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Discussion of Paper by W. A. Heiskanen, 'The Latest Achievements of Physical Geodesy'

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In his recent report on physical geodesy, *Heiskanen* [1960] replied to a criticism by *O'Keefe* [1959]. He wrote, 'O'Keefe has assumed that the flattening value of 1:299.8 corresponds to the hydrostatic equilibrium, and he uses this flattening value when he computes the spherical harmonics of the gravity anomalies and the undulations N of the geoid. Because the hydrostatic equilibrium is *not* complete, one cannot use this value, and Figure 2, page 2931, of *O'Keefe's* [1959] publication will not explain anything.'

In view of objections raised by Professor Heiskanen, it appears necessary to state more fully the reasoning leading to the adoption of the value 1/299.8 for the flattening of the reference ellipsoid. The underlying idea is to use as reference the figure that the earth would assume if it were in a state of fluid equilibrium. The advantage of this figure is that the stresses in the interior of the earth can be expected to be proportional to the amplitude of harmonics in the gravitational field that measure deviations from this ellipsoid. For this reason, namely, the interest to the geophysicist concerned with the structure of the earth, we have stressed the importance of the figure of fluid equilibrium.

There is, in addition, significance for geodesy in the disclosure that the geoid differs systematically and extensively from the figure of fluid equilibrium. These differences contradict the principle formulated by *Heiskanen and Vening Meinesz* [1958] as the basic hypothesis of geodesy, namely, that deviations from the ellipsoid of fluid equilibrium would be found to be small. The actual deviations have been found to be between 5 and 30 times those anticipated. For this reason a firm adherence to the ellipsoid of fluid equilibrium as the reference for calculations of anomalies is useful to the geodesist by keeping before his eyes the real size of the deviations which may exist in those parts of the world that he cannot measure.

The value 1/299.8 is based on a slight modification of the work of *Henriksen* [1960]. Henriksen's argument is summarized by the block diagram, Figure 1 [*O'Keefe, Roman, Yaplee, and Eckels*, 1959], which presents the relations among the physical constants related to the flattening of the earth. From the precession of the equinoxes, the value $H = 3.274 \times 10^{-3}$ is derived for the quantity $(C - A)/C$. On the other hand, from observations of satellites the value $J_2 = 1.0825 \times 10^{-3}$ is found for the quantity $(C - A)/Ma^2$, whence $C/Ma^2 = 3.306$ [cf. *Cook*, 1959]. The point of Henriksen's paper is that the hydrostatic value of the flattening appropriate to a given configuration of the earth can be directly calculated from this quantity C/Ma^2. That this is so is directly evident from the first-order equations of *Jeffreys* [1952], especially equation 47, from which we can determine the quantity η, and equation 50, which relates η to the flattening. These equations constitute the heart of the problem. However, a slightly more refined treatment can be made either by the equation of *de Sitter* [1924] which Henriksen used or by the equations given by *Jones* [1954]. The numerical result is 1/299.8.

By way of comparison, the method used before the advent of the satellites for this purpose

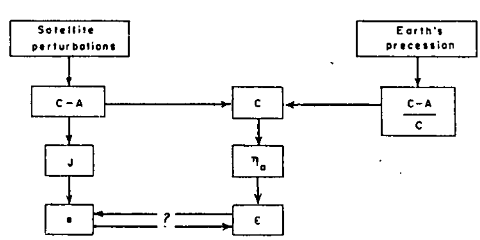

Fig. 1. Relationship or quantities involved in the flattening of the earth.

was somewhat roundabout. Lacking an accurate determination of $(C - A)/Ma^2$, the assumption was made that this quantity had precisely the value that it would have in the case of fluid equilibrium. This equation, together with the condition arising from the moon's precession, was sufficient to determine the value of the flattening; and in this way the old hydrostatic value of 1/297.3 was found [*Jeffreys*, 1952]. Since we now know that the earth is not in hydrostatic equilibrium, it is clearly no longer legitimate to calculate in this way.

It may be asked whether we can assume that the value of C which we obtained from the observations is identical with the value which the earth would have if it should go to hydrostatic equilibrium. The answer is that this is in all probability true to four significant figures. A change of C or A by one part in 100,000 is enough to change $(C - A)/Ma^2$ in the third significant figure. Thus the earth can change its flattening to agree with the requirements of hydrostatic equilibrium without seriously altering its moment of inertia. It is these considerations that justify the flattening of 1/299.8. It will be seen that they take into account fully the fact that hydrostatic equilibrium is not complete.

Heiskanen further asserts that he has 'not claimed that complete hydrostatic equilibrium prevails.' On this point there is considerable agreement. It seems to me, however, that the new numerical data mean that in estimating the contributions of the unexplored portions of the earth's gravitational field one should assume nonhydrostatic contributions in accordance with those found from satellites, i.e., approximately 10 times the contributions assumed by *Heiskanen and Vening Meinesz* [1958] in their formulation of the basic hypothesis of geodesy.

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