National Aeronautics and Space Administration

CAUCHY DRAG ESTIMATION FOR LOW EARTH ORBITERS

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code 595

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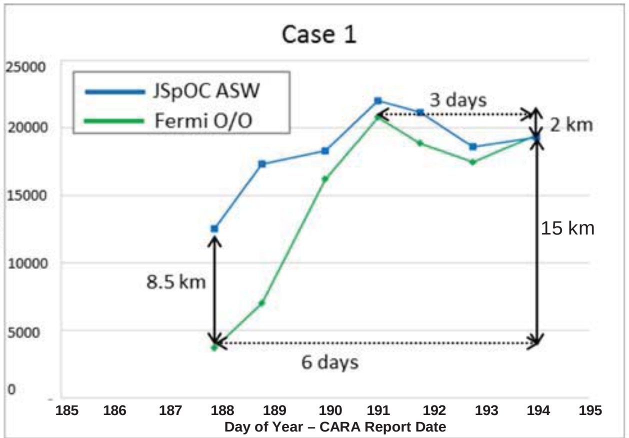
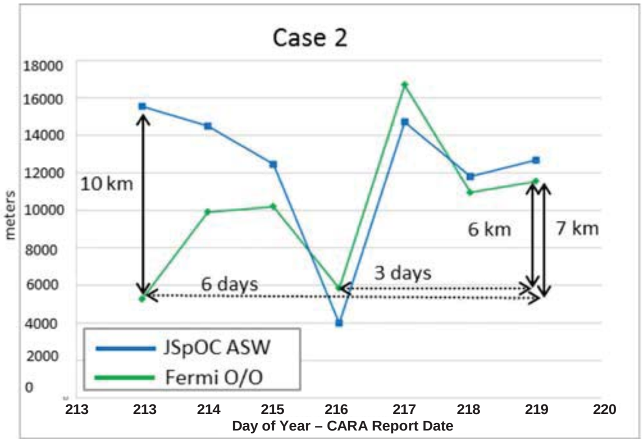
GPS-based Owner/Operator Predictions are Sometimes Inferior to JSpOC's

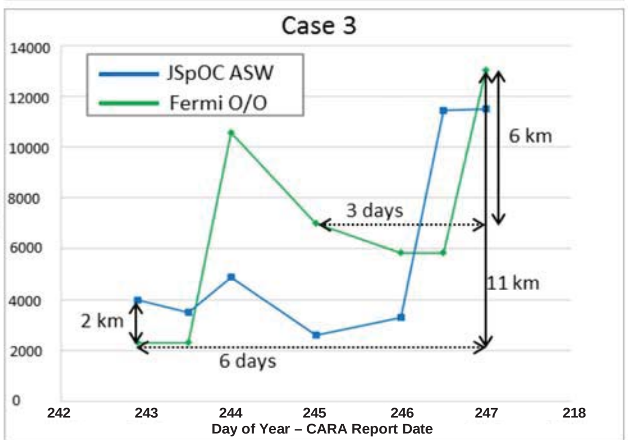
Initial JSpOC prediction ~2x closer to final consensus at Time of closest approach (TCA)

