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A Microwave Radiometric Method to Obtain the Average Path Profile of Atmospheric Temperature and Humidity Structure Parameters and its Application to Optical Propagation System Assessment

Free-Space Laser Communication and Atmospheric Propagation XXVII
Session 1: Atmospheric Propagation

Robert M. Manning, Brian E. Vyhnalek
NASA Glenn Research Center
Cleveland, OH

www.nasa.gov

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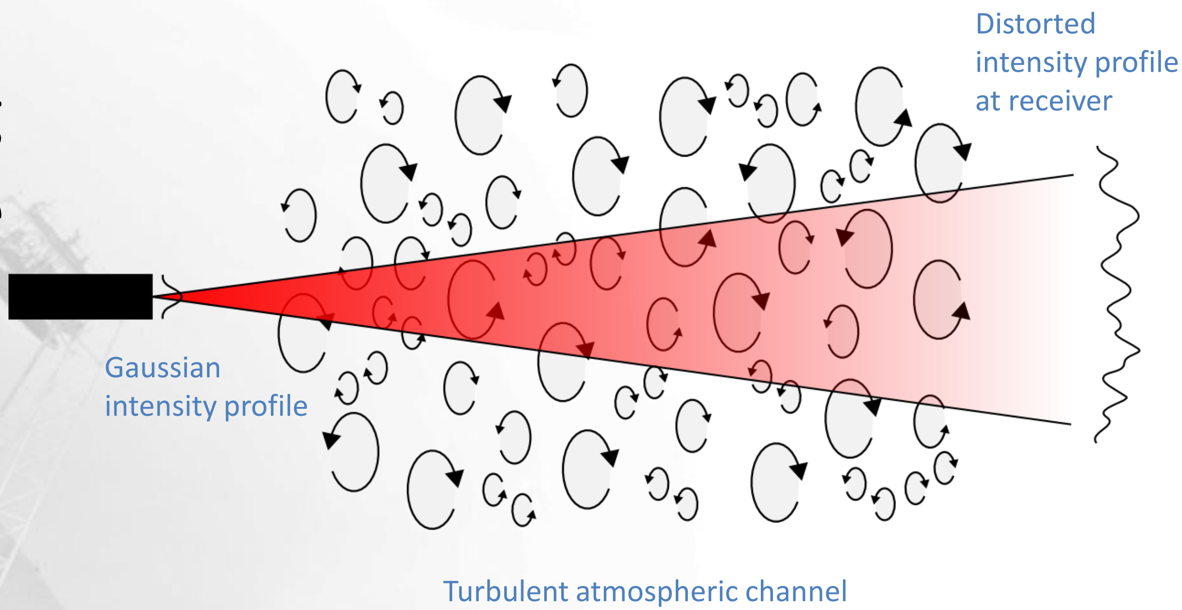
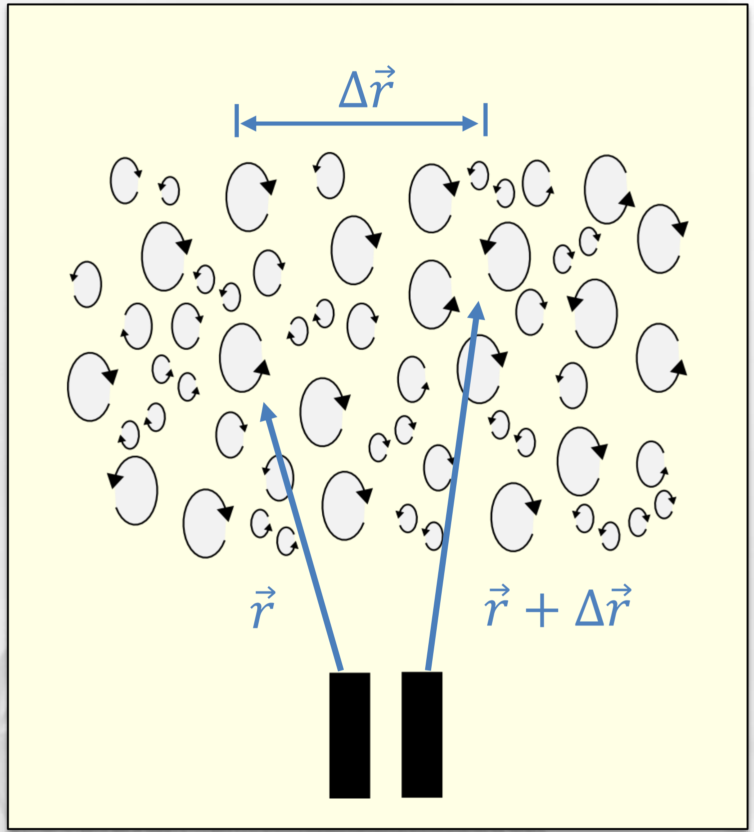
Turbulence Effects on Optical Beams

