CRITERIA FOR YIELDING OF DISPERSION-STRENGTHENED ALLOYS*

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A dislocation model is presented in order to account for the yield behavior of alloys with a finely dispersed second-phase. The criteria for yielding used in the model, is that appreciable yielding occurs in these alloys when the shear stress due to piled-up groups of dislocations is sufficient to fracture or plastically deform the dispersed second-phase particles, relieving the back stress on the dislocation sources.

Equations derived on the basis of this model, predict that the yield stress of the alloys varies as the reciprocal square root of the mean free path between dispersed particles. Experimental data is presented for several SAP-Type alloys, precipitation-hardened alloys and steels which are in good agreement with the yield strength variation as a function of dispersion spacing predicted by this theoretical treatment.

CRITERE DE RUPTURE POUR UN ALLIAGE DURCI PAR DISPERSION

Un modèle de dislocation a été présenté pour expliquer le comportement de la rupture des alliages à phase secondaire finement dispersée. Le critère de rupture employé dans ce modèle est qu'une déformation appréciable se produit dans ces alliages quand la tension de cisaillement due à des empilements de dislocations, est suffisante pour causer une rupture ou une déformation plastique des particules de la phase secondaire dispersée relachant la tension arrière sur les sources de la dislocation.

Des équations dérivées sur la base de ce modèle prédisent que la limite élastique des alliages varie en raison inverse du carré des libres parcours moyens entre les particules dispersées. Des valeurs expérimentales sont connues pour quelques alliages du type SAP, des alliages à précipitation structurale et des aciers, qui sont en bon accord avec la variation de la limite élastique en fonction de la distance entre particules dispersées prédites par ces considérations théoriques.

FLIEßKRITERIEN FÜR DISPERSIONSGEHÄRTETE LEGIERUNGEN

Ein Versetzungsmodell, welches dem Fließverhalten von Legierungen mit einer feindispersen zweiten Phase Rechnung trägt, wird ausgearbeitet. Das in diesem Modell benützte Fließkriterium besagt, daß in solchen Legierungen ausgiebiges Fließen einsetzt, wenn die von aufgestauten Versetzungsgruppen herrührende Schubspannung ausreicht, um die dispersen Teilchen der zweiten Phase zu durchbrechen oder plastisch zu verformen; die Rückspannung auf die Versetzungsquelle wird dann verringert.

Die auf Grund dieses Modells abgeleiteten Gleichungen sagen voraus, daß sich die Fließspannung wie die reziproke Quadratwurzel aus der mittleren freien Weglänge zwischen den dispersen Teilchen verhält. Experimentelle Ergebnisse an verschiedenen SAP-Legierungen, ausscheidungsgehärteten Legierungen und Stählen werden mitgeteilt, bei welchen die Variation der Fließspannung als Funktion der Teilchenabstände gut mit den Voraussagen der theoretischen Behandlung übereinstimmt.

INTRODUCTION

Plastic deformation in crystals is due to movement of dislocations. Yielding takes place when many dislocations move large distances through the lattice. Dislocations are nucleated at sources in the lattice due to an applied stress. If the stress required to nucleate dislocations is greater than the stress necessary to move dislocations appreciable distances along a slip plane, the yield stress of the material will equal the stress necessary to propagate dislocations from a source. In alloys where a continuous three-dimensional dislocation network provides Frank–Read sources, this stress is equal to

$$\text{nucleating stress } N = \frac{\mu b}{L} \tag{1}$$

where μ is a shear modulus, b is the Burgers vector and L is the linear distance between nodes in the network. For unresolved stresses and strains the right hand side of this equation should be multiplied by two.

In a dispersion-strengthened alloy, however, the stress necessary to move dislocations appreciable distances along a slip plane may be higher than the stress necessary to nucleate dislocations from a source. In this case, the yield stress of the alloy is determined by the stress required to move dislocations freely in a crystal lattice containing a uniformly dispersed second-phase. This paper presents a model for calculating this stress.