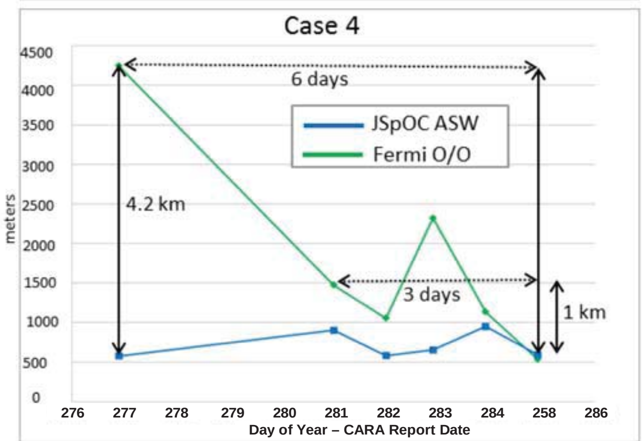




Initial JSpOC prediction ~3x closer to final consensus at TCA

Initial predictions similar; GPS moving toward consensus sooner

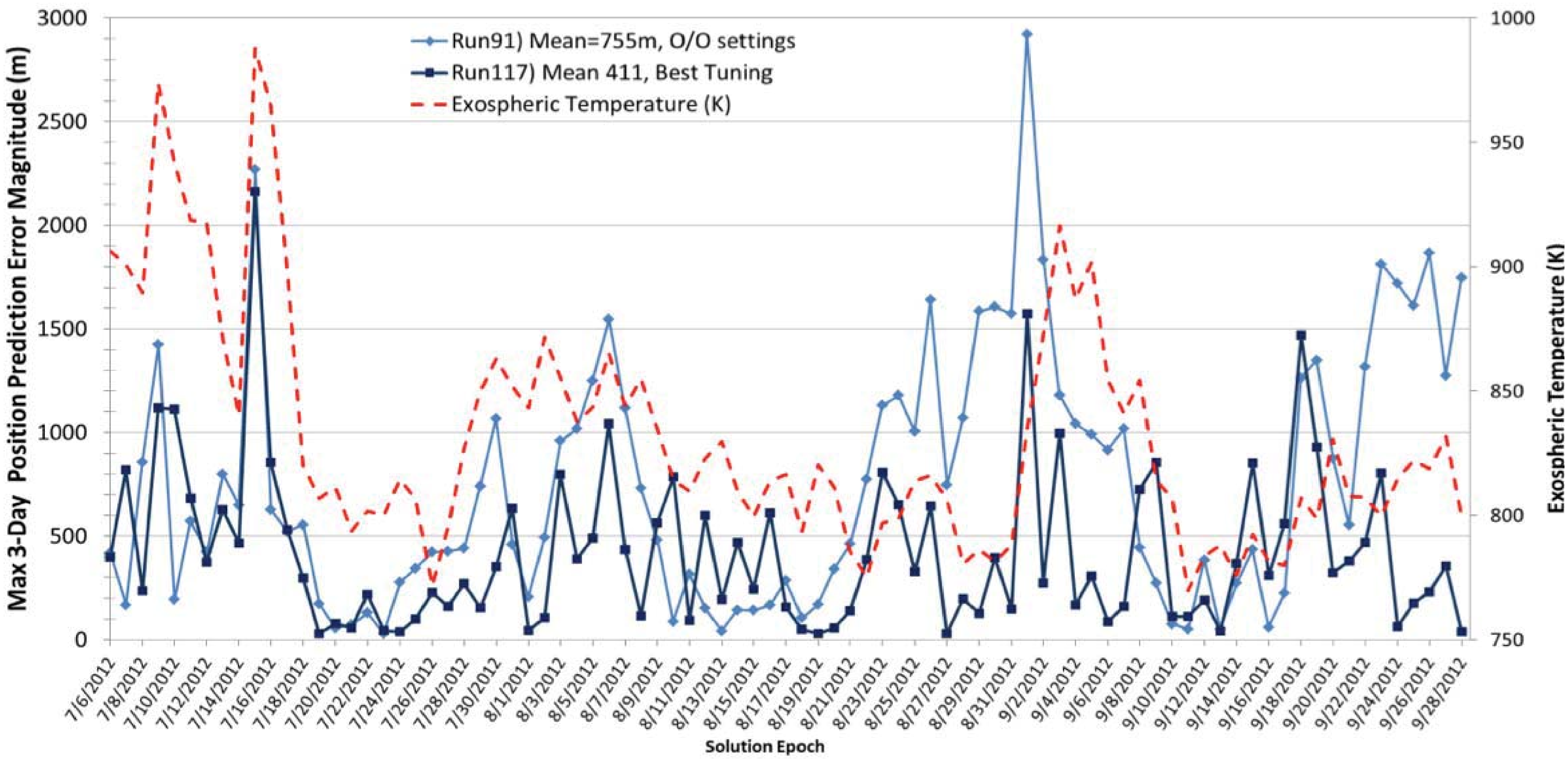




Initial JSpOC prediction within 500 of final consensus at TCA

Ref: M.A. Vavrina, C.P. Newman, S.E. Slojkowski and J.R. Carpenter, "Improving Fermi Orbit Determination and Prediction in an Uncertain Atmospheric Drag Environment" *Proceedings of the 24th International Symposium on Space Flight Dynamics*, www.issfd.org, 2014

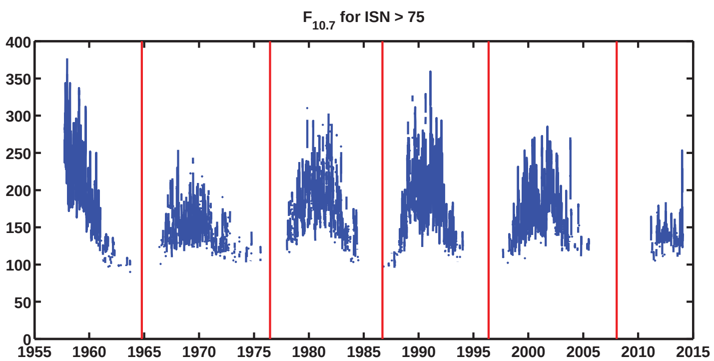
Tuning GPS EKF Yields Marginal Improvement in Prediction Robustness to Density Variation

