

CONCEPTUAL AND PROCEDURAL UNDERSTANDING OF ALGEBRA
CONCEPTS IN THE MIDDLE GRADES

A Thesis

by

HEATHER KYLE JOFFRION

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

December 2005

Major Subject: Curriculum and Instruction

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ABSTRACT

Conceptual and Procedural Understanding of Algebra Concepts in the Middle Grades.

(December 2005)

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Chair of Advisory Committee: Dr. Gerald Kulm

In this study, the balance between conceptual and procedural teaching and its effect on the development of algebraic reasoning was examined.

Participants included two seventh grade mathematics teachers and their students in targeted classes ($N = 33$). One video taped lesson from each teacher was selected for in-depth analysis of the balance between conceptual teaching, procedural teaching, and classroom time that included neither. Student participants took pretest and posttest algebra tests. Distribution of student responses and scores were analyzed for the degree of conceptual understanding demonstrated by students and then related to observed instructional practices.

It was concluded that the students of the teacher with a more explicit conceptual emphasis in her lessons performed better on the test and were better able to exhibit flexible reasoning in unfamiliar contexts. Students whose teacher focused more heavily on procedural instruction without conceptual connections were less flexible in their reasoning and unable to apply some of the procedures taught in class.

DEDICATION

I dedicate this work to those teachers who give everything they have to make a positive impact on the lives of their students.

ACKNOWLEDGEMENTS

There are many more individuals whose help I might acknowledge in this section than space will allow. The support that I have experienced during this challenging year has been incredible and humbling. Foremost, I acknowledge that without the strength provided by the Lord, I would not be submitting this manuscript. I also thank Him for providing such a supportive family to surround me. I'm grateful to my encouraging parents, a loyal brother, a supportive husband, and true and caring friends. I am thankful also for the challenges and encouragement constantly provided by the faculty and students of Navasota High School.

In finishing this manuscript, I would like to acknowledge most sincere gratitude to the faculty and graduate students in the Mathematics Education department at Texas A&M University. Specifically, I acknowledge the efforts of Dr. Gerald Kulm, Dr. Mary Margaret Capraro, Dr. Robert M. Capraro, Dr. Victor Willson, Dr. Oi Man Kwok, Shirley Matteson, Tamara Carter, and Margaret Mohr in helping this project come to fruition.

Finally, I am grateful for the involvement of the teacher and student participants in the Middle School Mathematics Project. This work is partially funded by the National Science Foundation IERI (Interagency Educational Research Initiative) grant #REC-0129398: Improving Mathematics Teaching and Achievement through Professional Development. However, all opinions and conclusions expressed in this manuscript are solely my own.

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CHAPTER I

INTRODUCTION

Statement of Problem

For all high school mathematics courses, algebra is widely regarded as a “gatekeeper.” Students in the United States fail mathematics more frequently than any other subject (Jacobson, 2000). The world of higher level mathematics and the opportunities that come with it are closed to students who do not succeed in high school algebra. Preparation for algebra in the middle grades is critical to student success in high school mathematics (Silver, 2000).

Using symbolic algebra to represent and solve linear equations is one of the expectations under the *Algebra* content strand for 6-8 grade mathematics in the National Council of Teachers of Mathematics (NCTM) *Principles and Standards for School Mathematics* (2000). Understanding of linear equations and algebraic relationships is fundamental to preparing students for success in future algebra experiences. Silver (2000) asserted, “In the middle grades, students need to develop a thorough understanding of, and representational facility in, dealing with linear functions and equations” (p. 22). Researchers have examined a variety of strategies for teaching the solving of simple linear equations. In the present study, the relationship between classroom emphases on conceptual understanding and/or procedural knowledge of linear algebra concepts and student achievement in algebraic reasoning were examined.

This thesis follows the style of the *Journal for Research in Mathematics Education*.

Rationale

Mathematics teachers in modern society are called to address the needs of many different learning styles. In the past, not all students were expected to take Algebra in high school. One semester of mathematics study in the ninth grade, limited to operations with positive rational numbers, was required for high school graduation, according to a bulletin published by the Los Angeles City Schools (Butler, 1968). This bears sharp contrast to expectations for today's high school students. The NCTM *Principles and Standards* (2000) state that all students should be enrolled in enriched, meaningful mathematics courses during each of the four years of high school, receiving necessary support to be successful. Texas now requires Algebra I and Geometry for all students, and three years of mathematics study for graduation (Texas Education Agency, 1998). This leaves today's teachers, especially those who serve middle school and high school students, with a tremendous responsibility to make algebra accessible for all students.

Functions, equations, graphs, and algebraic relationships and vocabulary receive attention in middle grades mathematics curricula. Students' understanding of these concepts, even before a formal Algebra course, prepares them for future success. Middle grades teachers need to know the most effective ways to enable their students' algebraic reasoning. Teachers use both conceptual and procedural methods of instruction when teaching students to solve problems requiring algebraic reasoning. Successfully balancing conceptual and procedural emphases in classroom instruction will support students as they begin to develop the algebra skills needed for success in high school mathematics classes.

Research Questions

Given these suppositions, the following will be investigated.

1. How does the degree to which a teacher emphasizes conceptual understanding of solving equations and the amount of emphasis placed on procedural knowledge correspond with students' ability to solve algebraic equations?

2. How does the degree to which a teacher emphasizes conceptual understanding and the amount of emphasis placed on procedural knowledge correspond with students' ability to model equations from verbal problem representations?

3. How does the degree to which a teacher emphasizes conceptual understanding and the amount of emphasis placed on procedural knowledge correspond with students' ability to recognize algebraic relationships in tabular representations?

4. How does the degree to which a teacher emphasizes conceptual understanding and the amount of emphasis placed on procedural knowledge correspond with students' demonstration of conceptual understanding and flexibility in problems requiring an application of algebraic concepts?

Educational Significance

The findings of this study will contribute to a body of research addressing the need for conceptual and procedural understanding and their connection. In order to provide mathematical foundations for student success, research-based pedagogical methods are mandated by the No Child Left Behind Act of 2001 (NCLB). By researching some effective teaching practices for critical pre-algebra skills, this study

will deepen understanding of this subject in the field of mathematics education, and better equip those who plan curriculum and teach students to meet the needs of middle grades mathematics students.

Definition of Terms

The following terms are listed in alphabetical order with the definition as will be used in this study:

1. Conceptual understanding/knowledge: understanding of ideas and generalizations that connects mathematical constructs (Ashlock, 2001) and is rich in relationships (Hiebert & Lefevre, 1986). Specifically in this study, conceptual understanding relates to the meaning of equations, equality, relationships between quantities, and variable.

2. Procedural understanding/knowledge: understanding that is focused “on skills and step-by-step procedures without explicit reference to mathematical ideas” (Ashlock, 2001, p. 8)

CHAPTER II

LITERATURE REVIEW

Conceptual and Procedural Understanding

In order to succeed in algebra, students need to develop both conceptual understanding of numbers and relationships and procedural skills in using them efficiently. With the completion of the Third International Mathematics and Science Study (TIMSS), much attention has come to procedural teaching and conceptual teaching. The video analysis portion of this study included a random sample of eighth grade classrooms across the United States, Germany, and Japan. Japanese students consistently rank in the top of international studies of mathematics achievement, and German students also typically outscore students in the United States. The TIMSS created a window into classrooms that revealed very different treatment of procedural knowledge between the U. S., Germany, and Japan. In Japan, students are encouraged to invent their own procedures for solving demanding problems. Instruction is problem-based, student-centered, and carefully structured to encourage students to arrive at desired outcomes. In the United States, teachers tend to present definitions and procedures and students are expected to practice them. German classrooms also revealed an emphasis on procedure, but the level of rigor far exceeded that of the American curriculum (Stigler & Hiebert, 1999).

Not all teaching in high-achieving countries is as open and consistently conceptual as that of Japan. Reflecting on the high achievement of East Asian students, Leung and Park (2002) presented teachers with arithmetic problems, asking them to

explain to the researcher how to solve it in the same way that they would explain it to a second-grader. The sample of teachers came from Korea, Hong Kong, and the United States. Across the study, nearly all the teachers emphasized procedure, and few were able to explain the concepts behind the procedures clearly. From this study, the researchers concluded, “. . . the assumption that one must first understand before one can have meaningful practice may not be valid. The process of learning very often begins with gaining competence in the procedure, and then, through repeated practice, students begin to learn the concepts behind the procedures” (p. 127). Leung and Park continued, asserting that procedural learning could only be the vehicle for understanding if it was contained in a curriculum based on sound concepts, sequenced to help students learn concepts as they develop procedures.

Boaler (1998) reported on a study that centered on the differing effects of problem-based conceptual instruction and traditional, more procedural instruction. Two schools were examined, one employing a traditional curriculum and the other employing solely activity-based instruction. After three years, the researchers determined that students whose instruction had been primarily procedural were unable to apply mathematical knowledge and problem-solving skills in unfamiliar situations. Students who learned in a problem-based environment were more flexible and better able to apply their mathematical understanding in a variety of academic and non-school situations.

