Rapid Generation of Optimal Asteroid Powered Descent Trajectories Via Convex Optimization

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Goal

- Goal: Design an optimal powered descent trajectory on-board the spacecraft in order to softly land on an asteroid.
  - Algorithm would need to be autonomous, reliable, robust and efficient.
  - Designing on-board facilitates easy change of parameters.

- Convex optimization is efficient and reliable.
  - Guarantees global minimum in finite number of steps, if problem is feasible.
  - Especially true in subclasses, such as Second Order Cone Programming.

- ***Can convex optimization be used to design the asteroid powered descent trajectory?***

Original Problem Statement

- Asteroid powered descent fuel optimal problem is nonlinear and nonconvex.

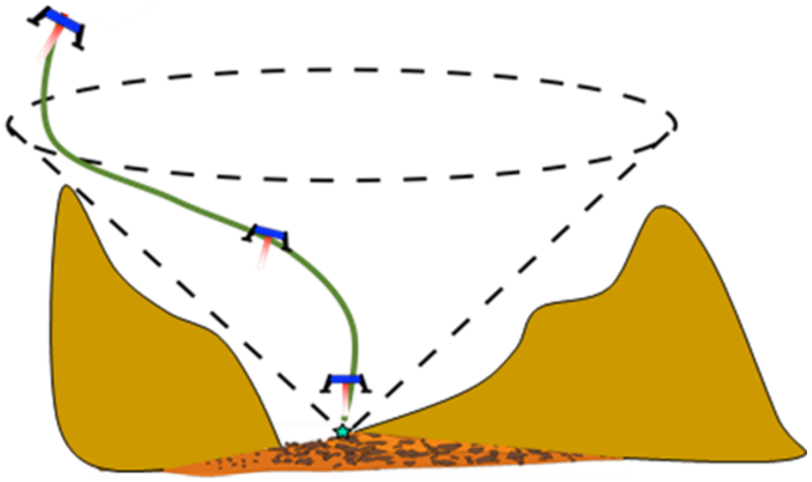
$$\min -m(t_f)$$

$$s.t.\ \ \dot{\vec{r}} = \vec{v},\ \dot{\vec{v}} = \frac{\vec{T}}{m} - 2\vec{\omega}\times\vec{v} - \dot{\vec{\omega}}\times\vec{r} - \vec{\omega}\times(\vec{\omega}\times\vec{r}) + \nabla U(\vec{r}),\ \dot{m} = -\frac{1}{v_{ex}}\left\|\vec{T}\right\|$$

$$T_{min} \le \left\|\vec{T}\right\| \le T_{max},\ \|\vec{r}\|\, cos\theta - \vec{r}^{\,T}\hat{n} \le 0,\ m \ge m_{dry}$$

$$\vec{r}(0) = \vec{r}_0,\ \vec{v}(0) = \vec{v}_0,\ m(0) = m_{wet},\ \vec{r}(t_f) = \vec{r}_f,\ \vec{v}(t_f) = \vec{v}_f,\ t_f\ \text{given}$$

  - Highlighted terms are not permissible for a convex optimization problem.



Relaxation

- Original Problem

$$\min -m(t_f)$$

$$s.t.\ \dot{\vec{r}} = \vec{v}$$

$$\dot{\vec{v}} = \frac{\vec{T}}{m} - 2\vec{\omega} \times \vec{v} - \dot{\vec{\omega}} \times \vec{r} - \vec{\omega} \times (\vec{\omega} \times \vec{r}) + \nabla U(\vec{r})$$