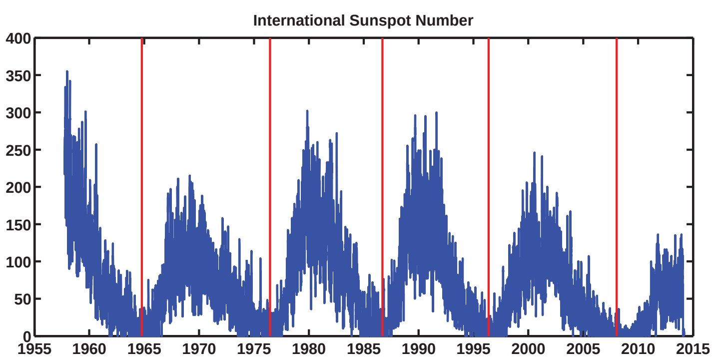


Ref: M.A. Vavrina, C.P. Newman, S.E. Slojkowski and J.R. Carpenter, "Improving Fermi Orbit Determination and Prediction in an Uncertain Atmospheric Drag Environment" *Proceedings of the 24th International Symposium on Space Flight Dynamics*, www.issfd.org, 2014

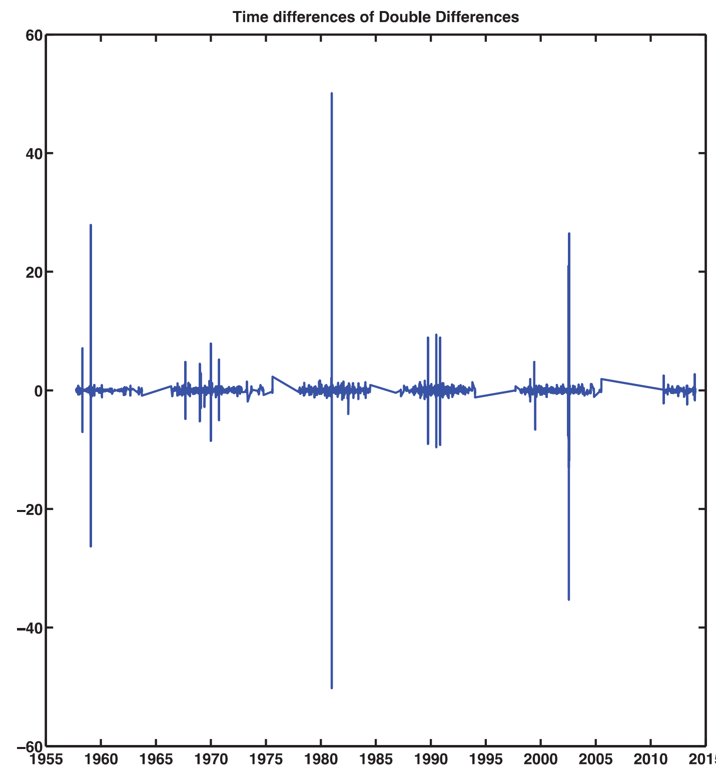
Solar Flux Variations During Higher Solar Activity Intervals are Not Gaussian

