Towards an Understanding of Atmospheric Balance

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Why is the extra-tropical atmosphere approx. quasi-geostrophic ?

The stability of quasi-geostrophic flow with respect to ageostrophic perturbations

Derivation of (2-layer, f_plane) PE in terms of Normal Modes

Errico JAS 1981

$$\frac{d}{dt} b_{\mathbf{K}} = \sum_{\mathbf{L},\mathbf{M}} [C_1 b^*_{\mathbf{L}} b^*_{\mathbf{M}} + C_2 g^*_{\mathbf{L}} g^*_{\mathbf{M}} + C_3 g^*_{\mathbf{L}} a^*_{\mathbf{M}} + C_3^* g^*_{\mathbf{L}} d^*_{\mathbf{M}} + C_4 a^*_{\mathbf{L}} a^*_{\mathbf{M}} + C_4^* d^*_{\mathbf{L}} d^*_{\mathbf{M}} + C_5 a^*_{\mathbf{L}} d^*_{\mathbf{M}}],$$

$$\frac{d}{dt} g_{\mathbf{K}} = \sum_{\mathbf{L},\mathbf{M}} [C_6 b^*_{\mathbf{L}} g^*_{\mathbf{M}} + C_7 b^*_{\mathbf{L}} a^*_{\mathbf{M}} + C_7^* b^*_{\mathbf{L}} d^*_{\mathbf{M}}],$$

$$\frac{d}{dt} a_{\mathbf{K}} = i\omega_K a_{\mathbf{K}} + \sum_{\mathbf{L},\mathbf{M}} [C_8 b^*_{\mathbf{L}} g^*_{\mathbf{M}} + C_9 b^*_{\mathbf{L}} a^*_{\mathbf{M}} + C_{10} b^*_{\mathbf{L}} d^*_{\mathbf{M}}],$$

$$d_{\mathbf{K}} = a^*_{-\mathbf{K}}.$$

$$C_1 = -\tfrac{1}{2}\mathbf{L} \times \mathbf{M}(L^{-2} - M^{-2}),$$

$$C_2 = -\tfrac{1}{2}\mathbf{L} \times \mathbf{M}(\omega_L^{-2} - \omega_M^{-2}),$$

$$C_3 = -(M^2\omega_M^2\omega_L^2)^{-1}[\mathbf{L} \times \mathbf{M}(M^2 - L^2) - i\omega_M(L^2\mathbf{M}\cdot\mathbf{K} + M^2\mathbf{L}\cdot\mathbf{K})],$$

$$C_4 = \tfrac{1}{2}\omega_M^{-2}\omega_L^{-2}[\mathbf{L} \times \mathbf{M}(M^{-2} - L^{-2})(1 + \omega_M\omega + i(\omega_L + \omega_M)(M^{-2}\mathbf{M}\cdot\mathbf{K} + L^{-2}\mathbf{L}\cdot\mathbf{K})],$$

$$C_5 = \omega_M^{-2}\omega_L^{-2}[\mathbf{L} \times \mathbf{M}(M^{-2} - L^{-2})(1 - \omega_M\omega_L) + i(\omega_L - \omega_M)(M^{-2}\mathbf{M}\cdot\mathbf{K} + L^{-2}\mathbf{L}\cdot\mathbf{K})],$$

$$C_6 = -\mathbf{L} \times \mathbf{M} L^{-2}\omega_M^{-2}(\omega_M^2 - \omega_L^2 + 1),$$

$$C_7 = \omega_M^{-2}M^{-2}(\mathbf{L} \times \mathbf{M} + i\omega_M \mathbf{K}\cdot\mathbf{M}),$$

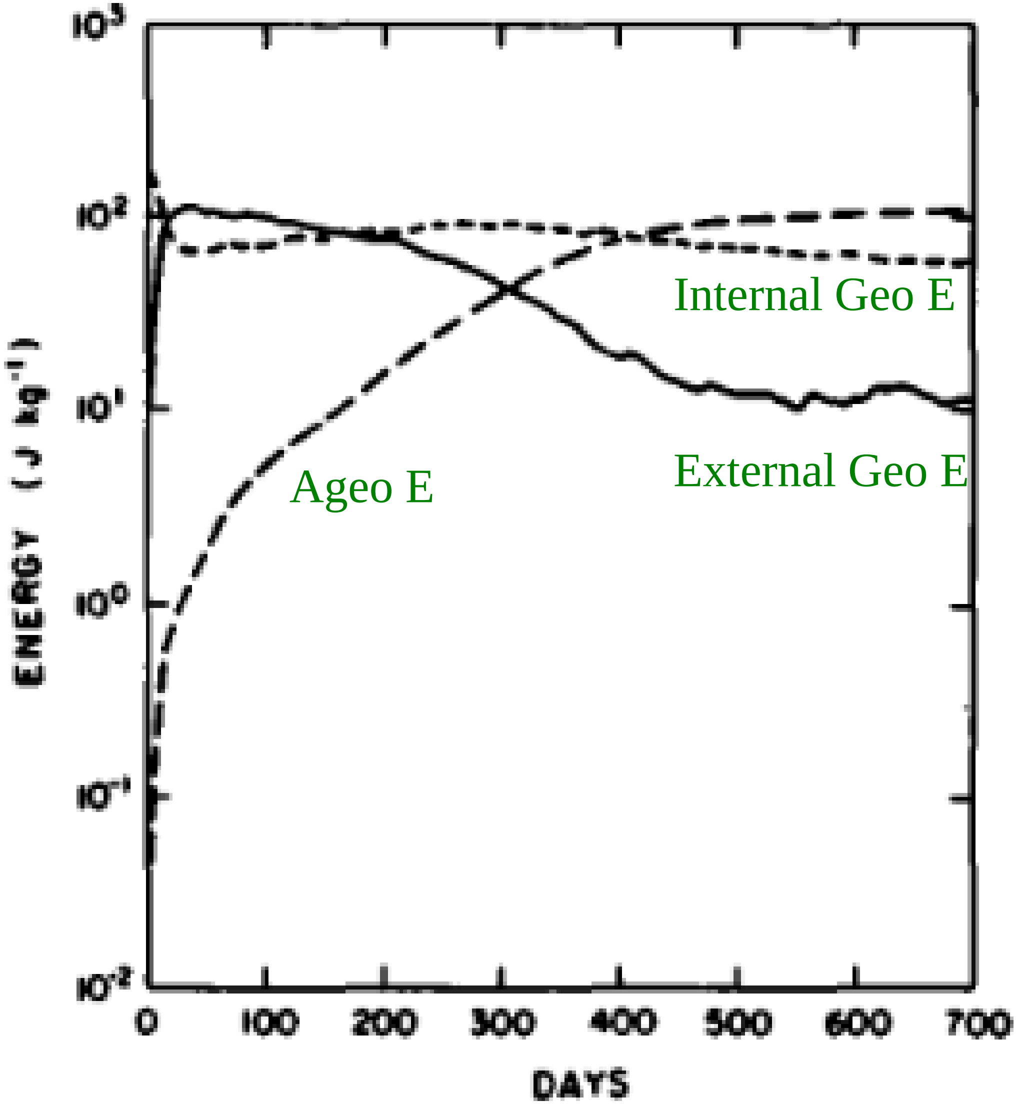
$$C_8 = -\omega_M^{-2}L^{-2}\mathbf{L} \times \mathbf{M}(\mathbf{K}\cdot\mathbf{L} - i\omega_K \mathbf{L} \times \mathbf{M}),$$

$$C_9 = (2\omega_M^2 L^2 M^2)^{-1}\{\mathbf{L} \times \mathbf{M}[L^2(1 - \omega_K\omega_M) - \omega_K(\omega_K M^2 - \omega_M K^2)] + i[2\omega_K(\mathbf{L} \times \mathbf{M})^2 + \omega_M L^2 \mathbf{K}\cdot\mathbf{M}]\},$$

$$C_{10} = (2\omega_M^2 L^2 M^2)^{-1}\{\mathbf{L} \times \mathbf{M}[L^2(1 + \omega_K\omega_M) - \omega_K(\omega_K M^2 + \omega_M K^2)] + i[2\omega_K(\mathbf{L} \times \mathbf{M})^2 - \omega_M L^2 \mathbf{K}\cdot\mathbf{M}]\}.$$

