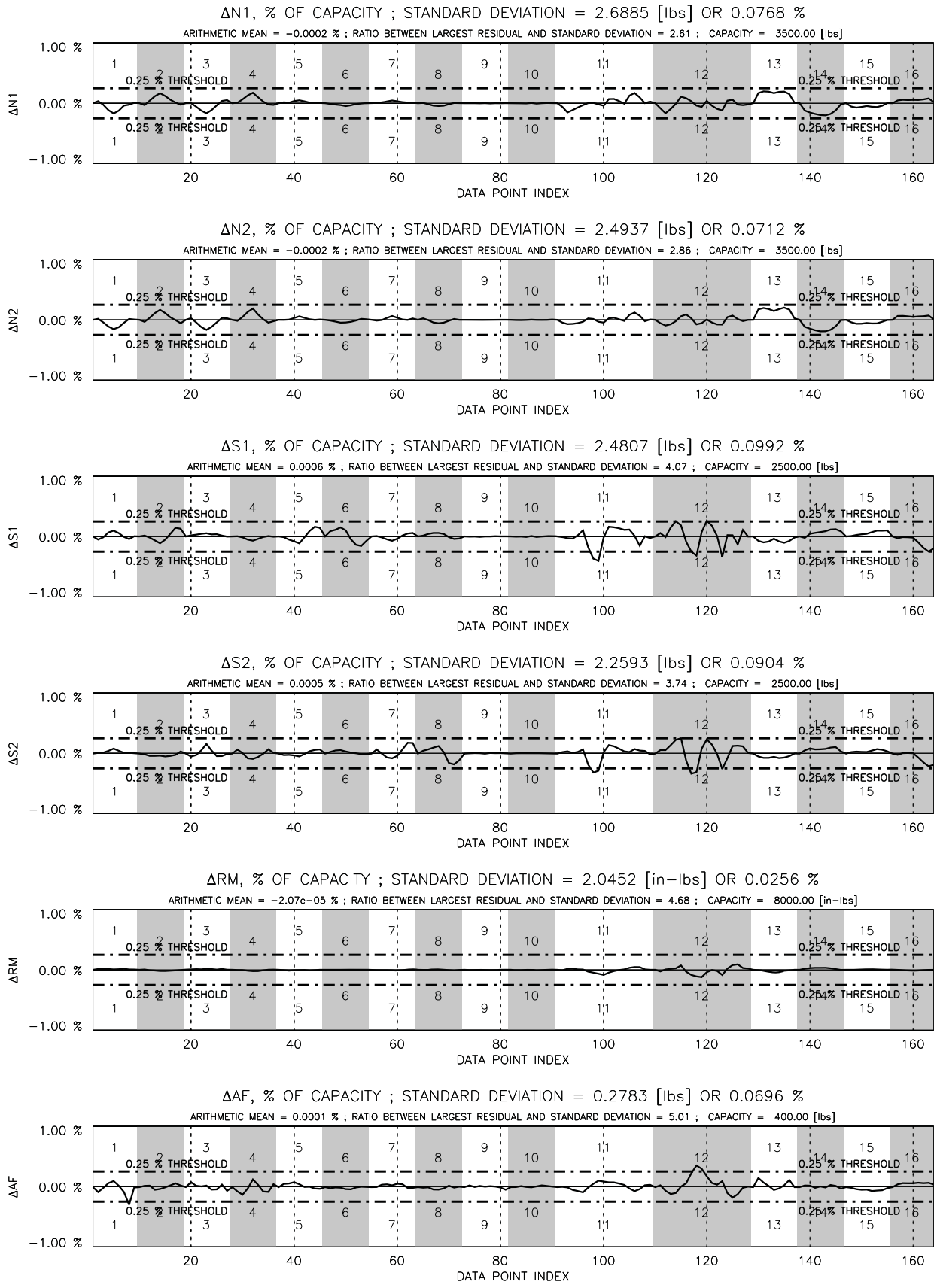


Fig. 3 Calibration load residuals for the selected math models.

