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Comparison of Turbulent Heat-Transfer Results for Uniform Wall Heat Flux and Uniform Wall Temperature

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Introduction

THE PURPOSE of this note is to examine in a more precise way how the Nusselt numbers for turbulent heat transfer in both the fully developed and thermal entrance regions of a circular tube are affected by two different wall boundary conditions. The comparisons are made for: (*a*) Uniform wall temperature (UWT); and (*b*) uniform wall heat flux (UHF). Several papers which have been concerned with the turbulent thermal entrance region problem are given in references [1 to 4].[2] Although these analyses have all utilized an eigenvalue formulation for the thermal entrance region (in contrast to reference [6] which used a boundary layer approach), there were differences in the choices of eddy diffusivity expressions, velocity distributions, and methods for carrying out the numerical solutions. These differences were also found in the fully developed analyses. Hence when making a comparison of the analytical results for uniform wall temperature and uniform wall heat flux, it was not known if differences in the Nusselt numbers could be wholly attributed to the difference in wall boundary conditions, since all the analytical results were not obtained in a consistent way. To have results which could be directly compared, computations were carried out for the uniform wall temperature case, reference [4*c*], using the same eddy diffusivity, velocity distribution, and digital computer program employed for uniform wall heat flux in references [4*a* and *b*]. In addition, the previous work was extended to a lower Reynolds number range so that comparisons could be made over a wide range of both Reynolds and Prandtl numbers.

The analysis of heat transfer in the turbulent thermal entrance and fully developed regions has already been thoroughly treated in references [4] and hence need not be presented here in detail. The calculations were carried out under the assumption of equal eddy diffusivities for heat and momentum. For the region near the tube wall, the diffusivity was evaluated from Deissler's formula [5], while for the region away from the wall, the diffusivity was found by differentiating the logarithmic velocity expression and using a linear variation of shear stress. For convenience, the Appendix gives a few of the final analytical expressions, and Table 2 provides numerical data for the cases which have not been previously tabulated in references [4].

In the next section, we then proceed directly to an examination of the Nusselt number results.

Nusselt Number Comparisons

Pr = 0.7. The results for Pr = 0.7 and four different Reynolds numbers are given in Table 1(*a*) as a function of the length-diameter ratio (x/D) from the tube entrance. The per cent difference is defined as $(Nu_{UHF} - Nu_{UWT})/Nu_{UWT}$. The largest differences are found in the low Reynolds number range, and for each case the differences become smaller at larger distances from the tube entrance. For the ranges of variables considered, the effect of the two different wall boundary conditions is never larger than 10 per cent.

Pr = 10 and Pr = 100. Results for the higher Prandtl numbers are given in Tables 1(*b*) and (*c*). For this range there is essentially no influence of the two different boundary conditions. Since the per cent difference does not become significant even for small x/D, the details in the thermal entrance region are only given as an example for one Reynolds number. Some of the Nusselt number results for uniform wall temperature are very slightly in excess of those for uniform wall heat flux. This is a somewhat surprising finding, and it may be inferred that the differences of a few tenths of a per cent are probably due to accuracy limitations of the calculations.

Conclusions. From these tabulations we may then conclude that for turbulent flow the heat-transfer mechanism in the thermal entrance and fully developed regions is quite insensitive to the two wall boundary conditions which were examined, at least for the range $Pr \geq 0.7$. This conclusion is in qualitative agreement with the findings of reference [6] for Pr = 0.73 (thermal entrance region calculations using a boundary layer model) and of reference [7] for Pr = 1.0 (fully developed region). The effect of other wall boundary conditions can be examined by the superposition techniques described in references [2 and 4*b*].

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APPENDIX

Summary of Analytical Results

Uniform Wall Temperature. The temperature distribution within the fluid is given by

$$\frac{T - T_w}{T_0 - T_w} = \sum_{n=1}^{\infty} C_n \Phi_n(r) e^{-4\beta_n^2 x/DRe}$$

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[2] Numbers in brackets designate References at end of paper.