MODEL

Dislocation loops are considered to be formed at some source under the action of an applied stress. The

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nature of the source is not critical in considering the model. As the dislocation loops expand from the source, they are either blocked from further motion by the dispersed second-phase particles, or they continue to move by bowing about the dispersed particles leaving residual dislocation loops surrounding each particle. The stress required to bow dislocation loops about the dispersed particles is the yield stress in the Orowan criterion,[1,2] which predicts the yield stress of dispersion strengthened materials to be inversely proportional to the dispersed-particle spacing. Several investigators[3-5] have tried to apply this relationship to their experimental data. However, in our model, it is postulated that even when the dislocations move past the dispersed particles, leaving residual loops surrounding the particles, yielding does not result. This postulate can be supported by the following argument. The first dislocation nucleated at a source moves in the slip plane until it is blocked from further movement by its interaction with dislocations nucleated from other sources. In single phase materials this blockage of the lead dislocation is overcome by the increase of stress on the dislocation due to the pile-up of subsequently nucleated dislocations behind the lead dislocation. In a dispersion strengthened alloy, however, the lead dislocation remains blocked because (1) the stress field of the residual loops—as in the Fisher *et al.*[6] work hardening model—decreases the effective stress on the dislocation source. Therefore, fewer dislocations are nucleated at each source; (2) the stress field of the residual loops interacts with the piled-up dislocation group changing the pile-up spacing. Both of these factors decreases the stress on the lead dislocation making it easier to be blocked. Therefore, the plastic strain, ε, of the dispersion strengthened alloy is

$$\varepsilon = MN\pi R^2 b \tag{2}$$

where M is the dislocation source density, N is the number of dislocations nucleated at each source, R is the average radius of the dislocation loops and b is the Burgers vector of the dislocation. Assuming reasonable values for these: $M = 10^9$ sources/cm^3, $N = 10$ dislocations/source, $R = 1/2$ $(M^{-1/3})$ and $b = 2 \times 10^{-8}$ cm^2, the resultant strain is about 10^{-4}. This is much less than the strain usually associated with yielding. Therefore, plastic deformation stops and yielding has not occurred when the back stress on the dislocation source, due to an array of either blocked dislocations or of residual loops around the particles, exceeds the applied stress.

Under these conditions, in alloys with fine dispersions, no apparent yielding has yet occurred. In order to cause such yielding, the shear stress due to the dislocations piled-up around or against the particles must fracture or plastically deform the dispersed second-phase particles. This relieves the back stress on the dislocation source and also increases the stress on the lead dislocation. The dislocations then can sweep out areas on the slip plane which are large with respect to the dispersion spacing.

The fineness of a second-phase dispersion necessary to make its fracture or plastic deformation the critical requirement for yielding, depends upon the density of dislocation sources in the alloy. Even at one half or more of the absolute melting temperature of the matrix metal, fracture or plastic deformation of the second-phase particles should be necessary for appreciable yielding unless recovery takes place. Recovery can occur either by climb of piled-up dislocations at a rate exceeding the applied strain rate, or by cross slip of piled-up dislocations out of the slip plane if the geometry of the dispersed second-phase particles permits. The possibility of recovery is not considered in the following calculation.

THEORY

On the basis of the preceding model, the yield strength of a dispersion strengthened alloy is now evaluated. In this evaluation the shear stress on the dispersed particles due to dislocations piled-up against or residual loops piled-up around the particles is calculated for straight dislocation segments piled-up against a straight barrier. This calculation is applicable to dispersion-strengthened alloys which contain dispersed particles of such a size and shape, e.g. flat plates and large spheres, that the piled-up dislocations have a large radius of curvature and can be considered straight. This is the case for many of these alloys, e.g. SAP-type alloys and most steels. When the radius of curvature is small this calculation of the shear stress on the particles no longer holds and the Fisher *et al.*[6] treatment becomes applicable. These two approaches are then compatible, each being the limiting case of the other. In this treatment the shear stress, τ, on a dispersed second-phase particle due to a piled-up array of dislocations can be considered to be equal to

$$\tau = n\sigma, \tag{3}$$

where n is the number of dislocation loops piled-up against or around a dispersed particle and σ is the applied stress. The number of dislocations, n, acting on a particle depends on the space between the particles, by

$$n = \frac{2\lambda\sigma}{\mu b}, \tag{4}$$

where λ is the spacing between dispersed particles and μ is a shear modulus of the matrix metal ($\mu \approx \sqrt{[\frac{1}{2}C_{44}(C_{11} - C_{12})]}$, for cubic metals, C_{ij} being the usual elastic constants). Combining equations (3) and (4), the shear stress, τ, on the particle is equal to

$$\tau = \frac{2\lambda\sigma^2}{\mu b}. \tag{5}$$

The dispersion strengthened alloy yields when the shear stress on the particle is equal to either the yield stress or fracture stress of the dispersed particle.

