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# A Case Study on the Application of a Structured Experimental Method for Optimal Parameter Design of a Complex Control System

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August 2015

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## **Abstract**

*This report documents a case study on the application of Reliability Engineering techniques to achieve an optimal balance between performance and robustness by tuning the functional parameters of a complex non-linear control system. For complex systems with intricate and non-linear patterns of interaction between system components, analytical derivation of a mathematical model of system performance and robustness in terms of functional parameters may not be feasible or cost-effective. The demonstrated approach is simple, structured, effective, repeatable, and cost and time efficient. This general approach is suitable for a wide range of systems.*

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# 1. Introduction

This report documents a case study on the application of Reliability Engineering techniques to achieve an optimal balance between performance and robustness by tuning the functional parameters of a complex non-linear control system. The system of interest is the nonlinear pendulum control system reported by Torres-Pomales and Gonzalez [2]. The following subsection examines fundamental systems concepts and establishes a general context for the case study.

## 1.1. Background Systems Concepts

An engineered **system** is an entity with a purpose that is achieved by the interactions (i.e., relations) of its internal components (i.e., sub-entities) with each other and with the external environment (i.e., surroundings). The concept of system is circular and recursive such that, in general, both the environment and the components of a system are themselves systems. A system of interest (SOI) (or system in focus) is always contained within a larger super-system (or meta-system) consisting of the SOI and its environment.

The **purpose** (i.e., a goal) of an engineered system is defined in terms of the desired (i.e., intended) effect on its environment achieved by the flow of matter, energy, or information between the system and its environment. The system purpose is the basis for the specification of the desired functional and quality attributes (i.e., properties) of the system. The purpose of system components is to contribute to accomplishing the larger goal of the containing system. Thus, in general, there is a dependence (i.e., a relation) between a system and its environment. Avizienis et al. define system **dependability** as “the ability to deliver service that can justifiably be trusted” and also “the ability to avoid service failures that are more frequent and more severe than is acceptable” [1]. Dependability is a function of **criticality**, which is a measure of the importance (i.e., degree or strength of influence) of a factor in achieving a larger goal. The failure or variability in performance of a high criticality component can have a severe effect on the operation of the containing system. Dependability is also a function of trust (i.e., confidence) in the service delivered by a system. **Assurance** is the level of confidence in the properties of a system. This confidence is bounded by the various forms of uncertainty (both epistemic and aleatoric) that threaten the ability to achieve and ascertain the desired system properties. Uncertainties can originate internal to the system, or they can be external in nature and their effects propagate to the system (for example, at the inputs). Dependability is bounded by the ability to measure uncertainties and predict their effects on goals. In general, the ability to assess uncertainties and their effects diminishes as system complexity (i.e., number and variety of components, relations, and states) increases. The dependability of a component (or sub-system) can be seen as a measure of the level of associated **risk** (i.e., potential for unwanted outcome) with respect to the goals of the containing system.

Goals, dependencies, and uncertainties are interrelated. In engineering a system, we want to achieve a balance between **performance** (i.e., utility or value with respect to purpose) and **robustness** (i.e., insensitivity to uncertainties and undesired influences) in achieving the goals. The relation between goals and uncertainties is primarily determined by the dependencies defined in the system architecture and is a function of system parameters. Once a system architecture has been specified, we want to determine the combination of parameter values that results in the optimal balance between performance and robustness. For complex systems with intricate and non-linear patterns of interaction between system components, analytical derivation of a mathematical model of system performance and robustness in terms of functional parameters may not be feasible or cost-effective. Even if a model can be derived, closed-form solutions may not be possible. Without effective analytical means to determine the best combination of system parameters, the alternative option is experimentation. However, trial-and-error can be time consuming and the results can be unpredictable. We want a methodical, effective, repeatable, and cost and time efficient approach to tune the parameters of a system.

## 1.2. Overview of Content

The next section describes the system of interest, including the plant and the controller. This is followed by the system tuning problem statement. An overview of the Reliability Engineering concepts applied in this study is given. This is followed by a description of the approach used to tune the system. Detailed results are presented. Conclusions and final remarks end the report.

## 2. System Under Study

Figure 1 is the block diagram of the control system. The state of the plant  $X$  is controlled by the motor torque  $T_d$ . The system is designed to track the target arm position  $\theta_d$  while maintaining control of the pendulum above the horizontal. The swing-up problem was not part of this study.

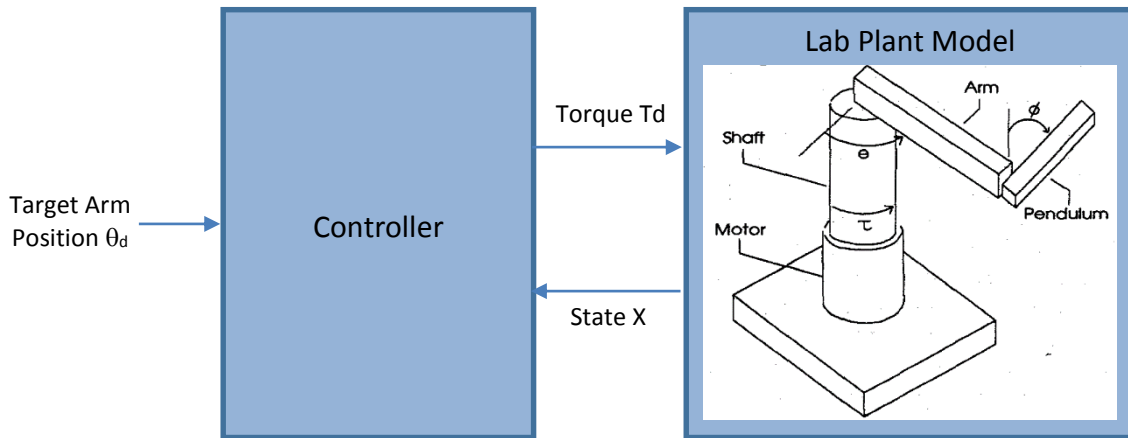


Figure 1: High-Level Block Diagram of Control System

### 2.1. Plant

The plant consists of a motor which generates a torque  $\tau$ , a vertical shaft, a horizontal arm with position and speed denoted  $\theta$  and  $\dot{\theta}$ , and a freely rotating pendulum with position and speed denoted  $\phi$  and  $\dot{\phi}$ . For this case study, it is assumed that the pendulum is always above the horizontal position, i.e.,  $\phi \leq \pm 90^\circ$ . Two plant models were developed. The Ideal Plant Model does not include motor dynamics and assumes that the desired motor torque equals the applied torque (i.e.,  $T_d = \tau$ ). The Laboratory Plant Model includes model dynamics, friction, and other hard nonlinearities.