Data Set: Observed $F_{10.7}$ Flux when ISN > 75

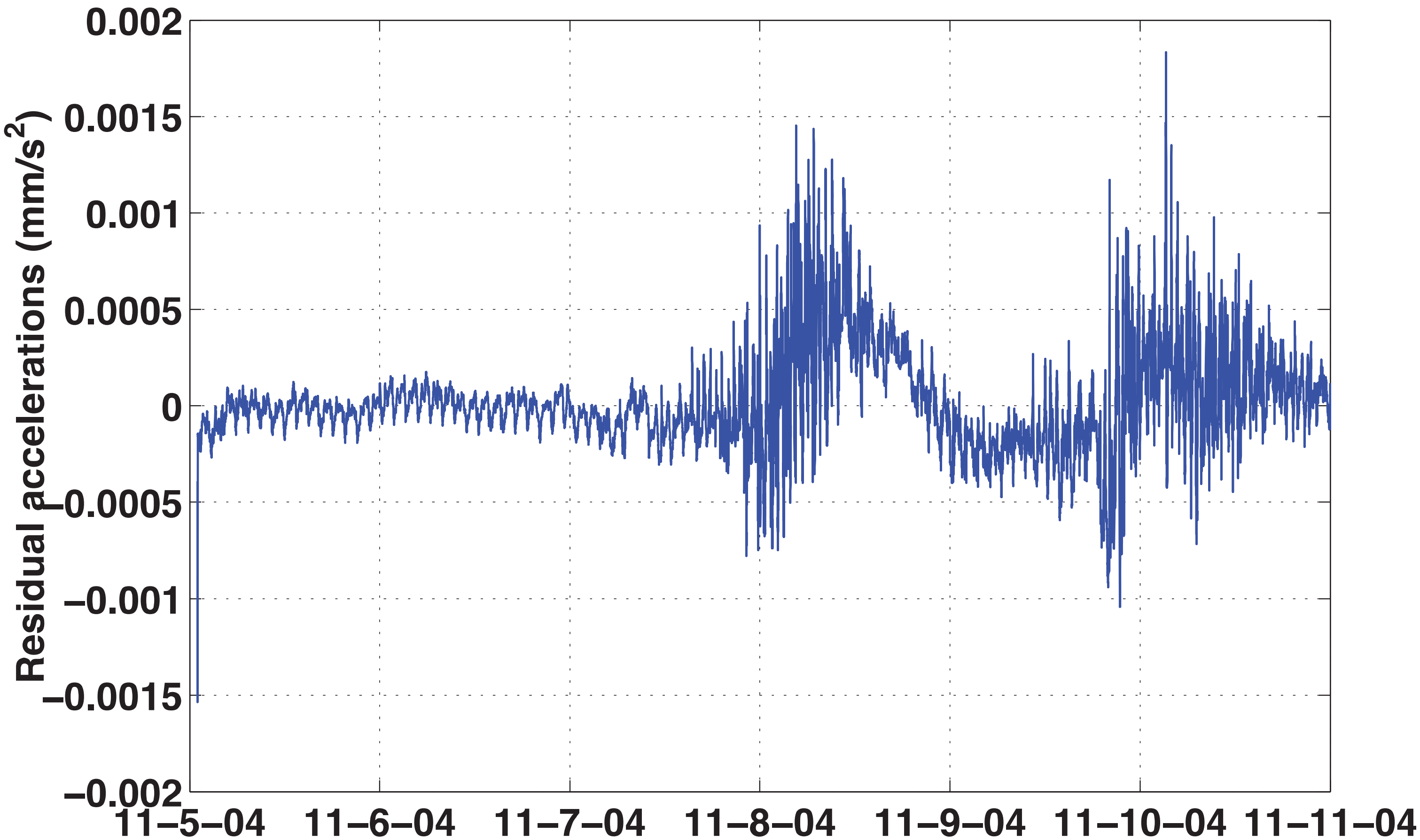




Variations in Data Set: Time differences of

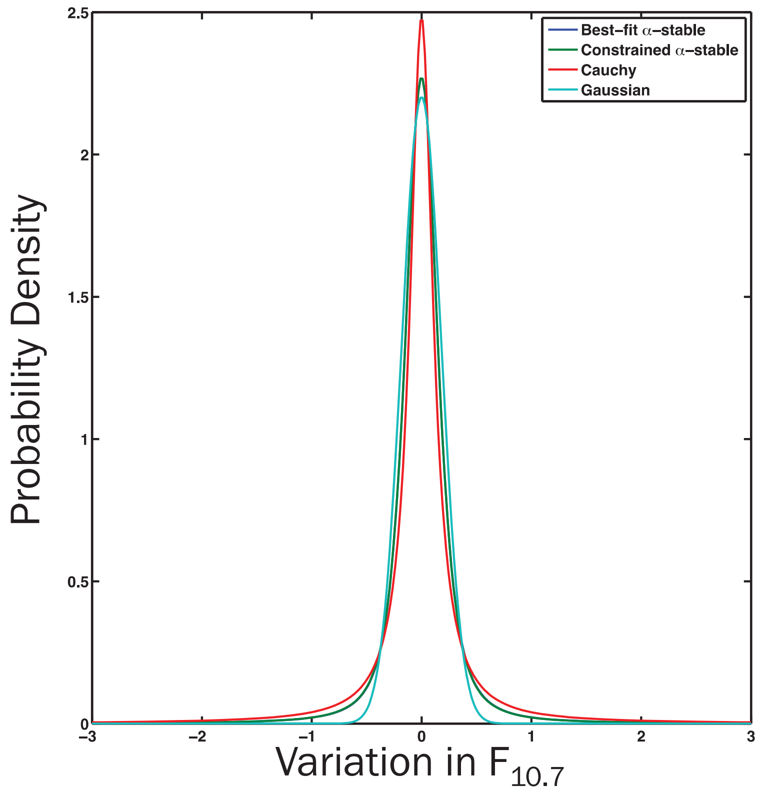


Drag Residuals from CHAMP are Not Gaussian



Fits to Stable Distributions Suggest a Cauchy Model

Fit PDFs for $F_{10.7}$ Data



Stable Distribution Fits

Data	Concen-tration*	Asym-metry	Scale
$F_{10.7}$	1.38	0.02	0.13
A_p	1.03	0.0014	3.46
CHAMP drag	1.34	0.6121	0.0001

*Gaussian = 2.0, Cauchy = 1.0

The Idan-Speyer Scalar Cauchy Estimator (ISCE)