There has been some debate over the relationship between conceptual and procedural knowledge and which type of understanding develops first as students encounter new mathematics (Gelman & Williams, 1998; Siegler, 1991; Siegler &

Crowley, 1994). Rittle-Johnson, Siegler, and Alibali (2001) proposed a mediating viewpoint, that, in fact, the two types of knowledge are not necessarily distinct, but rather opposite ends of a continuum and improvements in one type of understanding typically result from or result in improvements in the other type. They assert that the process of development of concept and procedure is iterative and closely intertwined.

In a similar study, Star (2002) reiterated Rittle-Johnson, Siegler, and Alibali's (2001) point that conceptual and procedural knowledge are not distinct entities. The researcher gave three examples of student solutions to a complicated single-variable equation. Each student was able to solve the equation successfully, but the degree of efficiency and sophistication of the solutions varied. The strategies and procedures employed by the students were very distinct and clearly reflected varying levels of conceptual understanding as manifested by their procedures. Star believes that the skill vs. concept debate has outlived its usefulness and that procedural competence should be bolstered with conceptual understanding, not replaced by it.

Jitendra, DiPipi, and Perron-Jones (2002) studied the effect of instruction that explicitly made conceptual connections on students' (specifically those with learning disabilities) abilities to solve mathematics word problems. They found that a schema-based strategy integrating elements of conceptual and procedural understanding effected a reasonably long-term improvement in students' abilities. These researchers used Anderson's (1989) definition for procedural knowledge as "organization of conceptual knowledge into action units" (p. 24). Without conceptual knowledge, this definition of

procedural knowledge is useless. Likely, this approach to procedural teaching contributed to the success of the students in the study.

Learning Trajectories

Central to much recent research in promoting conceptual understanding is the sequence in which algebra concepts are presented. Specifically, when should those concepts related to writing and solving simple equations be taught? The work of Carpenter, Fennema, Franke, Levi, and Empson (1999) in *Cognitively Guided Instruction*, and others who explored the problem solving methods and capabilities of young children inspired similar investigations into the process of problem solving in secondary students (Nathan & Koedinger, 2000). These authors suggest that students are able to solve many problems, especially those situated in familiar contexts, without explicit instruction in the operations required. Their investigations have shown students may be developmentally able to solve word problems before performing the associated symbolic operations.

The results of these investigations have challenged many of the beliefs about students' developmental stages in the learning of algebra. Nathan and Koedinger (2000) conducted a study to compare teachers' and researchers' beliefs about the learning trajectory associated with algebraic reasoning with the learning path actually demonstrated by student performance. These investigators examined the perceived difficulty of problems presented in symbolic formats (number sentences) and those presented in verbal formats (story problems), as well as problems of both types that

involved result-unknown (primarily arithmetic) or start-unknown (primarily algebraic) solutions. Discovering that teachers' and researchers' beliefs aligned with those implicit in algebra textbooks, the results suggest these professionals tend to believe that students' skills in symbolic manipulation precede students' abilities to apply those skills in problem-solving contexts. However, the data from this study revealed that students often were more successful in solving verbal, contextualized problems than those requiring symbolic manipulation only.

Koedinger and Nathan developed their ideas further in a more recent study (2004), determining that the previously observed trajectory could be attributed, in part, not to familiarity with situations presented in problems, but to difficulty with formal symbolic representation. This led to a conclusion that the differences in student learning were related to students' representation abilities. Problems situated in verbal contexts were easier for students to represent than those presented to the students in symbolic form.

In another study of early algebraic reasoning, Van Ameron (2003) determined that symbolizing and reasoning capabilities do not necessarily develop coincidentally. This study also revealed the value of encouraging students' informal understanding of algebraic reasoning and notations in helping students to bridge the gap between their arithmetic experiences and those in the realm of algebra. Student participants (sixth and seventh grade students) often demonstrated much stronger skills in solving formal and informal problems that require algebraic reasoning than in symbolizing equations.

Students' abilities to solve simple word problems with arithmetic can and should be connected to the formal algebraic symbolic notation.

Recent mathematics education reform stresses the importance of presenting students with opportunities to encounter problems that cannot be solved using routine methods (NCTM, 2000). Problem solving, a very important skill in mathematics, should be an important part of the algebra curriculum. Many teachers and curricula cling to a traditional view of mathematics learning that gives students a set of skills in symbol manipulation, followed by problem solving tasks that require application of those skills. Reformed mathematics suggests students may learn concepts through problem solving that may later be enhanced, supported, or refined by symbol manipulation and mathematical vocabulary (Latterell & Copes, 2003).

Symbol manipulation is a procedural skill. According to the study by Nathan and Koedinger (2000) mentioned above, traditional curriculum implicitly states that procedural skills precede students' conceptual understanding of material. However, their research suggests that this is likely not the case. This work and the others mentioned here lay foundation for the idea that students develop conceptual understanding before developing real procedural understanding.

Transition from Arithmetic to Algebra

Pre-algebra has been defined by some as the transition from the arithmetic of elementary school mathematics to the algebraic and more abstract skills required at the secondary level (Kieran & Chalouh, 1993). Stacey and MacGregor (1997) wrote of the

importance of laying a solid foundation for algebraic reasoning in the middle grades. They suggested that many aspects of algebraic reasoning can be developed through properties of arithmetic, and that teachers can help students develop their understanding of both simultaneously. They stressed the importance of teaching students to see processes and operations holistically, and stressing relationships between numbers instead of focusing primarily on the answer. Discussing the value and efficiency of informal approaches with students may help them make the transition from these intuitive approaches to more formal algebraic methods, but it is very important that students do make the transition. Students who make that transition in the middle grades will be more successful in their high school studies.

Van Dooren, Verschaffel, and Onghena (2003) conducted a study of pre-service primary teachers as well as pre-service secondary mathematics teachers. They sought to determine the kinds of strategies these pre-service teachers used in solving various problems. Participants focusing on secondary-level mathematics tended to use algebraic solution strategies, even when arithmetic strategies would have been simpler or more efficient. Participants studying primary education more frequently focused their solutions on arithmetic and were less likely to be able to solve more complicated problems that required algebraic reasoning. From this dichotomy, the researchers concluded that pre-service teachers need direct instruction in the transition between arithmetic and algebraic solution methods. They were concerned that primary teachers would be ill-equipped to help their students develop problem-solving skills and the underpinnings of algebraic reasoning and lower secondary school teachers (including

those in middle school) would be ill-equipped to facilitate students' transition from arithmetic to algebra.

Conceptual Understanding in Pre-Algebra

Concepts of equality. Students transitioning from arithmetic to algebra often struggle with misconceptions about the meaning of the *equals* sign. Recently the use of concrete models in teaching solving equations has become a more common practice to help students develop conceptual understanding of equality. In the majority of prior experiences, the *equals* sign was active. It indicated to a student that the “answer” should follow it. In algebra, students must see the *equals* sign as relational, denoting either side has equal value. Students as early as third grade can conceive of this aspect of equality when they are given experiences that feature the *equals* sign in situations that allow students to recognize quantitative sameness (Saenz-Ludlow & Walgumuth, 1998). Too often, children do not have such experiences with equality until formal algebra study.

The Balance Model for teaching about equality was described and studied by Vlassis (2002). This concrete model features the *equals* sign as the pivot on a balance scale. In order to maintain balance, whatever is added to or taken away from one side must be added or taken away from the other. This author explored the benefit of this concept in the supporting eighth graders' understanding as they solved linear equations. Citing various proponents and opponents of the use of manipulatives in teaching algebra concepts, Vlassis determined that the Balance Model and other concrete models for

solving equations were limited. Their usefulness seemed to disappear when modeling problems involving negative quantities.

Pirie and Martin (1997) were among the opponents of the Balance Model mentioned by Vlassis. They expressed concern with the model because of its apparent disconnect from the symbolic algebraic representation. Students often do not make the transition from weighing in a pan balance and drawing pictures of equations to the symbolic representation. These topics must be explicitly taught. In Pirie and Martin's study, some students persisted in believing that all variables represented weights or counters. These participants were not flexible in seeing the variables as quantities in word problems. They did recognize the value of concrete models in directing students to the need for symbolic representation and solving equations in the more conventional way.

Concept of variable. Pre-algebra students need to have a developed concept of the meaning of variable. This understanding should be rooted in experiences with patterns and generalizations. Variables are difficult, even for mathematics teachers, to describe in few words. The term takes on many different meanings in the study of algebra and therefore the concept is difficult for students. They should be treated as tools for expressing relationships and research suggests that it may be helpful for students to verbally express a generalization before attempting to represent it using symbols (Schoenfeld & Arcavi, 1988).

Misconceptions about variables are common among students who are learning to use them. The variable x has been mistaken for the multiplication symbol by many

students (Martinez, 2002). Wagner (1983) told of one of her students who stated that the next consecutive integer that followed x was y . In another investigation by Wagner (1977), she presented students with two equations, identical except for different letters were used as the single variable. The researcher received a variety of responses when she asked participants if different solutions would be obtained from solving both equations. Confusion was evident in responses including comments on which letter came first in alphabetical order. Other students believed it was impossible to determine whether or not they were the same until both had been solved. Though these studies took place long ago, there is little reason to believe confusion about the proper use of variables has been resolved.

