Optical alignment of the Global Precipitation Measurement (GPM) star trackers

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ABSTRACT

The optical alignment of the star trackers on the Global Precipitation Measurement (GPM) core spacecraft at NASA Goddard Space Flight Center (GSFC) was challenging due to the layout and structural design of the GPM Lower Bus Structure (LBS) in which the star trackers are mounted as well as the presence of the star tracker shades that blocked line-of-sight to the primary star tracker optical references. The initial solution was to negotiate minor changes in the original LBS design to allow for the installation of a removable item of ground support equipment (GSE) that could be installed whenever measurements of the star tracker optical references were needed. However, this GSE could only be used to measure secondary optical reference cube faces not used by the star tracker vendor to obtain the relationship information and matrix transformations necessary to determine star tracker alignment. Unfortunately, due to unexpectedly large orthogonality errors between the measured secondary adjacent cube faces and the lack of cube calibration data, we required a method that could be used to measure the same reference cube faces as originally measured by the vendor. We describe an alternative technique to theodolite autocollimation for measurement of an optical reference mirror pointing direction when normal incidence measurements are not possible. This technique was used to successfully align the GPM star trackers and has been used on a number of other NASA flight projects. We also discuss alignment theory as well as a GSFC-developed theodolite data analysis package used to analyze angular metrology data.

Keywords: optical alignment, theodolite autocollimation, metrology, GPM, optical metrology

1. INTRODUCTION

Theodolite autocollimation metrology continues to be an important part of spacecraft optical alignment. Spacecraft optical alignment is an art as well as a science for using optical instruments to place or determine the orientation and envelope of critical components on space flight hardware. It commonly employs the use of theodolites, alignment telescopes, autocollimators, tilting levels and optical transits, but may also include the use of coordinate measurement systems such as the laser tracker, laser radar, and coordinate measurement machine (CMM). The subject components are typically scientific instruments and attitude control system components such as star trackers, gyroscopes, sun and earth sensors, thrusters, or any feature that can be characterized with a unique pointing direction in space.

The end-to-end optical alignment of NASA's Global Precipitation Measurement (GPM) Spacecraft, which is scheduled to launch in early 2014, is an example of how spacecraft optical alignment is currently practiced by the Alignment, Integration and Test (AI&T) Group of the Optics Branch at NASA's Goddard Space Flight Center (GSFC). This paper specifically addresses the theory of theodolite autocollimation data analysis from measurements on 0.75 and 1.0 inch optical reference cubes (although other size reflectors are often used) as a preface to understanding an innovative application used on GPM and other NASA missions.

2. THEODOLITE AUTOCOLLIMATION MEASUREMENTS

A theodolite is a small, movable telescope that is mounted within two perpendicular axes of rotation, one vertical and one horizontal. The circles of rotation are precisely calibrated to mark the angle of rotation about each axis, thus providing the angular orientation of the telescope. The horizontal (azimuth) and vertical (elevation) circles of a theodolite are graduated from 0 to 360°. For theodolites used by the GSFC AI&T Group, the zero reading of the elevation (vertical circle) indicates the direction exactly opposite to that of gravity, while an elevation reading of 90° or 270° is the direction perpendicular to gravity. However, other theodolite conventions are possible. Furthermore, theodolites may be first order, having a resolution of 0.1 arc sec, or second order, having a resolution of 1 arc sec. Spacecraft optical alignment at the GSFC makes use of first order theodolites, exclusively.

Theodolite autocollimation measurements are used to determine the relative alignment between various components on a test object with respect to a common coordinate system. Generally, the optical axis of each component has been previously related to an external optical reference surface, such as a mirror or an optical reference cube, mounted rigidly to the component. The theodolite autocollimation measurement of any optical reference mirror or cube face requires that the theodolite be properly positioned at a vertical and horizontal position that allows the theodolite to be pointed, using its angular adjustments, along a line that intersects the cube face normal to its surface. Autocollimation occurs when collimated light emanating from the theodolite is returned along the same path after its reflection from the reflective surface (mirror or cube face). A level of skill is required by theodolite operators to gain line-of-sight and to autocollimate on various reflective surfaces at various heights and angles that may be required to measure all required cube faces of components on a given test object. For a large spacecraft like GPM, a theodolite system consisting of three to four theodolites and two to three skilled operators are typically employed. Each theodolite in the system must be critically leveled with respect to gravity before a measurement can be made and every measurement must be referenced to the "primary" theodolite, which acts as the facility or laboratory azimuth reference for all measurements in the system. The theodolite from which light is actually autocollimated on a cube face is called the "subject" theodolite for that measurement. Often, a subject theodolite cannot be referenced directly to the primary theodolite. The go-between is another theodolite called a "secondary" theodolite. The details of the data analysis that lead to the cube face pointing directions will be discussed later.