NAME OF RESIDUAL	MAXIMUM OF ABS(RESID.)	LOAD SERIES	N1 (LOAD)	N2 (LOAD)	S1 (LOAD)	S2 (LOAD)	RM (LOAD)	AF (LOAD)
ΔN1	0.17 %	1	-0.90	+0.00	+0.00	+0.00	+0.00	+0.00
ΔN1	0.17 %	2	-0.94	+0.00	+0.00	+0.00	+0.00	+0.00
ΔN1	0.17 %	3	+0.00	-0.94	+0.00	+0.00	+0.00	+0.00
ΔN1	0.17 %	4	+0.00	-0.93	+0.00	+0.00	+0.00	+0.00
ΔN1	< 0.1 %	5	+0.00	+0.00	+0.89	+0.00	+0.00	+0.00
ΔN1	< 0.1 %	6	+0.00	+0.00	+0.96	+0.00	+0.00	+0.00
ΔN1	< 0.1 %	7	+0.00	+0.00	+0.00	+0.95	+0.00	+0.00
ΔN1	< 0.1 %	8	+0.00	+0.00	+0.00	+0.93	+0.00	+0.00
ΔN1	< 0.1 %	9	+0.00	+0.00	+0.00	+0.00	+0.00	-0.42
ΔN1	< 0.1 %	10	+0.00	+0.00	+0.00	+0.00	+0.00	+0.24
ΔN1	0.17 %	11	+0.04	+0.04	+0.00	+0.00	-0.58	+0.00
ΔN1	0.17 %	12	+0.06	+0.06	+0.00	+0.00	+0.32	+0.00
ΔN1	0.20 %	13	+0.78	+0.78	+0.00	+0.00	+0.00	+0.00
ΔN1	0.20 %	14	+0.92	+0.92	+0.00	+0.00	+0.00	+0.00
ΔN1	< 0.1 %	15	+0.00	+0.00	-0.72	-0.72	+0.00	+0.00
ΔN1	< 0.1 %	16	+0.00	+0.00	-0.57	-0.57	+0.00	+0.00
ΔN2	0.15 %	1	-0.93	+0.00	+0.00	+0.00	+0.00	+0.00
ΔN2	0.17 %	2	-0.94	+0.00	+0.00	+0.00	+0.00	+0.00
ΔN2	0.16 %	3	+0.00	-0.93	+0.00	+0.00	+0.00	+0.00
ΔN2	0.19 %	4	+0.00	-0.88	+0.00	+0.00	+0.00	+0.00
ΔN2	< 0.1 %	5	+0.00	+0.00	+0.85	+0.00	+0.00	+0.00
ΔN2	< 0.1 %	6	+0.00	+0.00	+0.89	+0.00	+0.00	+0.00
ΔN2	< 0.1 %	7	+0.00	+0.00	+0.00	+0.69	+0.00	+0.00
ΔN2	< 0.1 %	8	+0.00	+0.00	+0.00	+0.83	+0.00	+0.00
ΔN2	< 0.1 %	9	+0.00	+0.00	+0.00	+0.00	+0.00	-0.67
ΔN2	< 0.1 %	10	+0.00	+0.00	+0.00	+0.00	+0.00	+0.09
ΔN2	0.12 %	11	+0.02	+0.02	+0.00	+0.00	-0.50	+0.00
ΔN2	< 0.1 %	12	+0.09	+0.09	+0.00	+0.00	+0.23	+0.00
ΔN2	0.20 %	13	+0.67	+0.67	+0.00	+0.00	+0.00	+0.00
ΔN2	0.19 %	14	+0.93	+0.93	+0.00	+0.00	+0.00	+0.00
ΔN2	< 0.1 %	15	+0.00	+0.00	-0.86	-0.86	+0.00	+0.00
ΔN2	< 0.1 %	16	+0.00	+0.00	-0.56	-0.56	+0.00	+0.00
ΔS1	< 0.1 %	1	+0.74	+0.00	+0.00	+0.00	+0.00	+0.00
ΔS1	0.15 %	2	+0.77	+0.00	+0.00	+0.00	+0.00	+0.00
ΔS1	< 0.1 %	3	+0.00	+0.97	+0.00	+0.00	+0.00	+0.00
ΔS1	< 0.1 %	4	+0.00	+0.88	+0.00	+0.00	+0.00	+0.00
ΔS1	0.16 %	5	+0.00	+0.00	-0.67	+0.00	+0.00	+0.00
ΔS1	0.16 %	6	+0.00	+0.00	-0.41	+0.00	+0.00	+0.00
ΔS1	< 0.1 %	7	+0.00	+0.00	+0.00	-0.84	+0.00	+0.00
ΔS1	< 0.1 %	8	+0.00	+0.00	+0.00	-0.80	+0.00	+0.00
ΔS1	< 0.1 %	9	+0.00	+0.00	+0.00	+0.00	+0.00	-0.32
ΔS1	< 0.1 %	10	+0.00	+0.00	+0.00	+0.00	+0.00	+0.29
ΔS1	0.40 %	11	+0.02	+0.02	+0.00	+0.00	-0.45	+0.00
ΔS1	0.34 %	12	-0.02	-0.02	+0.00	+0.00	+0.21	+0.00
ΔS1	0.10 %	13	-0.49	-0.49	+0.00	+0.00	+0.00	+0.00
ΔS1	0.12 %	14	-0.29	-0.29	+0.00	+0.00	+0.00	+0.00
ΔS1	0.10 %	15	+0.00	+0.00	+0.00	+0.00	+0.00	+0.00
ΔS1	0.25 %	16	+0.00	+0.00	-0.31	-0.31	+0.00	+0.00
ΔS2	< 0.1 %	1	+0.89	+0.00	+0.00	+0.00	+0.00	+0.00
ΔS2	< 0.1 %	2	+0.83	+0.00	+0.00	+0.00	+0.00	+0.00