Demonstartion of Equipartition

Errico Tellus 1984

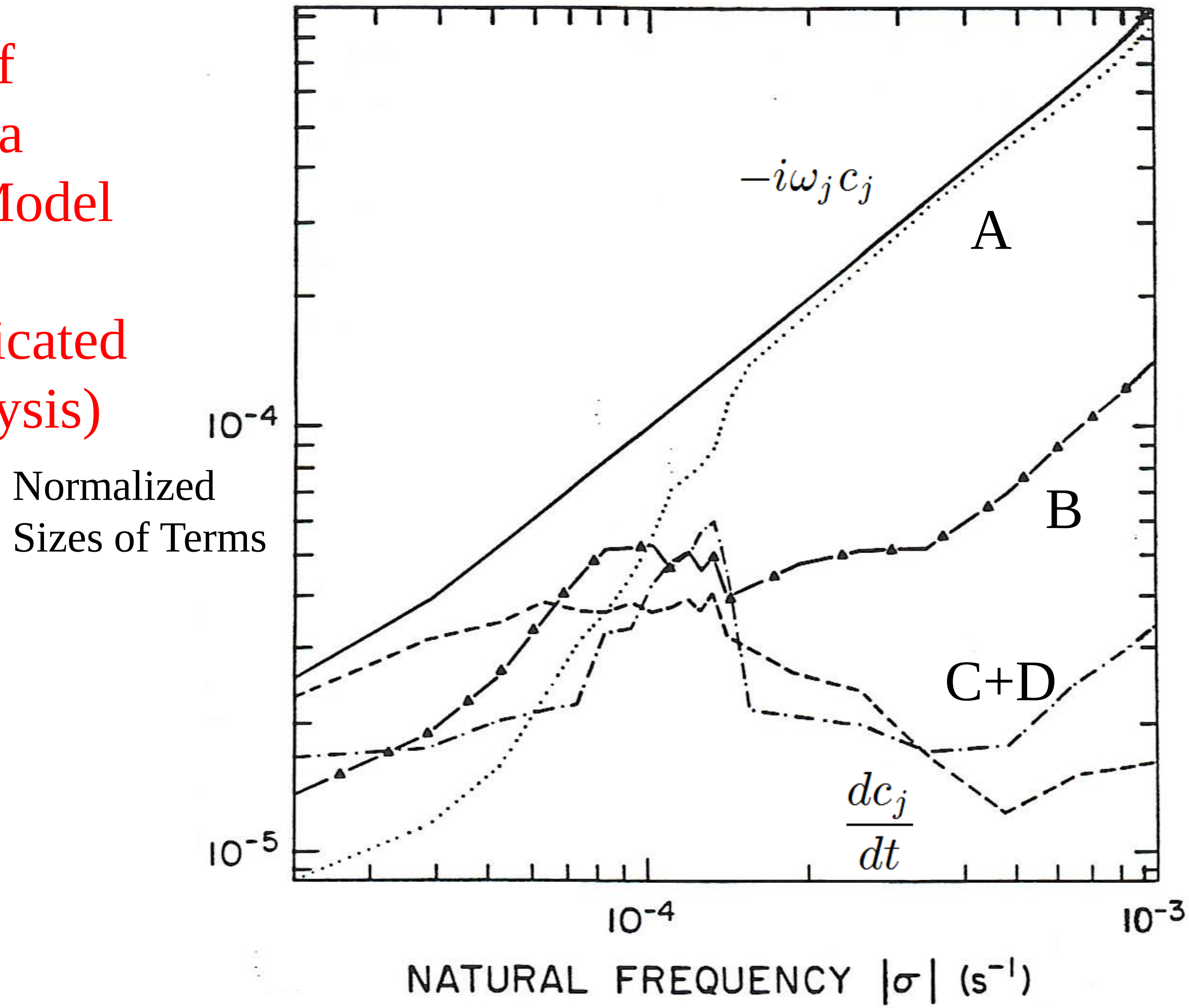


Examination of Balance

$$\frac{dc_j}{dt} = -i\omega_j c_j + A(r,r) + B(r,g) + C(g,g) + D$$

Balance of Modes in a Climate Model

(a sophisticated scale analysis)

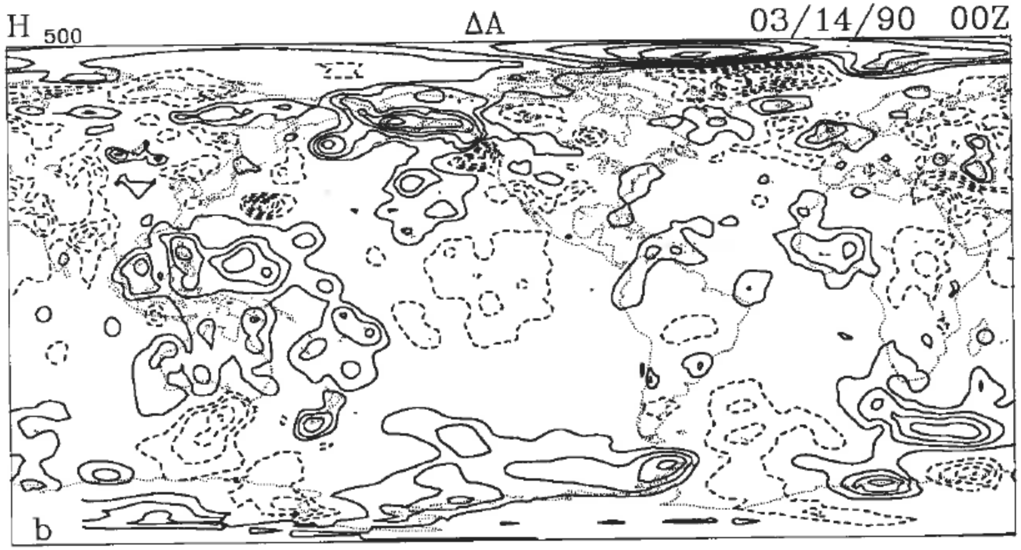


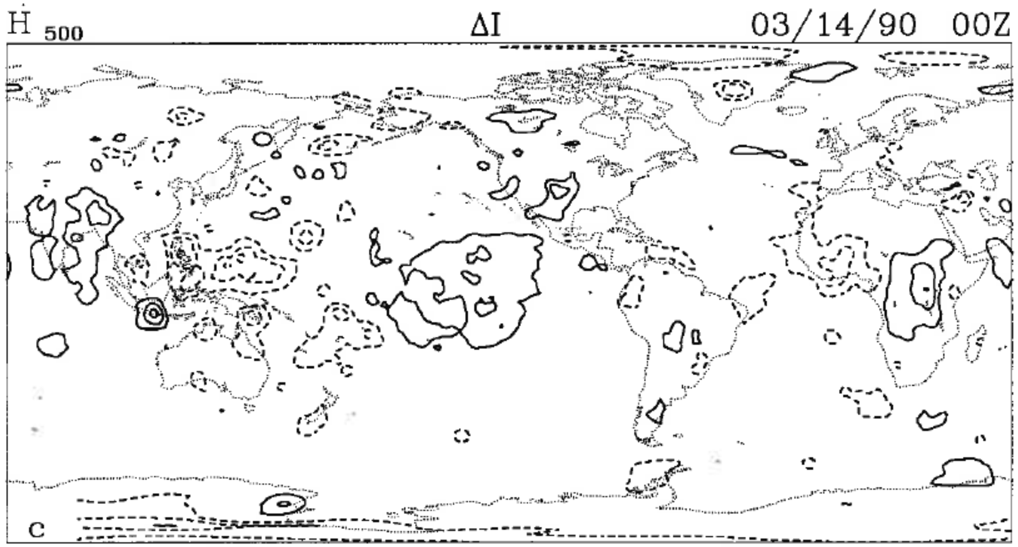
Errico 1984, 1990 *MWR*; Errico et al. *1988* MWR

Diabatic Balance ?