- Scintillation
- Beam broadening
- Spatial coherence
- Angle of arrival
- Temporal pulse stretching



Temporal and spatial intensity fluctuations at the receiving aperture results in power surges and fades

Turbulence Characterization



Requires simultaneous measurements at $\vec{r}$ and $\vec{r} + \Delta\vec{r}$

- Temperature (or humidity) structure function

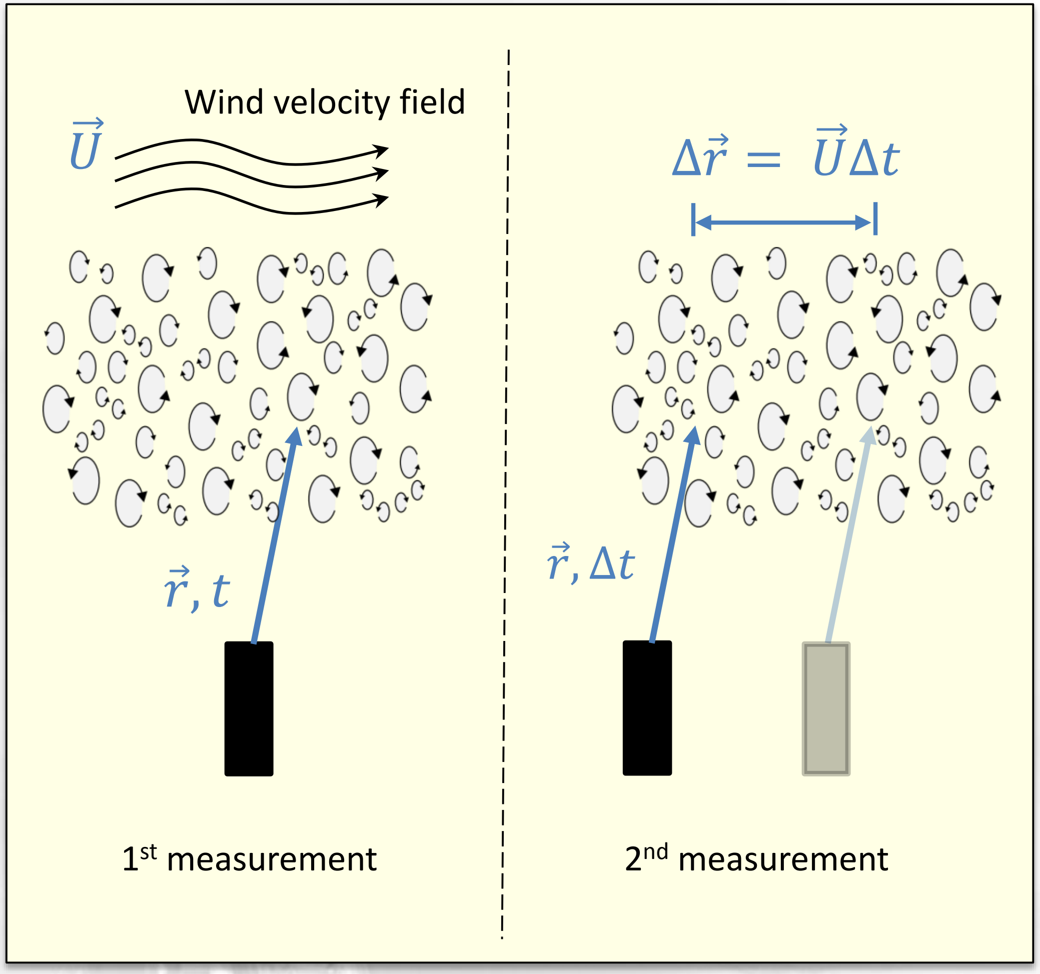
$$D_T(\vec{r}, \vec{r} + \Delta\vec{r}) = \left\langle \left[T(\vec{r}) - T(\vec{r} + \Delta\vec{r}) \right]^2 \right\rangle$$