The limiting stress, F, that will either plastically deform or fracture the dispersed particles is proportional to a shear modulus, μ^*, of the particle. Therefore

$$F = \frac{\mu^*}{C}, \tag{6}$$

where C is a constant of proportionality dependent upon the degree of lattice perfection of the dispersed particles. One would not expect to find any dislocations within particles whose volumes are less than 10^{-15} cm^3. Even larger particles of refractory materials will not deform plastically under simple shear stresses except at very high temperatures, e.g. 1200°C for Al_2O_3.[7] For these cases, the yield stress of the dispersed particles is approximately equal to the fracture stress, and it can theoretically be shown that the constant of proportionality, C, in equation (6) is somewhere in the neighborhood of thirty.[8] For larger particle sizes and for refractory particles at high temperatures, the yield stress is much less than the fracture stress. For these cases, the constant of proportionality, C, in equation (6) is experimentally found to be equal to 10^4 for most metals.[9]

Combining equations (5) and (6) gives the maximum stress that can be applied to the alloy before yielding occurs.

The yield stress of the alloy is therefore equal to

$$\text{yield stress} = \sqrt{\frac{\mu b \mu^*}{2\lambda C}}. \tag{7}$$

If the distribution of second-phase particles is such that the stress calculated from equation (7) is less than the stress necessary to cause a dislocation source to nucleate dislocations, the equation is no longer applicable. In this case, the yield stress of the alloy should be calculated from this dislocation nucleating stress, which is the yield stress of the matrix metal without a dispersed second-phase. If a continuous three-dimensional dislocation network provides Frank–Read dislocation sources, this stress is given by equation (1).

This model is based on yielding occurring when the area swept out per dislocation loop is large as compared to the dispersed particle spacing. The particles act to hinder dislocation motion. The derivation is similar to that given by Petch[10], where grain boundaries are the blocking structure.

EXPERIMENTAL VERIFICATION AND DISCUSSION

With the model outlined, it should be possible to predict the yield strength of an alloy containing a finely dispersed second-phase. In most of these alloys it is not possible to evaluate yield strength quantitatively from equation (7) for two reasons: (i) the value of the constant of proportionality, C, in equation (6) is very approximate since it depends upon what assumptions are made in its calculation; and (ii) the value of the shear modulus of the dispersed phase may not be known, or if it is known for the phase in bulk form, it may not apply to the fine particles in the dispersion because of differences in structure and composition. The model predicts, however, the variation of the yield strength with the degree of dispersion in these alloys, and under certain circumstances, the variation of yield strength with temperature.

According to equation (7) the yield strength should vary linearly with the reciprocal of the square root of the dispersion spacing. The line should extrapolate to zero for dispersions with an infinite spacing. In order to verify this relation, data are necessary for these alloys on the spacing of the second-phase and their yield strengths. The yield strength predicted by the model is the stress required to produce apparent yielding in the alloy. In single crystals this stress can be identified with the critical resolved shear stress. In polycrystalline materials this stress corresponds most closely to the elastic limit, an experimentally difficult property to determine. In place of the elastic limit, the stress required to produce 0.2 per cent offset may be used. This stress is assumed to be a constant amount greater than the elastic limit for a given series of alloys, and therefore should also vary with the dispersion spacing as predicted by equation (7). In this case, the intercept for alloys with an infinite dispersion spacing is no longer zero, but is some positive stress. Roberts *et al.*[11] determined the lower yield strength of several hypoeutectoid, eutectoid and hypereutectoid steels, some of them with a pearlitic, others with a spheroidized structure. In a few cases the elastic limit was also measured. For this same series of steels, the authors determined the mean ferrite path, which they defined as the mean distance between carbide particles or pearlite patches.