The interplay of analysis and initialization

Errico et al. *MWR* 1993





Tribbia
Daley
Williamson
Fillion
Courtier
ECMWF

Gravitational modes considered as forced and damped harmonic oscillators

Define $g(t)$ as the complex amplitude of a gravity–wave like mode at each time t, and let R and G be the sets of Rossby– and gravity–wave like modes. Then

$$\frac{dg}{dt} = -i\lambda g + N(R) + N(R, G) + N(G) + D(R, G) - \nu g$$

Consider $N(R) = F(t)$ as the dominant nonlinear term. Approximately then

$$\frac{dg}{dt} = -i\lambda g + F(t) - \nu g$$

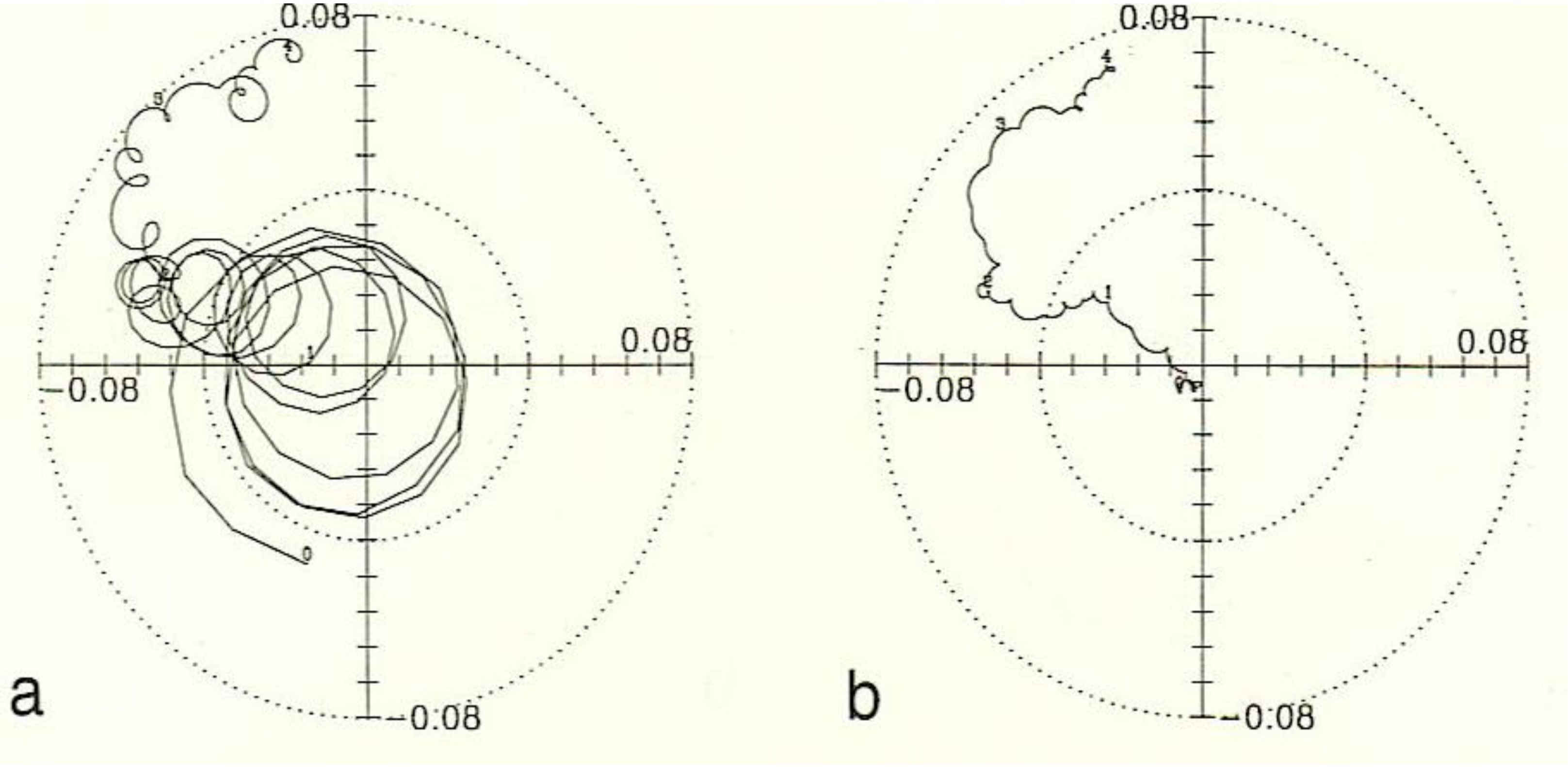
Consider $F(t) = F(0)\exp(-i\mu t)$. Then

$$g(t) = \left[g(0) - \frac{F(0)}{i\lambda - i\mu + \nu}\right]\exp(-(i\lambda + \nu)t) \;+\; \frac{F}{i\lambda - i\mu + \nu}$$

Errico 1981 *JAS*, 1984 *MWR*, 1997 *J Japan MS*

Harmonic Dial for External m=4 Mode, Period=3.7h

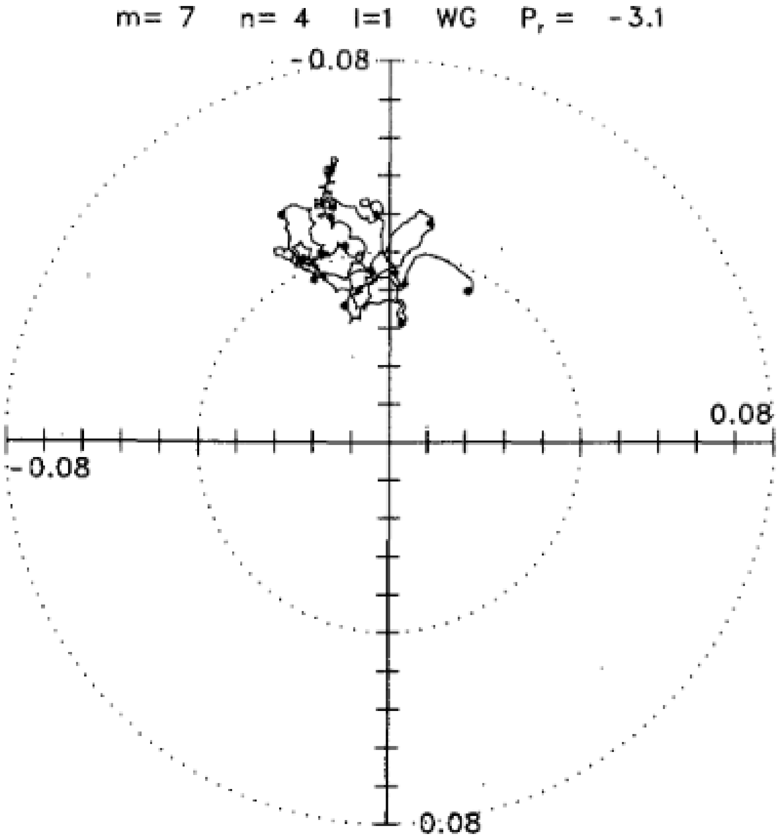
Without NNMI With NNMI

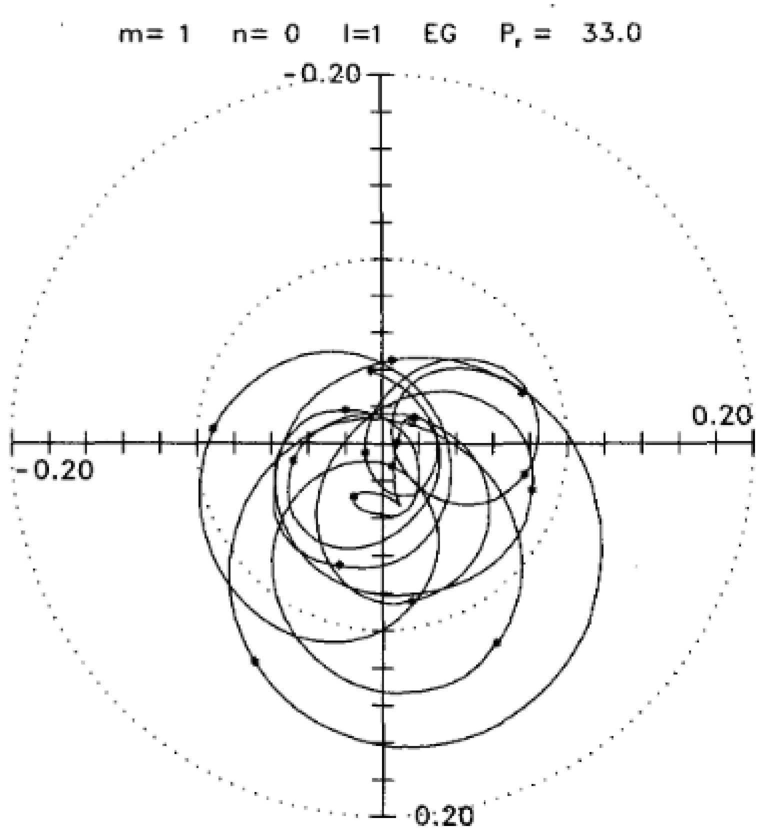


Errico 1997 *J Japan Met Soc*

Behavior of gravitational modes in a climate model:
Time series (harmonic dials) of complex mode amplitudes
Errico MWR 1989