- Contains the spatial statistics of the temperature field
- Within a range of certain $\Delta\vec{r}$, the well-known Kolmogorov "2/3" law holds

$$D_T(\vec{r}, \vec{r} + \Delta\vec{r}) = C_T^2 \left| \Delta\vec{r} \right|^{2/3}$$

Temperature structure parameter

Single Radiometer Turbulence Characterization



- Temperature (or humidity) measurements are taken at a fixed location
- Wind velocity field shifts the turbulent air mass by a distance $\Delta\vec{r} = \vec{U}\Delta t$ between measurements
- For a particular altitude, the structure function is now

$$D_T(t, t + U\Delta t) = \left\langle \left[T(t) - T(t + U\Delta t) \right]^2 \right\rangle$$

Due to the radiometer integration time Δt, Kolmogorov-Obukhov turbulence theory and the Taylor frozen flow hypothesis must be modified

Energy Transfer Spectra

- From the Boussinesq approximation (eddy viscosity model) of the Navier-Stokes equations, it is possible to obtain equations involving the Fourier spectra of the turbulent energy, wind velocity and temperature fluctuations*

$$\begin{cases} F(k) - \phi_{13}(k)\frac{d\bar{U}}{dz} + \beta\phi_{3T}(k) - 2\nu k^2\phi(k) = 0 \\ F_{TT}(k) - \phi_{3T}(k)\frac{d\bar{T}}{dz} - 2\nu_T k^2\phi_{TT}(k) = 0 \end{cases}$$

Energy transfer spectra due to wind velocity

Energy spectrum related to Reynolds stresses

Buoyancy

Kinematic viscosity

Turbulent energy fluctuations spectrum

Energy transfer spectra due to temperature fluctuations

Energy spectrum related to heat flux

Thermal diffusivity

Temperature fluctuations spectrum

*Tchen, C.M., "On the Spectrum of Energy in Turbulent Shear Flow," *J. Res. Natl. Bureau of Standards* **50** (1), 51-62 (1953)

Modified Turbulence Spectrum

- Case 1: Near the boundary surface
  - Significant stratification and shear

$$\phi_{TT}(k) = Ak^{-1}$$

$$A = 2^{1/2}\left|\frac{d\bar{U}}{dz}\right|\left|\frac{d\bar{T}}{dz}\right|^{-2}\gamma^{-1}b^{-2}N^{2}\epsilon^{-1}$$

TKE dissipation (ϵ)

Heisenberg constant ≈ 1 (γ)

Thermal diffusivity/ kinematic viscosity (b)

Thermal dissipation (N)

- Case 2: Free atmosphere
  - No stratification or shear

$$\phi_{TT}(k) = Bk^{-5/3}$$

$$B = \left(\frac{2}{3}\right) 4^{1/3}\gamma^{-2/3}b^{-1}N\epsilon^{-1/3}$$

- General:
  - Asymptotically reduces to either Case 1 or Case 2

$$V_{TT}(k) = \frac{AB}{B|k| + A|k|^{5/3}}$$

Relating to the Structure Functions

$$D_T(\Delta t) = 2\int_{-\infty}^{\infty} \left(1 - \left\langle e^{-ik(\bar{U}+v)\Delta t} \right\rangle\right) V_{TT}(k)dk$$

Average wind velocity

Wind velocity fluctuations

The connection of the temporal statistics of the temperature $T(t)$ to the spatial spectrum $V_{TT}(k)$ is through the Fourier-Stieltjes transform

Evaluation of the integral is analytical in terms of Meijer G functions, however two useful series expansions can be obtained for the asymptotic cases

$$D_T(\Delta t) \approx C_T^2 (\bar{U}\Delta t)^{2/3} \left[1 - 0.11 \frac{\langle v^2 \rangle}{\bar{U}^2}\right]$$