- Given a linear scalar system

$$x_{k+1} = \phi_k x_k + w_k$$

$$y_k = H_k x_k + v_k$$

- With Cauchy inputs and initial condition

$$\mathrm{p}_{\mathsf{x}_0}(x_0) = \frac{\alpha/\pi}{(x_0 - \bar{x}_0)^2 + \alpha^2}$$

$$\mathrm{p}_{\mathsf{w}_k}(w_k) = \frac{\beta/\pi}{w_k^2 + \beta^2}$$

$$\mathrm{p}_{\mathsf{v}_k}(v_k) = \frac{\gamma/\pi}{v_k^2 + \gamma^2}$$

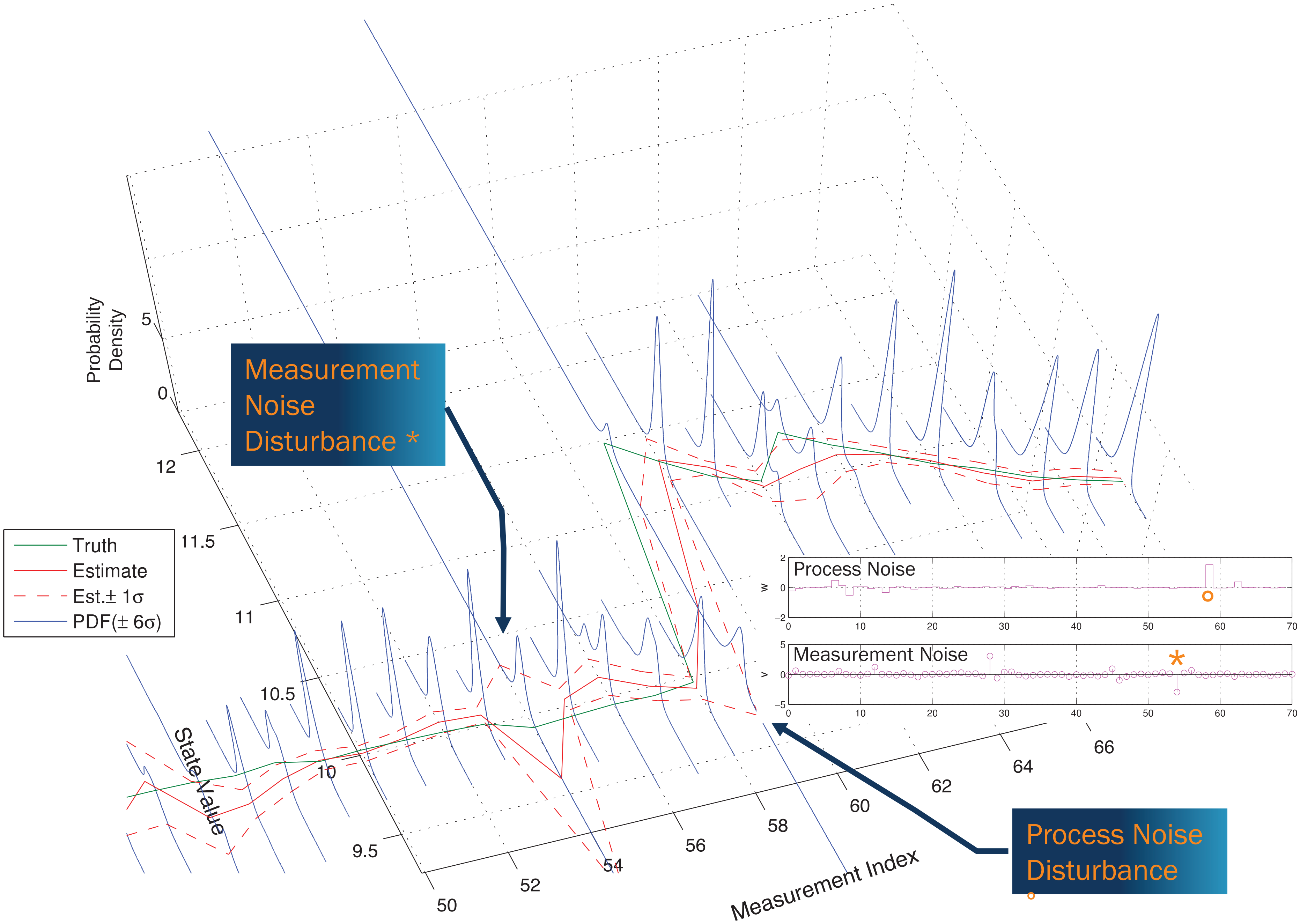
- Posterior PDF

$$\mathrm{p}_{\mathsf{x}_k|\mathbb{Y}_k}(x_k|\mathbb{Y}_k) = \sum_{i=1}^{k+2} \frac{a(i)_{k|k} x_k + b(i)_{k|k}}{(x_k - \sigma(i)_{k|k})^2 + \omega(i)_{k|k}^2}$$