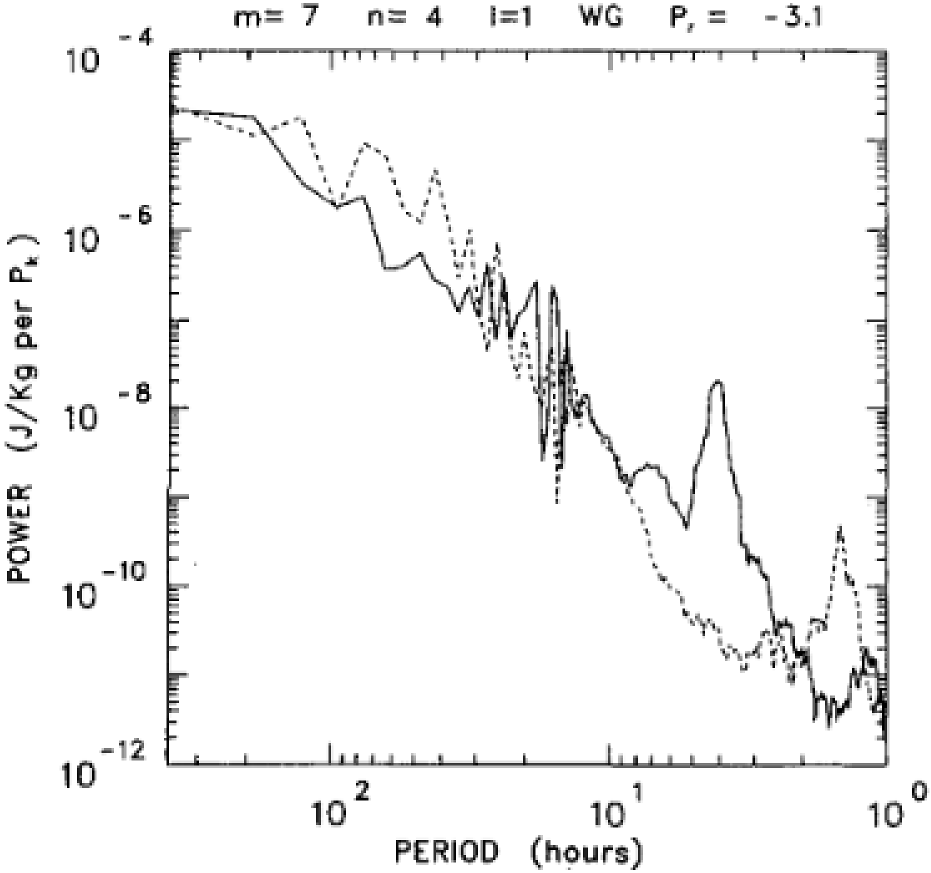
16 days shown

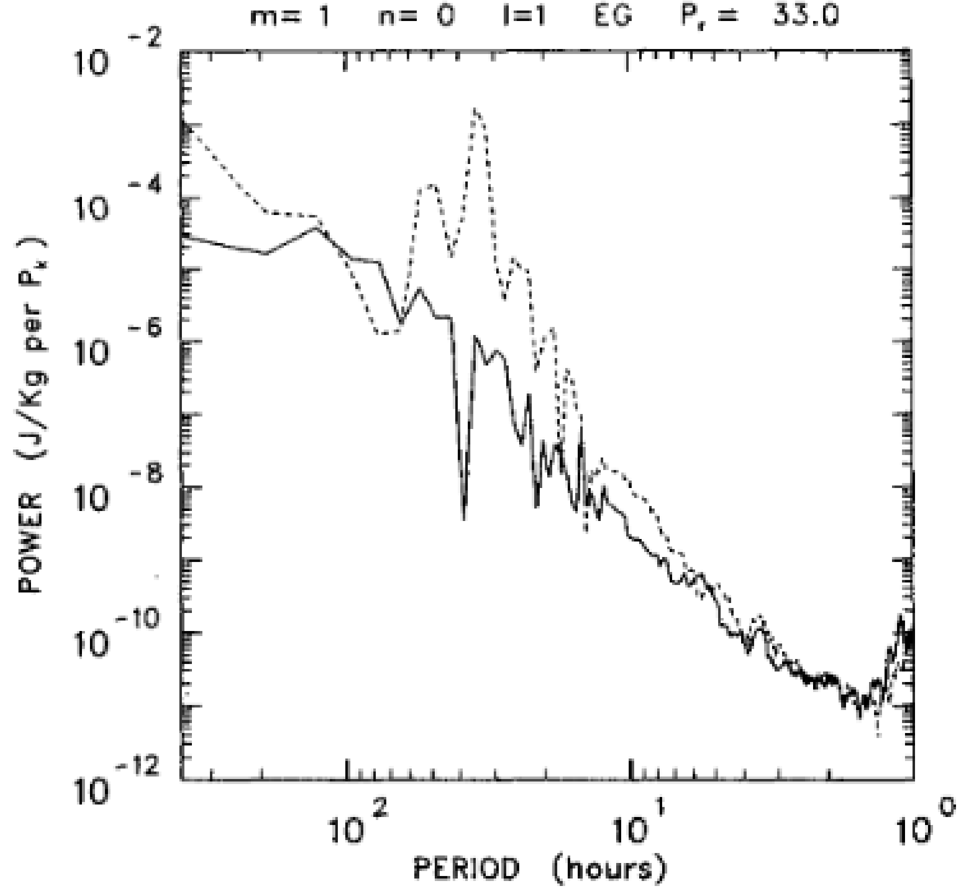




Behavior of gravitational modes in a climate model:
Power spectra of complex mode amplitudes

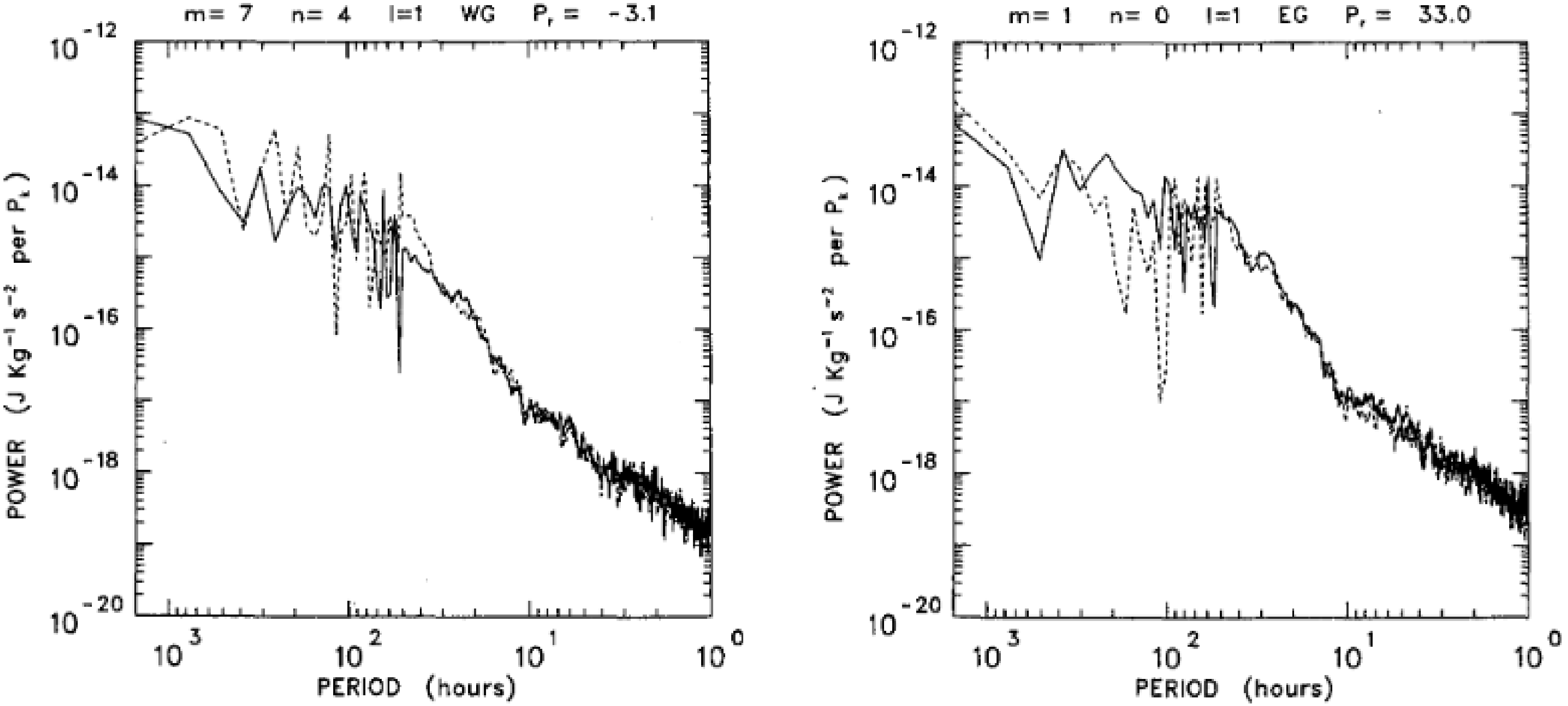
Errico MWR 1989





Solid: Westward propagating Dashed: Eastward propagating

Behavior of gravitational modes in a climate model:
Power spectra of convective heating
Errico MWR 1989