Moseley and Brenner (1997) discovered that students who were instructed in the use of variables with multiple representations demonstrated a more profound understanding of their usefulness. Placing participants' work on a continuum from arithmetic-based to algebraic, they determined that the use of multiple representations was critical in helping students bridge the conceptual gaps between arithmetic and algebra and learn to use variables to generalize relationships.

Linear Equations

Solving equations. Notably, learning to solve equations should involve more than memorizing a set of rules. Students who understand only an algorithmic method of solving equations will experience difficulty when they encounter equations in different forms, solving for different variables, and working with non-linear equations later in

their mathematics careers. Perso (1996) expressed concern that students who solve equations only by a set of memorized rules tend to have misconceptions about solving equations. It is common practice, Perso claimed, for frustrated teachers to teach students using rules instead of encouraging conceptual understanding of algebraic processes. When the idea of inverse operations, for example, is eclipsed by the memorized rule “Change the side, change the sign,” students will not likely see that inverse operations can be performed without changing the equality of the equation.

Conceptual understanding of solving equations may result in part from effective implementation of multiple representations. In a study of six pre-algebra classes, Brenner et al. (1997) developed an assessment tool to evaluate middle grades pre-algebra students’ skills in problem solving. Three of the participating classes were assigned to a treatment group and were taught about equations via a problem-based, reformed instructional unit that emphasized representation before symbol manipulation and solving equations. The other three classes followed the traditional pre-algebra curriculum set forth by the textbook used in all the participating classes. They determined that the reformed curriculum did not necessarily produce students more capable in symbol manipulation and solving equations. However, students who participated in the reformed curriculum did exhibit stronger problem representation skills, a critical skill for success in algebra.

Solving equations is not limited to finding a solution. It is important, as Wagner and Parker (1993) claim, to encourage students to check their work. Understanding that a variable in the original equation can be replaced by the value determined by the solution

is a powerful tool and demonstrates knowledge of the nature of variable and the purpose of solving equations. Fostering metacognition, students should recognize and correct errors and monitor their own work, very important skills in preparing students to be more independent learners.

Modeling equations from verbal representations. Among students' greatest difficulties in pre-algebra is modeling equations from problem situations. Translating from verbal relational statements to symbolic equations, or from English to "math," causes students of all ages a great deal of confusion. In a study Rosnick (1981) conducted in his college-level statistics class, 40% of sophomore and junior business majors were unable to select the meanings of the variables in a direct variation equation written from a simple English statement.

Like the solving of equations, modeling equations can be taught with a procedural or conceptual emphasis. Lodholz (1990) observed that writing equations from word problems is often a skill taught in contrived situations or in isolation. Mechanical word problems that require students to write an expression that represents "5 more than 3 times a number," when taught apart from opportunities for application, can cause students difficulty when interpreting meaningful sentences later. This gives students a procedural method for doing what, by its nature, should be conceptual.

Children may translate English sentences to mathematical expressions, simply moving from left to right. "Three less than a number" is interpreted by many students as " $3 - x$ " since the words "less than" (which mean to subtract, they have always been told) follow the 3. Teachers must be aware of these misconceptions and address them in

instruction (Lodholz, 1990). MacGregor and Stacey (1993) explored this well-documented hypothesis and their data suggested deeper cognitive reasons for students reversing variables or putting terms in the wrong order. The students in their study actually did make an attempt to understand the situation being described in a problem, but were unable to represent their cognitive model symbolically.

Even still, writing equations from word problems is a difficult skill for middle grades students, whether caused by cognitive misconceptions or literal translation. Their inclination to translate directly from English sentences to algebraic expressions may be augmented by the procedural method many teachers use when addressing this topic in class. It is not uncommon for teachers to encourage students to look for “key words” in word problems that signify a particular operation. Wagner and Parker (1993) stated, “Though looking for key words can be a useful problem-solving heuristic, it may encourage over-reliance on a direct, rather than analytical, mode for translating word problems into equations” (p. 128).

Recent research has demonstrated that teaching lower-achieving students a strategy for checking their symbolic representation once generated can be very useful in improving student understanding (Pawley, Ayres, Cooper, & Sweller, 2005). The checking method used by the students in this study required students to ask themselves, after finishing the problem, which quantity was bigger according to the verbal representation and then which quantity in their equation would be bigger. Though at first this strategy appeared to disadvantage and confuse students, after practice and acquisition, low-level students improved their representation skills greatly.

Encouraging conceptual understanding of this skill is typically done by fostering mathematical communication. Mathematics is a language for communication and a tool for new discovery. Like any language, it has grammar rules and syntax structure that can be difficult for students to master (Esty, 1992). Students must have skills in reading comprehension and reasoning before an algebraic expression or equation can be derived. The use of language in classrooms is critical in developing these skills with middle grades students. Students benefit from instruction that includes many types of mathematical and verbal communication, including writing and solving word problems, discussing solution strategies and concepts, and journaling (Esty, 1992; Johanning, 2000; Pugalee, 2004). Before students learn to represent algebraic situations symbolically, they should have opportunities to discuss them in easily understood, everyday language, thus developing their conceptual understanding (Kieran & Chalouh, 1993).

Conclusions

Mathematics education researchers have made significant strides toward understanding the balance of procedural and conceptual knowledge in the classroom, but many have come to conflicting conclusions about their relationship and the trajectory in which they typically develop. The call for decreased emphasis on procedure in American classrooms, however, is clear (Stigler & Hiebert, 1999). Many researchers believe that an increased emphasis on conceptual understanding will begin to minimize the gap that exists between Western countries and much of the rest of the world. Others continue to assert that we must not eliminate procedural teaching from the curriculum. Many agree

that both are necessary and integrating the two types of understanding is critical for greater student understanding.

CHAPTER III

METHOD

This investigation is part of the Middle School Mathematics Project, a collaborative effort between the University of Delaware and Texas A&M University, supported by Project 2061 of the American Association for the Advancement of Science (AAAS). This is a five-year IERI grant whose purpose is to examine curriculum materials, factors affecting student learning, and professional development support for teachers in middle grades mathematics. The study design was primarily qualitative.

Participants

Two seventh grade teachers and their students in targeted classes were examined in this study. Data were collected during the 2003-2004 school year. Each participating teacher was filmed three times teaching algebra lessons over the course of the year. One of the participating teachers is employed in a suburban district. She will hence be referred to as Teacher A. The other teacher, Teacher B works with a diverse population in a rural district. Both classes were general seventh grade mathematics classes. Neither contained exclusively advanced students nor students requiring remediation. Both teachers' curriculum was Glencoe's *Mathematics: Applications and Connections, Course 2*, though the teachers relied on and employed the textbook to varying degrees.

Together, Teacher A and Teacher B had 43 students complete the pretest, and 33 complete the posttest. Of these 33 students, 20 were students of Teacher A and 13 were

students of Teacher B. Only students who completed both the pretest and posttest were considered in the analysis.

Data Collection

During September 2003, parallel forms of the algebra test, containing similar questions presented in slightly different order were administered to student participants. A posttest identical to the pretest was administered to students in May at the end of the school year. The test consisted of 15 multiple choice and short answer questions, as well as one extended response item. Short and extended response items were scored according to a rubric by certified graders.

In addition to these data, one lesson video from each of the teachers was analyzed in depth for its degree of conceptual and procedural emphases. Selected lessons targeted learning goals related to representing linear equations and functions. The classroom videos selected for this analysis were approximately the same length. The taped segment for Teacher A featured 42 minutes and 10 seconds of classroom footage. The segment for Teacher B contained 38 minutes and 30 seconds of footage, but sound was unavailable for 1 minute and 40 seconds. Only 36 minutes, 50 seconds were analyzed for Teacher B.

The lesson videos were divided into ten-second intervals. The videos of Teachers A and B contained 254 and 221 ten-second intervals, respectively. Each interval was carefully watched at least twice by the researcher, then coded as C (Conceptual), P (Procedural), or N (Neither) for the type of understanding emphasized in the segment. A

brief description of the interactions, teacher questions, and student behavior was noted in the spreadsheet as well. This included notes of the researcher.

In order to increase the consistency of the coding scheme, the researcher sought specific indicators of conceptual or procedural teaching present during each time interval. These indicators were extracted from the rich descriptions of conceptual and procedural understanding found in Hiebert & Lefevre's chapter in Hiebert's pioneer work in the area (1986). The indicators of conceptual teaching and procedural teaching can be found in Appendix A. Before applying the analysis tool to the videos of participating teachers, it was used to analyze another lesson video. An iterative process was used to refine and add detail to the indicators as needed.