#### 2.1.1. Ideal Plant Model

The ideal plant model was derived using the Euler-Lagrange equation. Variable  $m$  denotes the pendulum mass,  $J_1$  is the moment of inertia of the shaft and arm relative to the shaft's main axis,  $J_2$  denotes the moment of inertia of the pendulum with respect to its center of mass,  $L_1$  is the length of the arm,  $L_2$  is the length of the pendulum, and  $g$  is the gravitational constant.



$$A\ddot{\theta} + B\ddot{\phi} + C = \tau \quad (1)$$

$$D\ddot{\theta} + E\ddot{\phi} + F + G = 0 \quad (2)$$

where:

$$A = J_1 + mL_1^2 + (1/4) mL_2^2 \sin^2(\phi)$$

$$E = J_2 + (1/4) mL_2^2$$

$$B = (1/2) mL_1 L_2 \cos(\phi)$$

$$F = - (1/8) mL_2^2 \sin(2\phi) (\dot{\theta})^2$$

$$C = - (1/2) mL_1 L_2 \sin(\phi) (\dot{\phi})^2 + (1/4) mL_2^2 \sin(2\phi) (\dot{\phi} \dot{\theta})$$

$$G = - (1/2) mgL_2 \sin(\phi)$$

$$D = B$$

### 2.1.2. Laboratory Plant Model

The laboratory plant model was developed starting from the ideal model and adds motor dynamics, friction, and some hard nonlinearities.

$$A_e \ddot{\theta} + B_e \ddot{\phi} + C_e = T_d \quad (3)$$

$$D_e \ddot{\theta} + E_e \ddot{\phi} + F_e + G_e + H_e = 0 \quad (4)$$

where:

$$A_e = A + J_m$$

$$E_e = E$$

$$B_e = B$$

$$F_e = F$$

$$C_e = C + a_m \dot{\theta}$$

$$G_e = G$$

$$D_e = D$$

$$H_e = (1/2) b_p \dot{\phi}$$

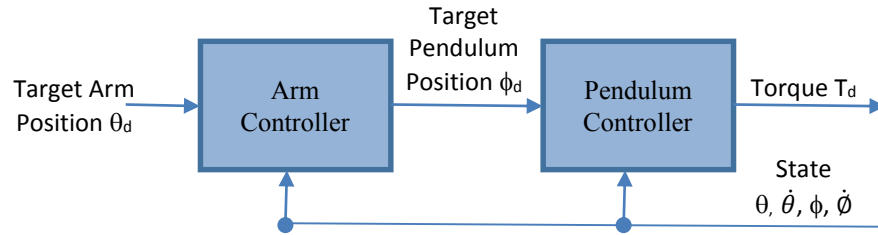
Here  $T_d$  denotes the torque developed by the motor,  $J_m$  denotes the rotational inertia of the motor, and  $a_m$  and  $b_p$  are viscous friction terms. In addition, the Laboratory Plant Model includes torque saturation  $T_{d,max}$ , motor angular speed saturation  $\dot{\theta}_{d,max}$ , static and dynamic motor friction  $T_{static}$  and  $T_{dynamic}$ , and input torque dead zone  $T_{d,dead}$ . In addition, scaling factors were used to more closely match the response of the model to the physical plant. Table 1 lists the model parameter values.

**Table 1: Parameter Values for Laboratory Plant Model**

Parameter	Value	Parameter	Value	Parameter	Value
$J_1$	0.04699 kg-m <sup>2</sup>	$a_m$	0.01932 N-m-s	$dA_e$	1.20
$J_2$	0.00780 kg-m <sup>2</sup>	$b_p$	0.01 N-m-s	$dB_e$	1.12
$J_m$	0.03790 kg-m <sup>2</sup>	$T_{d,max}$	9.8 N-m	$dC_e$	1.05
$L_1$	0.36 m	$\dot{\theta}_{d,max}$	28.27 rad/s	$dD_e$	1.12
$L_2$	0.28 m	$T_{static}$	0.10 N-m	$dE_e$	0.98
$m$	1.1564 kg	$T_{dynamic}$	0.08 N-m	$dF_e$	1.00
$g$	9.8 m/s <sup>2</sup>	$T_{d,dead}$	0.147 N-m	$dG_e$	1.08
				$dH_e$	5.50

## 2.2. Controller

Figure 2 illustrates the structure of the system controller. This is a dual-loop architecture in which the inner loop is responsible for the state of the pendulum and the outer loop generates pendulum position commands based on the target arm trajectory.



**Figure 2: Block Diagram of System Controller**

The system control laws were developed based on a Simplified Plant Model with scaling factors added to minimize the modeling error relative to the Laboratory Plant Model. The equations of the Simplified Plant Model are:

$$\ddot{\phi} = \bar{H}_a + \bar{L}_a T_d \quad (5)$$

$$\ddot{\theta} = \bar{Q}_a + \bar{R}_a T_d \quad (6)$$

where:

$$\begin{aligned}
\bar{H}_a &= -[A_a(F_a + G_a)]/(A_aE_a - B_aD_a) & C_a &= 0 \\
\bar{L}_a &= -D_a/(A_aE_a - B_aD_a) & D_a &= D \\
\bar{Q}_a &= B_a(F_a + G_a)/(A_aE_a - B_aD_a) & E_a &= J_2 + (1/4) mL_2^2 \\
\bar{R}_a &= E_a/(A_aE_a - B_aD_a) & F_a &= - (1/8) mL_2^2(\dot{\theta})^2 \\
A_a &= J_1 + mL_1^2 & G_a &= - (1/2) mgL_2(\phi) \\
B_a &= (1/2) mL_1L_2
\end{aligned}$$

The scaling factors in Table 2 were applied to the Simplified Plant Model.

**Table 2: Scaling Factors for Simplified Plant Model**

Parameter	Value	Parameter	Value
dA <sub>a</sub>	1.35	dE <sub>a</sub>	0.98
dB <sub>a</sub>	1.00	dF <sub>a</sub>	1.05
dC <sub>a</sub>	0.00	dG <sub>a</sub>	1.00
dD <sub>a</sub>	1.00	dL <sub>a</sub>	1.05

### 2.2.1. Pendulum Controller

The pendulum control law used the sliding controller theory presented by Slotine and Li [2]. The sliding surface is:

$$s = \left( \frac{d}{dt} + \lambda \right)^2 \left( \int_0^t \tilde{\phi} dt \right) = \tilde{\phi} + 2\lambda\dot{\tilde{\phi}} + \lambda^2 \int_0^t \tilde{\phi} dt$$

where  $\tilde{\phi} = \phi - \phi_d$ . The desired motor torque is:

$$T_d = (-\bar{H}_a + V + W)/\bar{L}_a \quad (7)$$

with:

$$V = \ddot{\phi}_d - 2\lambda\dot{\tilde{\phi}} - \lambda^2\tilde{\phi} \quad \Phi = \varepsilon\lambda$$

$$W = -K \text{ sat}(s/\Phi) \quad \beta = \sqrt{\bar{L}_u/\bar{L}_1}$$

$$K = \beta[\hat{F} + \eta] + (\beta - 1)|\hat{u}|$$

$$\hat{F} = |\bar{H}_u - \bar{H}_l|$$

$$\hat{u} = -\bar{H}_a + V$$

Factors  $\bar{H}_u$ ,  $\bar{H}_l$ ,  $\bar{L}_u$ , and  $\bar{L}_l$  are upper (u) and lower (l) bounds for  $\bar{H}$  and  $\bar{L}$  of the Laboratory Plant Model. These factors are given by the corresponding factors in the Simplified Plant Model using the scaling factors in Table 3. The tuning parameters for the Pendulum Controller are  $\lambda$ ,  $\varepsilon$ , and  $\eta$ .