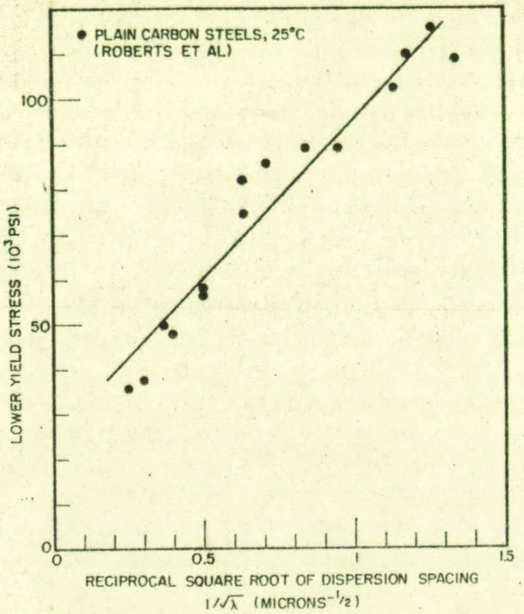

FIG. 1. The lower yield points of several hypoeutectoid, eutectoid and hypereutectoid steels are plotted versus the reciprocal square root of dispersion spacing. The line represents the least-squares fit of the data.

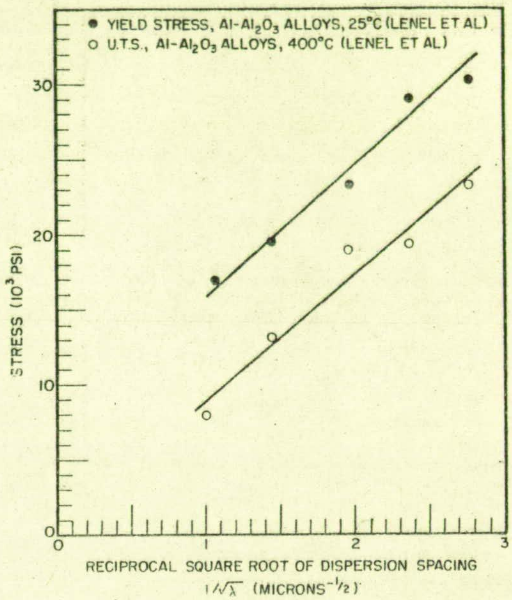

FIG. 2. The room temperature yield strengths and the ultimate tensile strengths at 400°C for several Al–Al_2O_3 SAP-type alloys are plotted versus the reciprocal square root of the dispersion spacing. Each line represents the least-squares fit of the data.

They plotted the lower yield point, and where available the elastic limit, versus the logarithm of the mean free path. This relationship, which was first proposed by Gensamer *et al.*[12] for flow stress is empirical however. The explanation Gensamer and co-workers proposed was based upon an assumed relationship between the rate at which dislocations are generated and the applied stress. This explanation no longer seems adequate in the light of modern dislocation theory. In accordance with the model postulated in this paper, the lower yield points of Roberts *et al.* were re-plotted in Fig. 1 versus the reciprocal of the square root of the spacings. A least-squares line has been drawn assuming a linear relationship. The fit is as satisfactory as that shown in the original plot of Roberts *et al.* Unfortunately, too few values were available for establishing a valid correlation between elastic limit and spacing.

Lenel *et al.*[13] determined, by quantitative electron microscopy, the average spacing between the plate-like oxide particles for a series of flake aluminum powder extrusions. For these same extrusions, Lenel and co-workers[4] determined the room temperature yield strength at 0.2 per cent offset and the ultimate tensile strength at 400°C. At 400°C, the ultimate tensile strength and the tensile yield strength are almost equal.[14] In Fig. 2, the strength values at the two temperatures are again plotted vs. the reciprocal of the average spacing, with the lines representing the least-squares fit of the data. This plot does not exhibit any more scatter than the empirical Gensamer type of plot suggested by Lenel for his data.