$$\dot{m} = -\frac{1}{v_{ex}} \left\| \vec{T} \right\|$$

$$T_{min} \leq \left\| \vec{T} \right\| \leq T_{max}$$

$$\|\vec{r}\|\, cos\theta - \vec{r}^{\,T} \hat{n} \leq 0$$

$$m \geq m_{dry}$$

$$\vec{r}\,(0) = \vec{r}_0,\ \vec{v}\,(0) = \vec{v}_0,\ m\,(0) = m_{wet}$$

$$\vec{r}\,(t_f) = \vec{r}_f,\ \vec{v}\,(t_f) = \vec{v}_f,\ t_f \text{ given}$$

- Relaxed Problem

$$\min -m(t_f)$$

$$s.t.\ \dot{\vec{r}} = \vec{v}$$

$$\dot{\vec{v}} = \frac{\vec{T}}{m} - 2\vec{\omega} \times \vec{v} - \dot{\vec{\omega}} \times \vec{r} - \vec{\omega} \times (\vec{\omega} \times \vec{r}) + \nabla U(\vec{r})$$

$$\dot{m} = -\frac{1}{v_{ex}} T_m$$

Slack Variable

$$\left\| \vec{T} \right\| \leq T_m$$

New Constraint

$$T_{min} \leq T_m \leq T_{max}$$

$$\|\vec{r}\|\, cos\theta - \vec{r}^{\,T} \hat{n} \leq 0$$

$$m \geq m_{dry}$$

$$\vec{r}\,(0) = \vec{r}_0,\ \vec{v}\,(0) = \vec{v}_0,\ m\,(0) = m_{wet}$$

$$\vec{r}\,(t_f) = \vec{r}_f,\ \vec{v}\,(t_f) = \vec{v}_f,\ t_f \text{ given}$$

Proved the optimal solution of the relaxed problem is the optimal solution of the original.

Successive Solution Method

- Solve a series of convex optimization problems.
- Equations of motion can be arranged as:

$$\dot{\vec{x}}^{(k)} = A\left(r^{(k-1)}\right)\vec{x}^{(k)} + B\vec{u}^{(k)} + c\left(r^{(k-1)}\right)$$

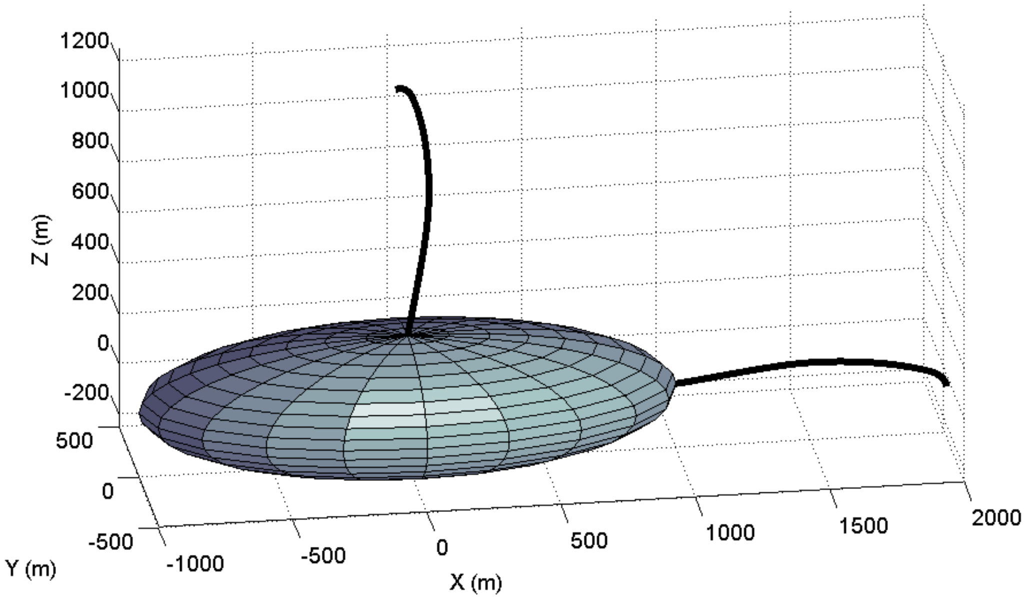
- A and c are evaluated using the previous solution (k-1).
- In the $(k)^{th}$ iteration, dynamics are linear and time varying.
- Iterations continue until two successive trajectories are within a set tolerance.

