Progress on Green's Function Calculations for Non-Axisymmetric Jets

Stewart J. Leib
Ohio Aerospace Institute

Acoustics Technical Working Group
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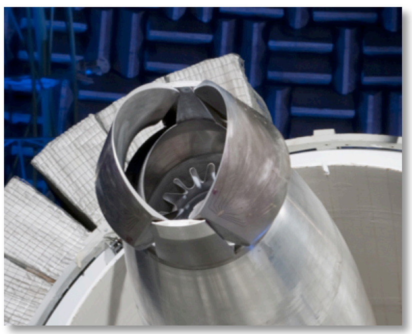
Presentation Outline

I. Motivation
II. Acoustic Analogy Approach
    i. Problem for Green's function
III. Previous Work
IV. Status of Green's function solver
V. Ongoing and Future Work

Jet Noise Prediction Needs

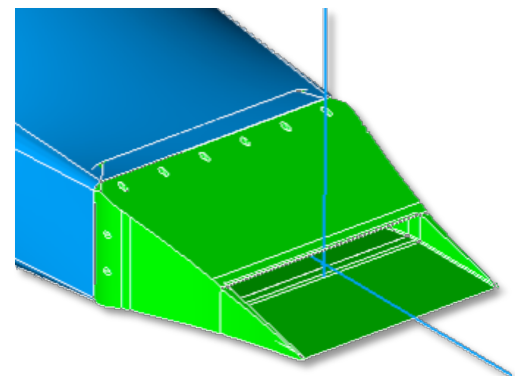
Next generation aircraft will involve complex exhaust system geometries

Non-circular exits



Multiple Streams

Nearby Solid Surfaces



In order to make noise predictions for Next GEN exhaust systems, need physics-based prediction methods that can handle:

- Non-axisymmetric mean flows
- Interactions with solid surfaces

Acoustic Analogy Approach

- Current work based on Goldstein (2003) formulation.

Formula for the Acoustic Spectrum

$$I_\omega(\boldsymbol{x}) = 2\pi \int_V \int_{-\infty}^{\infty} \int_V \Gamma_{\lambda j}(\boldsymbol{x} \mid \boldsymbol{y};\omega)\Gamma^*_{\kappa l}(\boldsymbol{x} \mid \boldsymbol{y}+\eta;\omega) e^{-i\omega\tau} \mathcal{R}_{\lambda j \kappa l}(\boldsymbol{y},\eta,\tau)\, d\eta\, d\tau\, d\boldsymbol{y}$$

Propagator Function (Green's Function) $\Rightarrow \Gamma_{\lambda j}$ -- Computed

$$\Gamma_{\lambda j}(\boldsymbol{x} \mid \boldsymbol{y};\omega) \equiv \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\omega(t-\tau)} \left[\frac{\partial g^a_{\lambda 4}(\boldsymbol{y},\tau \mid \boldsymbol{x},t)}{\partial y_j} - (\gamma - 1)\frac{\partial \tilde{v}_\lambda}{\partial y_j} g^a_{44}(\boldsymbol{y},\tau \mid \boldsymbol{x},t) \right] d(t-\tau)$$

Source Terms $\Rightarrow \mathcal{R}_{\lambda j \kappa l} = \varepsilon_{\lambda j,\sigma m} R_{\sigma m \gamma n} \varepsilon_{\kappa l,\gamma n}$; $\varepsilon_{\lambda j,\sigma m} \equiv \delta_{\lambda\sigma}\delta_{jm} - \frac{\gamma-1}{2}\delta_{\lambda j}\delta_{\sigma m}$

$R_{\sigma m \gamma n}$ Reynolds Stress Auto-Covariance Tensor -- Modeled

Computation of the Green's function is generally the most time-consuming part of the prediction.

Acoustic Analogy Approach Computation of Green's function

- Reduced-order models.
  - Relatively quick turn-around time for design and concept evaluation.
- Locally parallel mean flow, observer in the far field.

Problem for adjoint scalar Green's function

$$\frac{\partial}{\partial y_j}\frac{\widetilde{c^2}\left(\boldsymbol{y}_\perp\right)}{\left[1-M\left(\boldsymbol{y}_\perp\right)\cos\theta\right]^2}\frac{\partial g\left(\boldsymbol{y}_\perp;\varphi,\theta:\omega\right)}{\partial y_j}+\omega^2\left\{1-\frac{\left(\widetilde{c^2}\left(\mathbf{y}_\perp\right)/c_\infty^2\right)\cos^2\theta}{\left[1-M\left(\mathbf{y}_\perp\right)\cos\theta\right]^2}\right\}g\left(\mathbf{y}_\perp;\varphi,\theta:\omega\right)=0 \qquad \text{j=2,3}$$

$$g\left(\boldsymbol{y}_\perp;\varphi,\theta:\omega\right)\to\frac{\left(\omega/c_\infty\right)^2 e^{-i\omega/c_\infty\sin\theta\, y_\perp\cos\left(\varphi-\varphi_0\right)}e^{i\pi/4}}{2\left(2\pi\right)^2\sqrt{2\pi\sin\theta\,\omega/c_\infty}}+\text{outgoing waves}$$

$$\text{as } y_\perp\to\infty$$

Previous Work

- Noise predictions for rectangular jets[1].
  - Conformal mapping to elliptical coordinates for Green's function.
  - Hybrid (space-time/frequency wavenumber) source model[2].
  - Comparisons with acoustic data (Extensible Rectangular Nozzles) for cold subsonic jets over a range of aspect ratios, jet exit Mach numbers .
- Other applications for conformal mapping method[3].
- Not all nozzle geometries amenable to conformal mapping method.
  - Need more general method.

1. Leib, S.J., 2013 Noise Predictions for Rectangular Jets Using a Conformal Mapping Method, *AIAA Journal*, Vol. 51 No. 3, pp. 721-737.
2. Leib, S.J. & Goldstein, M.E. 2011 A Hybrid Source Model for Predicting High-Speed Jet Noise, *AIAA Journal*, Vol. 49 No. 7, pp. 1324-1335.
3. Leib, S.J., Green's Functions for Prediction of Noise From Non-Axisymmetric Jets, Acoustics Technical Working Group, April 2012.

Green's Function: Reduced-Order Models Expansion in Orthogonal Functions

- Represent mean flow quantities in the governing equation for the adjoint Green's function by sum of orthogonal functions in an appropriate coordinate system.
  - For computational efficiency, a relatively small number of functions is desired.
- Expand Green's function in series of these orthogonal functions
- Solve system of coupled ordinary differential equations for Green's function modes:
  - Iterative solution (Mani).
  - Direct solution of banded system.