**Table 3: Scaling Factors for Error Bound Models**

Parameter	Value	Parameter	Value
dA <sub>u</sub>	1.25	dA <sub>l</sub>	1.60
dB <sub>u</sub>	1.00	dB <sub>l</sub>	1.00
dC <sub>u</sub>	0.00	dC <sub>l</sub>	0.00
dD <sub>u</sub>	1.00	dD <sub>l</sub>	1.00
dE <sub>u</sub>	0.90	dE <sub>l</sub>	0.98
dF <sub>u</sub>	1.30	dF <sub>l</sub>	0.85
dG <sub>u</sub>	1.30	dG <sub>l</sub>	0.80
dL <sub>u</sub>	0.90	dL <sub>l</sub>	0.90

### 2.2.2. Arm Controller

The arm control law is a PID law (Proportional, Integral, Derivative) with a nonlinear scaling factor to map the desired arm angular acceleration to the desired pendulum position.

$$\phi_d = [-K_1 \ddot{\theta} - K_2 \dot{\theta} - K_3 \int_0^t \tilde{\theta} dt] / [(\frac{1}{2})(L_1/L_2)\dot{\theta}^2 + (g/L_1)] \quad (8)$$

where  $\tilde{\theta} = \theta - \theta_d$ . The tuning parameters for the Arm Controller are  $K_1$ ,  $K_2$ , and  $K_3$ .

## 3. System Tuning Problem

The purpose of the control system is to move the arm following the trajectory given by the Target Arm Position  $\theta_d$  while balancing the pendulum. Ideally, the arm would perfectly track the target position with no effects due to variations in the characteristics of the plant. In real life, it is not possible to mitigate completely the effects of delays and functional coupling between system elements. Thus, the goal is to achieve an optimal balance between arm tracking performance and robustness. The robustness attribute

would account for plant modeling errors and variability in the characteristics of the physical plant. Note that swing-up of the pendulum is not within the scope of this arm-tracking problem. The applicable system initial conditions include those where the pendulum is above the horizontal position ( $\phi \leq \pm 90^\circ$ ) and the combined values of the plant state variables  $\phi$ ,  $\dot{\phi}$ ,  $\theta$ , and  $\dot{\theta}$  are within the stable region of the control system. It is also assumed that deriving a closed-form analytical derivation of optimal parameter settings is not feasible.

## 4. Reliability Engineering Concepts

Statistical techniques from the field of Reliability Engineering enable learning about and characterization of products and processes by means of systematic experimentation and modeling. The following sections present brief overviews of these topics.

### 4.1. Design of Experiments

Design of Experiments (DOE) theory is applied in the development of strategies to investigate the effects of independent controlled variables on system performance measured at selected variables of interest [4] [5]. The system of interest (SOI) is modeled as a black box to study the relations between Control, Input Signal, and Noise Factors (i.e., variables, parameters) and the Response Variable, as illustrated in Figure 3. The Noise Factors represent sources of system variability. In this model, the value of the response variable is a function of the Control, Input Signal, and Noise variables:  $y = f(X, S, Z)$ . We seek efficient experiments that yield insightful knowledge about the system with as few resources as possible.

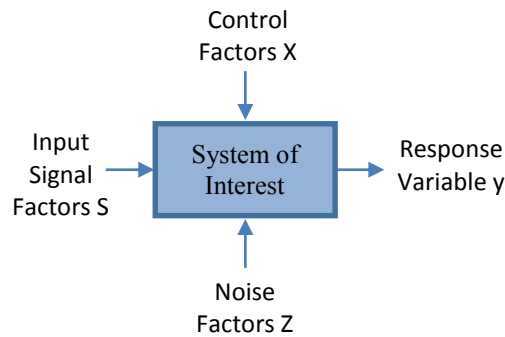


Figure 3: High-Level Black-box Model of the System of Interest (SOI)

In a basic experiment, we study the relation between the Control Factors (denoted  $x_1, x_2, \dots, x_k$ ) and the Response Variable (denoted  $y$ ) while the Input Signal Factors are given and the Noise Factors are not controlled. An experiment consists of a series of test points in which the Control Factors are assigned particular values and the Response is measured. In a **two-level experiment**, the Control Factors are tested at two different values. For a quantitative variable, the levels are the extremes of the value range, denoted  $x_{\max}$  and  $x_{\min}$ . For a qualitative variable, which has discrete values, the two levels are different values in the variable's value set. A **Full Factorial (Experiment) Design** consists of all possible combinations of the control variables. This is a complete and systematic study of the design space defined by the Control Factors. A two-level full factorial design consists of  $2^k$  test points. A **Fractional Factorial Design** includes a subset of test points in a Full Factorial Design. There are  $2^{k-p}$  test points in a two-level fractional factorial design, which consists with  $k-p$  independently assigned control factors and  $p$  **confounded factors** that are

assigned values as function of the independent control factors.

In designing experiments, there is a tradeoff between the quality of data and the cost needed to generate it. In general, we want to maximize the accuracy of the characterization made possible by the data (i.e., the benefit) while minimizing the number of test points (i.e., the cost). The simplest way to increase the quality of information generated by an experiment is to increase the number of levels of the control factors. A three-level fractional factorial design consists of  $3^{k-p}$  test points. The obvious problem with this approach is that the number of test points (and thus, the cost of the experiment) increases exponentially. A **Central Composite Design** (CCD) consisting of full or factorial experiment portion, a center portion, and an axial portion offers a more favorable tradeoff between quality and size of an experiment [6]. Figure 4 illustrates the structure of a two-factor CCD experiment. The values  $-1$  and  $+1$  are standardized representations of the variables values,  $x_{\min}$  and  $x_{\max}$ , respectively. In this figure, the four points at the corners of the box are the **factorial portion** of the experiment:  $(\pm 1, \pm 1)$ . Point  $(0, 0)$  is the **central portion**. The points at the ends of the central axes  $(0, \pm\alpha)$  and  $(\pm\alpha, 0)$  are the **axial portion**. Notice that this is five-level experiment with the control factors taking values  $0, \pm 1$ , and  $\pm\alpha$ . When  $\alpha = 1$ , the experiment is called a **Face-Centered Design** and the factors take only three possible values:  $0$  and  $\pm 1$ . The number of test points in a CCD experiment is  $2^{k-p} + 2k + 1$ .

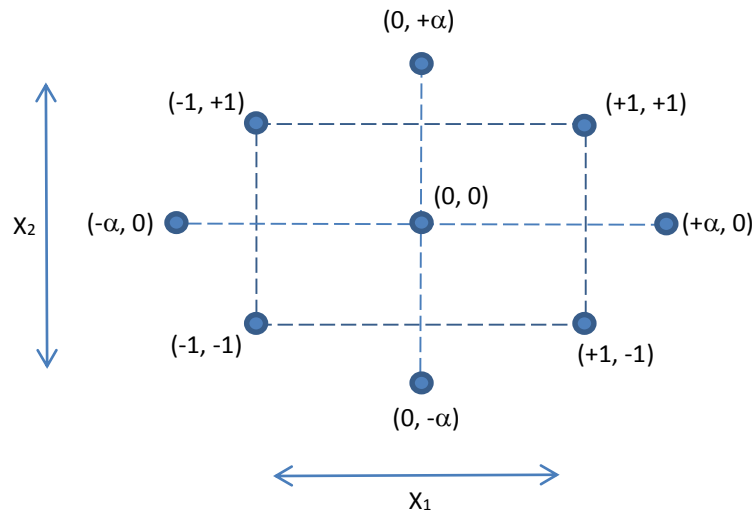


Figure 4: Structure of a Central Composite Design (CCD)