To ensure reliability of coding results, another graduate student in mathematics education was selected by the researcher. With the researcher, this graduate student viewed video clips and discussed the coding indicators, as applied to segments of the lesson. After an hour of this training, the second student coded three minute segments from each video. The percentage of agreement between the researcher and second graduate student was 92.1%. After the second graduate student had coded the three minute sections, the second graduate student viewed each ten-second interval for which the appropriate coding was unclear to the researcher in the original viewing (four total intervals). Each interval was discussed and the researcher made the final decision for the appropriate coding.

The two other lessons available for each teacher were also watched by the researcher. Instances of instruction related to the research questions and specific test

items were noted by the researcher and revisited as needed during the analysis.

Transcripts of specific interactions and descriptions of teacher behavior that addressed research questions and test items were recorded.

Data Analysis

Responses on multiple choice items selected by students of each teacher were compiled and entered into a spreadsheet. Scores for each student provided by scorers on the free response and extended response items were also entered into the spreadsheet. A total score on all items was calculated for each student for the pretest and posttest. Two *t*-tests were calculated ($\alpha = .05$) to examine the differences in the performance of the classes on the pretest and on the posttest.

Selected items from the test were grouped according to the student competency they assessed and the research question they addressed. Each item selected for this study can be found in Appendix B. The first research question required an assessment of students' ability to solve equations and Items 1 and 15 were selected to address this question. The second research question addressed students' ability to model equations from a verbal representation. Items 2, 3, and 8 were chosen by the researcher to address this student ability. The third research question addressed students' ability to analyze relationships within a table. To assess this student ability, items 5, 7, 11, and 16B were analyzed. The final research question required an assessment of students' ability to apply algebraic concepts. To do this, items 4, 9, and 16C were selected.

Each item and its answer choices were analyzed for content and potential misconceptions. Student responses were also analyzed, including correct responses and incorrect answers given by many students. In the case that an incorrect answer choice was selected by many students, teaching methods contributing to student misconceptions were sought from the analyzed videos.

The degree to which conceptual knowledge or procedural knowledge was needed to answer given items was noted. The type of understanding needed to answer an item correctly was related to student responses and teachers' emphasis in delivery of instruction. Conceptual and procedural teaching approaches contributing to student success or misconceptions on particular items were explored and described.

Results of this study were connected to past findings on the importance of conceptual understanding in enhancing students' procedural knowledge of verbal and symbolic algebraic expressions. In light of the current importance of using multiple representations to convey algebraic ideas, this research supported the importance of those ideas and revealed the types of questions students whose teachers emphasize different aspects of understanding are better able to answer.

CHAPTER IV

RESULTS

The balance between conceptual and procedural emphasis in classroom lessons is central to each of the four research questions of this study. The first section of this chapter addresses the results of the video analysis that were used to examine conceptual and procedural emphases in the participating classrooms. A section describing the overall student achievement of the participating classes follows. The final section containing the analyses of the test items and student responses is divided into four sub-sections, each addressing one of the four research questions, which follow:

1. How does the degree to which a teacher emphasizes conceptual understanding of solving equations and the amount of emphasis placed on procedural knowledge correspond with students' ability to solve algebraic equations?
2. How does the degree to which a teacher emphasizes conceptual understanding and the amount of emphasis placed on procedural knowledge correspond with students' ability to model equations from verbal problem representations?
3. How does the degree to which a teacher emphasizes conceptual understanding and the amount of emphasis placed on procedural knowledge correspond with students' ability to recognize algebraic relationships in tabular representations?
4. How does the degree to which a teacher emphasizes conceptual understanding and the amount of emphasis placed on procedural knowledge correspond with students' demonstration of conceptual understanding and flexibility in problems requiring an application of algebraic concepts?

Video Analysis

Teacher A. In the lesson selected for Teacher A, students explored the differences between two cellular phone plans. One included a free phone and slightly higher monthly rate, while the other charged the customer for a phone but the monthly rate was lower. The lesson observed was the second day that the students had been interacting with this problem. On the first day, students had worked in groups to make graphs of the total cost of each plan and the number of months that had passed. Throughout the lesson observed, the teacher and students made reference to the hand-drawn graphs. During this lesson, students wrote equations for their lines, used graphing calculators to compare the linear graphs, used a table of values generated by hand, and then used a table made by the calculators to make decisions about the best plan for different needs.

Teacher A asked questions that required students to reason flexibly and encouraged conceptual understanding. In one series of questions, she asked the following: a) At three months, which plan is better? b) At nine months, which plan is better? c) Is there a time when both plans cost the same amount? d) If you have \$300 set aside to pay for your phone, when will you run out of money on plan 1? e) When would you run out of money on plan 2? f) How much would using the phone for two years cost? By asking a variety of questions, students must interpret the graph or table, recognize when the independent variable is given and dependent is unknown, and identify the value for the dependent variable when the independent variable is given.

Teacher A's support of conceptual understanding was not limited to her questioning techniques. She constantly encouraged students to make connections between graphs, tables, and equations. Connections between different representations and connections to prior knowledge are one of the most prominent characteristics of conceptual understanding. Teacher A never executed a procedure without explaining (or asking a student to explain) why the technique was mathematically necessary and valid. She also consistently connected the calculator activity to the work the students had done by hand the day before.

Of the 254 ten-second intervals in Teacher A's lesson video, 72.7% were conceptual, 14.2% were procedural, and 13.0% were classified as promoting neither type of understanding. See Table 1 for a summary of these results and a comparison with Teacher B.

The other two algebra lessons for Teacher A were also watched by the researcher. The continuity in the lessons was notable. Each of the three lessons included students creating a table of values that represented something they had experienced or measured, working as a group to represent their table with a graph and an equation, presenting their work to the class, and reinforcing their work at the end by using graphing calculators. In each lesson, the concepts of proportional and non-proportional relationships, unit rate (rate of change), and connections between each representation were emphasized.

Teacher B. The lesson selected for Teacher B included two distinct topics. Students were studying roller coasters and amusement parks. Students spent the first

twenty minutes of class creating a graph that represented the total number of passengers that could ride on a particular roller coaster after a given number of hours if, each hour, 760 passengers can ride. The teacher led the class in creating the graph. This lesson was chosen for analysis because of this portion's alignment to the content presented by Teacher A. In the second half of the lesson, students interpreted a table that gave track lengths of different coasters and time needed for the coaster to travel the track. Students created a bar graph of average speeds after calculating a speed in feet per second for each coaster. Though the second half of the lesson was less focused on explicit algebra topics, there were still many opportunities to promote algebraic reasoning.

The lesson presented by Teacher B had potential to be very conceptual, but her delivery was only minimally conceptual. Questions asked by Teacher B were often sufficiently answered with a simple "yes" or "no," and often the correct answer could be determined by her tone of voice or a statement that led up to the question. During the part of the lesson in which students wrote and graphed an equation to represent the passengers on the roller coaster, the teacher told the students what the appropriate equation was, without reference to the concept of unit rate or what the variables x and y represented. Teacher B then led the students through the creation of a table of values, telling students what operations to do as they filled their tables. Student input here was limited to calculations called for by the teacher.

After creating the table, many opportunities for analysis presented themselves. The teacher explained how to read the completed table, that after one hour, 760 people could ride, after two hours, 1520 people could ride, and so on. She did ask students how

many people can ride in five hours and then in ten hours, but she never asked students for the number of hours needed to accommodate a particular number of riders. In other words, all of her questions asked for the value of the dependent variable when given an independent value and never the other way. This bears contrast to the type of reasoning required to answer the questions of Teacher A, who gave known values for independent and dependent values.

The rest of the lesson was very procedurally focused. Students were frequently distracted by where to write answers on their worksheets. The teacher gave explicit directions for how to label their graphs and later what numbers and operations to enter into the calculators. In one particular segment that caused significant student confusion, students were filling out a table that included track length (in feet), time of ride (in minutes and seconds), and a blank column for speed (in feet per second). The teacher instructed students to begin by calculating the total number of seconds needed for each ride. Because there was no specified place for this in the table, students became very confused, making comments like, “I thought we were supposed to find speed,” and “I don’t know where to write this.” This confusion stemmed from a lack of understanding of the reason for calculating the total number of seconds needed for each ride. Although the teacher did briefly mention it at the beginning of the exercise, it was clear that most students did not understand why they needed this information. This may have been assuaged by finding total seconds and speed for each ride, one at a time, instead of finding total seconds for each ride and then calculating speed as a separate step. The lack of connection between the step of finding total seconds and the following step of

calculating speed indicated a lack of conceptual understanding and an emphasis on procedural knowledge.

The majority of instruction delivered by Teacher B was procedural. Of the 221 ten-second intervals in Teacher B's lesson video, 18.6% were conceptual, 60.6% were procedural, and 20.8% were classified as promoting neither type of understanding. Table 1 presents a comparison between the two teachers.

The other two algebra lesson videos were also watched for Teacher B. These lessons were significantly more discrete than those of Teacher A. The first of the algebra lessons focused on translating verbal expressions to symbolic expressions and equations. This skill was the only skill addressed in the entire lesson, without reference to any other algebra concepts. Instruction on this topic was limited to very procedural translation based on rules and "key words," without any contextualized examples. In the second of the algebra lessons, after an example by the teacher, students worked independently to create scatterplots of tables of values they had been given. The plots did not create linear relationships, but students were to note the trend of the data. The scatterplots resulted from the execution of a series of steps that included labeling the graph, writing ordered pairs from the table, and plotting them on the graph. Again, in this lesson, there were no explicit references to other algebra topics, with an emphasis on procedure.