Fig. 4a Correlations coefficients 1 to 300 of the residual and load combinations.

NAME OF RESIDUAL	MAXIMUM OF ABS(RESID.)	LOAD SERIES	N1 (LOAD)	N2 (LOAD)	S1 (LOAD)	S2 (LOAD)	RM (LOAD)	AF (LOAD)
ΔS2	0.16 %	3	+0.00	+0.70	+0.00	+0.00	+0.00	+0.00
ΔS2	< 0.1 %	4	+0.00	+0.76	+0.00	+0.00	+0.00	+0.00
ΔS2	< 0.1 %	5	+0.00	+0.00	-0.63	+0.00	+0.00	+0.00
ΔS2	< 0.1 %	6	+0.00	+0.00	-0.67	+0.00	+0.00	+0.00
ΔS2	0.17 %	7	+0.00	+0.00	+0.00	-0.74	+0.00	+0.00
ΔS2	0.18 %	8	+0.00	+0.00	+0.00	-0.54	+0.00	+0.00
ΔS2	< 0.1 %	9	+0.00	+0.00	+0.00	+0.00	+0.00	+0.09
ΔS2	< 0.1 %	10	+0.00	+0.00	+0.00	+0.00	+0.00	-0.82
ΔS2	0.32 %	11	-0.01	-0.01	+0.00	+0.00	-0.43	+0.00
ΔS2	0.34 %	12	-0.04	-0.04	+0.00	+0.00	+0.15	+0.00
ΔS2	< 0.1 %	13	-0.83	-0.83	+0.00	+0.00	+0.00	+0.00
ΔS2	0.10 %	14	-0.53	-0.53	+0.00	+0.00	+0.00	+0.00
ΔS2	< 0.1 %	15	+0.00	+0.00	+0.19	+0.19	+0.00	+0.00
ΔS2	0.22 %	16	+0.00	+0.00	-0.42	-0.42	+0.00	+0.00
ΔRM	< 0.1 %	1	+0.50	+0.00	+0.00	+0.00	+0.00	+0.00
ΔRM	< 0.1 %	2	+0.81	+0.00	+0.00	+0.00	+0.00	+0.00
ΔRM	< 0.1 %	3	+0.00	+0.65	+0.00	+0.00	+0.00	+0.00
ΔRM	< 0.1 %	4	+0.00	+0.93	+0.00	+0.00	+0.00	+0.00
ΔRM	< 0.1 %	5	+0.00	+0.00	-0.39	+0.00	+0.00	+0.00
ΔRM	< 0.1 %	6	+0.00	+0.00	-0.41	+0.00	+0.00	+0.00
ΔRM	< 0.1 %	7	+0.00	+0.00	+0.00	-0.84	+0.00	+0.00
ΔRM	< 0.1 %	8	+0.00	+0.00	+0.00	-0.57	+0.00	+0.00
ΔRM	< 0.1 %	9	+0.00	+0.00	+0.00	+0.00	+0.00	-0.57
ΔRM	< 0.1 %	10	+0.00	+0.00	+0.00	+0.00	+0.00	+0.37
ΔRM	< 0.1 %	11	+0.07	+0.07	+0.00	+0.00	-0.23	+0.00
ΔRM	0.12 %	12	+0.11	+0.11	+0.00	+0.00	+0.07	+0.00
ΔRM	< 0.1 %	13	-0.97	-0.97	+0.00	+0.00	+0.00	+0.00
ΔRM	< 0.1 %	14	-0.96	-0.96	+0.00	+0.00	+0.00	+0.00
ΔRM	< 0.1 %	15	+0.00	+0.00	+0.90	+0.90	+0.00	+0.00
ΔRM	< 0.1 %	16	+0.00	+0.00	+0.94	+0.94	+0.00	+0.00
ΔAF	0.30 %	1	+0.47	+0.00	+0.00	+0.00	+0.00	+0.00
ΔAF	< 0.1 %	2	+0.62	+0.00	+0.00	+0.00	+0.00	+0.00
ΔAF	< 0.1 %	3	+0.00	+0.05	+0.00	+0.00	+0.00	+0.00
ΔAF	0.14 %	4	+0.00	-0.29	+0.00	+0.00	+0.00	+0.00
ΔAF	< 0.1 %	5	+0.00	+0.00	+0.58	+0.00	+0.00	+0.00
ΔAF	< 0.1 %	6	+0.00	+0.00	+0.74	+0.00	+0.00	+0.00
ΔAF	< 0.1 %	7	+0.00	+0.00	+0.00	+0.81	+0.00	+0.00
ΔAF	< 0.1 %	8	+0.00	+0.00	+0.00	+0.65	+0.00	+0.00
ΔAF	< 0.1 %	9	+0.00	+0.00	+0.00	+0.00	+0.00	+0.53
ΔAF	< 0.1 %	10	+0.00	+0.00	+0.00	+0.00	+0.00	+0.09
ΔAF	< 0.1 %	11	+0.04	+0.04	+0.00	+0.00	-0.34	+0.00
ΔAF	0.35 %	12	-0.08	-0.08	+0.00	+0.00	+0.07	+0.00
ΔAF	0.14 %	13	-0.38	-0.38	+0.00	+0.00	+0.00	+0.00
ΔAF	< 0.1 %	14	-0.12	-0.12	+0.00	+0.00	+0.00	+0.00
ΔAF	< 0.1 %	15	+0.00	+0.00	-0.70	-0.70	+0.00	+0.00
ΔAF	< 0.1 %	16	+0.00	+0.00	-0.68	-0.68	+0.00	+0.00

Fig. 4b Correlations coefficients 301 to 576 of the residual and load combinations.