Green's Function: Reduced-Order Models Governing Equation

$$\frac{\partial^2 g}{\partial r^2} + \frac{1}{r}\frac{\partial g}{\partial r} + \frac{1}{r^2}\frac{\partial^2 g}{\partial \phi^2} + \mathcal{R}\frac{\partial g}{\partial r} + \mathcal{F}\frac{\partial g}{r\partial \phi} + \omega^2 \mathcal{H} g = 0$$

Mean flow dependent coefficients

$$\mathcal{R} = \left\{ \frac{2\cos\theta}{\left[1 - M(\boldsymbol{y}_\perp)\cos\theta\right]}\frac{\partial M}{\partial r} + \frac{1}{\widetilde{c^2}(\boldsymbol{y}_\perp)}\frac{\partial \widetilde{c^2}(\boldsymbol{y}_\perp)}{\partial r} \right\} \quad \mathcal{H} = \left\{ \frac{\left[1 - M(\boldsymbol{y}_\perp)\cos\theta\right]^2}{\widetilde{c^2}(\boldsymbol{y}_\perp)} - \frac{\cos^2\theta}{c_\infty^2} \right\} \quad \mathcal{F} = \left\{ \frac{2\cos\theta}{\left[1 - M(\boldsymbol{y}_\perp)\cos\theta\right]}\frac{\partial M}{r\partial \phi} + \frac{1}{\widetilde{c^2}(\boldsymbol{y}_\perp)}\frac{\partial \widetilde{c^2}(\boldsymbol{y}_\perp)}{r\partial \phi} \right\}$$

Fourier series expansion:

$$\mathcal{R}(r,\phi) = \sum_{l=-L}^{L} \mathcal{R}_l(r)e^{il\phi} \quad ; \quad \mathcal{F}(r,\phi) = \sum_{l=-L}^{L} \mathcal{F}_l(r)e^{il\phi} \quad ; \quad \mathcal{H}(r,\phi) = \sum_{l=-L}^{L} \mathcal{H}_l(r)e^{il\phi}$$

$$g(r,\phi) = \sum_{n=-N}^{N} g_n(r)e^{in\phi}$$

Green's Function: Reduced-Order Models Governing Equation

System of ODEs for Fourier components of Green's function:

$$\frac{d^2 g_n}{dr^2} + \frac{1}{r}\frac{dg_n}{dr} - \frac{n^2}{r^2} g_n + \sum_{l=-L}^{L} R_l \frac{\partial g_{n-l}}{\partial r} + \sum_{l=-L}^{L} F_l \frac{i}{r}(n-l) g_{n-l} + \omega^2 \sum_{l=-L}^{L} H_l g_{n-l} = 0 \quad ; \quad -N \le n \le N$$

Boundary conditions:

(I) Far-Field:

$$\frac{dg_n}{dr} + \kappa_n g_n \to \left(\frac{d}{dr} + \kappa_n\right) \frac{(\omega / c_\infty)^2 e^{i\pi/4}}{4(2\pi)^2 \sqrt{2\pi \sin\theta \omega / c_\infty}} e^{-in(\varphi+\pi/2)} H_n^{(2)}\left(\frac{\omega}{c_\infty} r \sin\theta\right) \quad \text{as } r \to \infty$$

(II) Centerline:

$$g_n(0;\varphi,\theta:\omega) = 0, n \ne 0$$

$$\frac{dg_0(0;\varphi,\theta:\omega)}{dr} = 0$$

Green's Function: Reduced-Order Models Numerical Methods

- Replace radial derivatives with second-order central differences.
- Form a system of algebraic equations for the Fourier modes of the Green's function at discrete grid points is formed.

$$\boldsymbol{B}_j \boldsymbol{g}^{j-1} + \boldsymbol{A}_j \boldsymbol{g}^{j} + \boldsymbol{C}_j \boldsymbol{g}^{j+1} = 0 \quad ; \quad j = 1, J$$

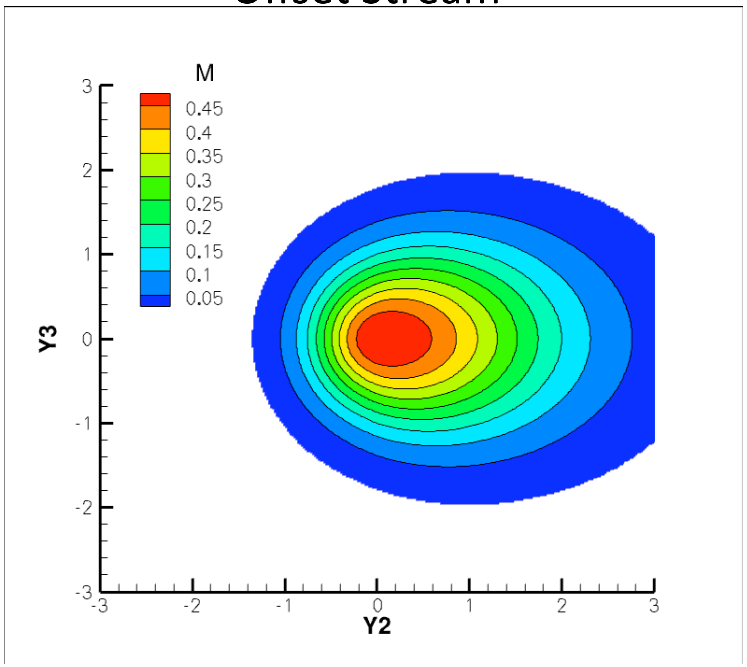
Solution Vector: $$\boldsymbol{g}^j = \left\{ \begin{matrix} g_{-N}^j & g_{-N+1}^j & \cdot & \cdot & \cdot & g_0^j & \cdot & \cdot & \cdot & g_{N-1}^j & g_N^j \end{matrix} \right\}^T$$

  - Solve (banded system) directly using a sparse system algorithm
- Compute Fourier modes of coefficients and sum series for the Green's function using FFTW.

Validation and Test Cases

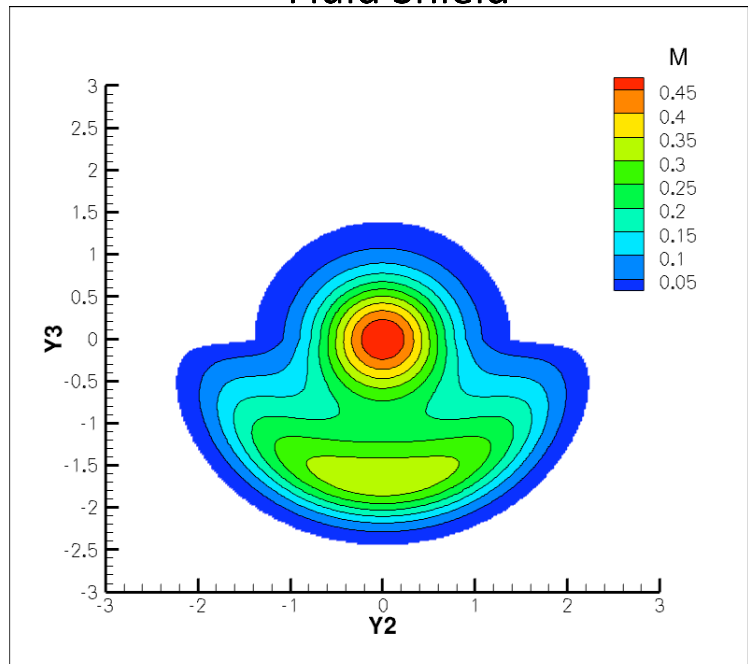
- Reproduce round jet results. ✔
- Consider two non-circular test cases:

Offset Stream



$$M(r,\phi) = M_C e^{-(1+\alpha-2\alpha\cos\phi)r^2}$$

Fluid Shield

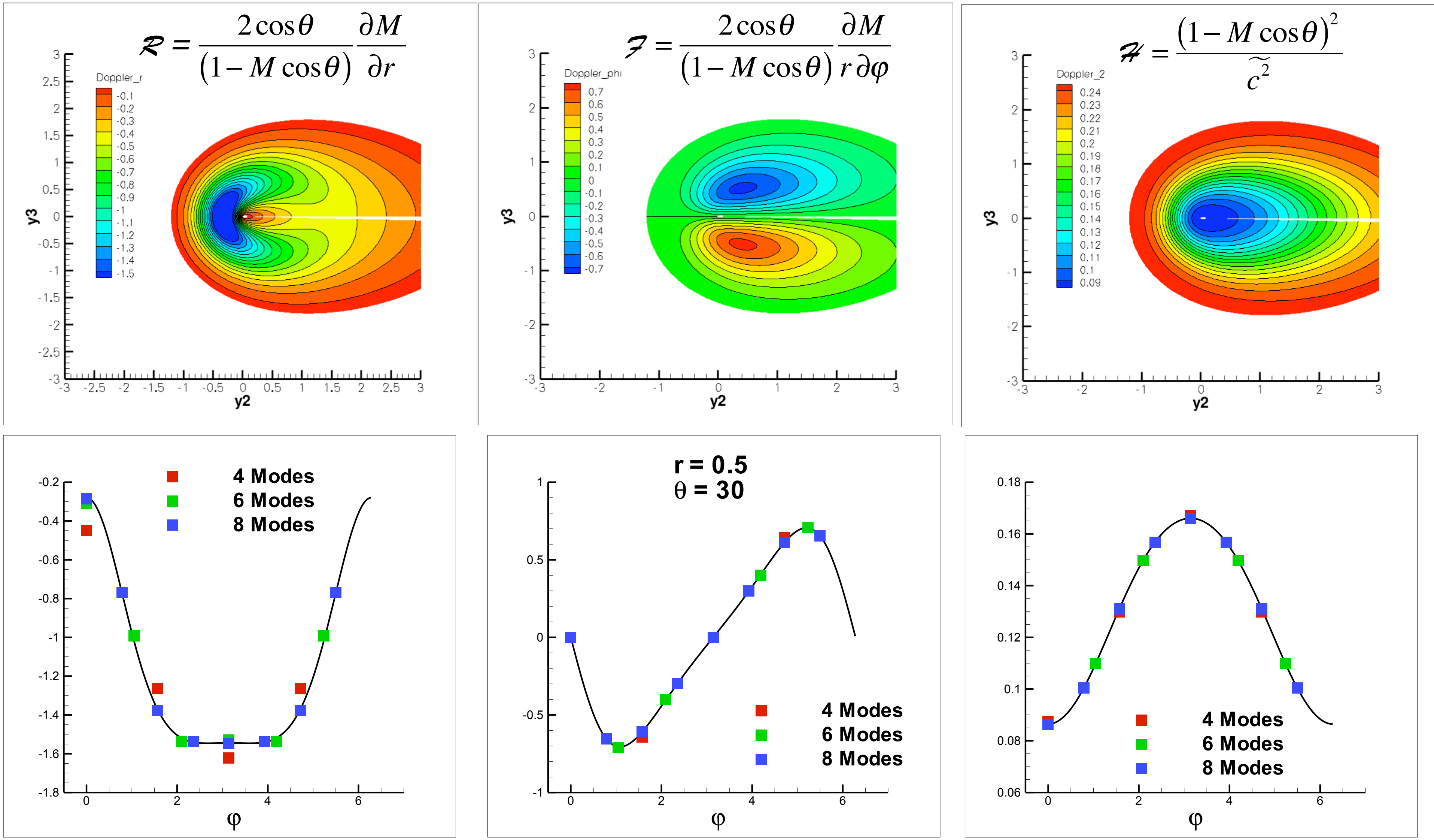


$$M(r,\phi) = M_C\left[e^{-ar^2} + br^2 e^{-c(r-1)^4} h(\phi)\right]$$

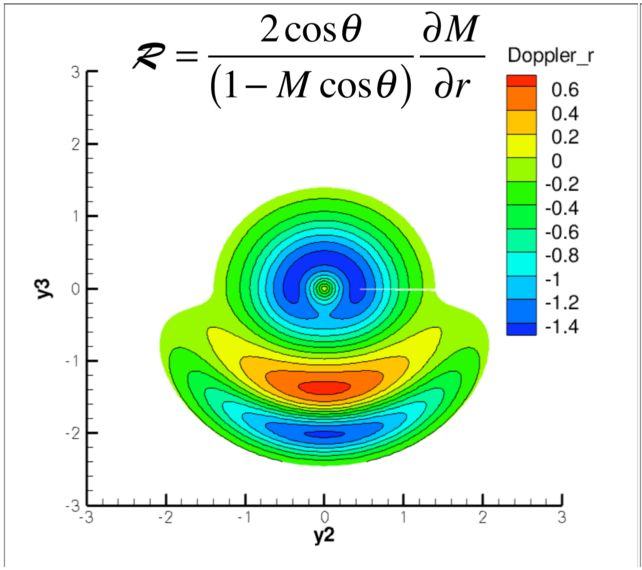
$$h(\phi) = \begin{cases} 0 & , \quad 0 \le \phi < \pi \\ -\sin\phi & , \quad \pi \le \phi < 2\pi \end{cases}$$