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Table 1(a); Pr 0.7

x/D	Re = 10,000			Re = 50,000		
	Nu_{UHF}	Nu_{UWT}	% diff.	Nu_{UHF}	Nu_{UWT}	% diff.
2	42.83	39.28	9.0	131.6	125.3	5.1
5	36.90	34.68	6.4	116.7	111.9	4.2
10	34.15	32.44	5.3	108.7	104.8	3.7
20	32.72	31.32	4.5	103.8	100.5	3.3
30	32.42	31.11	4.2	102.4	99.40	3.0
∞	32.32	31.06	4.1	101.8	98.95	2.8

x/D	Re = 100,000			Re = 500,000		
	Nu_{UHF}	Nu_{UWT}	% diff.	Nu_{UHF}	Nu_{UWT}	% diff.
2	220.2	211.6	4.1	757.5	740.2	2.3
5	196.4	190.2	3.3	686.7	672.7	2.1
10	183.6	178.4	2.9	644.9	633.6	1.8
20	175.2	170.8	2.5	616.3	607.1	1.5
30	172.8	168.8	2.3	606.5	598.4	1.4
∞	171.4	167.7	2.2	599.7	592.9	1.1

Table 2(a) Uniform wall temperature

(1) Re = 10,000; Pr = 0.7

n	β_n^2	$(d\Phi_n/dr^+)_{r_o^+}$	C_n
1	44.371	-0.036269	1.2664
2	454.26	.022654	-.49152
3	1137.4	-.018059	.43507
4	2106.2	.016892	-.41660
5	3346.0	-.016687	.41254
6	4864.0	.017637	-.41692
7	6617.8	-.018532	.41912

(2) Re = 50,000; Pr = 0.7

n	β_n^2	$(d\Phi_n/dr^+)r_o^+$	C_n
1	141.36	-0.030622	1.2267
2	1676.2	.017838	-.42513
3	4184.6	-.013381	.39001
4	7679.4	.011209	-.37987
5	12167.0	-.0098511	.37320
6	17667.0	.0090539	-.36768
7	24167.0	-.0085049	.36286

Table 1(b); Pr 10

x/D	Re = 100,000		
	Nu_{UHF}	Nu_{UWT}	% diff.
2	733.6	732.0	0.2
5	711.3	710.4	.1
10	697.7	697.2	.07
20	688.5	688.3	.03
30	685.5	685.4	.01
∞	683.9	683.9	0

Fully developed Nusselt numbers

Re	Nu_{UHF}	Nu_{UWT}	% diff.
50,000	381.0	379.7	0.3
500,000	2722	2730	-.3

Table 1(c); Pr 100

x/D	Re = 100,000		
	Nu_{UHF}	Nu_{UWT}	% diff.
2	1554	1556	-0.2
5	1543	1545	-.2
10	1536	1539	-.2
20	1531	1534	-.2
30	1530	1532	-.2
∞	1529	1532	-.2

Fully developed Nusselt numbers

Re	Nu_{UHF}	Nu_{UWT}	% diff.
50,000	836.6	834.9	0.2
500,000	6352	6386	-.5

Table 2(b) Uniform wall heat flux

Re = 10,000; Pr = 0.7

n	β_n^2	A_n
1	361.38	-0.21646
2	984.83	-.11776
3	1863.8	-.090430
4	2990.7	-.073326
5	4364.4	-.059857
6	6004.1	-.044084
7	7946.8	-.032717

For Re = 10,000; $r_o^+ = 324$

where T_w is the wall temperature, T_0 the entering fluid temperature, C_n are constants, β_n and Φ_n the eigenvalues and eigenfunctions, x the distance from the tube entrance, and r the radial coordinate. The Reynolds number is defined by $\bar{u}D/\nu$, where $\bar{u}$ is the mean fluid velocity.

The Nusselt number is given by

$$\mathrm{Nu_{UWT}} = \mathrm{Pr}\,\frac{\displaystyle\sum_{n=1}^{\infty} C_n \left(\frac{d\Phi_n}{dr}\right)_{r_0} e^{-4\beta_n^2 x/DRe}}{\displaystyle\sum_{n=1}^{\infty} \frac{C_n}{\beta_n^2} \left(\frac{d\Phi}{dr}\right)_{r_0} e^{-4\beta_n^2 x/DRe}}$$

Uniform wall heat flux. The fluid temperature distribution is given by

$$\frac{T - T_0}{q\dfrac{r_0}{k}} = \frac{2}{\mathrm{RePr}}\,\frac{x}{D} + G(r) + \sum_{n=1}^{\infty} C_n\Phi_n(r)e^{-4\beta_n^2 x/DRe}$$

where q is the wall heat flux per unit area, r_0 the tube radius, and $G(r)$ is the fully developed temperature distribution. The Nusselt number is given by

$$\mathrm{Nu_{UHF}} = \frac{2}{G(r_0)}\,\frac{1}{1 + \displaystyle\sum_{n=1}^{\infty} A_n e^{-4\beta_n^2 x/DRe}}$$

where $A_n = C_n\Phi_n(r_0)/G(r_0)$. The Φ_n, C_n, and β_n for uniform heat flux are different from those for uniform wall temperature.

For a detailed description of the analytical method, the reader is referred to references [4].

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