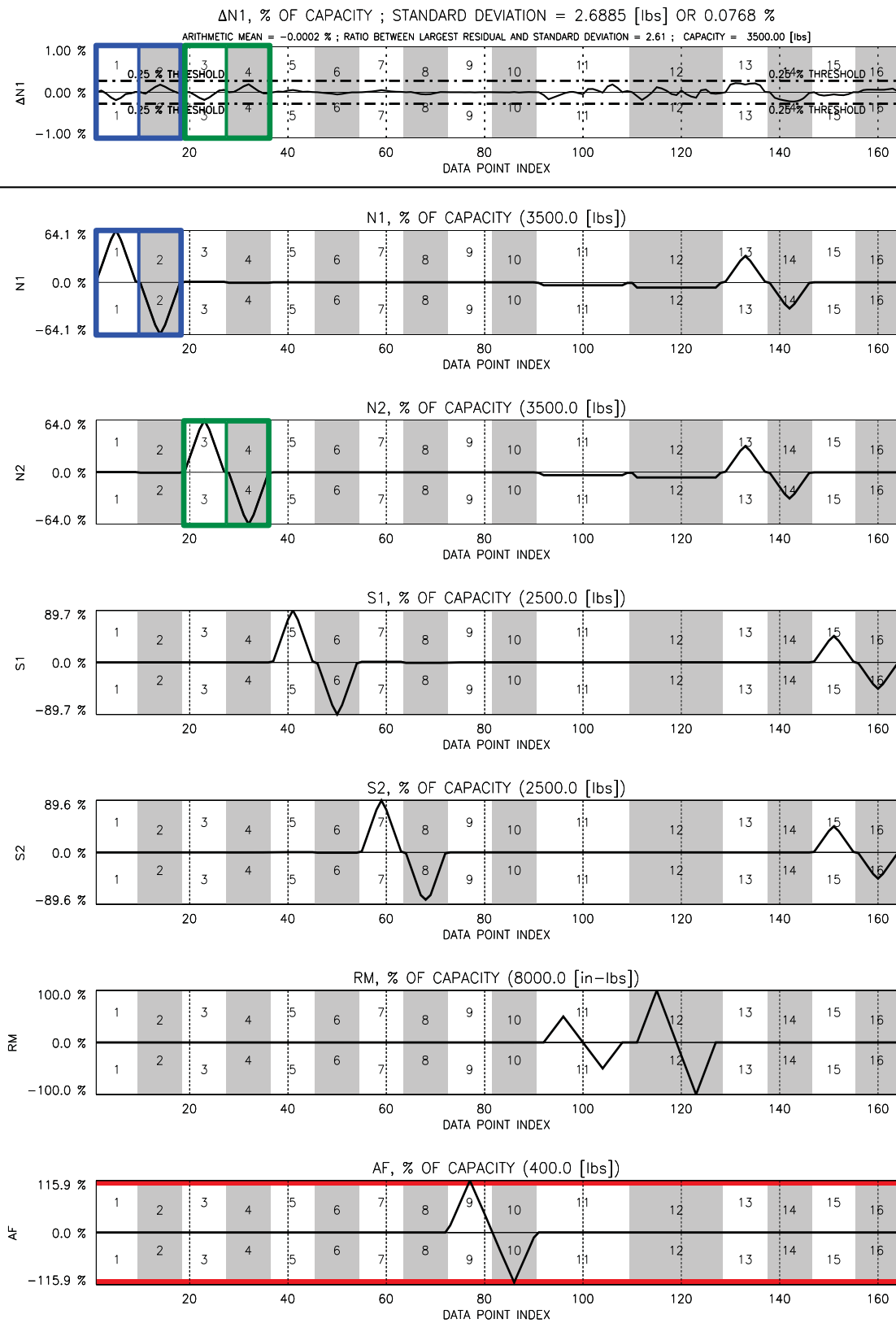


Fig. 5 Forward normal force residuals plotted versus the tare corrected loads.