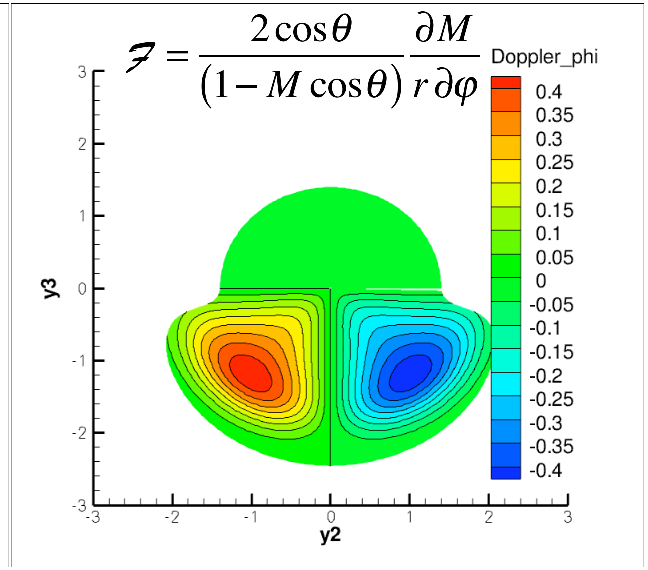
Test Case : Offset Stream

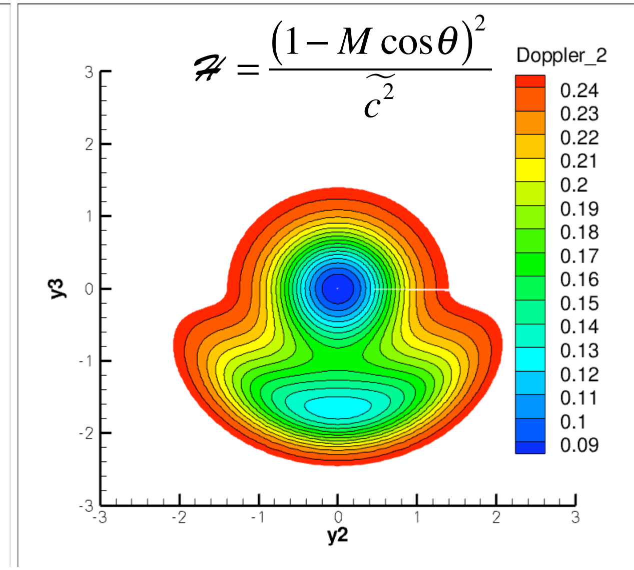
Fourier Representation of Coefficients

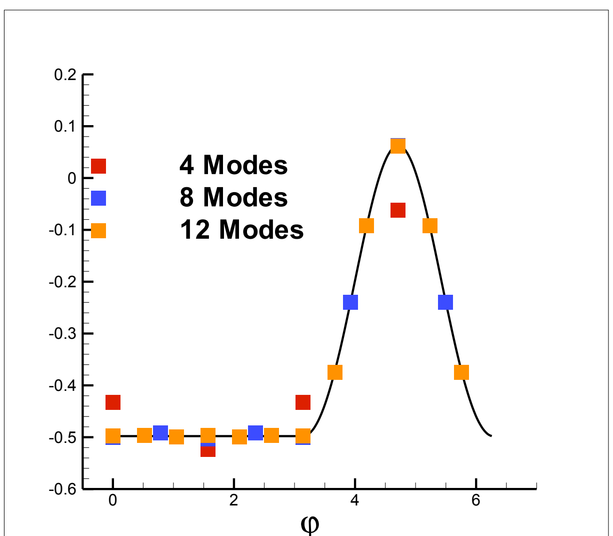


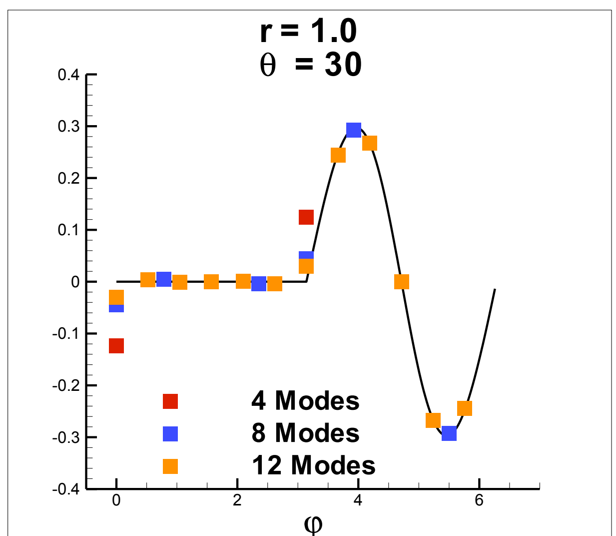
Test Case : Fluid Shield
Fourier Representation of Coefficients

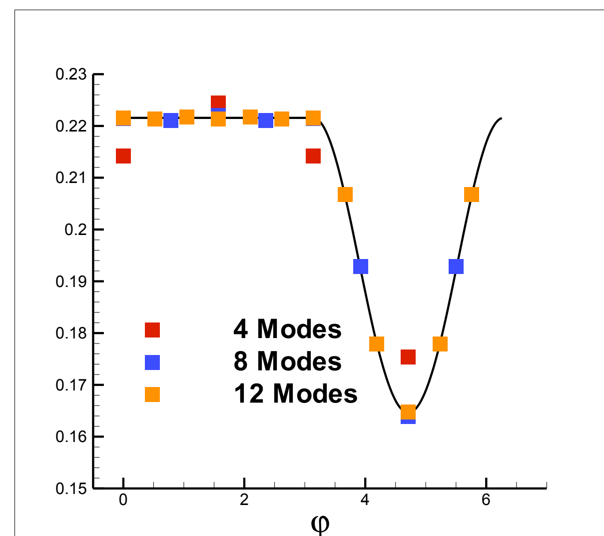












Tests of Green's Function Solver

- Numerical parameters:

$$N = 16, \quad \Delta = 0.005, \quad y_T^{\max} = 5.0$$

- Green's function results to be presented:
  - Effect of number of mean flow modes. $L = 4,6,8$
  - High-frequency behavior:

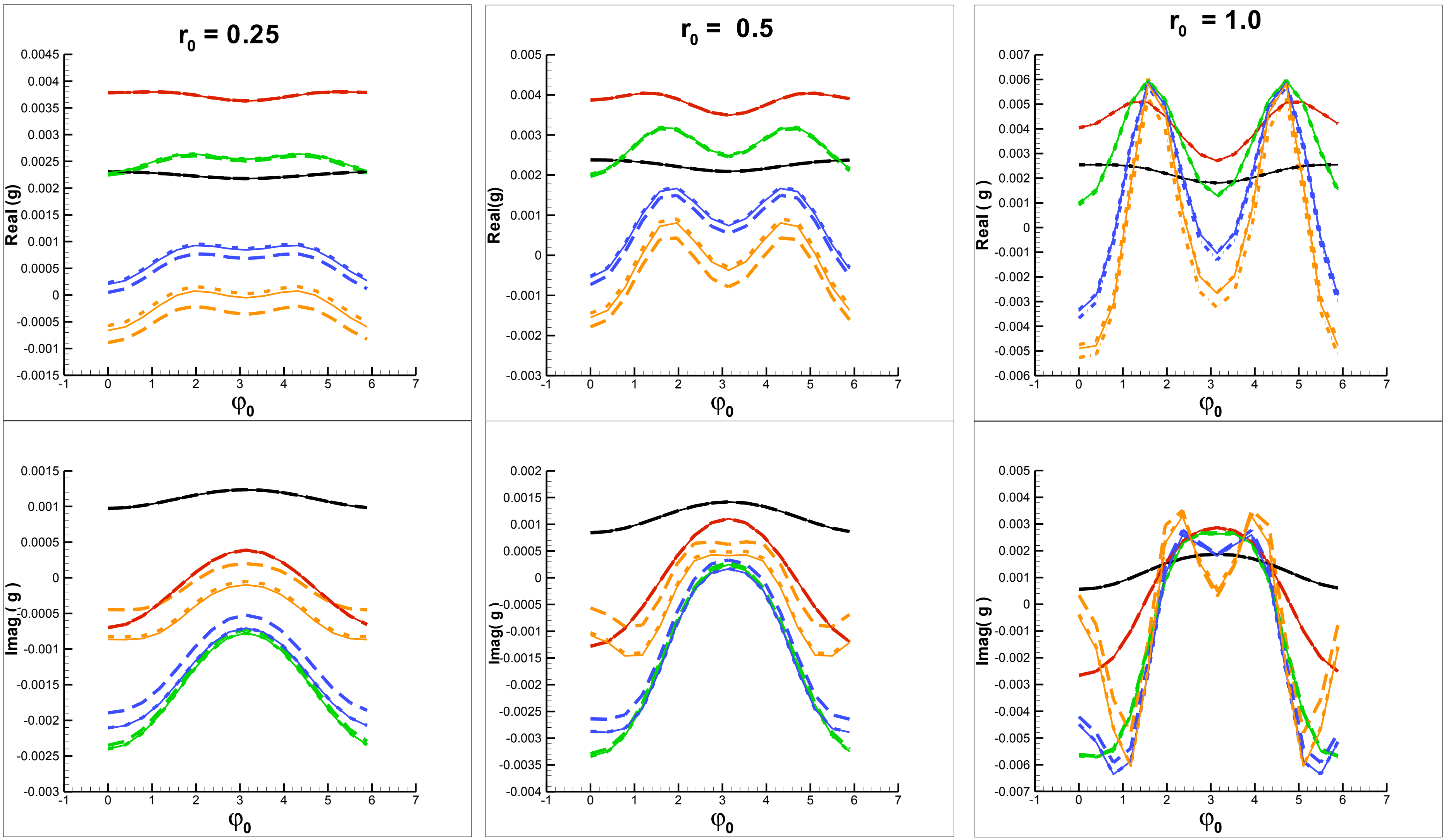
$$\omega^{-3/2} |g| \quad \text{independent of frequency as } \omega \to \infty$$