## 4.2. Response Modeling

We want to use the results of an experiment to generate a mathematical model of the relation between the control factors  $x$  and the response variable  $y$ . Here, we consider only polynomial models. A two-level experiment that tests each factor only at two values contains enough information to build a linear model. In addition to the direct relation between each control factor and the response variable, we want to consider possible interactions between control factors. The general expression for a **linear polynomial model** with two-factor interaction terms is:

$$y = b_0 + \sum_i b_i x_i + \sum_i \sum_j b_{ij} x_i x_j \quad (9)$$

where  $1 \leq i \leq k$ ,  $1 \leq j \leq k$ , and  $i \neq j$ . If the variables are assigned values in the  $\pm 1$  as in Figure 4, the

magnitude of the coefficients  $b_i$  is a measure of the **strength** of the effect (and hence, importance) of the corresponding factor  $x_i$  or interaction  $x_i x_j$ . Note that higher-order interactions are possible, but as a rule of thumb, the strength of interactions decreases as their order (i.e., number of interacting factors) increases. For a three-level experiment such as a CCD, the system response can be modeled with a second-order polynomial.

$$y = b_0 + \sum_i b_i x_i + \sum_i \sum_{j, i \neq j} b_{ij} x_i x_j + \sum_i b_{ii} x_i^2 \quad (10)$$

This model can achieve better accuracy for systems with non-linear responses.

The basic relation between modeling and experiment design is that high-order models can achieve better accuracy but require experiments with higher granularity and, thus, higher cost. We need models that have adequate fidelity for the intended use at a reasonable cost. Analysis of the application and careful judgment are required to determine the best combination of experiment design and response model.

### 4.3. Optimal and Robust Parameter Design

Once a model has been developed, an optimization tool can be used to determine the combination of control factors that results in the best system response. There are three categories of optimization problems: the-smaller-the-better, the-larger-the-better, and the-closer-to-nominal-the-better (i.e., as close as possible to a target). The quality of system performance is determined not only on achieving the desired target value under ideal conditions, but must also take into accounts the effects of variability in uncontrolled noise factors. To achieve a high **quality** system design, both the mean and the dispersion in system performance must be controlled. An optimal parameter design achieves the best balance between performance and robustness (i.e., insensitivity to variability). For this, we need a parameter optimization approach that can account for the mean value  $\bar{y}$  and deviation  $\sigma_y$  of the system response. One approach to this is the use of **Dual Response Surfaces**, in which models are developed for the mean response and the standard deviation, and an optimizer is used to minimize the deviation with a constraint on the error of the mean response relative to the desired target [7]. Another approach described below is to optimize signal-to-noise ratios, which is the approach used in the case study reported here.

To enable a robust parameter design, the experiment must generate data suitable for the characterization of the effects of the noise factors  $z$  (see Figure 3). Note that the categorization of factors as either control or noise can be a matter of judgment based on the goals of the parameter design activity. Factors with weak effects on the response variable could be classified as noise in order to simplify the design activity by focusing on the strongest control factors. Because of this, it is possible that some noise factors are controllable in an experiment and others may really be uncontrollable. For simulation-based experiments such as the one considered in the case study reported here, the set of controllable noise factors may be larger than for real physical experiments. Figure 5 illustrates the high-level structure of an experiment intended to characterization and optimization of system performance and robustness. The inner and outer arrays are independent experiments for control and noise factors as described in preceding sections. The complete outer array experiment is executed for each test point of control factors in the inner array. A response measure is made for every test point of control and noise factors, and robustness measures (such as mean response and deviation) are made by combining the results of a complete noise factors experiment for a particular control factors test point.

The signal-to-noise ratio is a robustness measure intended to capture with a single metric both the deviation of the average response from the target response  $y_m$  and the dispersion of the response. The mean square deviation of the response is given by:

$$\Delta_{msd} = (\bar{y} - y_m)^2 + \sigma_y^2 \quad (11)$$

The signal-to-noise ratios for the optimization problems of the-smaller-the-better, the-larger-the-better, and the-closer-to-nominal-the-better are given in Table 4. This robustness measure is computed for each test point of the inner array in Figure 5. At the completion of the experiment, a model can be built to relate the control factors to the signal-to-noise ratio SN. A control factor optimization based on the maximization of the signal-to-noise ratio gives the system configuration with the best balance of performance and robustness.

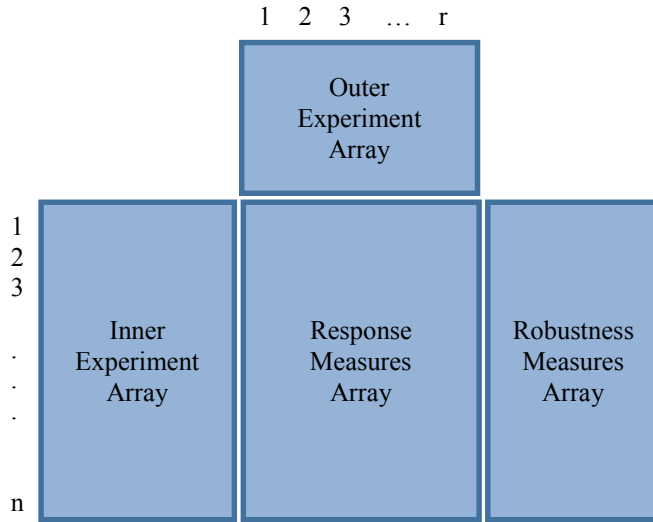


Figure 5: Structure of Robust Parameter Design Experiment

Table 4: Formulas for Signal-to-Noise Ratio

Case	Signal-to-Noise Ratio (SN)
the-smaller-the-better	$-10 \log_{10}[(1/r) \sum_i y_i^2]$
the-larger-the-better	$-10 \log_{10}[(1/r) \sum_i (1/y_i^2)]$
the-closer-to-nominal-the-better	$10 \log_{10}[\bar{y}^2/\sigma_y^2]$

## 5. System Tuning Approach

This section describes the elements of the approach followed to tune the parameters of the control system. This case study was intended as a proof of concept and demonstration, and as such, the tuning problem was scoped to be representative but without excessive complexity. MATLAB was used to model the system and perform simulation experiments.



## 5.1. Control and Noise Factors

Table 5 lists the chosen control and noise factors. The baseline values for the noise factors are the nominal values given in Table 1. The baseline values for the Arm Controller and the Pendulum Controller factors were determined by trial-and-error to result in a stable system (i.e., small state perturbations are mitigated asymptotically) with the region of convergence for the pendulum state given in Table 6 (i.e., if the initial pendulum state is in this region, the plant state will converge the zero state  $(\phi, \dot{\phi}, \theta, \text{ and } \dot{\theta}) = (0, 0, 0, 0)$ , assuming  $\theta_a = 0$ ). The ranges of the control and noise factors were specified as percentages of the baselines values.