The critical resolved shear stress of a series of overaged high-purity aluminum–copper alloys and the spacing between the second-phase particles in these alloys was determined by Dew-Hughes and Robertson[5]. They interpreted the data as supporting Orowan's yield strength theory. A least-squares analysis of the data plotted according to Orowan's theory, however, shows neither predicted linear variation of critical resolved shear stress with the reciprocal of the dispersion spacing, nor a line intercept of zero for an infinite spacing. On the other hand, if their values for critical resolved shear stress are plotted vs. the reciprocal of the square root of the particle spacing as is shown in Fig. 3, a better fit is obtained. The line representing the least-squares fit of the data goes through the origin. This indicates that the proposed model, in which yielding takes place when the second-phase particles shear, would also apply to aluminum alloys containing a dispersion of the theta phase. Dew-Hughes and Robertson consider this possibility, but conclude from an examination of the micrograph of a fractured sample near its fracture surface that the particles do not shear during plastic deformation of the matrix. However, we do not believe that the shear of the particles can actually be detected by this type of examination.

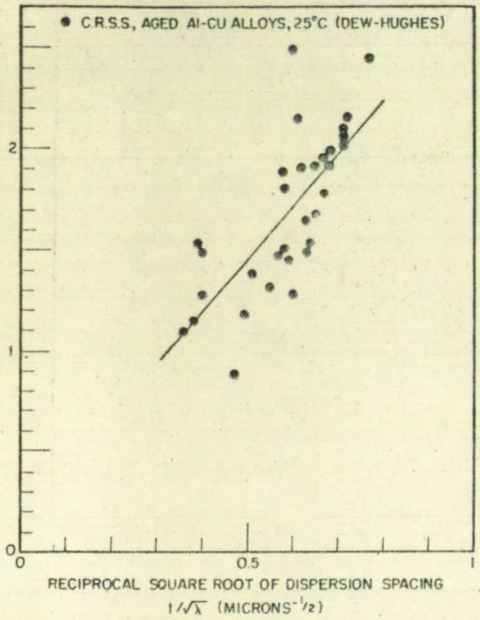

FIG. 3. The critical resolved shear stress of several overaged Al–Cu alloys are plotted versus the reciprocal square root of the dispersion spacing. The line represents the least-squares fit of the data.

Inspection of equation (7) shows that of the terms which determine yield stress, only the shear moduli have an appreciable temperature dependence. Therefore, if the temperature dependence of these shear moduli is known, or a reasonable approximation can be made, the variation of yield strength with temperature should be predictable according to the equation

$$\sigma_T = \sigma_{25}(\mu_T\mu_T^*/\mu_{25}\mu_{25}^*)^{1/2} \tag{8}$$

in which σ is the yield stress, μ and μ^* the shear moduli of the matrix metal and dispersed phase, respectively; the subscripts T and 25, refer to the values of the properties at the test temperature and at 25°C.

This predicted temperature dependence can only be checked by determining the temperature dependence of the elastic limit, not that of the offset yield strength. Although it is reasonable to assume that the off set yield strength for a given series of alloys at any given temperature is a constant amount greater than the elastic limit, this amount is a function of temperature. Therefore, the temperature dependence of the elastic limit cannot be deduced from that of the off set yield strength. Unfortunately, no data on the variation of the elastic limit with temperature for any dispersion strengthened alloy are currently available.

CONCLUSIONS

(1) The model presented appears to explain the yielding behavior of a series of dispersion-strengthened alloys for which no other theory seems satisfactory.

(2) The strengthening effect given by equation (7), due to the dispersed second-phase will be apparent only if it is greater than the yield stress of the matrix metal. This indicates that the fineness of dispersion required for strengthening is a function of the shear moduli of the matrix metal and of the dispersed second-phase and is thus dependent upon the particular alloy system and test temperature.

(3) When treated in terms of this model, coherency effects of the dispersed second-phase particles are only important where the range of additional lattice strain is of the order of the dispersed particle spacing. Under these conditions, the effect of a coherent second-phase would be to change the dispersion spacing.

(4) Particle geometry is important in two ways: For determining the mode of particle shear which causes yielding, and preventing recovery by cross-slip and climb.

(5) Much additional work must be done in order to verify the model, such as quantitatively evaluating from fundamental constants and determining the temperature dependence of the yield strengths for several alloy systems. The difficulties involved in these areas have been mentioned previously, but these should not prove insurmountable.

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