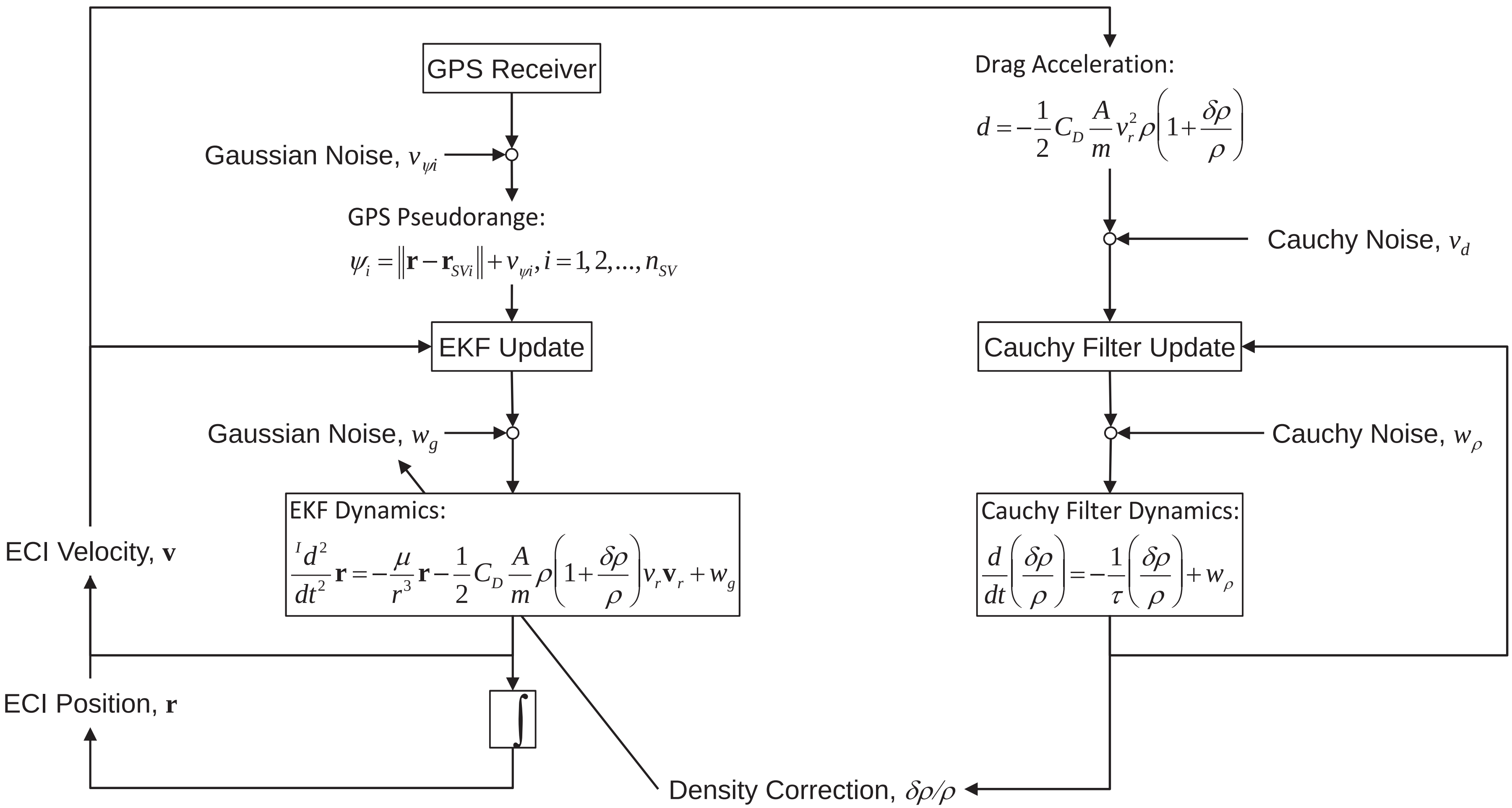
- Has Finite Moments

$$\hat{x}_{k|k} = \mathrm{E}\left[\mathsf{x}_k|\mathbb{Y}_k\right] = \int_{-\infty}^{\infty} \xi_k \, \mathrm{p}_{\mathsf{x}_k|\mathbb{Y}_k}(\xi_k|\mathbb{Y}_k)\,\mathrm{d}\xi$$

$$= \pi \sum_{i=1}^{k+2} \frac{a(i)_{k|k}\left(\sigma(i)_{k|k})^2 - \omega(i)_{k|k}^2\right) + b(i)_{k|k}\sigma(i)_{k|k}}{\omega(i)_{k|k}}$$


Measurement Noise Disturbance *
Process Noise Disturbance
Probability Density
5
0
12
11.5
11
10.5
10
9.5
State Value
50
52
54
56
58
60
62
64
66
Measurement Index
Truth
Estimate
Est.± 1σ
PDF(± 6σ)
Process Noise
w
Measurement Noise
v

The ISCE May Be Embedded Within the EKF for Density Estimation



Schmidt-Kalman Consider States Encapsulate the ISCE Moments

$$K_s = \begin{bmatrix} P_{ss_{k|k-1}} & P_{sc_{k|k-1}} \end{bmatrix} \begin{bmatrix} H_s^{\mathsf{T}} \\ H_c^{\mathsf{T}} \end{bmatrix} \left(\begin{bmatrix} H_s & H_c \end{bmatrix} \begin{bmatrix} P_{ss_{k|k-1}} & P_{sc_{k|k-1}} \\ P_{cs_{k|k-1}} & P_{cc_{k|k-1}} \end{bmatrix} \begin{bmatrix} H_s^{\mathsf{T}} \\ H_c^{\mathsf{T}} \end{bmatrix} + R_k \right)^{-1}$$