Test Case : Offset Stream Green's Function Results

$\theta = 30 \;\; ; \;\; \varphi = 0$

St =0.2,0.4,0.6 ,0.8,1.0

Long-Dash $L = 4$, Short-Dash $L = 6$, Solid $L = 8$



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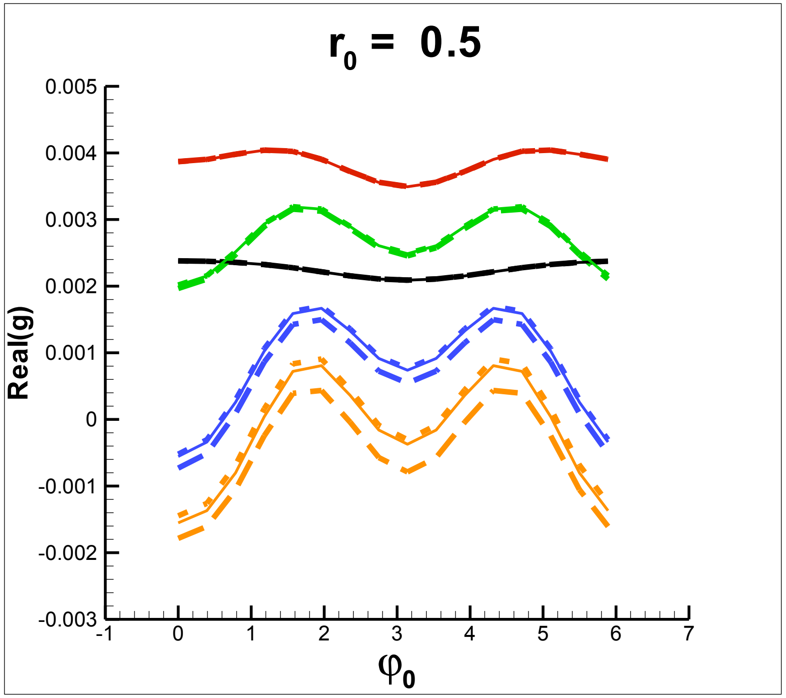
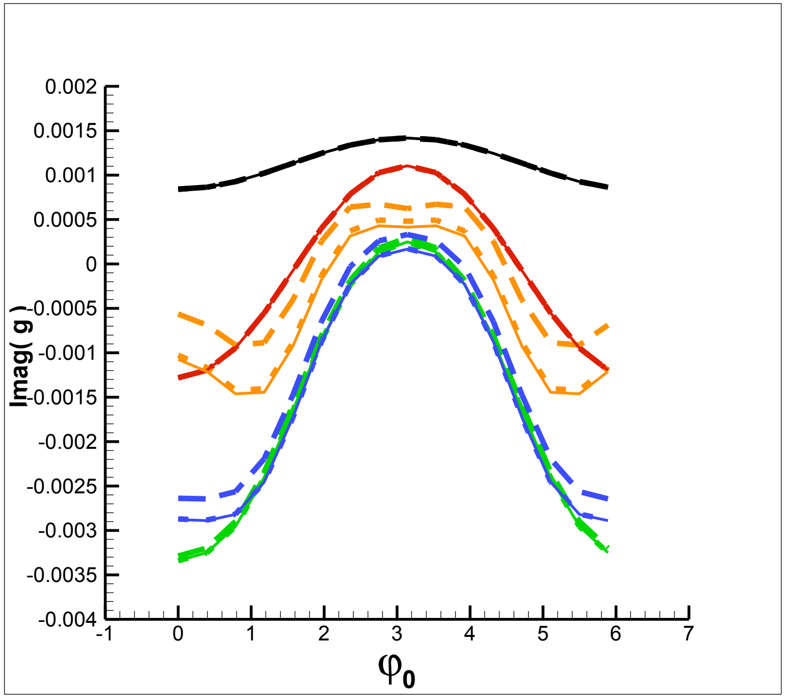
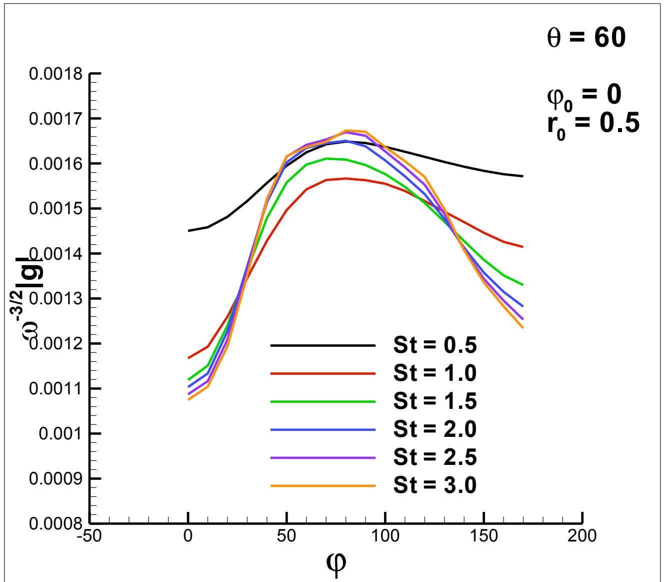




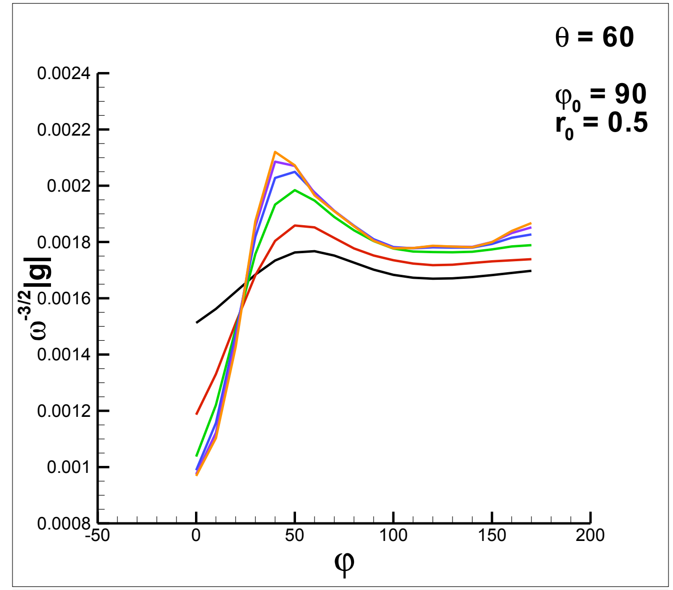
Test Case : Offset Stream Green's Function Results