**NO CORRELATION BETWEEN
AXIAL FORCE RESIDUALS
AND “N1” IS DETECTED**

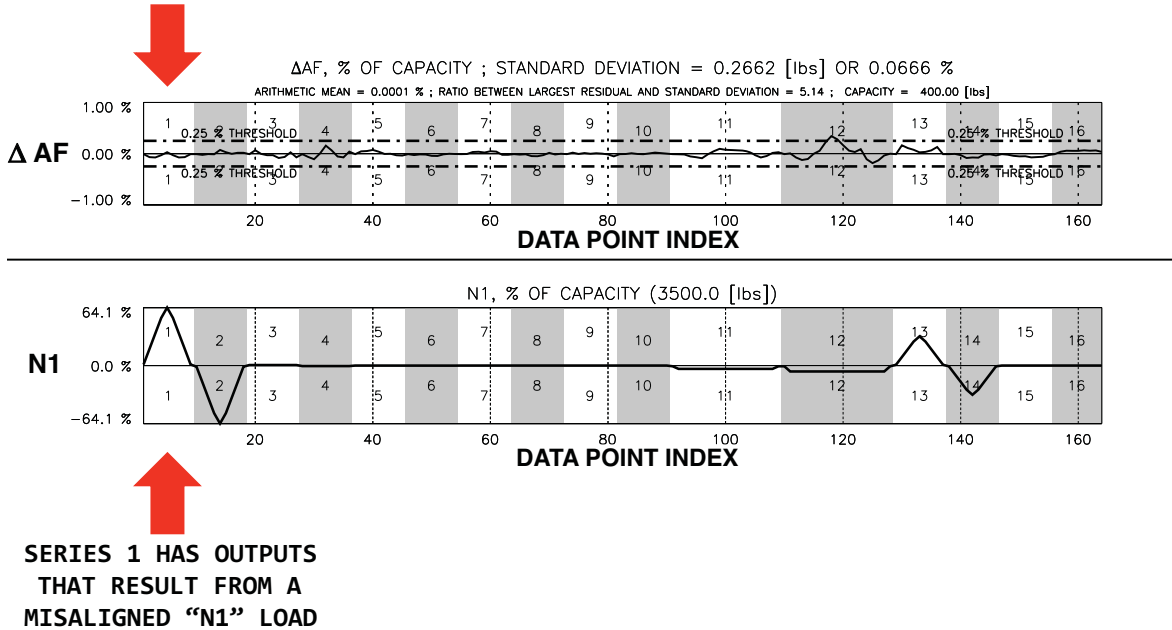


Fig. 6 Axial force residual plotted versus the tare corrected forward normal force; a repeat of series 1 with the original gage outputs is not included.