Solve-For States: $\hat{s}_{k|k} = \hat{s}_{k|k-1} + K_s \left\{ z_k - h\left(\begin{bmatrix} \hat{s}_{k|k-1} \\ \hat{c}_{k|k-1} \end{bmatrix} \right) \right\}$

Consider States: $\hat{c}_{k|k} = \hat{x}_{k|k} = \text{ISCE mean}$

$$P_{ss_{k|k}} = \left(I - K_s \begin{bmatrix} H_s & H_c \end{bmatrix}\right) P_{ss_{k|k-1}} - K_s H_c P_{cs_{k|k-1}}$$

$$P_{sc_{k|k}} = P_{cs_{k|k-1}}^{\mathsf{T}} = \left(I - K_s \begin{bmatrix} H_s & H_c \end{bmatrix}\right) P_{sc_{k|k-1}} - K_s H_c P_{cc_{k|k-1}}$$

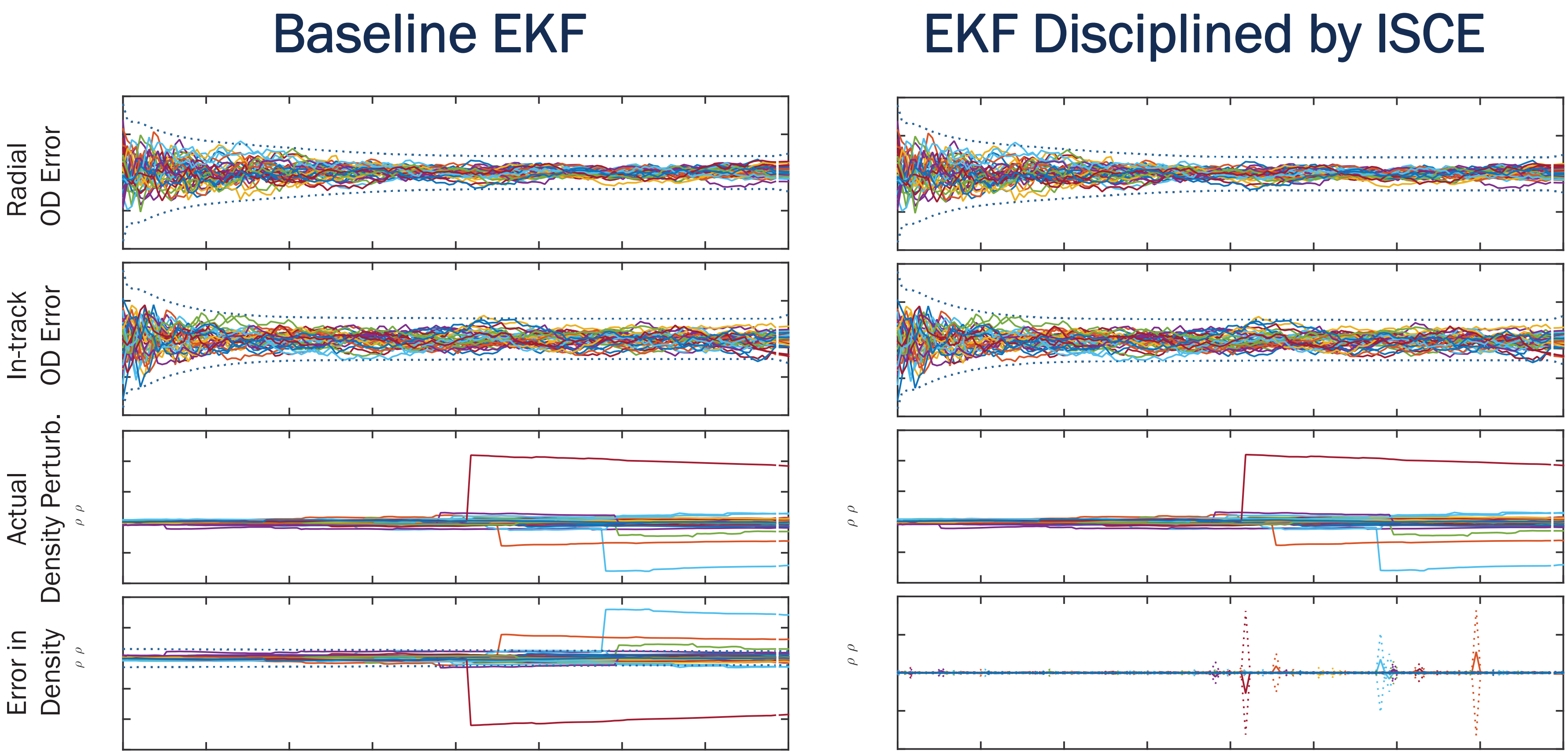
$$P_{cc_{k|k}} = p_{x_{k|k}} = \text{ISCE variance}$$