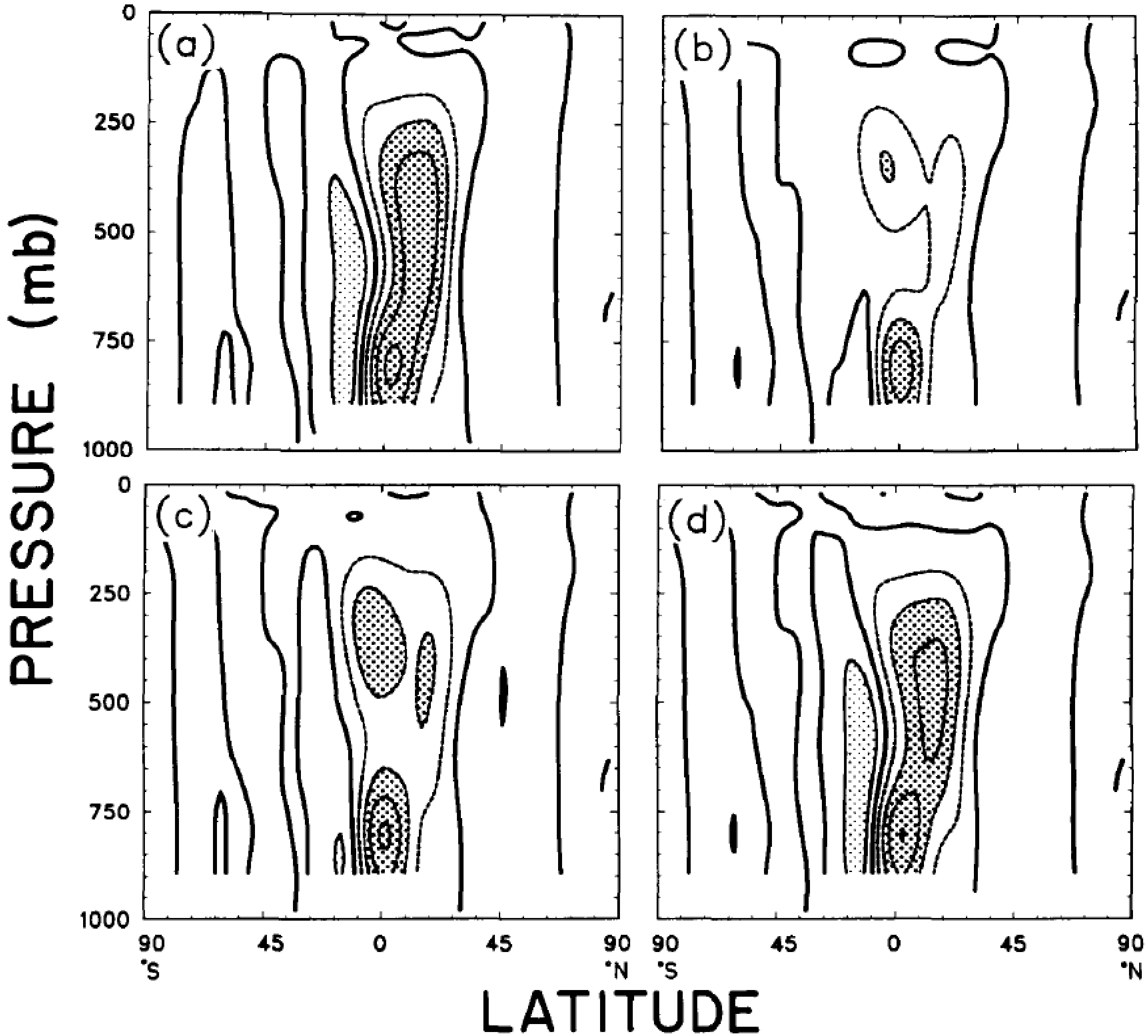


Diabatic balance vs appropriate cutoff

Errico and Rasch *Tellus* 1988

Truth (natural balance)