**Table 5: Control and Noise Factors for the Control System**

Category	System Element	Factor	Baseline Value	Range (%)
Control Factors	Arm Controller	$K_1$	2.50	$\pm 50$
		$K_2$	2.00	$\pm 50$
		$K_3$	0.10	$\pm 50$
	Pendulum Controller	$\lambda$	20.00	$\pm 50$
		$\varepsilon$	0.017	$\pm 50$
Noise Factors	Laboratory Plant Model	$L_1$	0.36	$\pm 15$
		$L_2$	0.28	$\pm 15$
		m	1.1564	$\pm 15$

**Table 6: Region of Convergence of Baseline Control System**

Initial Pendulum Position $\phi$ ( $^\circ$ )	Range of Initial Pendulum Speeds $\dot{\phi}$ ( $^\circ/s$ )	Initial Pendulum Position $\phi$ ( $^\circ$ )	Range of Initial Pendulum Speeds $\dot{\phi}$ ( $^\circ/s$ )
+35	(-600, +30)	-35	(-30, +600)
+25	(-500, +190)	-25	(-190, +500)
+15	(-525, +340)	-15	(-340, +525)
+5	(-525, +500)	-5	(-500, +525)

## 5.2. Stimulus Tracking Signal

The chosen Arm Position Target  $\theta_d$ , which corresponds to the Input Signal in Figure 3, was a ramp waveform from  $0^\circ$  to  $90^\circ$  with a  $30^\circ/\text{s}$  slope from seconds 2 to 5. This is illustrated in Figure 6.

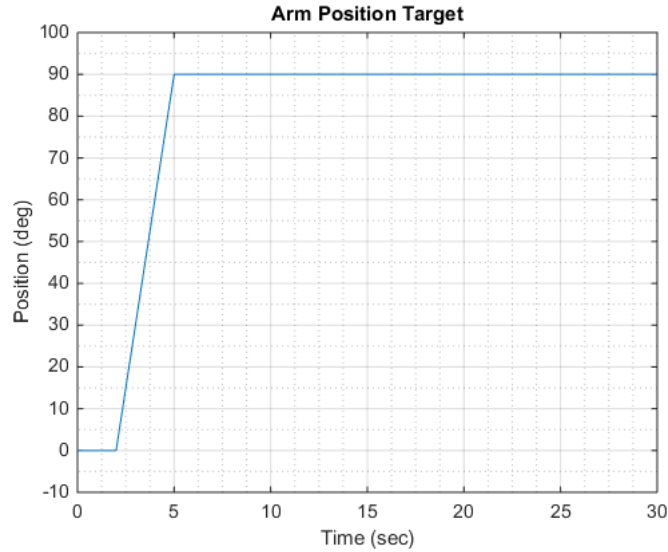


Figure 6: Arm Position Tracking Target

## 5.3. Response Metrics

Each experimental test point consisted of a 30 second run in which the control and noise factors were held constant and the stimulus signal was applied. Each run was divided into two phases: a transient phase (denoted  $l$ ) from 0 to 15 seconds, and a steady-state phase (denoted  $u$ ) from 15 to 30 seconds. The goal of the control system is to minimize the arm tracking error, defined as  $e = \tilde{\theta} = \theta - \theta_d$ , while balancing the pendulum. The performance metrics are given in Table 7, where  $T_l = 15$  seconds and  $T_u = 30$  seconds. Note that an outer array experiment consists of  $m$  test points (see Figure 5), and the signal-to-noise ratio is computed based on the results of one outer array experiment (i.e., one inner array row).

**Table 7: System Response Performance Metrics**

Experiment Segment	Phase	Description	Label	Formula
Test Point (i, j)	Transient	Mean Square Error (MSE)	$P_1$	$\left(\frac{1}{T_1}\right) \int_0^{T_1} e^2 dt$
		Maximum Square Error	$e_1^2$	$\max(e^2) \text{ for } 0 \leq t \leq T_1$
	Steady State	Mean Square Error (MSE)	$P_u$	$\left(\frac{1}{T_u - T_1}\right) \int_{T_1}^{T_u} e^2 dt$
		Maximum Square Error	$e_u^2$	$\max(e^2) \text{ for } T_1 \leq t \leq T_u$
	All	Combined Error	$E$	$w_1P_1 + w_2e_1^2 + w_3P_u + w_4e_u^2; w_q = 0.25$
Inner Array Row		Signal-to-Noise Ratio	SN	$-10 \log_{10} \left[ \left(\frac{1}{m}\right) \sum_{j=1}^m E_j^2 \right]$

## 5.4. Experiment Design

The Inner Experiment Array for the Control Factors was a Central Composite Design for the five factors  $K_1, K_2, K_3, \lambda,$  and  $\epsilon$ , which are denoted  $X_1, X_2, X_3, X_4,$  and  $X_5$ . The factorial portion of the Inner Array was a half-fractional factorial design with confounded factor  $X_5 = X_1X_2X_3X_4$ .

The Outer Experiment Array for the Noise Factors was a half-fractional factorial design for the three factors  $L_1, L_2,$  and  $m$  denoted  $N_1, N_2,$  and  $N_3$ . The confounded noise factor was  $N_3 = N_1N_2$ .

## 6. Results

As stated above, the experiment was performed using MATLAB. This section describes the experiment results and the parameter design with optimal signal-to-noise ratio.

### 6.1. Experimental Response Measures

Table 8 shows the results of the experiment. The combined error measures for each test point are given in the columns labeled E1 to E4. The signal-to-noise ratio for each row of control factors is given in the S/N column.

### 6.2. Response Model

The system response was modeled with a quadratic polynomial as in equation (10). Rather than including all the two-factor terms, an effects analysis was performed to identify the strongest interactions.

These were  $X_1X_2$ ,  $X_1X_3$ ,  $X_1X_4$ ,  $X_2X_3$ , and  $X_1X_5$ . The full model for the signal-to-noise ratio is:

$$SN = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5 + b_{12}X_1X_2 + b_{13}X_1X_3 + b_{14}X_1X_4 + b_{23}X_2X_3 + b_{15}X_1X_5 + b_{11}X_1^2 + b_{22}X_2^2 + b_{33}X_3^3 + b_{44}X_4^4 + b_{55}X_5^5$$

The model coefficients were determined using regression. The values are given in Table 9

**Table 8: Experiment System Response Measures**

						Noise Factors				
						N1/L1	-1	1	-1	1
						N2/L2	-1	-1	1	1
						N3/m	1	-1	-1	1
Control Factors										
X1/k1	X2/k2	X3/k3	X4/lambda	X5/epsilon		E1	E2	E3	E4	S/N
1	-1	-1	-1	-1	1	164.759	211.915	169.640	202.773	-104.769
2	1	-1	-1	-1	-1	92.331	105.338	98.999	101.663	-92.042
3	-1	1	-1	-1	-1	140.695	174.424	142.577	172.215	-101.287
4	1	1	-1	-1	1	101.038	102.994	138.310	98.582	-94.268
5	-1	-1	1	-1	-1	164.944	240.843	168.811	230.420	-106.383
6	1	-1	1	-1	1	96.915	105.989	107.866	101.518	-92.725
7	-1	1	1	-1	1	145.174	171.730	152.304	162.395	-101.280
8	1	1	1	-1	-1	91.764	103.789	125.609	99.935	-93.272
9	-1	-1	-1	1	-1	155.422	203.201	153.378	206.819	-104.023
10	1	-1	-1	1	1	82.765	97.702	87.739	94.618	-90.194
11	-1	1	-1	1	1	128.789	159.214	131.805	154.813	-99.436
12	1	1	-1	1	-1	83.854	98.019	109.475	98.034	-91.652
13	-1	-1	1	1	1	153.453	216.126	154.486	207.743	-104.435
14	1	-1	1	1	-1	83.720	101.371	85.523	99.925	-90.649
15	-1	1	1	1	-1	128.261	159.511	130.653	174.621	-100.152
16	1	1	1	1	1	83.644	94.659	110.132	91.651	-91.184
17	0	0	0	0	0	97.714	118.274	100.694	114.897	-93.690
18	-1	0	0	0	0	137.437	174.931	139.790	170.091	-101.060
19	1	0	0	0	0	83.531	96.579	90.889	93.246	-90.258
20	0	-1	0	0	0	100.726	124.102	103.103	120.556	-94.476
21	0	1	0	0	0	95.844	114.538	99.449	111.058	-93.176
22	0	0	-1	0	0	97.679	118.160	100.654	114.810	-93.677
23	0	0	1	0	0	97.759	118.347	100.751	114.936	-93.700
24	0	0	0	-1	0	104.244	122.517	109.246	117.315	-94.645
25	0	0	0	1	0	96.731	118.287	99.177	115.201	-93.600
26	0	0	0	0	-1	96.856	118.396	99.271	121.655	-93.938
27	0	0	0	0	1	98.500	118.874	102.004	114.817	-93.805