$$\begin{bmatrix} \hat{s}_{k+1|k} \\ \hat{c}_{k+1|k} \end{bmatrix} = \int_{t_k}^{t_{k+1}} f\left(\begin{bmatrix} \hat{s}(\tau) \\ \hat{c}(\tau) \end{bmatrix} \right) \mathrm{d}\tau$$

$$\begin{bmatrix} P_{ss_{k+1|k}} & P_{sc_{k+1|k}} \\ P_{cs_{k+1|k}} & P_{cc_{k+1|k}} \end{bmatrix} = \Phi(t_{k+1}, t_k) \begin{bmatrix} P_{ss_{k|k}} & P_{sc_{k|k}} \\ P_{cs_{k|k}} & P_{cc_{k|k}} \end{bmatrix} \Phi(t_{k+1}, t_k)^{\mathsf{T}} + Q(t_{k+1}, t_k)$$

Definitive OD is Similar, and Density Estimation is Superior (36 trials)

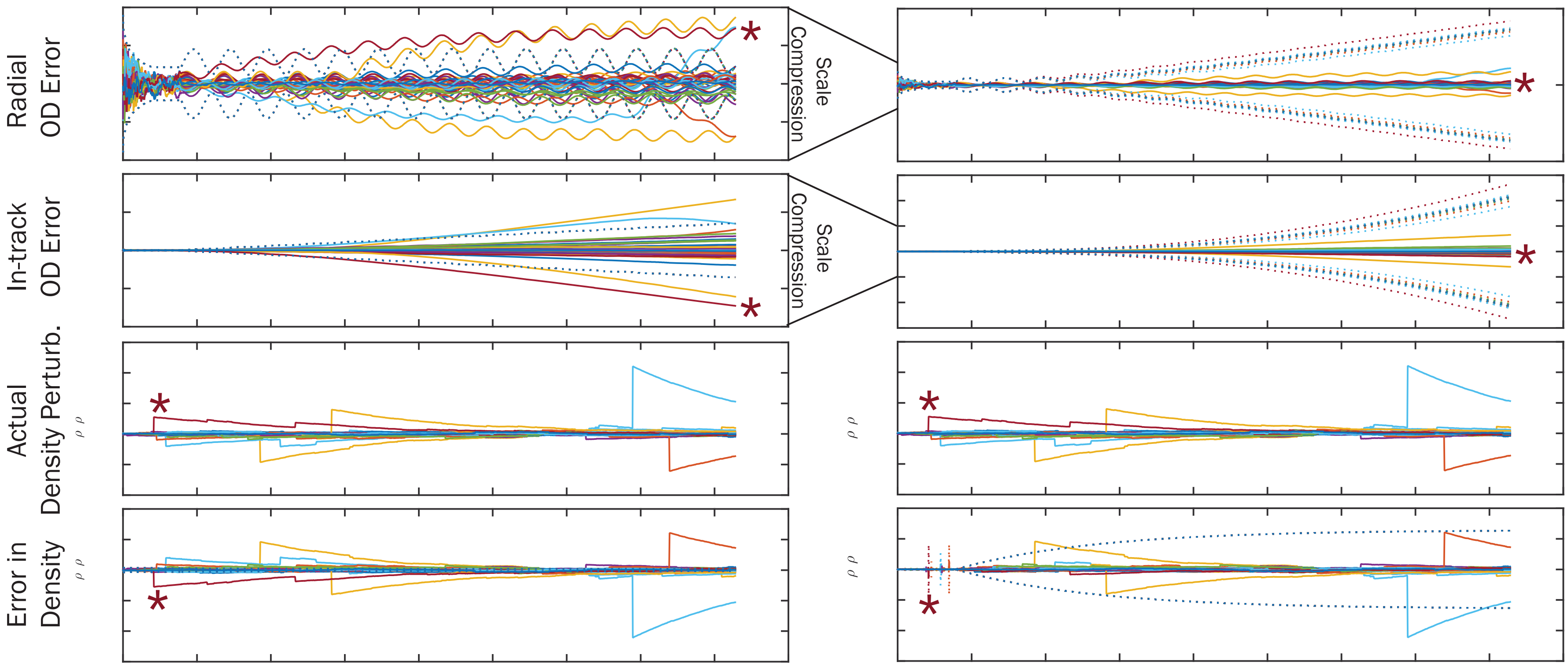
Dashed lines = ±3σ formal error



Predictive OD: Superior for Observed, and Robust to Unobserved, Density Dispersions

Dashed lines = ±3σ formal error
* Observed during definitive span

Baseline EKF ... EKF Disciplined by ISCE



Conclusions

- Space weather data show heavy-tailed characteristics that are better modeled by Cauchy than Gaussian
- Cauchy estimator (ISCE) may be embedded in EKF, using Schmidt-Kalman consider framework, for density estimation
- Definitive OD performance indistinguishable from EKF
- Predictive OD performance superior to EKF