Linear NMI P<48 hr

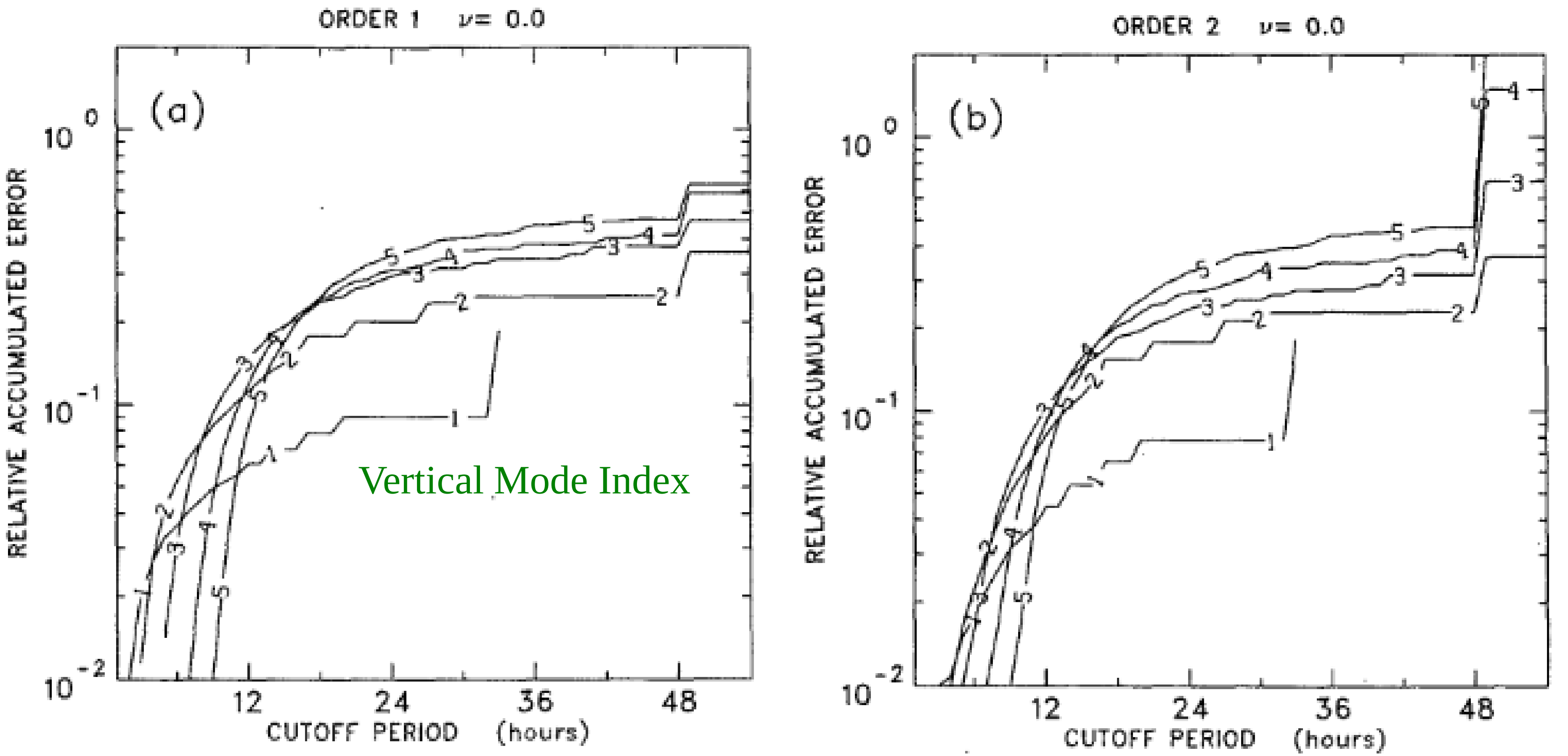


Diabatic NNMI P<48 hr

Adiabatic NNMI P<24 hr

Higher-order Machenhauer schemes

Errico *MWR* 1989

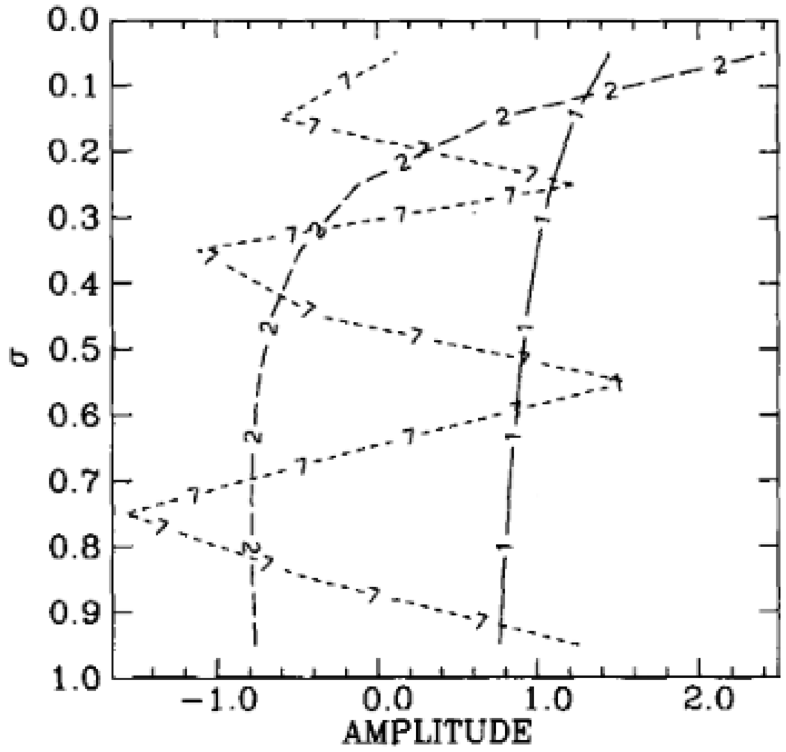


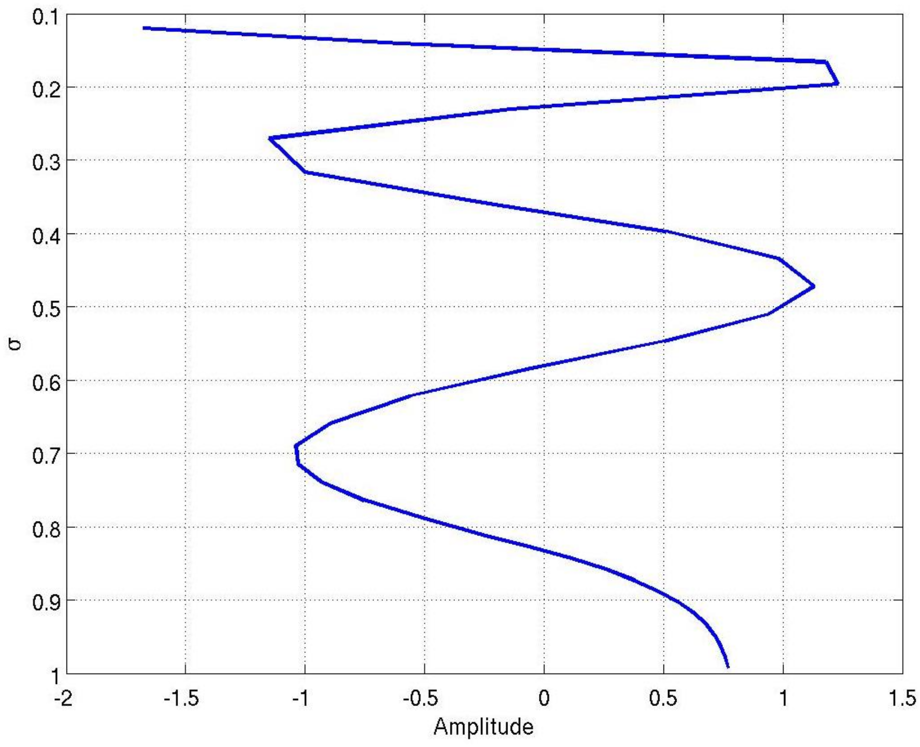
Other Issues

Vertical modes in discrete models

10 level MAMS
Modes 1, 2, 7 (H=10,000, 2050, 13 m)

72 level GEOS-5
Mode 29 (H=13m)
(Notice only plotted up to σ=0.1



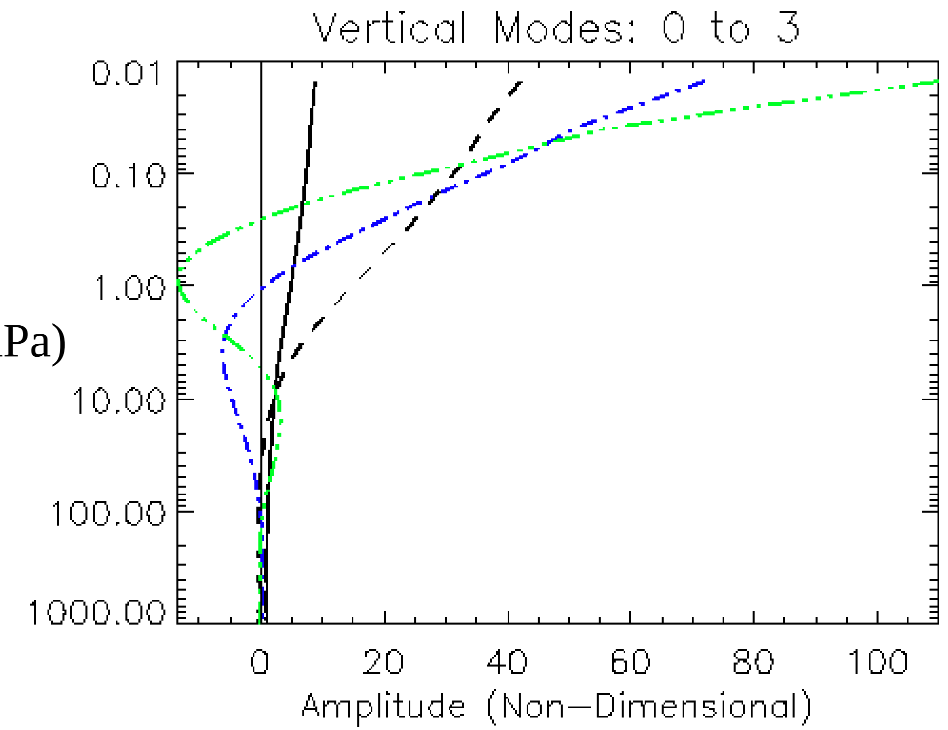


23 zero crossings above for σ<0.1

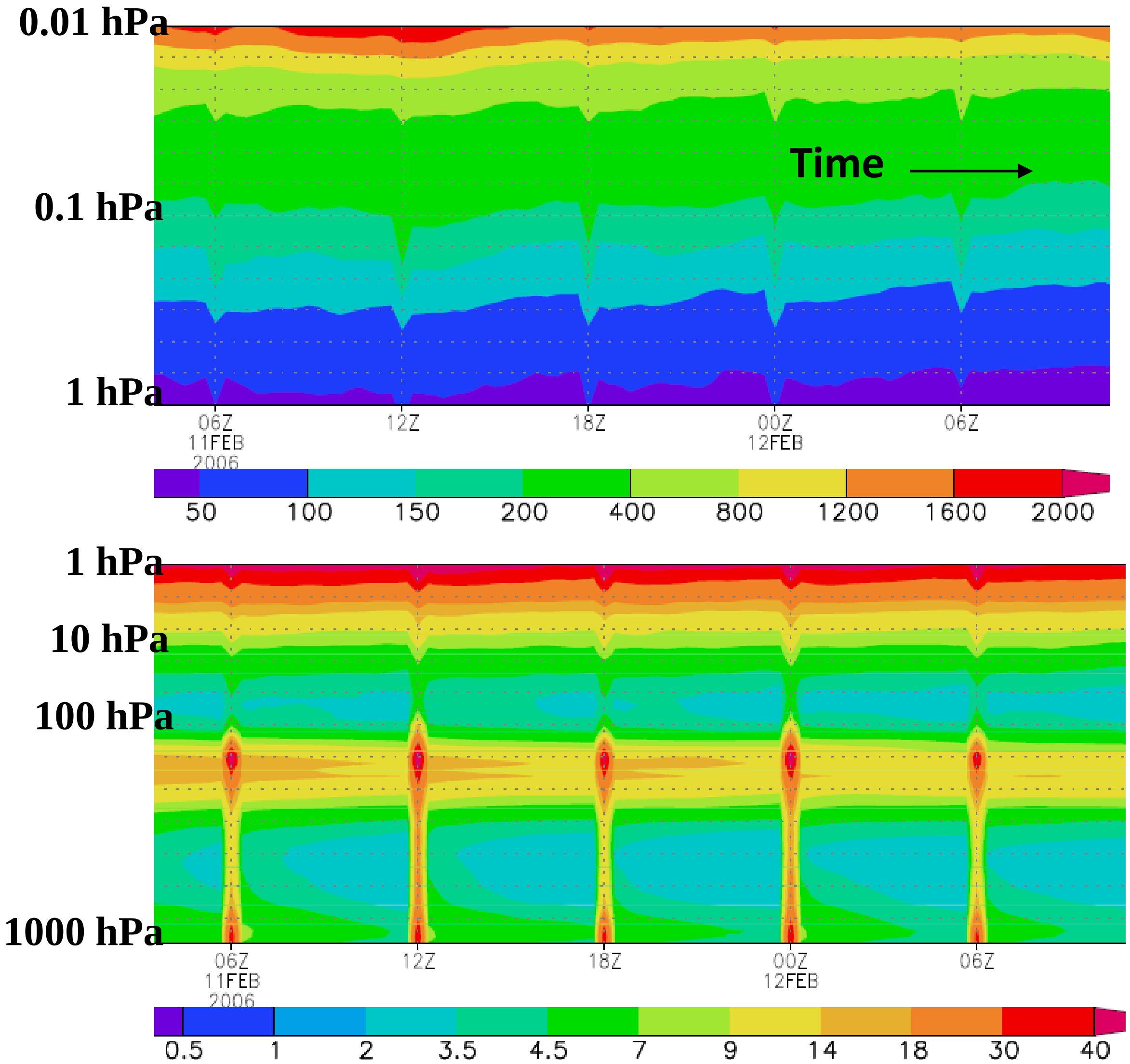
High amplitude modes in the upper atmosphere