**Table 9: Coefficients for Model of Signal-to-Noise Ratio of System Response**

Coefficient	Value	Coefficient	Value	Coefficient	Value	Coefficient	Value
b <sub>0</sub>	-93.4849	b <sub>4</sub>	0.8526	b <sub>14</sub>	0.1848	b <sub>22</sub>	-0.3668
b <sub>1</sub>	5.3657	b <sub>5</sub>	0.0723	b <sub>23</sub>	0.2449	b <sub>33</sub>	-0.2291
b <sub>2</sub>	0.7772	b <sub>12</sub>	-1.3888	b <sub>15</sub>	-0.1675	b <sub>44</sub>	-0.6631
b <sub>3</sub>	-0.1351	b <sub>13</sub>	0.1913	b <sub>11</sub>	-2.2001	b <sub>55</sub>	-0.4123

### 6.3. Optimal Parameter Design

The optimizing compiler in Excel was used to determine the maximum signal-to-noise ration within the range of the control factors using the model in the previous section. The optimal signal-to-noise ratio was  $SN_{\text{optimal}} = -89.62$  with the parameter values given in Table 10.

**Table 10: Optimal Values of the System Control Factors**

Control Factor	Value	Control Factor	Value
K <sub>1</sub>	3.750	$\lambda$	27.826
K <sub>1</sub>	1.035	$\varepsilon$	0.016
K <sub>1</sub>	0.080		

### 6.4. Verification

A brief experiment was run to verify the optimization results. For this verification experiment, the control factors were set as in Table 10 and noise factors were cycled as in the outer array in Table 8. The resulting signal-to-noise ratio was  $SN_{\text{verification}} = -90.082$ , which is a close match to the optimal value. Figure 7 to Figure 10 show the arm position tracking response for each verification test point. The steady-state oscillations are probably due to time delays in the system and the torque dead zone of the motor. Figure 11 shows the time history of all the plant state variables for the verification run with Noise Factors set to  $(N_1, N_2, N_3) = (+1, +1, +1)$ .

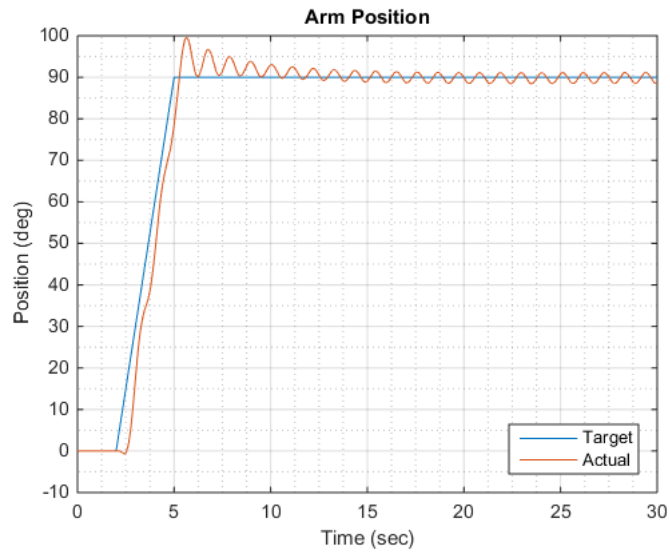


Figure 7: Verification Run for Noise Factors (N1, N2, N3) = (-1, -1, +1)

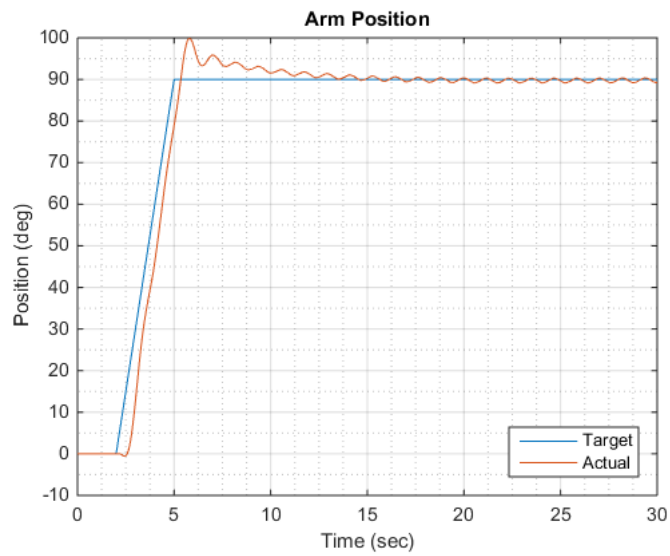


Figure 8: Verification Run for Noise Factors (N1, N2, N3) = (+1, -1, -1)