Table 1

Results of Video Analysis

	Teacher A		Teacher B	
	Intervals	Percent of total	Intervals	Percent of total
Conceptual (C)	184	72.7%	41	18.6%
Procedural (P)	36	14.2%	134	60.6%
Neither (N)	33	13.0%	46	20.8%

Student Achievement

On the whole algebra test, students of Teacher A had an average score of 7.75 on the pretest and 9.40 on the posttest. Students of Teacher B had an average score of 6.46 on the pretest and a score of 5.31 on the posttest. A *t*-test ($\alpha = .05$) was used to determine that the difference in the pretest means for the two teachers was not statistically significant (significance level .360), but that the difference in the posttest means was statistically significant (significance level .002).

Item Analysis

Items, student responses, and teaching behaviors addressing each research question were studied. The student competency central to the first research question is the ability to solve linear equations. The second research question addresses students' ability to model equations and expressions from verbal problem representations. The focus of the third research question is students' ability to assess algebraic relationships

among quantities in tables. To answer the final question, students' ability to apply algebra concepts and to employ flexible algebraic reasoning were examined.

Solving equations. To address the first research question, Items 1 and 15 were examined to study students' ability to solve linear equations (See Appendix B for test items). Student responses to these items may be found in Tables 2 and 3.

The first item on the test was the most basic and perhaps the most procedural in its nature. Students were given the equation $43 = \square - 28$ and asked to select the value of the box. This is the only item on the entire test on which Teacher B's students outperformed those of Teacher A. As displayed in Table 2, among the students in Teacher B's class, 11 (84.6%) chose correct answer choice, *D*) 71. Of Teacher A's students, 11 (55%) answered this item correctly. Only 2 of Teacher B's students answered this item incorrectly, selecting the distracter answer *A*) 15. Seven of Teacher A's students selected *A* as well. Answer choice *A* was likely selected by many of the students because they performed the operation $43 - 28$, seeing these two numbers and the subtraction operation. These students may have been careless or they may have an incomplete understanding of inverse operations or equality. Answer choice *C*) 61 was also selected by two of Teacher A's students, who may have found that answer as a result of an addition error (most likely, not regrouping).

Table 2

Student Responses for Item 1

Response	Teacher A	Teacher B
A	35%	15.4%
B	0%	0%
C	10%	0%
D	55%*	84.6%*

* Students giving the correct answer

In none of the available video footage did Teacher A solve a problem like this in class, so little is known about student instruction in solving a one-step linear equation like the one presented in item 1. In one small aside during her lesson on writing equations from verbal representations, Teacher B solved an equation much like this one. She mentioned it in review, quickly solving one of the equations they had written. Students seemed acquainted with the idea of performing the “opposite” operation. Though execution of the solution to the problem in the lesson video was very procedural, most of her students mastered the steps to executing the procedure and were able to answer this question correctly.

Item 15 required students to provide their own answers and show their method of finding their answer. They were asked to find the value(s) of x that made the equation $19 = 3 + 4x$ true. Student results can be found in Table 3. The two-step equation in this item was clearly difficult for the seventh-grade participants in this study. A brief description of the scoring rubric for this item and scores earned by students can be found in Table 3.

Students of Teacher A outperformed those of Teacher B on this item. Among Teacher A's students, 45% (9 students) gave an answer that was completely correct, including demonstrating a clear and mathematically sound method for selecting the value they chose. Among Teacher B's students, 15.4% (2 students) successfully did this.

Table 3

Student Scores for Item 15

Score and method	Teacher A	Teacher B
2 – Traditional use of inverse operations	0%	0%
2 – Use of guess and check	10%	7.7%
2 – Use of running equation or other valid method	35%	7.7%
1 – Correct answer without explanation or minor errors leading to incorrect conclusion	15%	23.1%
0 – Incorrect answer or explanation or no response	40%	61.5%

As indicated above, almost half of Teacher A's students answered this item correctly and supported their answers, receiving a score of 2. However, none of these students solved the equation using the traditional series of steps to isolate the variable. All nine students either employed a guess and check method or some kind of running

series of operations to solve the two-step equation. This demonstrates that these students had a flexible conceptual understanding of the function of the variable and could use their own methods for figuring its value. They were not dependent on an algorithm, and therefore could solve problems that they may not have “known how” to do. This corresponds with their teacher’s consistently conceptual teaching (see the description of Teacher A’s lessons in the section above) and her avoidance of strictly procedural teaching. Understanding concepts and not relying on memorized procedures enabled these students to flexibly apply their understanding in an unfamiliar situation and arrive at a valid solution.

Modeling equations. Items 2, 3, and 8 were examined to assess students’ ability to derive an algebraic equation or expression from a short verbal description of a numerical relationship in order to answer the second research question (see Appendix B). Table 4 displays student responses on items 2 and 3. Table 5 contains scores received by students on item 8.

Item 2 from the test required students to write an algebraic equation representing the situation, “Julie has 3 times as many trading cards as Mary. They have 36 trading cards in all.” Student responses to this item are displayed in Table 4. Of Teacher A’s students, 10 (50%) chose answer choice C) $x + 3x = 36$, the correct response. The correct answer was selected by 5 (38.5%) of Teacher B’s students. The most popular incorrect answer choice was A, selected by 10 of Teacher A’s students and 7 of Teacher B’s students. Selection of choice A indicates a reliance on a more procedural translation from verbal to symbolic representations. Students likely saw the phrase “3 times” in the

problem and thought to translate that as $3x$. The “36 trading cards in all” indicated to students that they should finish their algebra sentence $3x = 36$. Many students from both teachers made this mistake.

Item 3 was similar to Item 2, asking students to select an expression that could be used to represent the number of rows, if there were n girls all together and each row had 6 girls. The correct answer choice was D) $\frac{n}{6}$. As detailed in Table 4, of Teacher A’s students, 55% chose the correct answer. Of Teacher B’s students, 15.4% selected the correct choice. The most popular distracter answer choice in this item was choice C) $6n$, selected by 7 (35%) of Teacher A’s students and 8 (61.5%) of Teacher B’s students.

Table 4

Student Responses for Items 2 and 3

Response	Item 2		Item 3	
	Teacher A	Teacher B	Teacher A	Teacher B
A	50%	53.8%	0%	0%
B	0%	0%	5%	23.1%
C	50%*	38.5%*	35%	61.5%
D	0%	7.7%	55%*	15.4%*

* Students giving correct response

One of the additional videos watched for Teacher B was a lesson in translating verbal representations to symbolic algebra. Her instruction on this was entirely procedural. She began by leading the class in a creation of a list of key words that

indicated different operations. She instructed students to translate word for word from the text to the algebraic equation, remembering to “flip it” when using *less than* or *more than*. The reasons for any of this were never discussed and no contextualized examples were given to support the concept. When creating their list of key words, the word *each* was listed under both division and multiplication. This may contribute to Teacher B’s students’ confusion on this item. The word *each* does appear in the question for item 3. Many of her students chose multiplication as the necessary operation, when the situation actually called for the division of the total n by 6, the number of girls in each row.

The final item on the test that required students to represent a verbal statement with algebraic symbols and variables was item 8. Item 8 was a free response question asking students to generate an equation that represented the statement, “Tachi is exactly one year older than Bill” if T represented Tachi’s age and B , Bill’s age. Student scores on this item can be found in Table 5. Again, on this item, Teacher A’s students (45% correct) performed significantly better than those of Teacher B (15.4% correct). An additional 30% of Teacher A’s students transposed their variables, while none of Teacher B’s students committed this error. Eleven of Teacher B’s 13 students left this item blank or gave another answer.

Table 5

Student Scores for Item 8

Score and indicator	Teacher A	Teacher B
1 – Student gave $T = B + 1$ or an equivalent equation	45%	15.4%
0 – Student transposed T and B	30%	0%
0 – Student gave any other response or no response	25%	84.6%

It is surprising so few students answered this correctly. Meaningful conceptual knowledge may not be necessary to answer this question. A procedural approach would yield a correct answer. Translating this statement one word at a time would result in, “Tachi (T) is (=) exactly one (I) year older than (+) Bill (B)” or the algebraic sentence $T = I + B$. Given the description of Teacher B’s instruction on this topic, even if students did not remember her rule about “flipping” the order of the terms being added or subtracted (because addition is commutative), students should have been able to solve this problem easily. Yet almost all of Teacher B’s students answered this completely incorrectly. This demonstrates that her lesson on this was not internalized by students.