Scaling and Discretization

- Position, velocity, acceleration and time are scaled to keep the magnitudes near 1.
  - Position scaled by *smallest* semi-major axis
- Discretize the continuous equations to form equality constraints.
- **Now, the problem is in the form of a convex optimization problem.**
  - Linear equality constraints
  - Convex inequality constraints
- Used CVX, publically available Matlab based convex solver.
  - A 600 second trajectory, 300 nodes ~ 3300 variables per iteration, 4 iterations takes 3.5 minutes to complete.

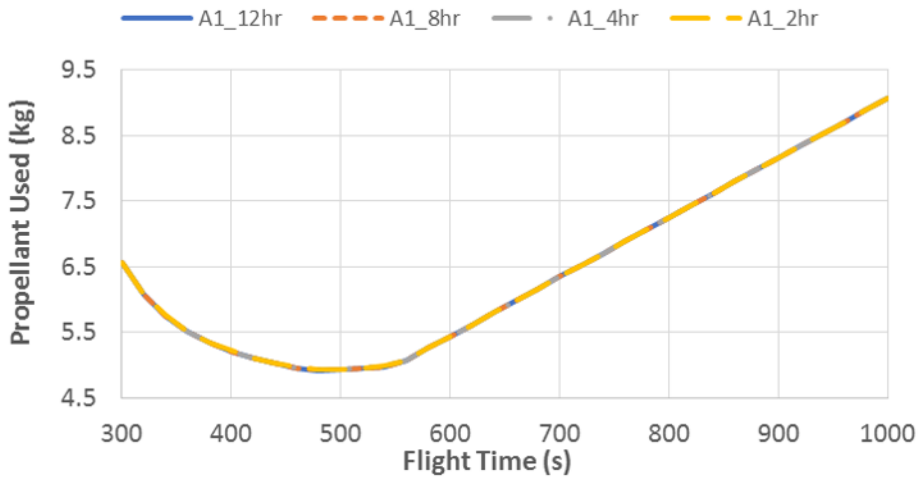
Asteroid Assumptions

- Two Trajectories
  - North Pole Trajectory (NP) lands on the z axis
  - Equatorial Trajectory (EQ) lands on the x axis
- Three Asteroid Sizes
  - A1: 1000 x 500 x 250 m
  - A2: 750 x 500 x 250 m
  - A3: 500 x 500 x 250 m
- Four Periods
  - 12 hr, 8 hr, 4 hr, 2 hr
  - Constant spin rate along z axis.