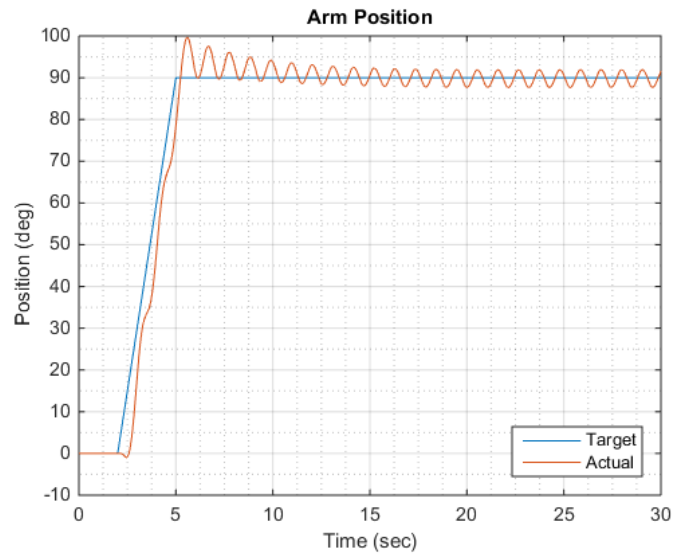


Figure 9: Verification Run for Noise Factors  $(N1, N2, N3) = (-1, +1, -1)$

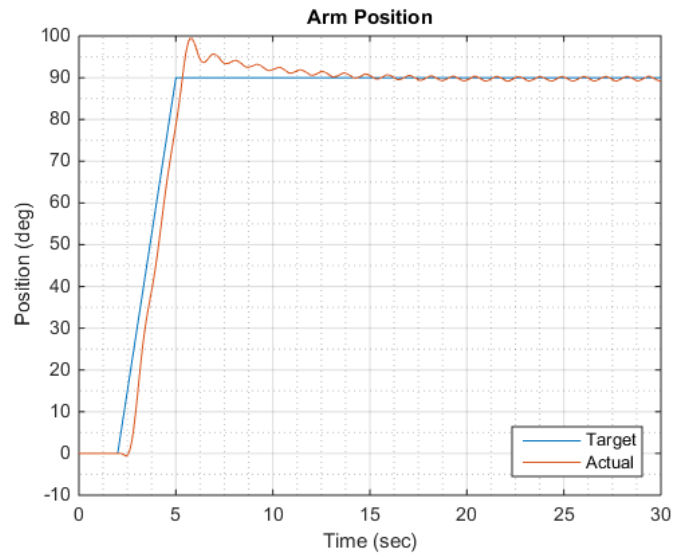


Figure 10: Verification Run for Noise Factors  $(N1, N2, N3) = (+1, +1, +1)$

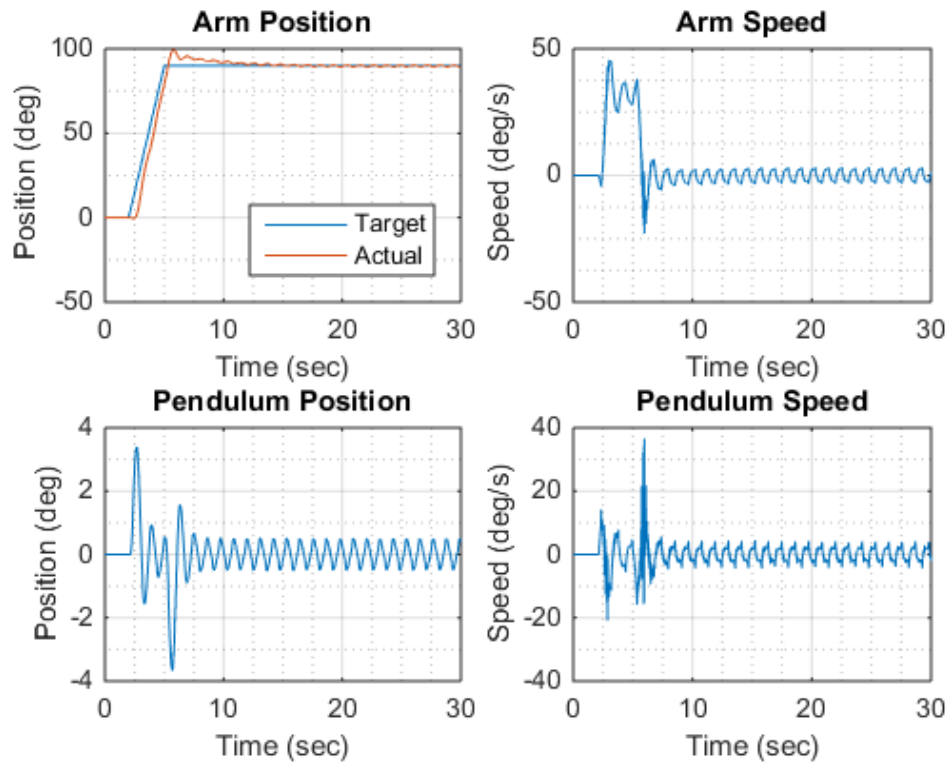


Figure 11: Plant State Variables for Verification Run with Noise Factors  $(N1, N2, N3) = (+1, +1, +1)$

## 7. Final Remarks

This case study has demonstrated the application of Reliability Engineering techniques for optimal tuning of a complex control system. These techniques pertain to experiment design and mathematical modeling of the effects of control and noise factors on system performance and robustness. The applied approach enables the generation of insight into the relation between goals, dependencies, and uncertainties in a system. This is especially useful when complexity limits our ability to derive closed-form analytical expressions of system performance. The examined system contained many complex features such a dual-loop control architecture, plant and controller state (i.e., memory), and a variety of nonlinear relations including quadratic, sinusoidal, and thresholds.

The demonstrated approach is simple, structured, effective, repeatable, and cost and time efficient. This general approach is suitable for a wide range of systems. The case study showed that the applied techniques are able to predict performance with high accuracy. Simulation-based experimentation in a flexible environment such as MATLAB enables quick turnaround in the generation, characterization, and tuning of virtual system prototypes.

There are several ways to expand the scope and depth of the case study presented here. Future case studies could increase the number of plant and controller parameters in the sets of control and noise factors. Also, instead of a single ramp input signal, a set of input signals could be applied such as steps, square waves, and sinusoidal waveforms. In that case the input signals would be handled as additional noise factors and considered in the design of the experiment outer array. This would expand the set of conditions in the



characterization of robustness. Additionally, in this case study, we chose to follow a trial-and-error approach to identify a stable system configuration that served as a baseline for the study of performance and robustness. In tuning a nonlinear system from a stability perspective, we are primarily interested in the size of the region of convergence. It should be possible to develop simple metrics for the size of the region of convergence. Given that, an experiment could be designed to explore the system design space and identify the proper tuning of parameters to achieve an optimal balance of stability, performance, and robustness.

## 8. References

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## Appendix A. MATLAB Script for Simulation Experiment

```
% Run Experiment on pendulum control system
% Notes:
% 1. Performance measure results are saved to file
%
echo on ;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Set filename for control and noise matrices
% => This one in MATLAB format
% => This one will be in Excel-loadable format
control_matrix_filename_TXT = 'control_matrix.dat' ;
noise_matrix_filename_TXT   = 'noise_matrix.dat' ;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Set filename for error matrix
% => This one in MATLAB format
% => This one will be in Excel-loadable format
E_matrix_filename_TXT      = 'E_matrix.dat' ;
Pl_matrix_filename_TXT     = 'Pl_matrix.dat' ;
Pu_matrix_filename_TXT     = 'Pu_matrix.dat' ;
e2l_matrix_filename_TXT    = 'e2l_matrix.dat' ;
e2u_matrix_filename_TXT    = 'e2u_matrix.dat' ;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Set filename for performance measures
% => This one in MATLAB format
perf_measures_filename_MAT = 'perf_measures_MAT' ;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Set filename for signal-to-noise measures
% => This one will be in Excel-loadable format
% ==> Make entries TAB separated and include end-of-line character
% ==> Can also include headings (for rows, columns, table caption, etc.)
SN_measures_filename_TXT = 'SN_measures.dat' ;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Set model information
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Specify model name
model_name = 'system_controller_n_plant_enhanced_v3p0' ;

% Run time (seconds)
t_final = 30.0 ;

% Transient interval duration (seconds)
t_Transient = 15.0 ;

% Set plant initial conditions
Initial_arm_angle_deg      = [0 0.0] ;
Initial_arm_speed_deg_sec  = [0 0.0] ;
Initial_pendulum_angle_deg = [0 0.0] ;
Initial_pendulum_speed_deg_sec = [0 0.0] ;
```

```

% Baseline controlled model parameters
k1_base      = 2.50 ; % X1
k2_base      = 2.00 ; % X2
k3_base      = 0.10 ; % X3
lambda_base  = 20.0 ; % X4
epsilon_base = 0.017 ; % X5
eta_base     = 1.0 ; % This one will remain constant

% Baseline noise (uncontrolled) model parameters
J1_kg_m2_base = 0.04699 ; % This one will remain constant
J2_kg_m2_base = 0.007800 ; % This one will remain constant
L1_m_base     = 0.36 ; % N1
L2_m_base     = 0.28 ; % N2
m_kg_base     = 1.1564 ; % N3

% Range percentage for model parameters (%)
r_control = 50.0 ;
r_noise   = 15.0 ;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Set experiment matrices
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Matrix for Controlled parameters
% X1, X2, X3, X4, X4, X5(= X1X2X3X4)
Control_matrix = ...
[
-1, -1, -1, -1, +1;
+1, -1, -1, -1, -1;
-1, +1, -1, -1, -1;
+1, +1, -1, -1, +1;

-1, -1, +1, -1, -1;
+1, -1, +1, -1, +1;
-1, +1, +1, -1, +1;
+1, +1, +1, -1, -1;

-1, -1, -1, +1, -1;
+1, -1, -1, +1, +1;
-1, +1, -1, +1, +1;
+1, +1, -1, +1, -1;

-1, -1, +1, +1, +1;
+1, -1, +1, +1, -1;
-1, +1, +1, +1, -1;
+1, +1, +1, +1, +1;

0, 0, 0, 0, 0;

-1, 0, 0, 0, 0;
+1, 0, 0, 0, 0;
0, -1, 0, 0, 0;
0, +1, 0, 0, 0;
0, 0, -1, 0, 0;
0, 0, +1, 0, 0;
0, 0, 0, -1, 0;
0, 0, 0, +1, 0;