**CORRELATION BETWEEN
AXIAL FORCE RESIDUALS
AND “N1” IS DETECTED**

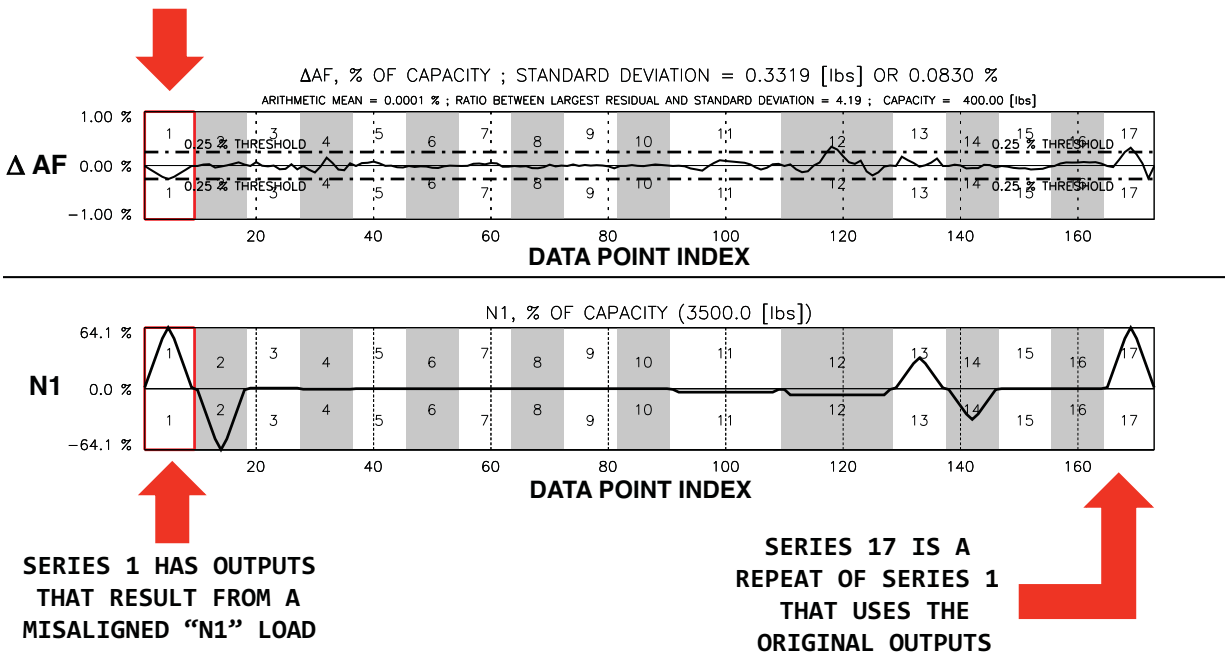


Fig. 7a Axial force residual plotted versus the tare corrected forward normal force; a repeat of series 1 with the original