$\bar{U}\Delta t \ll 1$

Kolmogorov "2/3" law

Crossover frequency
$\boldsymbol{k_C = (B/A)^{3/2}}$

$\bar{U}\Delta t \gg 1$

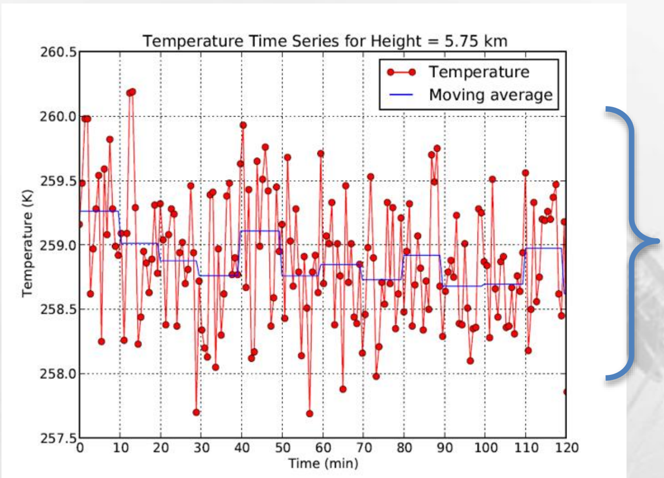
$$D_T(\Delta t) \approx \frac{C_T^2}{4} k_C^{-2/3} \left[0.57722 + \log(k_C) + \log(\bar{U}\Delta t) - \frac{1}{2}\frac{\langle v^2 \rangle}{\bar{U}^2}\right]$$

Instrumentation

- NASA TDRSS ground terminal site located at White Sands, NM
- Radiometrics MP-3000A
- 35 calibrated channels
  - 300 MHz bandwidth/channel
  - 21 K-band (22 to 30 GHz)
  - 14 V-band (51 to 59 GHz)
- 1.1 second integration time per channel
- Total $\Delta t \approx 40$ second sample period
- Temperature resolution ≈ 0.1 K