```

```

0, 0, 0, 0, -1;
0, 0, 0, 0, +1 ] ;

% Matrix for Noise parameters
% N1, N2, N3(= N1N2)
Noise_matrix = ...
[
-1, -1, +1;
+1, -1, -1;
-1, +1, -1;
+1, +1, +1 ] ;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Performance Measure matrices
weights          = [0.25, 0.25, 0.25, 0.25] ;
num_metrics      = 4 ;
num_ctrl_rows   = size(Control_matrix, 1) ;
num_ctrl_columns = size(Control_matrix, 2) ;
num_noise_rows  = size(Noise_matrix, 1) ;
num_noise_columns = size(Noise_matrix, 2) ;

P1 = zeros(num_ctrl_rows, num_noise_rows) ;
Pu = zeros(num_ctrl_rows, num_noise_rows) ;
e2l = zeros(num_ctrl_rows, num_noise_rows) ;
e2u = zeros(num_ctrl_rows, num_noise_rows) ;
U   = zeros(num_ctrl_rows, num_noise_rows, num_metrics) ;
E   = zeros(num_ctrl_rows, num_noise_rows) ;
SN  = zeros(num_ctrl_rows, 1) ;

%pause

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Loop over Control Matrix
for i = 1 : num_ctrl_rows
    % Factor values for current control point
    k1      = [0 (k1_base * ...
                (1 + Control_matrix(i,1)*r_control/100.0)) ] ;
    k2      = [0 (k2_base * ...
                (1 + Control_matrix(i,2)*r_control/100.0)) ] ;
    k3      = [0 (k3_base * ...
                (1 + Control_matrix(i,3)*r_control/100.0)) ] ;
    lambda  = [0 (lambda_base * ...
                (1 + Control_matrix(i,4)*r_control/100.0)) ] ;
    epsilon = [0 (epsilon_base * ...
                (1 + Control_matrix(i,5)*r_control/100.0)) ] ;
    eta     = [0 eta_base] ; % This one remains constant

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Loop over Noise matrix
for j = 1 : num_noise_rows
    % Factor values for current noise point
    J1_kg_m2 = [0 J1_kg_m2_base] ; % This one remains constant
    J2_kg_m2 = [0 J2_kg_m2_base] ; % This one remains constant
    L1_m      = [0 (L1_m_base * (1 +
                        Noise_matrix(j,1)*r_noise/100.0))] ;
    L2_m      = [0 (L2_m_base * (1 +
                        Noise_matrix(j,2)*r_noise/100.0))] ;

```

```

m_kg      = [0 (m_kg_base * (1 +
                Noise_matrix(j,3)*r_noise/100.0))] ;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Run the model
plant_state = sim(model_name, 'StopTime', 't_final') ;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Get the plant state histories
time        = plant_state.get('time') ;
arm_position_target_deg =
    plant_state.get('arm_position_target_deg');
arm_position_deg      = plant_state.get('arm_position_deg') ;
arm_speed_deg_sec    = plant_state.get('arm_speed_deg_sec') ;
pendulum_position_deg =
    plant_state.get('pendulum_position_deg') ;
pendulum_speed_deg_sec =
    plant_state.get('pendulum_speed_deg_sec') ;

%show_plant_state_v1

%pause

% Get number of time points in the simulation
num_time_points = length(time) ;

% Scan full length of simulation run
for k = 1 : num_time_points

    % For Lower/Transient time interval
    if (time(k) <= t_Transient)
        Pl(i,j) = Pl(i,j) + ...
            (arm_position_deg(k) - ...
                arm_position_target_deg(k))^2 ;
        e2l(i,j) = max([e2l(i,j), ...
            (arm_position_deg(k) -
                arm_position_target_deg(k))^2]) ;

    % For Upper/Steady-state time interval
    else
        Pu(i,j) = Pu(i,j) + ...
            (arm_position_deg(k) -
                arm_position_target_deg(k))^2 ;
        e2u(i,j) = max([e2u(i,j), ...
            (arm_position_deg(k) -
                arm_position_target_deg(k))^2]) ;
    end
end

% Finish up averaging sums of squares
Pl(i,j) = Pl(i,j)/num_time_points ;
Pu(i,j) = Pu(i,j)/num_time_points ;

% Store basic measures
U(i, j, 1) = Pl(i,j) ;
U(i, j, 2) = e2l(i,j) ;
U(i, j, 3) = Pu(i,j) ;

```

```

    U(i, j, 4) = e2u(i,j) ;

    % Computer composite measure
    for k = 1 : num_metrics
        E(i, j) = E(i,j) + weights(k)*U(i, j, k) ;
    end % k

    % Update signal-to-noise ratio with current composite measure
    SN(i) = SN(i) + E(i, j)*E(i, j) ;

%pause

    end % j

    % Compute Signal-to-Noise ratio for current control point
    SN(i) = -10*log(SN(i)/num_noise_rows) ;

%pause

end % i

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Save experiment control and noise matrices in Excel-loadable format
save(control_matrix_filename_TXT, 'Control_matrix', '-ascii', '-tabs') ;
save(noise_matrix_filename_TXT, 'Noise_matrix', '-ascii', '-tabs') ;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Save performance matrices in MATLAB format
save(perf_measures_filename_MAT, ...
    'P1', 'e2l', 'Pu', 'e2u', 'U', 'E', 'SN') ;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Save performance matrices in Excel-loadable format
save(P1_matrix_filename_TXT, 'P1', '-ascii', '-tabs') ;
save(Pu_matrix_filename_TXT, 'Pu', '-ascii', '-tabs') ;
save(e2l_matrix_filename_TXT, 'e2l', '-ascii', '-tabs') ;
save(e2u_matrix_filename_TXT, 'e2u', '-ascii', '-tabs') ;
save(E_matrix_filename_TXT, 'E', '-ascii', '-tabs') ;
save(SN_measures_filename_TXT, 'SN', '-ascii', '-tabs') ;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Finished
echo off ;

```

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