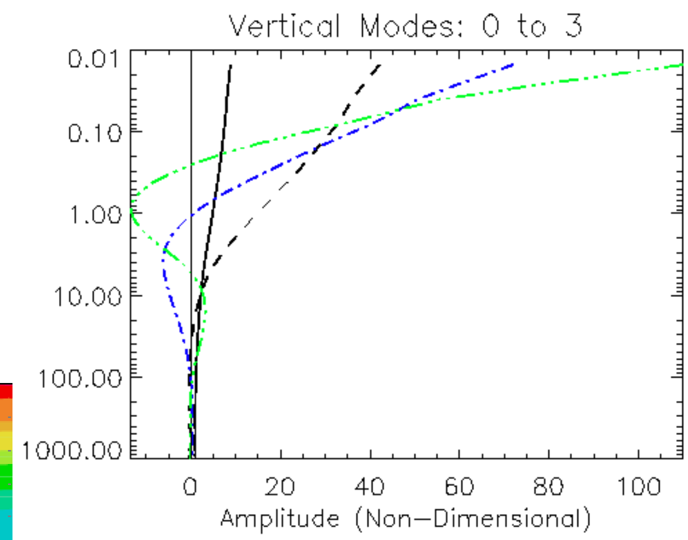
72 level GEOS-5 model with top at 0.01 hPa

P (hPa)



Global mean squared divergence tendency

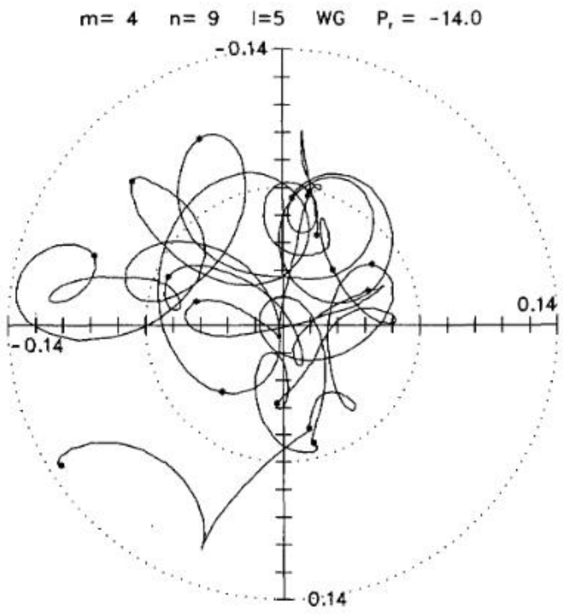




Structures of 3 largest scale vertical normal modes

GMAO-GSI 3DVAR
72 level model

Derivation of (2-layer) PE in terms of Normal Modes
Errico JAS 1981



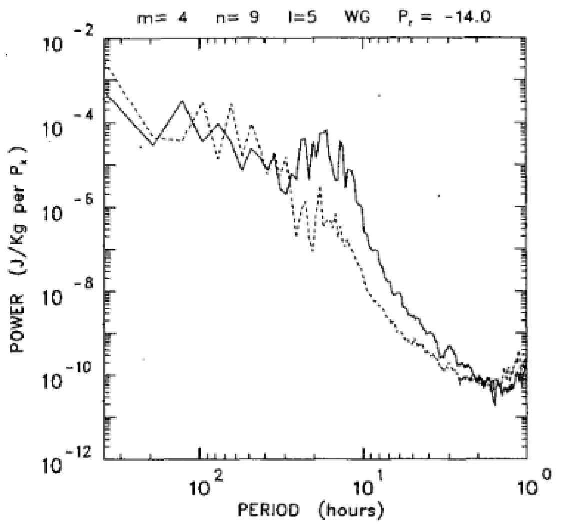
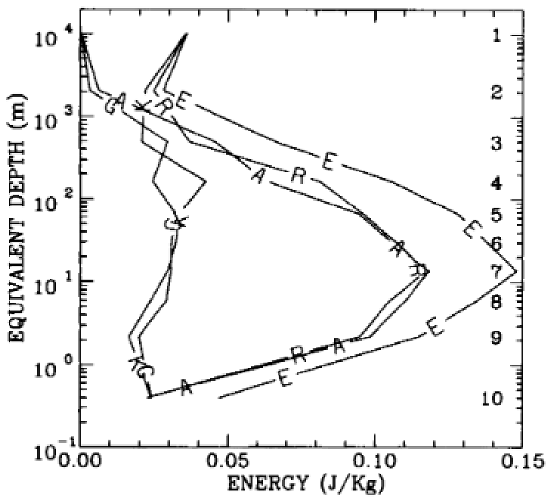


FIG. 3. Like Fig. 1 except for the fifth antisymmetric, zonal wavenumber 4, $l = 5$ WG mode. The progression in the dial is predominantly clockwise (westward propagation).

Derivation of (2-layer) PE in terms of Normal Modes

Errico JAS 1981



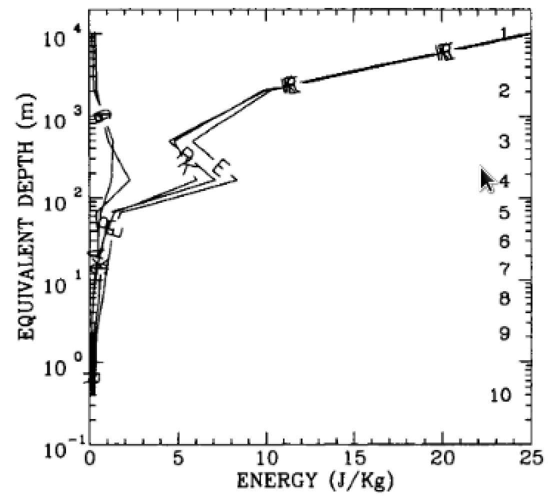


Figure 4. The kinetic energy (K), available potential energy (A), rotational-mode energy (R), gravitational-mode energy (G), and total energy (E) contributed by vertical modes of indicated equivalent depths at $t = 0$. The integers on the right-hand side indicate corresponding vertical-mode indices ℓ.