Examples of these skills being explicitly taught were evident in Teacher B’s lesson videos, but none of Teacher A’s videos included instruction in the translation from English sentences to algebraic representations. The primary focus of Teacher A’s lesson was mathematical communication. Students read understandable, familiar

problem situations, spoke with their group members about representing the ideas, interacted with their teacher, presented to the class, and wrote about the relationships between quantities. Topics such as translating verbal representations to symbolic algebra were not addressed as stand-alone topics, but were presented as a means to generating an equation, which was used to make a table and graph, which were used to answer meaningful questions.

Student analysis of tables. Items 5, 7, 11, and 16B required students to identify patterns in a given table of values, describe algebraic rules or patterns, or fill in missing values (see Appendix B). The ability to see relationships between values in tables is central to the third research question.

In item 5, students were given a complete table and required to select a rule that could be used to derive values in column B from the values given in column A. Table 6 contains a summary of student responses. The correct answer choice *D) Divide the number in column A by 4* was selected by the majority of the participants in the study, including 85% of Teacher A's students (17 students) and 53.8% of Teacher B's students (7 students). All 3 of Teacher A's students who answered incorrectly chose *C) Multiply the number in column A by 4*. Teacher B's students who answered incorrectly were divided among all three incorrect answer choices.

Item 7 also required students to assess relationships between values in a table, but instead of the relationship between x and y values, they were asked about the way that the x and y values changed. With the responses to item 5, responses given by students on item 7 can be found in Table 6. The correct answer choice *C) The y values*

increase by 2 and the x values increase by 1 was selected by 80% of Teacher A's students and 69% of Teacher B's students.

Table 6

Student Responses for Items 5 and 7

Response	Item 5		Item 7	
	Teacher A	Teacher B	Teacher A	Teacher B**
A	0%	7.7%	0%	7.7%
B	0%	15.4%	20%	0%
C	15%	23.1%	80%*	69%*
D	85%*	53.8%*	0%	15.4%

* Students giving correct response

** One student did not respond

Item 11 was a free response question that asked students to fill in a missing element in a table of values. Table 7 displays student scores on this item. From Teacher A's students, 12 students (60%) chose the correct answer, 48. Of Teacher B's students, 4 students (30.8%) gave the correct value. The relationship between columns A and B was quite difficult to determine, though A values increased by 4, then 8, then 12 and so on, while B values increased by 2, 4, 6 and so on. Students who answered this item correctly most likely used the pattern of the values in column A to fill the missing cell.

Table 7

Student Scores for Item 11

Score and indicator	Teacher A	Teacher B
1 – Student answered 48	60%	30.8%
0 – Student did not answer 48	40%	69.2%

The last question that required students to analyze a table was 16B, a part of an extended problem situation with multiple questions. Students were presented with a diagram showing several arrangements of pine trees surrounding a square arrangement of apple trees. The examples in the diagram showed arrangements with one, two, three, and four rows of apple trees. The table students were given in 16B included a column that represented the number of rows of apple trees (n), a column representing the number of apple trees, and one containing the number of pine trees. The table only included values up to $n = 5$. The first row ($n = 1$) was filled in for students, as was the number of apple trees in the next row ($n = 2$). The rest of the table was blank for the student to fill. Student scores on this item can be found in Table 8. Eleven of Teacher A's students (55%) filled in the table completely correctly, while an additional 3 students filled it mostly correctly, making only one error. Only four of Teacher B's students (30.8%) generated all the values correctly, while none of her students gave only one incorrect entry.

Table 8

Student Scores for Item 16B

Score and indicator	Teacher A	Teacher B
2 – All entries in tables are correct	55%	30.8%
1 – One incorrect entry in table	15%	0%
0 – Multiple mistakes or blank tables	30%	69.2%

This item could be answered by students in several ways. Of the cells to be filled in by students, only two of the values (the number of apple trees and pine trees in the row $n = 5$) could not be obtained by counting the example diagrams on the first page of the extended response item. Drawing an arrangement of five rows and counting the apple and pine trees would yield the two missing values. After beginning the table, they may also notice one of several patterns in the numbers and be able to continue the patterns without continuing to count trees in the diagram. Regardless of the method employed, students of Teacher A were better equipped to address this and all the other questions requiring recognition of patterns in tables than the students of Teacher B. This is likely due to the flexibility Teacher A encouraged by asking a variety of types of questions and pointing out many different relationships between numbers in the tables she presented in class. Teacher B did not do this in her class.

Teacher A connected the relationship between values in a table very explicitly and several times in her lesson videos. At the beginning of the analyzed video, Teacher A began by leading a class discussion in the comparison of two tables, one that

demonstrated a constant unit rate and a second in which a constant value was added to the independent values to obtain the dependent values. Students pointed out that, in the first table, each y value was obtained by multiplying the x value by 7. Their teacher then asked them what they could multiply by x in the second table to get the y values.

Students recognized that multiplication would not yield the values in the second table, and that another operation, in this case, addition, was needed. Comparing the two tables brought the type of question presented by item 5 to an even higher level of analysis.

Each time the class made a table in Teacher A's lessons, the class always returned to talk about the relationships within the table, including the rule that connected x and y values, as well as the changes in the x and y values.

In the first section of the analyzed lesson, Teacher B also created a table of values with her students. She gave students x values and told them what operations to do in order to obtain the y value. After the table was completed, Teacher B did not return to the table and emphasize that for each x value, the same rule was used to obtain the y value. She simply went on to plot the points and create the graph without emphasizing the consistency in her table. She did not note the consistency of the changes in the x or y values in the table either. Her failure to make conceptual connections and her focus on the procedure of creating a graph may have contributed to her students' performance on the items requiring students to analyze tables.

Applying algebraic reasoning. To answer the fourth research question and assess students' ability to apply their understanding of algebra concepts, including equality and variable, items 4, 9, and 16C were examined (see Appendix B).

Item 4 on the test presented a conceptual question in which students had to interpret an algebraic rule, rather than perform a calculation. Student responses on this item are summarized in Table 9. Fifteen of Teacher A's students (75%) answered the question correctly, while 7 of Teacher B's students (53.8%) chose the correct answer. Teacher A's students who answered the item incorrectly were divided among the other three answer choices, but 4 of Teacher B's students (30.7% of the whole class) who answered incorrectly chose answer choice *C*) *The sum of two whole numbers is a whole number.*

Table 9

Student Responses for Item 4

Response	Teacher A**	Teacher B
A	10%	7.7%
B	75%*	53.8%*
C	5%	30.8%
D	5%	7.7%

* Students giving the correct answer

** One student did not respond

To answer this item correctly, students must recognize that variables a and b can represent any numbers. They also must see the *equals* sign as a balance, a symbol that

does not connote action but states a relationship of quantitative sameness. Neither teacher directly addressed this topic in the lessons that were available; however, the treatment of key words in Teacher B's lesson on translating verbal to symbolic expressions may have contributed to many of her students selecting an incorrect choice in this item. In her lesson, she told the class that one of the key words indicating addition was *sum*. Seeing addition in the rule, it is possible that students automatically chose the answer choice that contained the word *sum*.

Item 9 presented students with the equation $a = b - 2$, and the pair of values $a = 3$ and $b = 5$ that satisfied it. Students were asked to find a different pair of values that satisfied the equation. This question presented difficulty for many students; only 45% of Teacher A's students and 15.4% of Teacher B's students were able to provide a pair that satisfied the equation. Many of the same skills needed to solve Item 4 were also needed for this item, though this item was more difficult for students, perhaps because students had to generate a response.

As indicated by their performance on Item 9 and Item 4, members of Teacher A's class appear to have a better understanding of variables as placeholders and equality than those of Teacher B, though many of her students do still lack a complete understanding. Teacher A consistently stressed relationships between quantities in her lessons. She demonstrated that, if you understand the relationship between two quantities, the value for the independent variable can be determined if the dependent is known and the value for the dependent variable can be determined if the independent variable is known. Teacher A used this vocabulary with her students and they were clearly comfortable

determining which quantity could be manipulated and which quantity was dependent on the other.

In only one brief mention of the relationships between quantities in Teacher B's lesson, she asked students to calculate values for the dependent variable, given several independent variables. She did not use the terms "independent" or "dependent" variables when asking students to perform these calculations. Again, her treatment of the topic was markedly procedural. The treatment of equality and relationships in equations by the two teachers may have contributed to their respective students' performance on Items 4 and 9.

The final item that was examined to assess students' levels of algebraic reasoning was 16C, the third part of the extended response item. Generally, student performance on this item was among the lowest on the entire test. Students were asked to find the value of n for which $n \cdot n$ is equal to $8n$ and explain how they found their answer. Solving the equation $8n = n^2$ is an advanced task for seventh graders, and probably unlike equations they have solved in class. None of the 33 students in the study solved the equation by algebraic methods, or even by guessing and checking possible values for n . Two students from Teacher A's class found the correct answer by continuing the table. Two additional students from Teacher A's class gave correct answers and fuzzy or incomplete explanations or gave slightly incorrect answers with a correct explanation. Two students from Teacher A's class also gave a correct response with an incorrect explanation. Altogether, 6 students from Teacher A's class displayed some degree of understanding of this problem (30%). Among Teacher B's students, no one gave correct

answers and correct or even incomplete explanations. One student did give a correct answer with no explanation and another gave a correct answer with an incorrect explanation. Two students from her class (15.4%) demonstrated some understanding of the question.