3. THEORY OF MEASUREMENT

3.1 Angular conventions

The initial goal of a theodolite measurement is to obtain the roll and zenith of the reflective surface in the coordinate system of the primary theodolite. The angular conventions used in data analysis are defined below and shown in the schematic of Figure 1. The +X axis is defined to be the direction anti-parallel to gravity and the azimuth circle of rotation for a theodolite is about the direction parallel to the +X axis. The word "counterclockwise" used in the definitions below and elsewhere in this document assumes a right handed coordinate system. To visualize a counterclockwise angle or rotation, one places the thumb of the right hand in the positive axial direction. The fingers can then curl about the direction of the thumb in the counterclockwise sense.

Zenith: The direct angle that the vector makes with the +X-axis and has a value between 0° and 180°.

Roll: The counterclockwise angle about the +X-axis made by the vector's projection in the YZ plane and measured with respect to the +Z-axis.

Pitch: The counterclockwise angle about the +Y-axis made by the vector's projection in the ZX plane and measured with respect to the +X axis.

Yaw: The counterclockwise angle about the +Z-axis made by the vector's projection in the XY plane and measured with respect to the +Y axis.

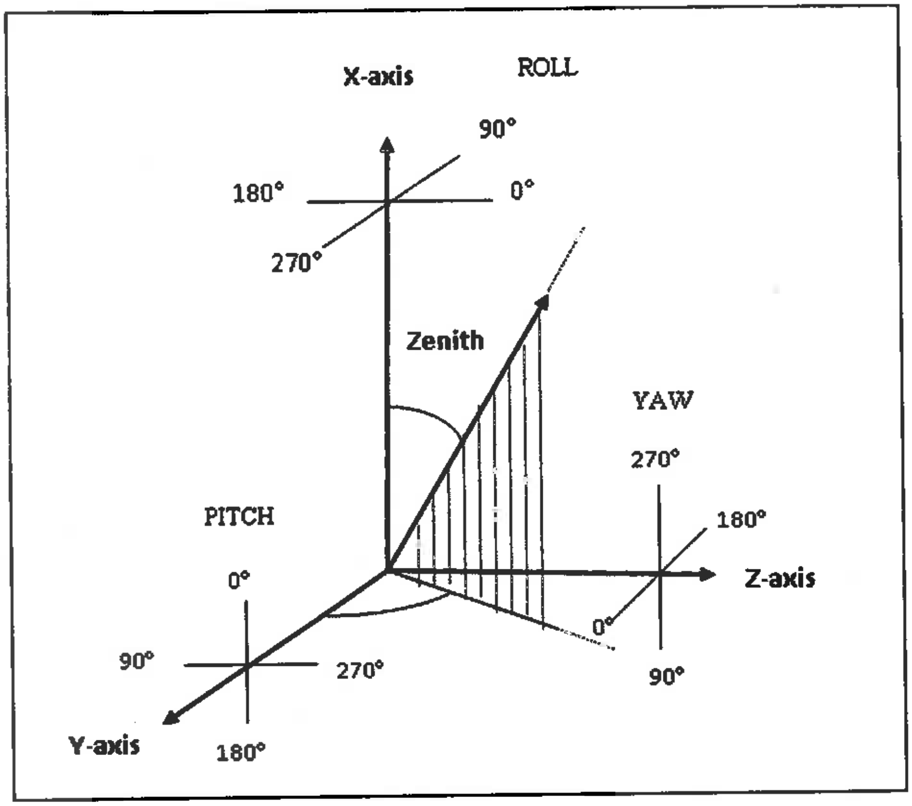

Figure 1. Optical Alignment Data Analysis Conventions

3.2 Elevation reference

To obtain meaningful data, the vertical and horizontal circles of a theodolite require references. Gravity is the natural, absolute reference for the vertical circle, but there is no such corresponding natural reference for the horizontal circle. In practice, the zenith reference for a theodolite only requires that the theodolite be leveled to gravity within a few arc seconds. When this leveling is accomplished, the calculation of the zenith for a given elevation measurement is straight forward and given below.