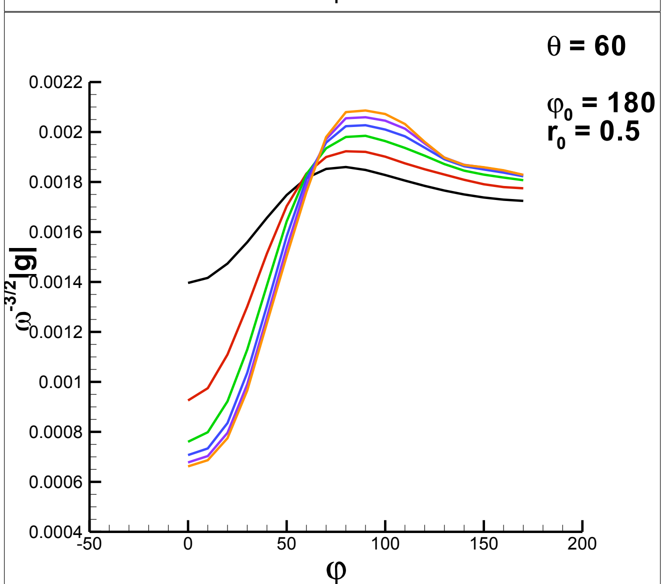


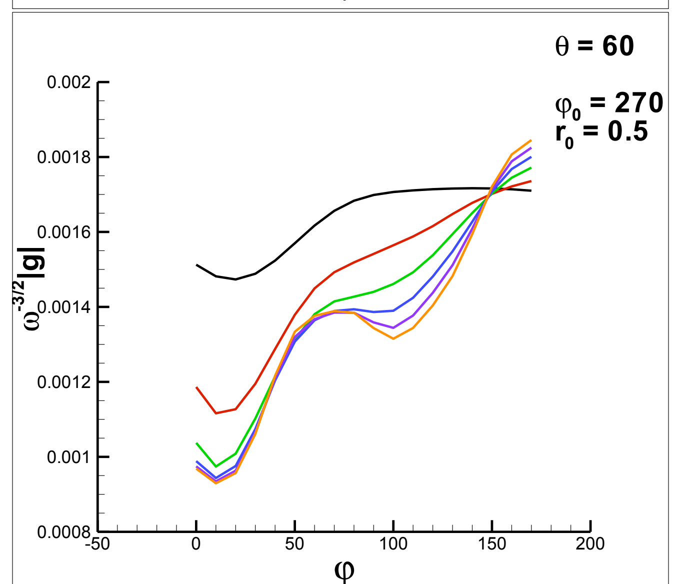
High- Frequency Collapse of the scaled Green's function:

$$\omega^{-3/2}\left|g\left(r_o,\varphi_0;\varphi,\theta:\omega\right)\right|$$

vs. φ





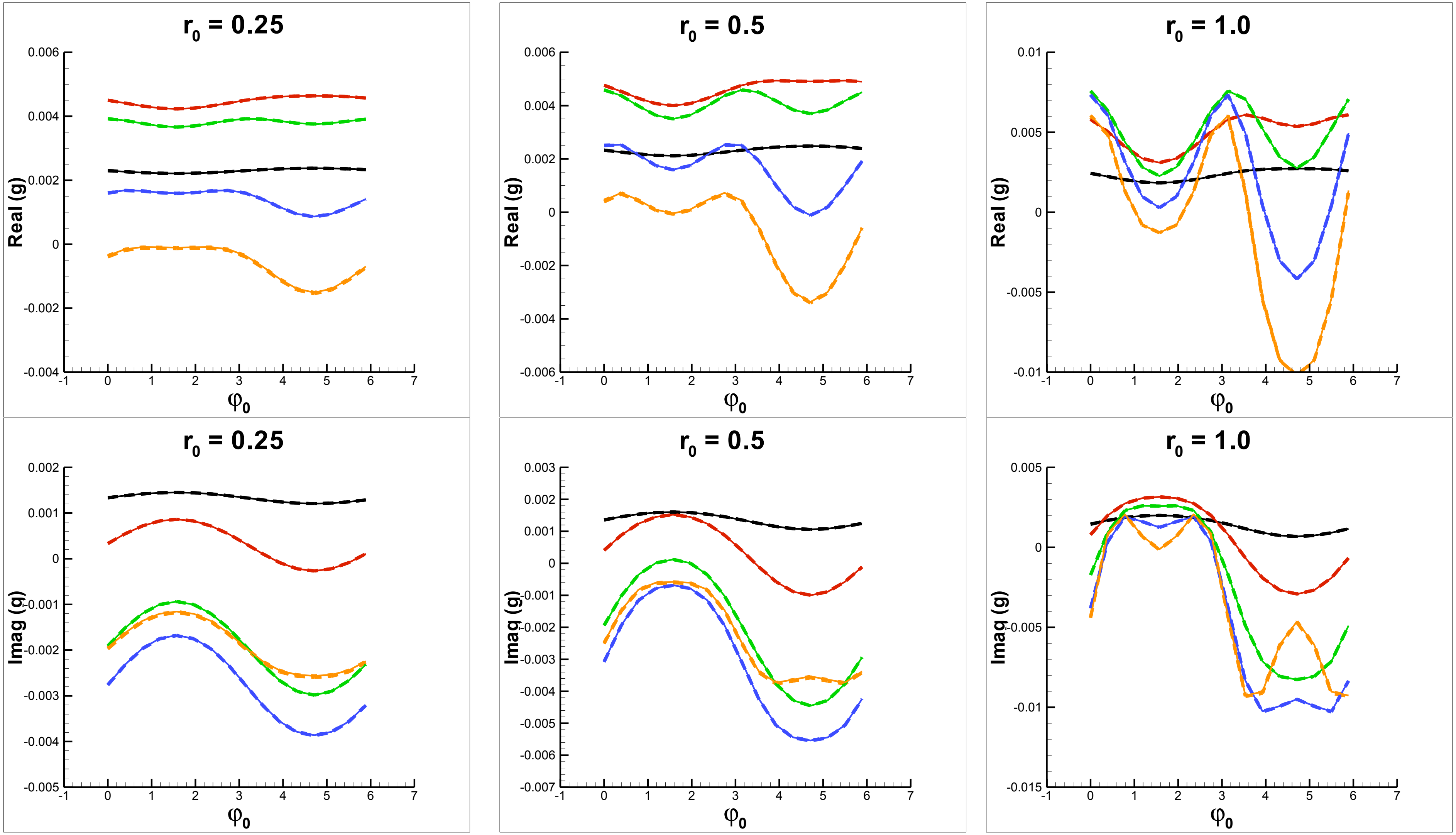


$\theta = 30 \;\; ; \;\; \varphi = \frac{3\pi}{2}$

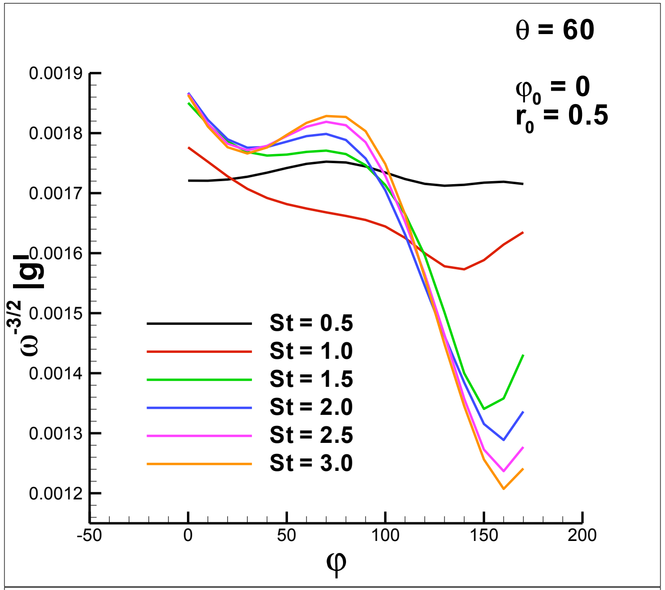
Test Case : Fluid Shield Green's Function Results