Being able to apply existing knowledge in new situations is a benefit of developing conceptual understanding. Procedural knowledge is limited to executing solution steps in situations resembling those in which the procedure was learned. The relative success of Teacher A's students on this item may be attributed to her more conceptual approach. Her students were confident enough to understand the problem and use their own methods to arrive at a solution, even in an unfamiliar situation.

CHAPTER V

DISCUSSION AND CONCLUSIONS

This chapter will elaborate on the results outlined in the last chapter, noting findings that support or seem to contradict existing research. Also, support for answers to each research question will be noted.

Video Analysis

The results of the video analysis presented a stark contrast between the two teachers' approaches. Teacher A focused on making conceptual connections, with almost three-quarters of her class time spent in enhancing conceptual understanding. Teacher B was largely focused on ensuring that her students carried out the procedures in the day's lesson, with the majority of her class time spent encouraging procedural understanding. This contrast was noticeable in watching the videos, but made more evident through the results of coding of the ten-second time intervals.

Teacher A. The conceptual emphasis of Teacher A was consistent with the recommendations of the TIMSS study. Although Teacher A's lesson was more teacher-directed than the lessons of Japanese teachers described in the TIMSS study (Stigler & Hiebert, 1999), her lesson did encourage students to use their own procedures for solving problems. Her lessons each contained an extended exploration of a problem presented in a familiar context, which made them more student-centered than lessons containing more abstract problems or "real-life" problems to which the students could not relate.

The most glaring element of her lessons was connections. Statements made and questions asked by Teacher A were laden with connections. She made explicit connections between solution and graphing methods (by hand and aided by technology), between different representations of the same problem, between prior learning and the current lessons, and even between different students' solutions. She also encouraged the students to make connections themselves, as well. Students were required to contrast different problem situations and make decisions. In this way, her lesson exemplified the description of conceptual teaching outlined by Hiebert (1986).

Teacher A also carried out the recommendations of many reformers and researchers by giving her students multiple opportunities during each class to engage in meaningful mathematical communication (Esty, 1992; Johanning, 2000; Pugalee, 2004). Students in Teacher A's class worked in groups, presenting to the class, interacted with the teacher, and wrote to answer questions requiring them to explain their reasoning. Teacher A solicited input from many students and always asked students to justify their thinking. The discourse in her classroom was not as student-centered as the discourse described by Sherin (2002), but her use of multiple students' input and student discussion created a classroom that, in some ways, resembled the discourse community described and advocated by this author. This likely contributed to student success on many of the items on the posttest.

Teacher B. Teacher B's manner of instruction was primarily procedural. The mathematical content of her lessons was not rich in explanations, but rather focused primarily on step-by-step methods of finding the answer to each problem encountered.

Connections between topics, concepts, and procedures were not emphasized often in her lessons. Procedures and topics were treated as stand-alone skills to be mastered, aligning the focus of her teaching with given definitions and indicators of procedural teaching (Ashlock, 2001; Hiebert, 1986).

Overall, the students of Teacher B did not perform as well as those of Teacher A after a year of instruction from their respective teachers. Some of the research mentioned in Chapter II supported varying degrees of procedural teaching (Jitendra et al., 2002; Leung & Park, 2002; Star, 2002). None of these researchers, however found that strictly procedural teaching without connections made to concepts resulted in improved student performance. Leung and Park (2002), whose study provided the greatest support for procedural teaching asserted the importance of practicing procedures as a vehicle to establish an understanding of concepts and also advocated the importance of teaching procedures in a curriculum sequenced to help students learn procedures as they learn concepts. These elements of more effective procedural teaching were missing in Teacher B's lessons. Her lessons consisted of procedural teaching, not completely devoid of concepts, but largely leaving conceptual connections unaddressed.

Teacher B also spent a significantly larger part of her lesson engaged in topics that were neither procedural nor conceptual than Teacher A. Many of the intervals designated as promoting neither type of understanding for her students were series of administrative directions. As mentioned in Chapter IV, her students were distracted by details involved in directions and she spent quite a bit of time telling students where to find the assignment in their textbooks, what they should find on given pages, where to

write information, and how to label their assignments. Her 46 ten-second intervals add up to almost 8 minutes in a 40 minute lesson spent giving directions. Her directions did not include any kind of academic transition; she never mentioned a connection between the activity completed and the upcoming activity. Over the course of a school year, spending much time addressing directions and administrative tasks could add up to a significant loss of student instructional time and may have contributed to the poor performance of her students.

The use of classroom instruction time is important. Large amounts of time spent in addressing discipline issues or administrative tasks take time away from meaningful instruction. A study conducted in Kentucky revealed that in schools with large achievement gaps between student populations, teachers spent more time addressing administrative tasks (Meehan, Cowley, Schumacher, Hauser, & Croom, 2003). Though the achievement gap between subpopulations was not measured in the present study, Teacher A's students outperformed those of Teacher B, who spent much more time in non-mathematical dialogues.

Student Achievement

As noted in the previous chapter, the two classes in the study began the year by performing roughly equally well on the pretest of the algebra test. The mean score of Teacher A's students increased 1.65 points, while the mean score of students in Teacher B's class actually decreased 1.15 points from the pretest to the posttest. The cause of the great difference between these two groups of students would be difficult to determine,

but the difference in teaching styles of Teachers A and B undoubtedly contributed to the relative success of Teacher A's students.

Item Analysis

Solving equations. On the first item of the test, Teacher B's students outperformed Teacher A's class. With the box instead of a variable, this item may have been easier than the others on the test, because it resembles problems students may have encountered in elementary school. This item was easily solved by a procedural approach. Almost all of Teacher B's students executed the proper procedure for finding the value of the box. Her procedural approach seems to have been internalized by students and they may even have corresponding conceptual understanding. Symbol manipulation is all that is required to solve this item.

However, when her students came to the unfamiliar problem presented in item 15, very few of her students were able to solve the equation. Though solving two-step equations is typically introduced in seventh grade curricula, including the textbook used by the two classes in this study, none of these students used the traditional series of steps typically employed to solve this equation. Teacher B's students were well-equipped with the procedure to solve the one-step equation, but were unsuccessful with a more difficult problem utilizing the same reasoning. Her procedural approach did not encourage viewing a problem holistically. Also in her lessons, students were never presented with a problem that they had not, immediately before, been taught how to solve. Thus, when

students encountered this problem they did not know how to solve, most of her students were unable to use their own approach to figure it out.

The order of Teacher B's lessons reflected the traditional viewpoint discussed by Nathan and Koedinger (2000). Teacher B's style of teaching indicated a belief that students need to receive instruction in a particular skill before encountering problems that require them to employ that skill. Had students been presented with non-routine problems, as suggested by the NCTM (2000), or encountered problems in which they had not received explicit instruction, her students may have been more comfortable using another method to solve some of the problems on this test that were unfamiliar.

Teacher A's instruction seemed to be more empowering. On item 15, almost half of her students were able to employ mathematically sound methods and obtain a correct value for x . Her problem-based instruction may have resembled the reformed curriculum implemented in the school described by Boaler (1998). The students who participated in the problem-based curriculum were more flexible in their reasoning abilities and in their ability to apply their knowledge in new situations. Similarly, many of Teacher A's students appeared to exhibit this kind of flexibility.

Modeling equations. The relative success of Teacher A's students on this group of items reflected much of what is suggested by the literature. As mentioned above, Teacher A encouraged different kinds of mathematical communication in her classroom. Her students expressed consistent numerical relationships and generalizations in words, before or after representing the generalization with variables and other symbols, as suggested by Schoenfeld and Arcavi (1988). Kieran and Chalouh (1993) also advocated

giving students the opportunity to express algebraic situations in easily understood language, as a means to developing conceptual understanding of a problem, before representing them symbolically.

The procedural approach to translating verbal representations to symbolic representations presented by Teacher B did not help her students succeed on the items that required students to apply this skill. Though taught this skill directly and explicitly, very few of her students were able to answer the items studied in this section. A skill taught in isolation, it is doubtful that students even recognized these questions as being connected to their lesson on translating expressions. Lodholz (1990) recognized the potential for student difficulty in encountering future need to apply this skill, when taught this skill in contrived situations.

Her students' failure to answer item 8 correctly, even when a procedural, word-for-word translation would have produced a correct answer mirrors a discovery made by MacGregor and Stacey (1993). They noted that, "In test items designed so that syntactic translation would produce a correct equation, most students did not translate words to symbols sequentially from left to right, but tried to express the meaning and wrote incorrect equations" (p. 217). Teacher B's lesson on modeling equations from problem situations, though not internalized or applied by her students, may not have been the source of the difficulties her students had with these items. Her students may have lacked the cognitive structures necessary for the level of abstraction required by this skill, as suggested by MacGregor and Stacey. Pawley et al. (2005) found that gradually increased exposure to problems requiring this skill, along with implementation of a

strategy to check the equation once written, can increase low-level students' ability to do this successfully. Had Teacher B repeatedly exposed her students to similar problems with decreased support for solving them, and taught her students a method of reflecting on the equation after writing it, her students may have been more successful with these items.