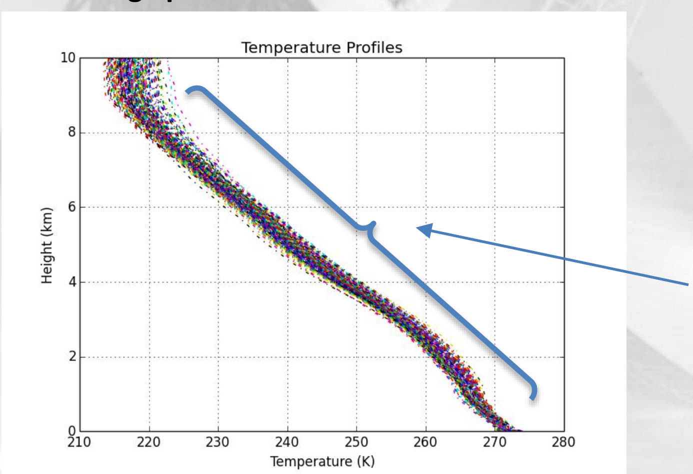
Temperature Data Analysis

Example fluctuations over a 2-hour time period



Fluctuation standard deviation, σ ≈ 0.5 K – 3 K

Average profiles for each 10 min window

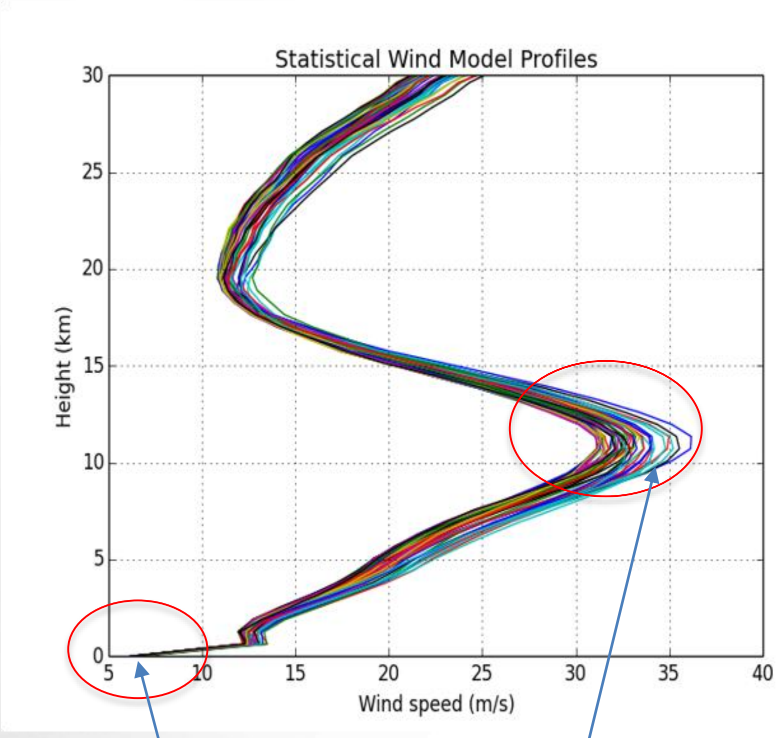


Average lapse rate of ≈ -5.474 K/km

- Measurements taken in January, 2013
- Dataset comprised of about 2100 profiles taken over a 24-hour period
- Each temperature time series divided into 10-minute moving average windows

Statistical Wind Model

- Vertical profiles of horizontal wind speed
- SPARC Data Center High-resolution radiosonde measurements at Santa Theresa, NM
- Statistics derived from 2376 wind profiles
- Principal component analysis (PCA) used for data reduction and retention of key features of the wind behavior

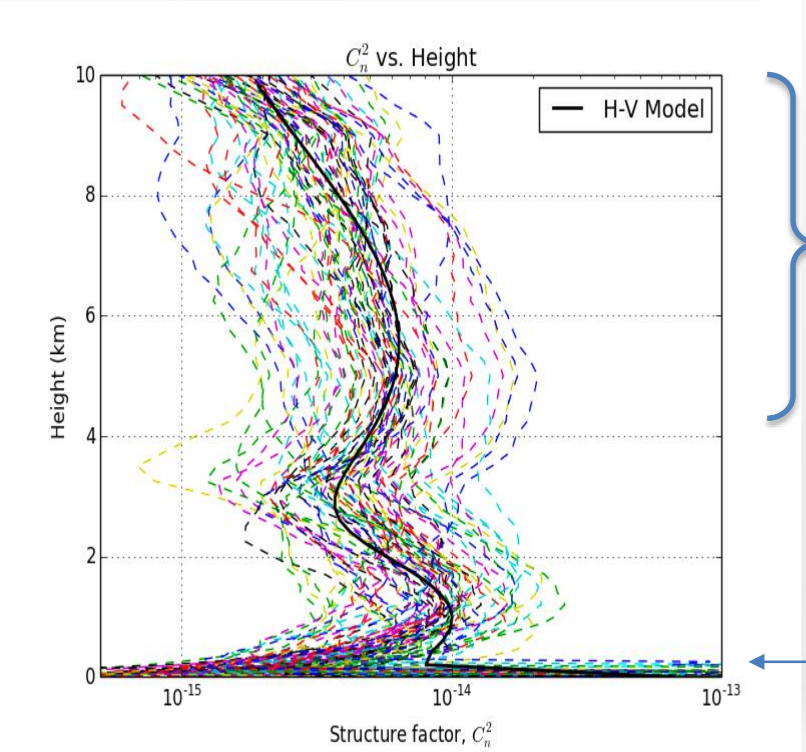


Ground wind speeds typically between 5 and 8 m/s

Max tropopause wind speeds typically between 30 and 40 m/s

Results for C_n^2

Refractive index structure parameter profiles



$$HV(h) = Ae^{-\frac{h}{H_A}} + Be^{-\frac{h}{H_B}} + Ch^{10}e^{-\frac{h}{H_C}} + De^{-\frac{(h-H_D)^2}{2d^2}}$$