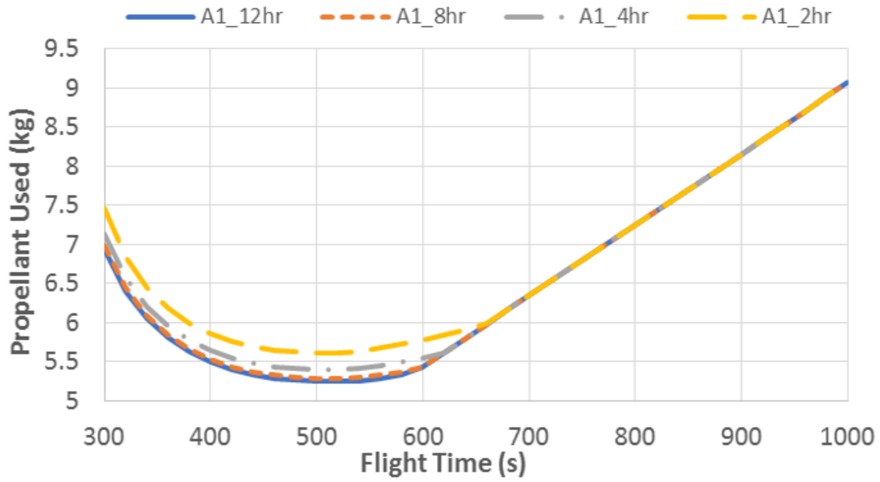


Propellant Usage Parameter Sweeps

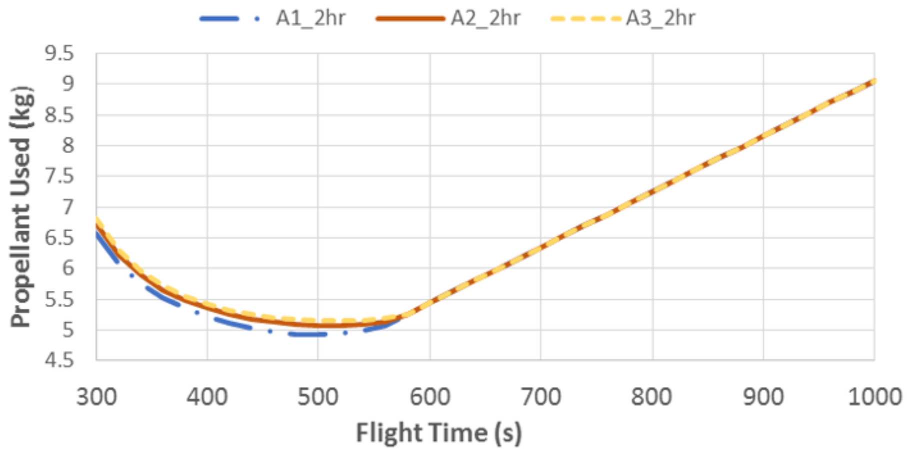
NP trajectories 1000 x 500 x 250 m



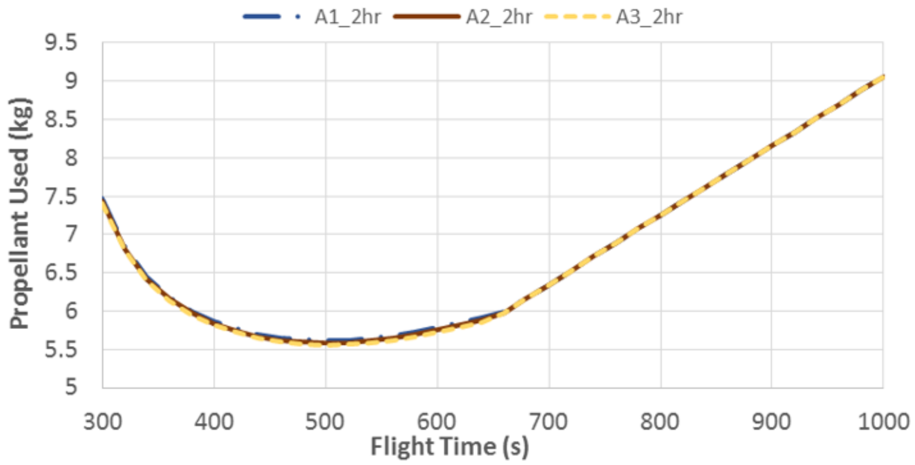
EQ trajectories 1000 x 500 x 250 m



NP trajectories 2 hr all asteroids

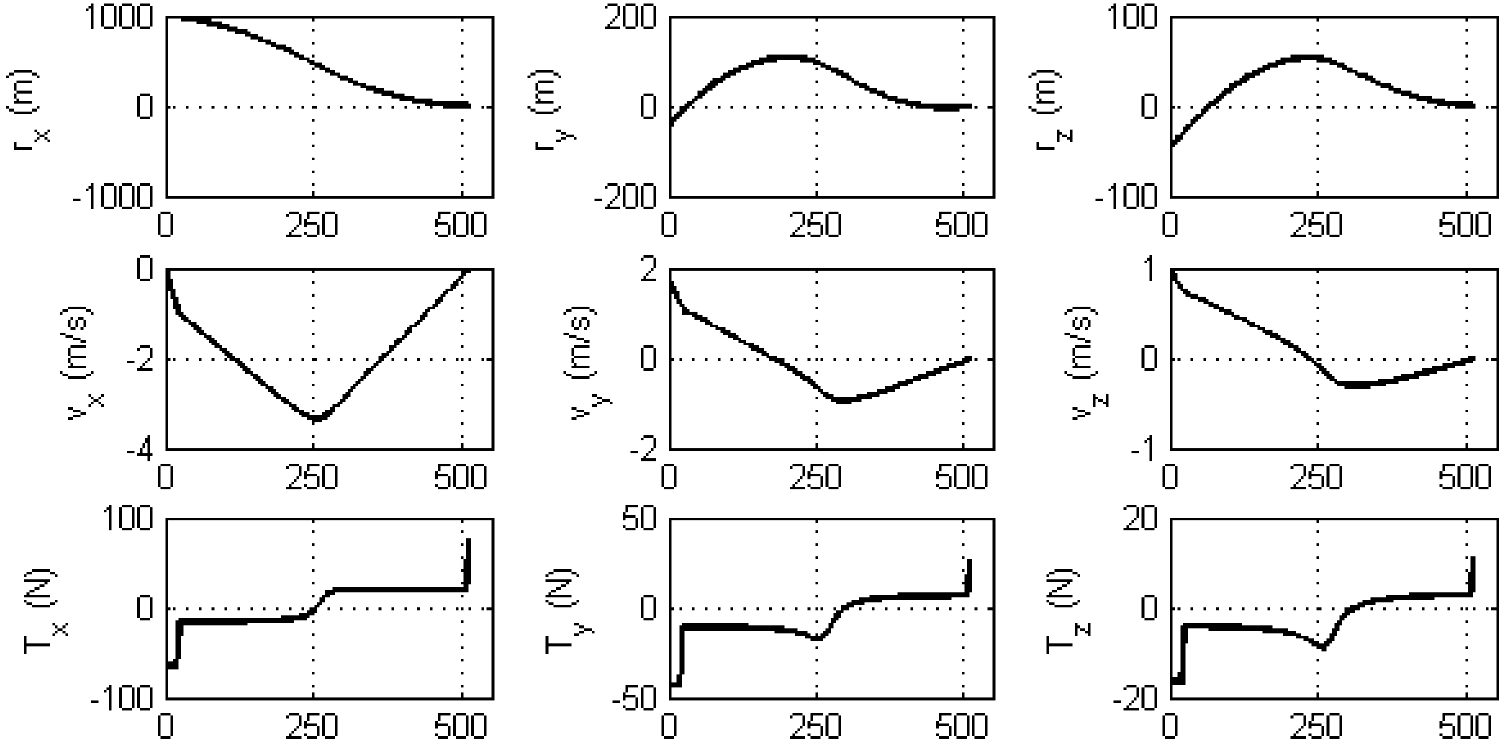