$$\text{Zenith} = 180° - \text{Elevation} \tag{1}$$

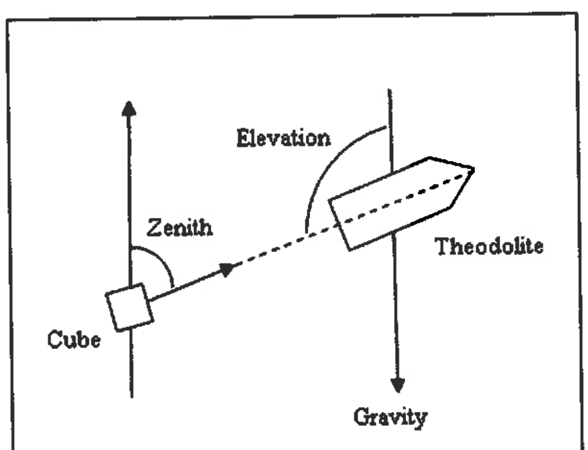

Figure 2. Zenith relationship to elevation

3.3 Azimuth references (roll references)

Though no natural reference for the theodolite horizontal circle (azimuth) exists, a number of solutions have been used at GSFC, such as a fixed reference mirror, a leveled dihedral in conjunction with a leveled rotary table, or another theodolite. Figure 3 shows the simplicity of using a reference mirror as a fixed azimuth reference. Accordingly, the measurements of Cube Face A and Cube Face B have the same pointing **reference**, the reference mirror. Therefore:

$$\alpha = \text{Subject Azimuth A} - \text{Azimuth Reference A}$$

$$\beta = \text{Subject Azimuth B} - \text{Azimuth Reference B}$$

$$\text{roll } \textbf{difference} \text{ (between Cube Face A and Cube Face B)} = \alpha - \beta \tag{2}$$

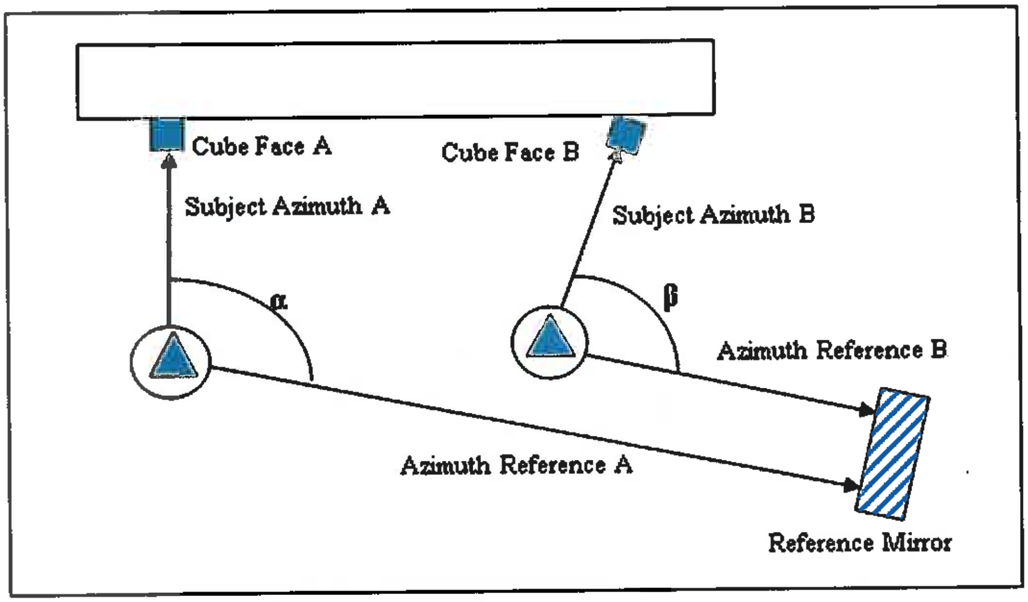

Figure 3. Reference mirror used as a fixed azimuth (roll) reference

However, the use of a theodolite as fixed azimuth reference is the most convenient and least limiting choice. This method was used for all measurements on GPM and is the method assumed in discussions that follow. For most measurements there are generally three geometries of concern, leading to three types of roll calculations. The most basic is when the primary theodolite is also the subject theodolite. The relationship between the roll and the azimuth of the primary theodolite (Prim Az) is given by the equation:

$$\text{Roll} = 360° - \text{Prim Az} \tag{3}$$

By definition (see section 3.1), the roll is a counterclockwise measure, while the azimuth of a theodolite is a clockwise measure.