General Hufnagel-Valley model

Free atmospheric estimates of C_n^2 about 10x – 100x larger than expected

Ground estimates of $C_n^2 \approx 10^{-13}$ m$^{-2/3}$

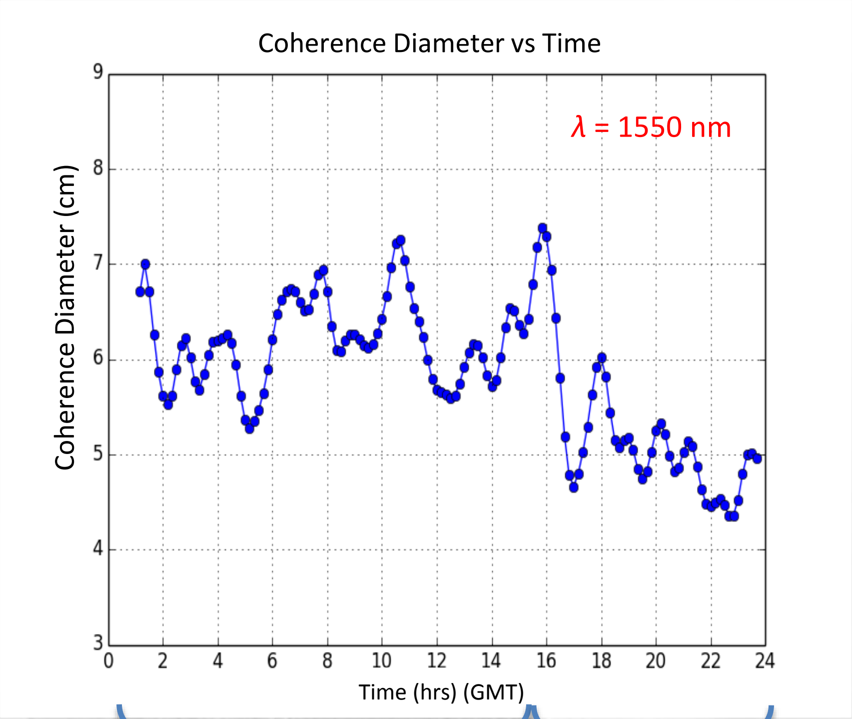
- At optical wavelengths the refractive index structure parameter is a function of C_T^2 only

$$C_n^2 = \left(10^{-6} \times \frac{77.689\langle P\rangle}{\langle T\rangle^2}\right)^2 C_T^2$$

Average prevailing pressure (hPa)

Specific atmospheric conditions during data compilation were not available, thus a nominal value of $k_c = 15\ \mathrm{m}^{-1}$ was assumed

Results – Coherence Diameter



Night time variation between 5.2 cm – 7.5 cm

Day time variation between 4.3 cm – 6 cm

- Coherence diameter, also known as the Fried parameter

$$r_0 = \left[0.423k^2 \int_0^H C_n^2(h)dh\right]^{-3/5}$$

- Determines resolution limitations of telescopes

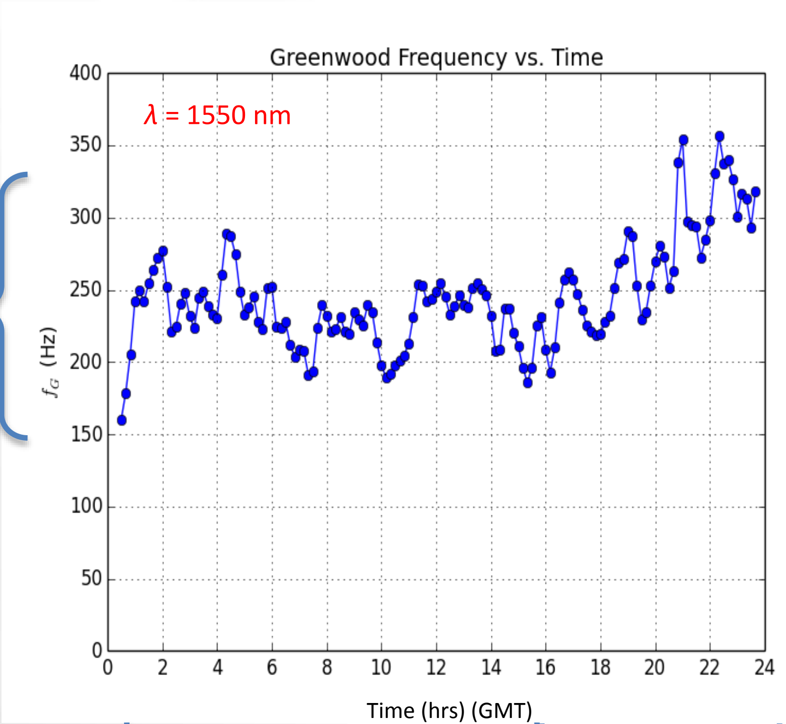
$D > r_0$ ⇨	atmosphere limited
$D < r_0$ ⇨	diffraction limited