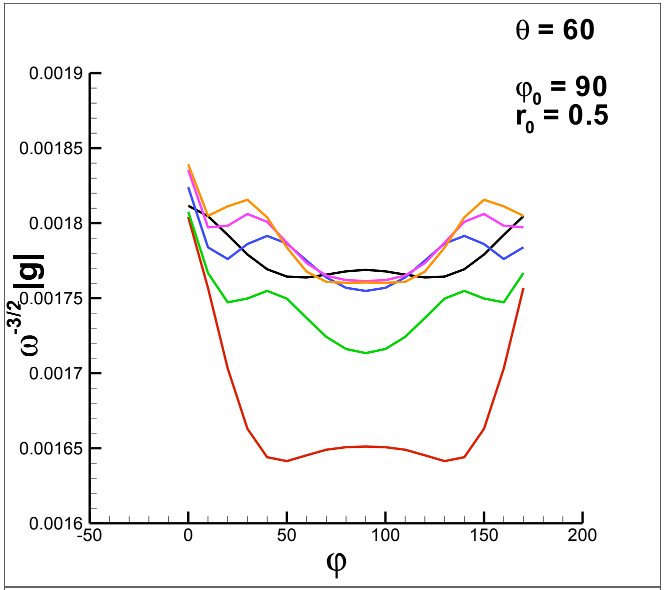
St =0.2,0.4,0.6 ,0.8,1.0

Long-Dash $L = 4$, Short-Dash $L = 6$, Solid $L = 8$



Test Case : Fluid Shield
Green's Function Results

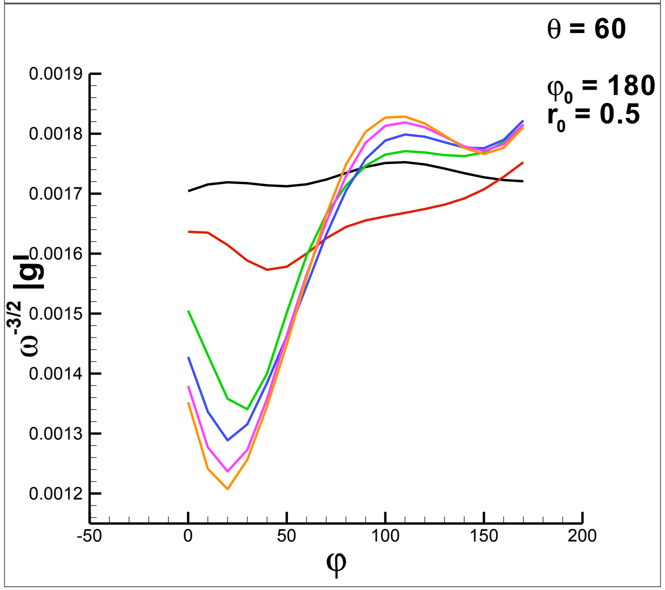


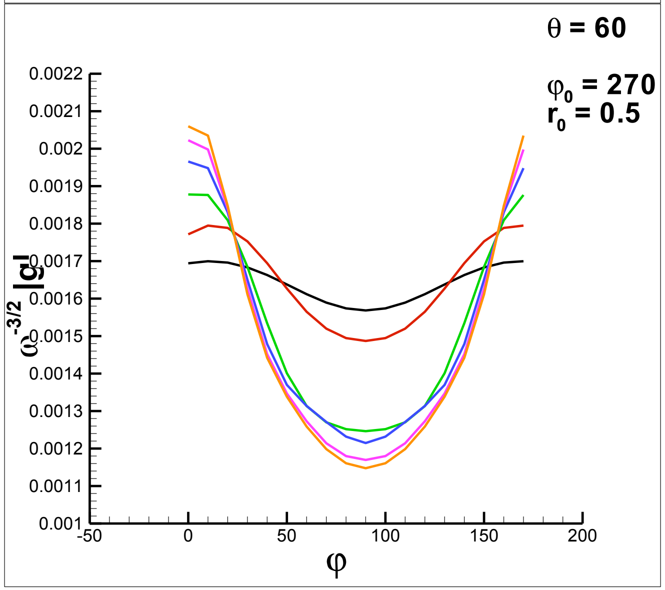


High- Frequency Collapse of the scaled Green's function:

$$\omega^{-3/2}\left|g\left(r_o,\varphi_0;\varphi,\theta:\omega\right)\right|$$

vs. φ





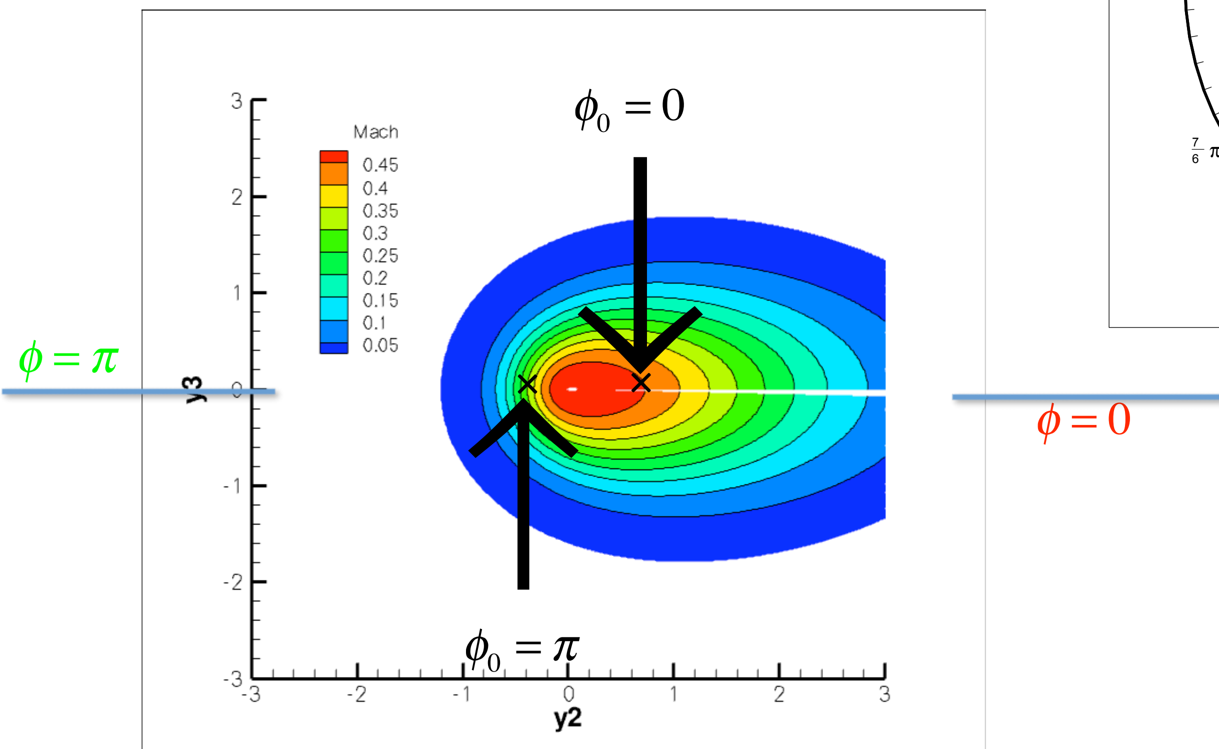
Noise Shielding by Non-Axisymmetric Mean Flows