gage outputs is included as series 17.

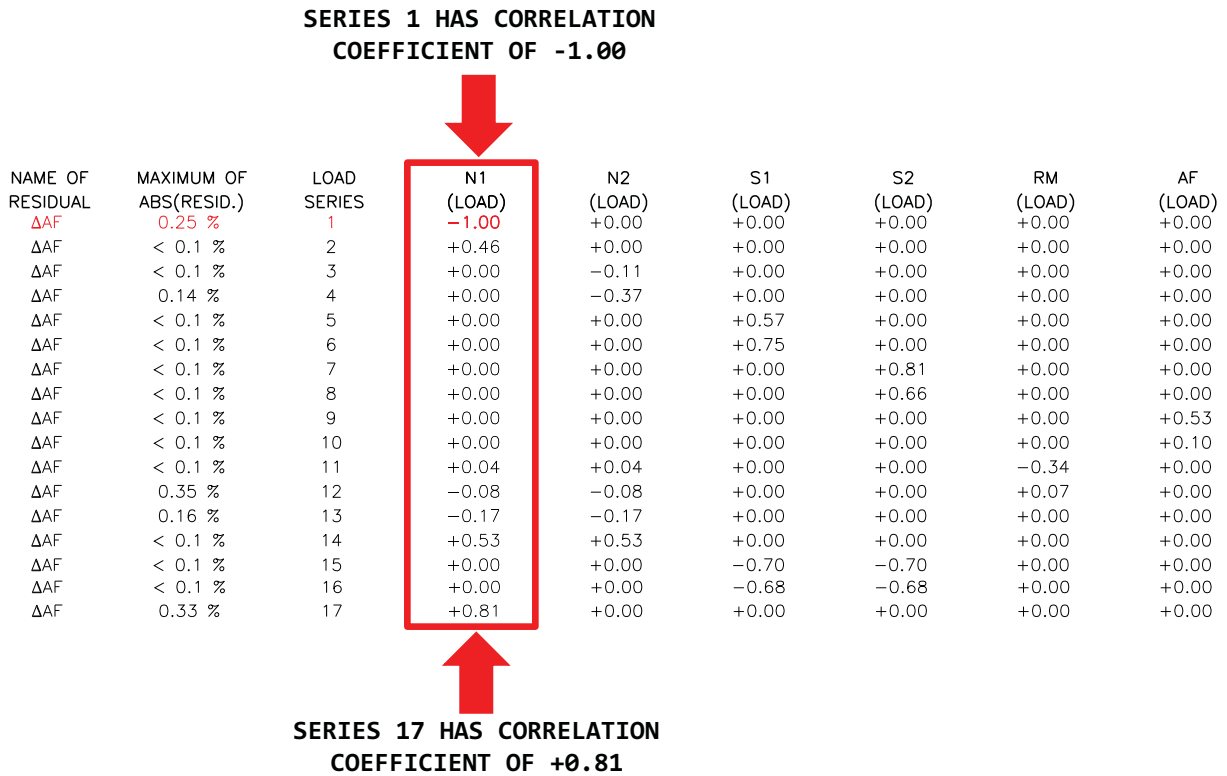


Fig. 7b Correlation coefficients of data pairs defined by the axial force residual and the tare corrected forward normal force of the 17 load series.