- Also determines the spacing of actuators in adaptive optical systems

Results - Greenwood Frequency

- The Greenwood frequency specifies the response characteristic required of an atmospheric adaptive optics system to mitigate the refractive index perturbations

$$f_G = 0.255 \left[k^2 \int_0^H C_n^2(h) \left(\bar{U}(h)\right)^{5/3} dh \right]^{3/5}$$

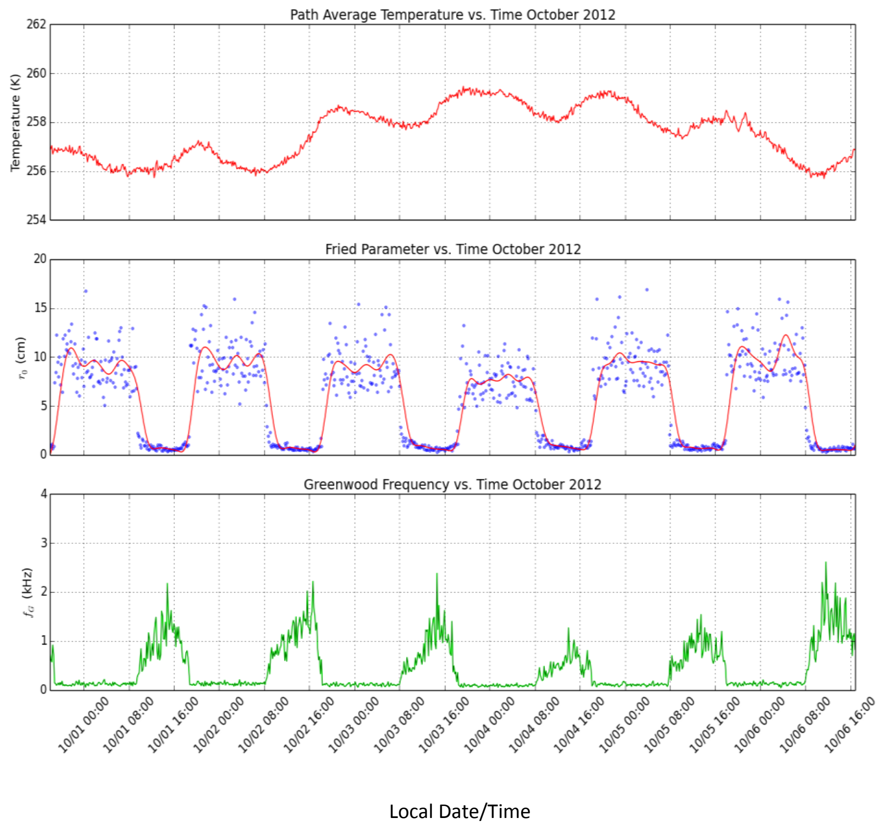


These values are about a factor of 3 larger than expected for the experimental site in January

Night time variation between 150 – 290 Hz

Day time variation between 200 – 350 Hz

Results – Five Days in October 2012



- Diurnal variations are easily resolved
- Verification that the resolution requirements of the radiometer are sufficient for this method

Coherence diameter varies between about 1 cm (day) and 10 – 18 cm (night)

Greenwood frequency varies between ≈ 2.5 kHz (day) and 200 Hz (night)

Summary

- Atmospheric remote sensing method using a single microwave profiling radiometer to obtain temperature and humidity turbulence structure parameters
- Augmented Kolmogorov turbulence theory to account for boundary effects in a general stratified atmosphere
- Test case shows promising results; however Greenwood frequencies and coherence diameters are over/under estimated
- A more rigorous turbulence spectrum derivation is required
- Ground-based measurements of the gradient Richardson numbers are required for better estimation of the crossover frequency k_c
- Concurrent radiosonde measurements of structure parameters along with the radiometer is needed for appropriate comparison