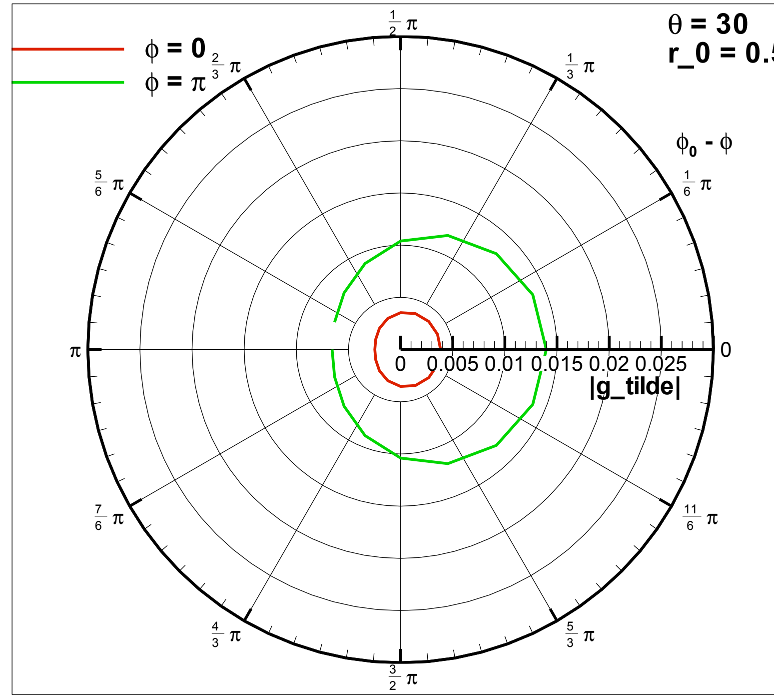
$St = 0.6$

Plot $\left| g\left(r_0, \phi_0 \mid \omega, \phi, \theta\right) \right|$ vs. $\left(\phi_0 - \phi\right)$

for observer locations:

$\left(\phi = 0, \theta = 30\right)$ and $\left(\phi = \pi, \theta = 30\right)$





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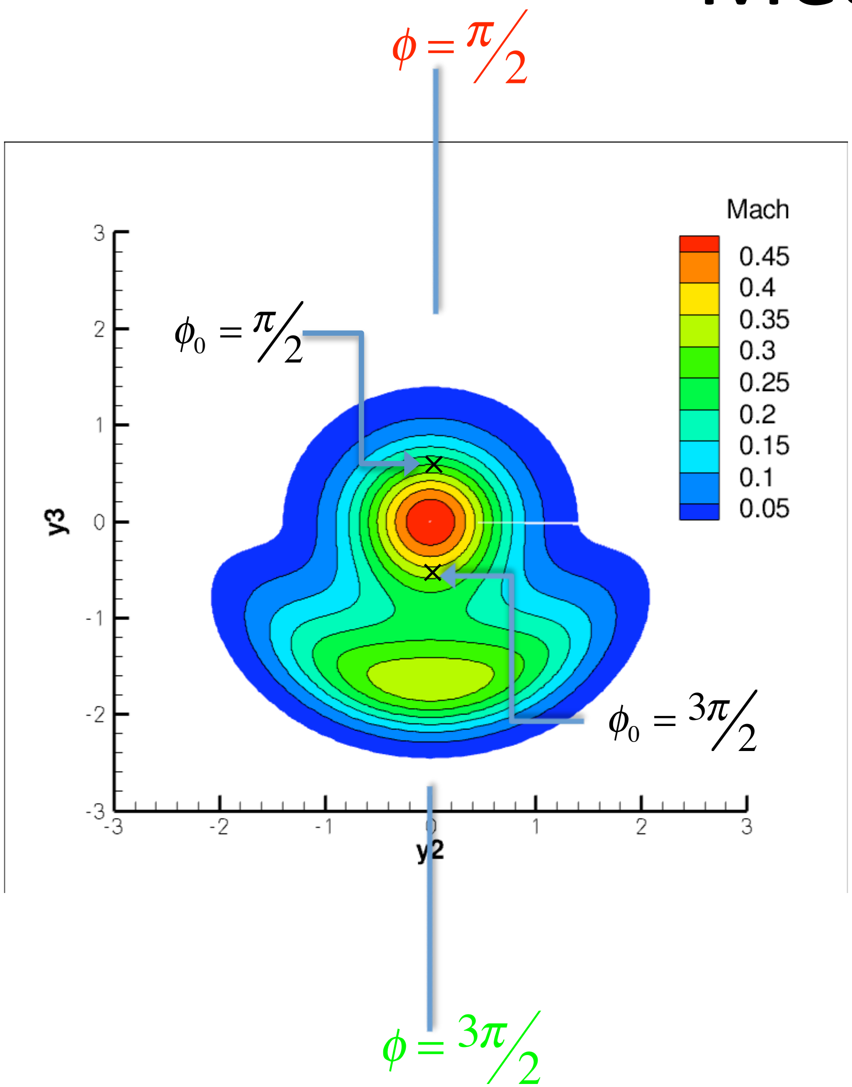
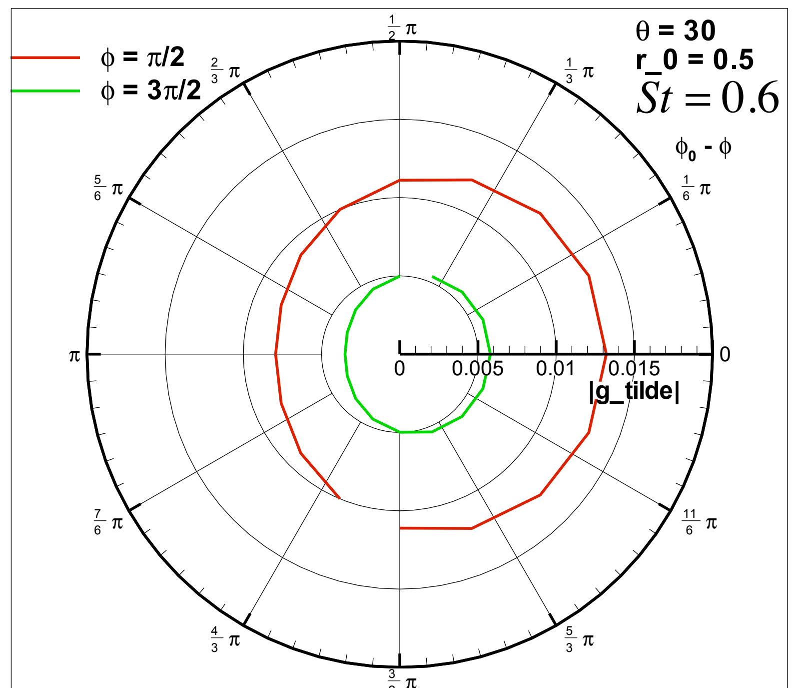


Plot $\left|g\left(r_0,\phi_0 \mid \omega,\phi,\theta\right)\right|$ vs. $\left(\phi_0 - \phi\right)$

for observer locations:

$\left(\phi = \pi/2, \theta = 30\right)$ and $\left(\phi = 3\pi/2, \theta = 30\right)$



Ongoing and Future Work

- Apply to non-axisymmetric mean flow obtained from RANS solution.
- Integrate with a source model for noise predictions.

- Continued development of a code for numerical solution of Green's function of acoustic analogy equations. (Collaboration with John Goodrich, GRC)
  - Validation of reduced-order models.
  - Study effects of non-parallel mean flow.
  - High-resolution calculations for cases of special interest.

STATUS:

- Exercising 2D LEE solver with non-uniform mean flow.
- Extend to 3D.
- Adapt for acoustic analogy equations.