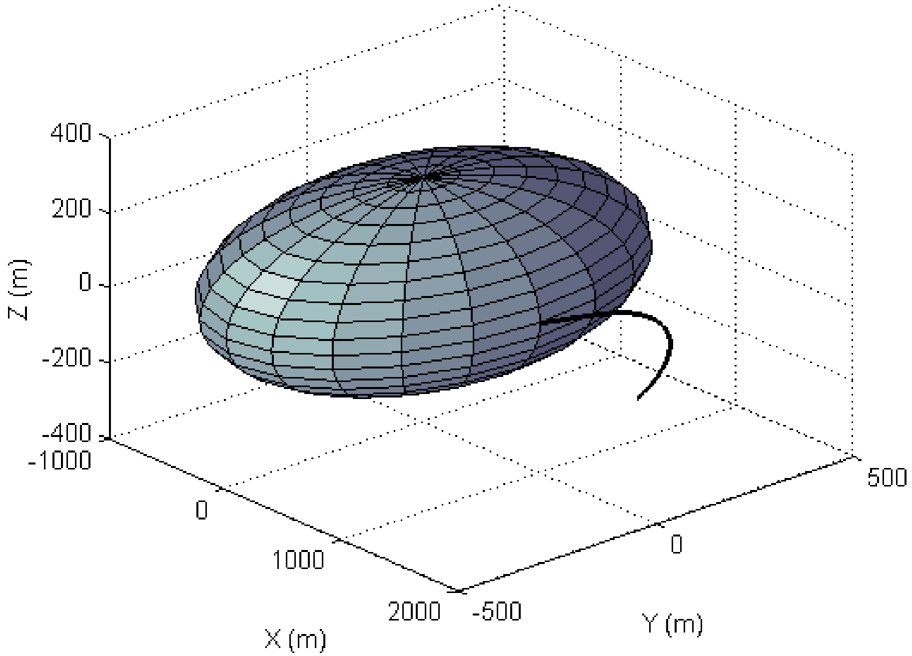
EQ trajectories 2 hr all asteroids

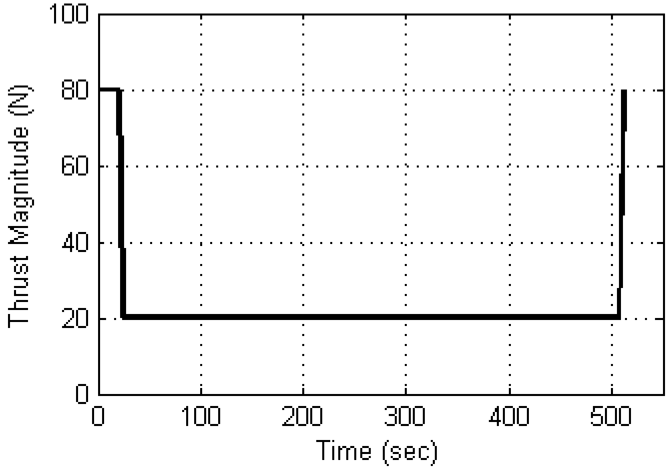


EQ Optimal Trajectory

- Minimum Propellant flight time 512 sec.
- 1000 x 500 x 250 m





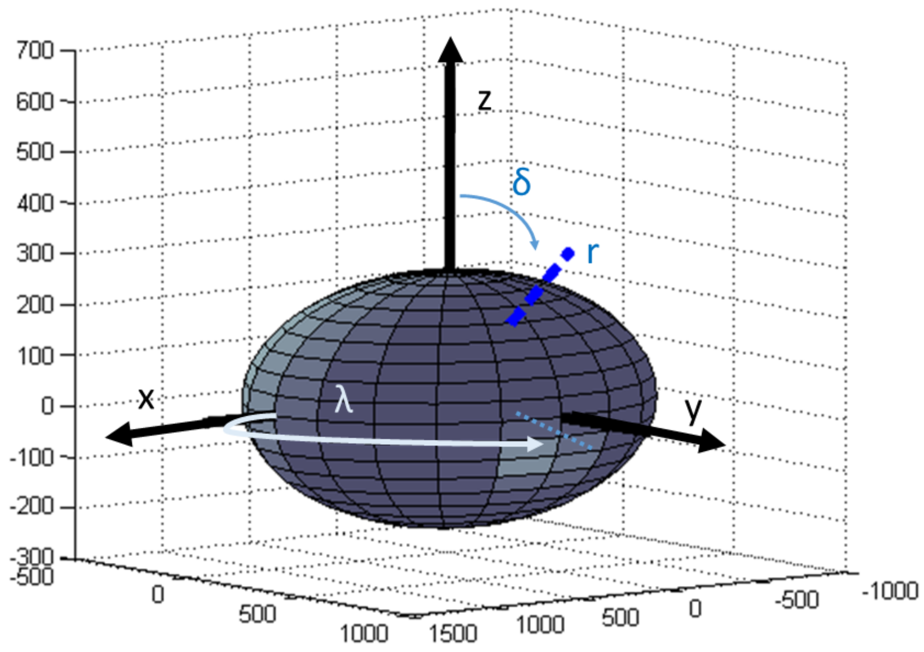


Conclusions

- Powered descent trajectory can be formulated as a convex optimization problem.
- Successive solution method is the key to handling nonlinear gravity model.
- Handles a wide range of parameters successfully.
- Viable algorithm for rapidly designing asteroid powered descent trajectories on-board the spacecraft.