Derivation of (2-layer) PE in terms of Normal Modes

Errico JAS 1981

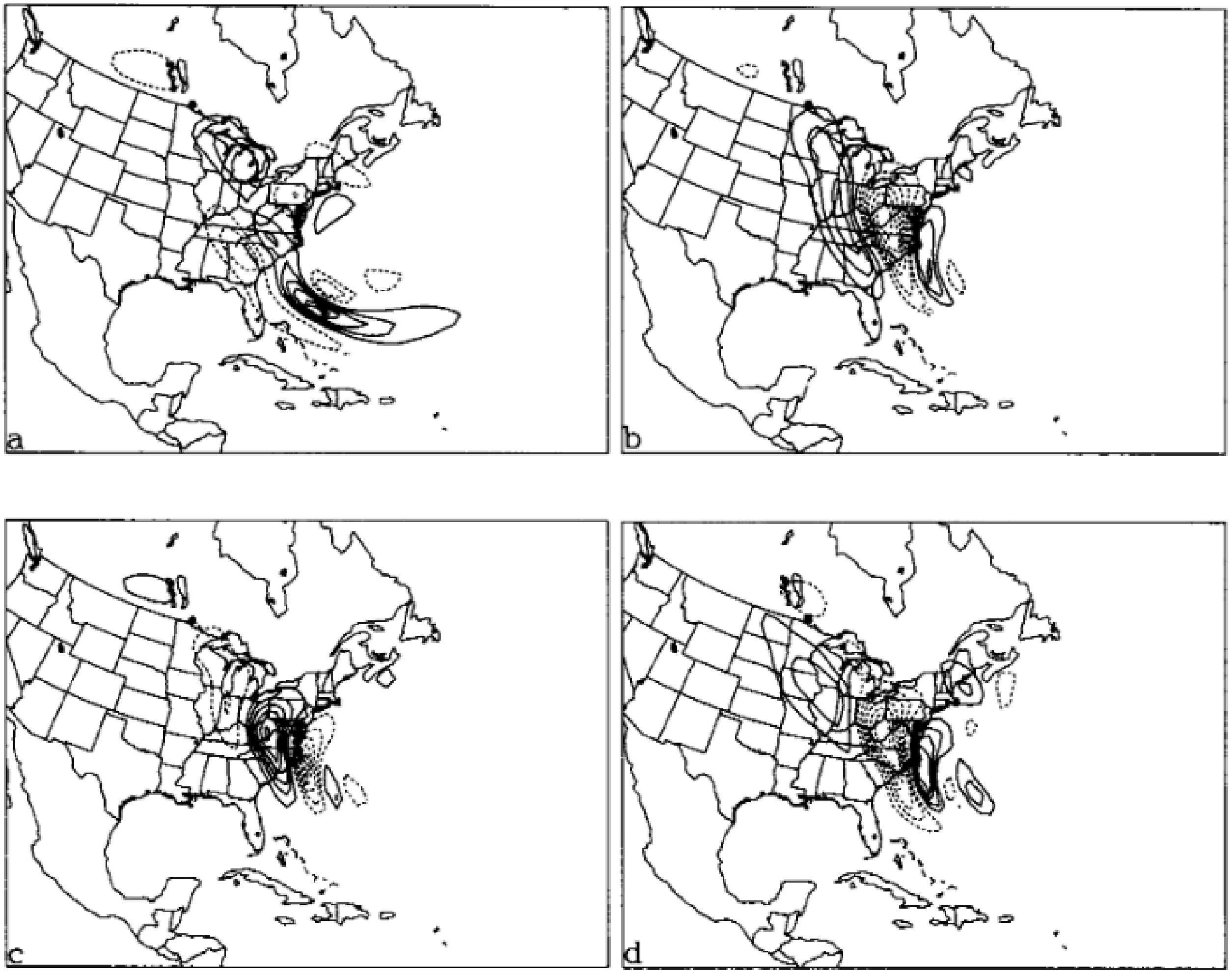


Figure 6. The (a) and (b) R and (c) and (d) its complement components of the (a) and (c) u' and (b) and (d) T' fields on $\sigma = 0.55$ at $t = 0$ for SV1 determined using the E norm applied to the dry form of the linearized model. Contour intervals are (a) and (c) 1 m s^{-1}, (b) 1 K, and (d) 0.5 K, with zero-contours omitted and negative values shown dashed. See text for further explanation.

Derivation of (2-layer) PE in terms of Normal Modes

Errico JAS 1981

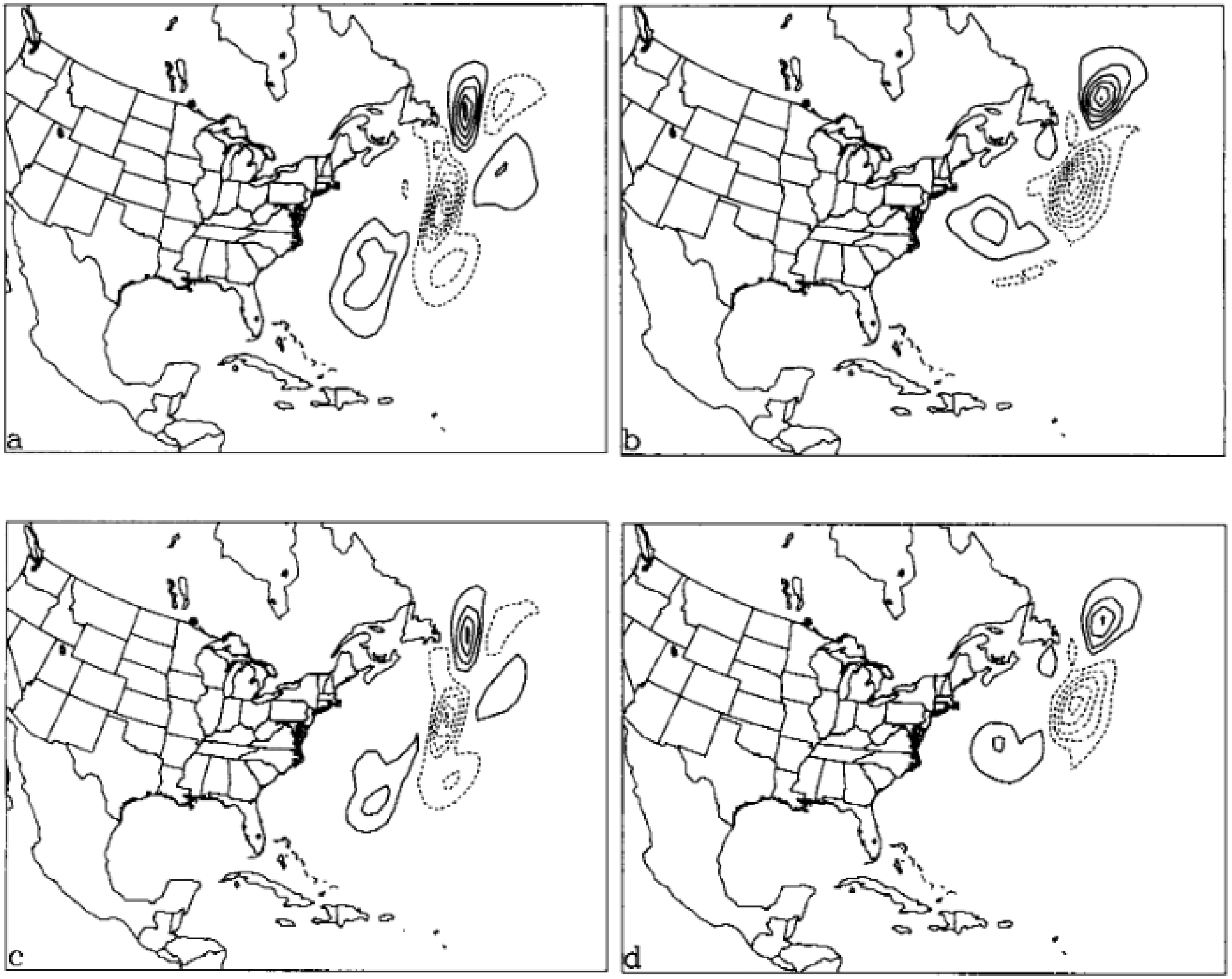


Figure 8. The (a) and (c) v' and (b) and (d) T' fields on $\sigma = 0.55$ at $t = 24$ h determined from the linearized evolutions begun from (a) and (b) R-mode components of SV1 and (c) and (d) their complement of SV1. Contour intervals are (a) 10 m s^{-1}, (b) 2 K, (c) 5 m s^{-1}, and (d) 1 K, with zero-contours omitted and negative values shown dashed. See text for further explanation.

Partitioning of analysis error energy in terms of normal modes:
(as inferred from an OSSE)
Errico et al. *Met Z.* 2007

Vert mode index	k	1	2	3	4	5	6	7	8	9
Equiv Depth	H(m)	10943	4444	1538	628	311	175	109	71	49
G-mode Energy	G(J/kg)	.18	.16	.22	.32	.31	.32	.29	.28	.25
R-Mode Energy	R(J/kg)	.82	.47	.38	.51	.52	.58	.56	.45	.33
Ratio G/TE	f_g	.18	.25	.37	.39	.37	.36	.34	.38	.43

Summary

1. Much can be learned from some old works
2. The standard Normal Modes provide useful concepts and tools
3. The standard Normal Modes have limitations
   a. the universality of vertical modes
   b. internal modes (when C approx = U)
   c. more realistic basic states (e.g. as for SVs)
4. Is Initialization still an issue ?
5. There is more to understand
   a. time scales of moist diabatic processes
   b. effects of top boundary conditions, non-hydrostatic behavior
   c. SV behavior