Student analysis of tables. In order to be successful on some of these items, students needed to be able to see relationships between numbers. On others, simply being able to detect and continue a pattern was sufficient. The relative success of Teacher A's students over Teacher B's students on all items requiring analysis or completion of tabular representations reflects the treatment of tables in algebra lessons.

Items 5 and 7 required similar skills. Teacher A's students answered both items with a similar percentage of correct responses. Two more of Teacher B's students answered item 7 correctly than answered item 5 correctly. Item 7 asked students to describe the change in the x column (values increasing by 1) and the change in the y column (values increasing by 2). Students could answer this item by recognizing the recursive patterns in each column without recognizing a generalized rule relating the columns in the table. Students have an inclination to look for recursive rules to describe the relationships between numbers in a table (Rubenstein, 2002). This basic skill of pattern identification is frequently addressed in elementary school mathematics.

The skill presented by item 5 was to verbalize a consistent relationship between values in two columns, or to write an explicit rule used to map the values of one column to the values in the other. Identifying a relationship between the columns of a table of

values requires algebraic reasoning in addition to pattern identification. According to Lannin (2003), finding a recursive pattern can lead to the development of an explicit rule, a more advanced skill requiring generalization. Thus, item 5 presented more difficulty for the students of Teacher B than item 7.

In Teacher A's lessons, each time a table was generated from a problem situation or from an equation, by calculator or by hand, Teacher A emphasized consistency in the table, its connection to the equation used to generate the table, as well as its effect on the graph. She asked her students to verbalize relationships between values in the two columns in their own words, as suggested by Kieran and Chalouh (1993) and Schoenfeld and Arcavi (1988). Her approach to teaching students to recognize these relationships was closely related to her instruction on the translation of verbal to symbolic representations. By explicitly noting connections between each representation of an equation (graphs, tables, verbal descriptions, and symbolic equations), her students became flexible in their expression and interpretation of each form. This practice contributed to her students' success, much like the students in the studies described by Brenner, et al. (1997) and Moseley and Brenner (1997).

Applying algebraic reasoning. The items studied to address this question assessed student understanding of the crucial concepts of *variable* and *equality*. These are fundamental to success in preparing for further algebra study. Again, Teacher A's students experienced more success on these items than the students of Teacher B. Two of the items answered most frequently incorrectly were included in this section of the

study, items 9 and 16C. These two questions were obviously difficult for students, largely unfamiliar to student participants.

A benefit of conceptual understanding is the ability to apply existing knowledge to new situations. Conceptual knowledge and procedural knowledge deeply embedded in conceptual connections are more likely to be recalled and applied in new situations (Hiebert & Lefevre, 1986). As demonstrated by the reformed curriculum studied by Boaler (1998), students who experience problem-based learning rich in conceptual connections are more flexible in their problem solving ability and can apply existing knowledge in new situations. By making strong conceptual concepts, Teacher A enabled many of her students to apply their existing knowledge to new situations. A much larger portion of her students demonstrated some understanding of the two difficult questions than the students of Teacher B. Even in her instruction on procedures, conceptual connections gave her students a real understanding of the reasons behind the procedures.

Concluding Remarks

This study examined only two teachers with very different instructional approaches. The students of the teacher who delivered conceptual instruction improved their algebra skills from the beginning of the year to the end. The students who received more procedural instruction, without the support of the conceptual network, showed little improvement over the course of the year. Their knowledge stood alone as individual pieces and they were not able to apply it in new situations. These students were not well equipped to solve problems or apply algebraic reasoning. The students of the more

conceptual teacher, on the other hand, were significantly better prepared to answer questions requiring algebraic reasoning.

As Robert Davis said, “Mathematics does not lie in its symbols, but in the ideas these symbols represent” (1986, p. 269). Algebraic relationships encountered in students’ everyday experience will not present themselves as matters requiring symbol manipulation. They will present themselves as decisions to make in the grocery store, at the gasoline pump, in financial planning, and when choosing service providers. Teachers must prepare students, not to carry out algebraic procedures for their own sake, but to use algebra as a tool to solve problems and represent situations. Without conceptual understanding, procedures mean almost nothing. Connections make mathematics meaningful, memorable, and powerful.

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APPENDIX A

For the analysis of classroom videos, instances of conceptual teaching and procedural teaching will be coded according to the following indicators (Hiebert & Lefevre, 1986).

Conceptual Teaching

- Relationships between numbers, topics, or representations explicitly pointed out
- Concepts are connected to students' current knowledge and future learning
- Explanations of the reasons for executing elements of the procedure are emphasized

Procedural Teaching

- Rules and algorithms are presented as a series of steps
- Solutions are presented as step-by-step and sequential
- Solutions are presented by the instructor as hierarchical and very structured
- Students are required to operate on objects or symbols
- Input/output model of student processes is evident or implied
- Steps being presented can be learned by rote

APPENDIX B

The test items found below and mentioned in this thesis are proprietary and may not be reproduced or used without permission.

1. What is the value of \square in this equation?

$$43 = \square - 28$$

- A. 15
- B. 25
- C. 61
- D. 71

2. Mary has some trading cards. Julie has 3 times as many trading cards as Mary. They have 36 trading cards in all.

Which of these equations represents their trading card collection?

- A. $3x = 36$
- B. $x + 3 = 36$
- C. $x + 3x = 36$
- D. $3x + 36 = x$

3. There are n Girl Scouts marching in a parade. There are 6 girls in each row. Which expression could you use to find out how many rows of Girl Scouts are marching in the parade?

- A. $n - 6$
- B. $\frac{6}{n}$
- C. $6n$
- D. $\frac{n}{6}$

4. Jacob writes the following rule:
If a and b represent any two numbers, $a + b = b + a$.

Which of the following describes Jacob's rule in words?

- A. Equals added to equals are equal.
- B. Order doesn't matter when adding two numbers.
- C. The sum of two whole numbers is a whole number.
- D. When adding three numbers, it doesn't matter how the numbers are grouped.

5.

A	B
12	3
16	4
24	6
40	10

What is a rule used in the table to get the numbers in column B from the numbers in column A?

- A. Add 9 to the number in column A.
- B. Subtract 9 from the number in column A.
- C. Multiply the number in column A by 4.
- D. Divide the number in column A by 4.

7. The table shows values for the equation $y = 2x + 5$.

x	y
1	7
2	9
3	11
4	13

Which sentence describes the change in the y values compared to the change in the x values?

- A. The y values increase by 6 as the x values increase by 1.
- B. The y values increase by 7 as the x values increase by 1.
- C. The y values increase by 2 as the x values increase by 1.
- D. The y values increase by 5 as the x values increase by 2.

8. Tachi is exactly one year older than Bill.

Let T stand for Tachi's age and B stand for Bill's age.

Write an equation to compare Tachi's age to Bill's age.

9. $a = b - 2$ is a true statement when $a = 3$ and $b = 5$.

Find a **different** pair of values for a and b that **also** make this a true statement.

$a =$ _____

$b =$ _____

11. The table represents a relationship between A and B.

A	B
8	3
12	5
20	9
32	15
?	23

Based upon this relationship, what is the missing number in column A? _____

15. Find the value(s) of x that make the equation true.
Show how you got your answer.

$$19 = 3 + 4x$$

Trees

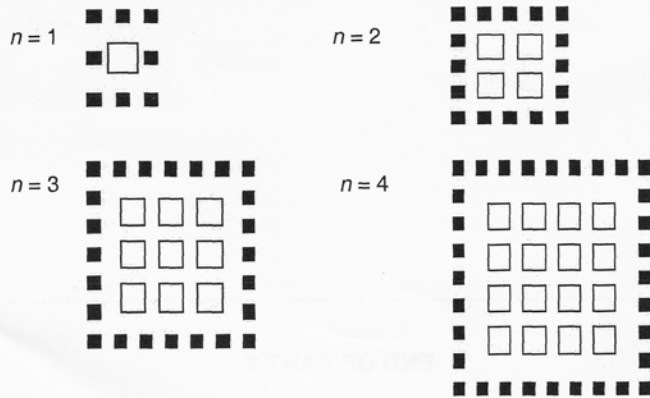
16. A farmer plants his orchard so that pine trees are all around the border and apple trees are in the center in a grid.

Here you see a diagram of this situation where you can see the pattern of apple trees and pine trees for any number of apple trees:

■ = pine tree

□ = apple tree

n = number of rows of apple trees



- (B) Complete the table. (n = number of rows of apples trees)

n	Number of apples trees	Number of pine trees
1	1	8
2	4	
3		
4		
5		

- (C) Look at the table. You might notice that the number of apple trees can be found by using the formula $n \cdot n$. The number of pine trees can be found by using the formula $8 \cdot n$. Remember, n is the number of rows of apple trees.

There is a value of n for which the number of apple trees equals the number of pine trees. Find that value of n . _____

Explain how you found your answer.

VITA

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