- Back-up

Coordinates



- Use an asteroid centered fixed coordinate system to determine the spacecraft location, gravity and equations of motion.
  - x aligned with the largest axis, z aligned with the smallest axis, y completes the orthogonality.
  - Radius (r) is the distance from asteroid center to spacecraft.
  - λ spacecraft longitude
  - δ spacecraft latitude

Gravitational Acceleration

- Equations

$$\frac{\partial U}{\partial r_x} = -\frac{\mu}{r^3} r_x + -\frac{\mu}{r^3}\left(G_1 r_x + G_2 \frac{r_x r_z}{\left(r_x^2+r_y^2\right)^{0.5}}\right) + \frac{\mu}{r} G_3 r_y$$

$$\frac{\partial U}{\partial r_y} = -\frac{\mu}{r^3} r_y + -\frac{\mu}{r^3}\left(G_1 r_y + G_2 \frac{r_y r_z}{\left(r_x^2+r_y^2\right)^{0.5}}\right) - \frac{\mu}{r} G_3 r_x$$

$$\frac{\partial U}{\partial r_z} = -\frac{\mu}{r^3} r_z + -\frac{\mu}{r^3}\left(G_1 r_z + G_2 \frac{r_z^2}{\left(r_x^2+r_y^2\right)^{0.5}}\right) + \frac{\mu}{r} \frac{G_2}{\left(r_x^2+r_y^2\right)^{0.5}}$$

- Where

$$r = \left(r_x^2 + r_y^2 + r_z^2\right)^{\frac{1}{2}}$$

$$\delta = \arcsin\left(\frac{r_z}{r}\right)$$

$$\lambda = \arctan\left(\frac{r_y}{r_x}\right)$$

$$G_1 = \frac{1}{r^2}\left\{3C_{20}\left[1.5sin^2\left(\delta\right) - 0.5\right] + 9C_{22}cos^2\left(\delta\right)cos\left(2\lambda\right)\right\}$$

$$G_2 = \frac{1}{r^2}\left\{1.5C_{20}sin\left(2\delta\right) - 3C_{22}sin\left(2\delta\right)cos\left(2\lambda\right)\right\}$$

$$G_3 = \frac{1}{r^2}\left\{\frac{1}{\left(r_x^2+r_y^2\right)}6C_{22}cos^2\left(\delta\right)sin\left(2\lambda\right)\right\}$$

A, B, c

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ \omega^2 - \frac{\mu}{r^3} & 0 & 0 & 0 & 2\omega & 0 & 0 \\ 0 & \omega^2 - \frac{\mu}{r^3} & 0 & -2\omega & 0 & 0 & 0 \\ 0 & 0 & -\frac{\mu}{r^3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^{(k-1)} \qquad B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\frac{1}{v_{ex}} \end{bmatrix}$$

$$c = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\frac{\mu}{r^3}\left(G_1 r_x + G_2 \frac{r_x r_z}{\sqrt{(r_x^2+r_y^2)}}\right) + \frac{\mu}{r} G_3 r_y \\ -\frac{\mu}{r^3}\left(G_1 r_y + G_2 \frac{r_y r_z}{\sqrt{(r_x^2+r_y^2)}}\right) - \frac{\mu}{r} G_3 r_x \\ -\frac{\mu}{r^3}\left(G_1 r_z + G_2 \frac{r_z^2}{\sqrt{(r_x^2+r_y^2)}}\right) + \frac{\mu}{r} \frac{1}{\sqrt{(x_k^2+y_k^2)}} G_2 \\ 0 \end{bmatrix}^{(k-1)}$$

Spacecraft Parameters

- Thrust 80 N – 20 N
  - Isp 225 sec
- Mass 1400 kg
- Initial Conditions

	${r_0}_x$(m)	${r_0}_y$(m)	${r_0}_z$(m)	${v_0}_x$(m/s)	${v_0}_y$(m/s)	${v_0}_z$(m/s)
NP	-50	-50	1000+γ	2	1	0
EQ	1000+α	-50	-50	0	2	1