Primary theodolite azimuth reference (subject to primary)

Figure 4 is a schematic of a theodolite system employing two theodolites, a primary and a subject, to measure two adjacent faces of cubes **A**, **B** and **C**. The primary theodolite acts as the fixed azimuth reference for all theodolite measurements in the theodolite system. The cube face viewed by the primary theodolite provides it with its own fixed azimuth reference. The subject theodolite can be moved to obtain measurements about the spacecraft, and for this case, would be repositioned to measure a second face for cubes **A** and **B** and two faces of cube **C**. Each time the theodolite is moved in space, it must again be referenced to gravity (re-leveled) and to the primary in order to obtain a meaningful measurement. Referencing the primary, referred to as "bucking the theodolites," means that the primary and subject theodolites have been aligned to each other such that each instrument views the opposing instrument's collimated light (in the shape of reticles) aligned with its fixed reticles. As designated in the geometry of Figure 4, the reading of the subject theodolite is called the **Subject Azimuth Reference** (Sub Az Ref) while the reading of the primary theodolite is called the **Primary Azimuth Reference** (Prim Az Ref).

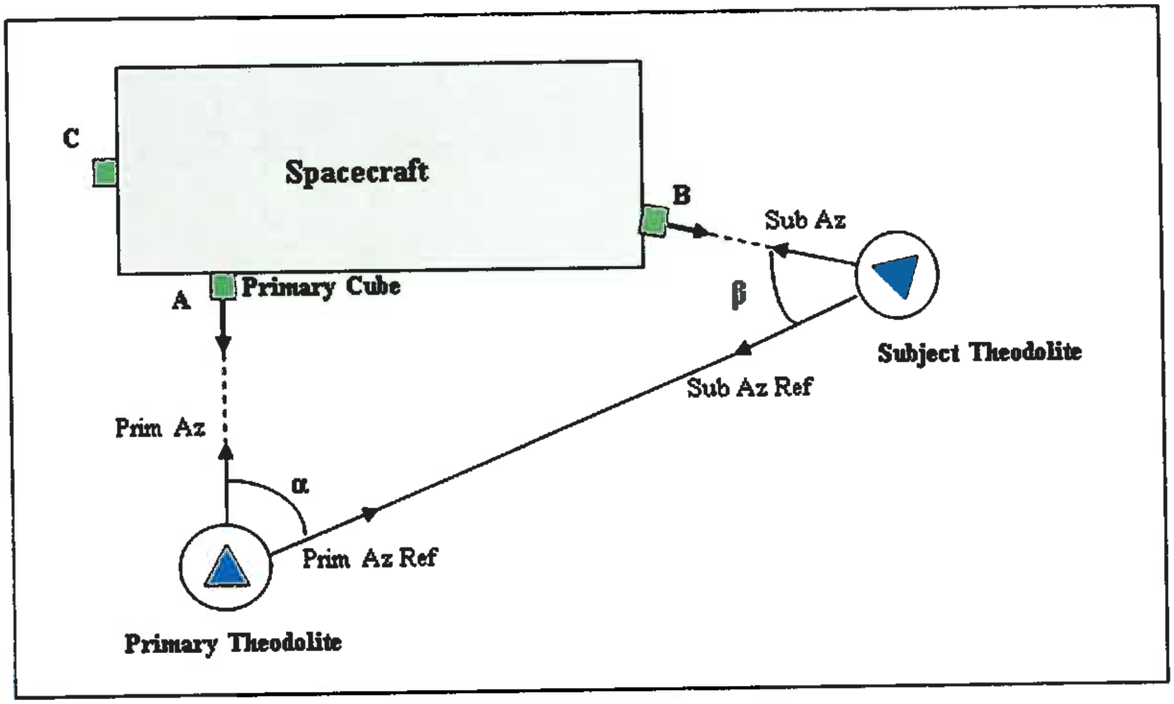

Figure 4: Primary and subject theodolite setup

Calculation of the roll (subject to primary)

Bucking the primary and subject theodolites provides the information necessary to calculate the Subject Azimuth (Sub Az) in the primary theodolite circle (see Figure 5). To calculate the subject azimuth theodolite reading in the primary theodolite azimuth circle, calculate the angular difference (Sub Az – Sub Az Ref) and then add the result to the direction that the Primary would read if it pointed in the direction of the subject theodolite when the theodolites were bucked (Prim Az Ref +180°). Therefore, the equation of the roll becomes:

$$\text{Roll} = 360° - ((\text{Sub Az} - \text{Sub Az Ref}) + \text{Prim Az Ref} + 180°) \tag{4}$$

Note that in the determination of the angular difference (Sub Az – Sub Az Ref), care must be taken when the theodolite is rotated through its zero mark.

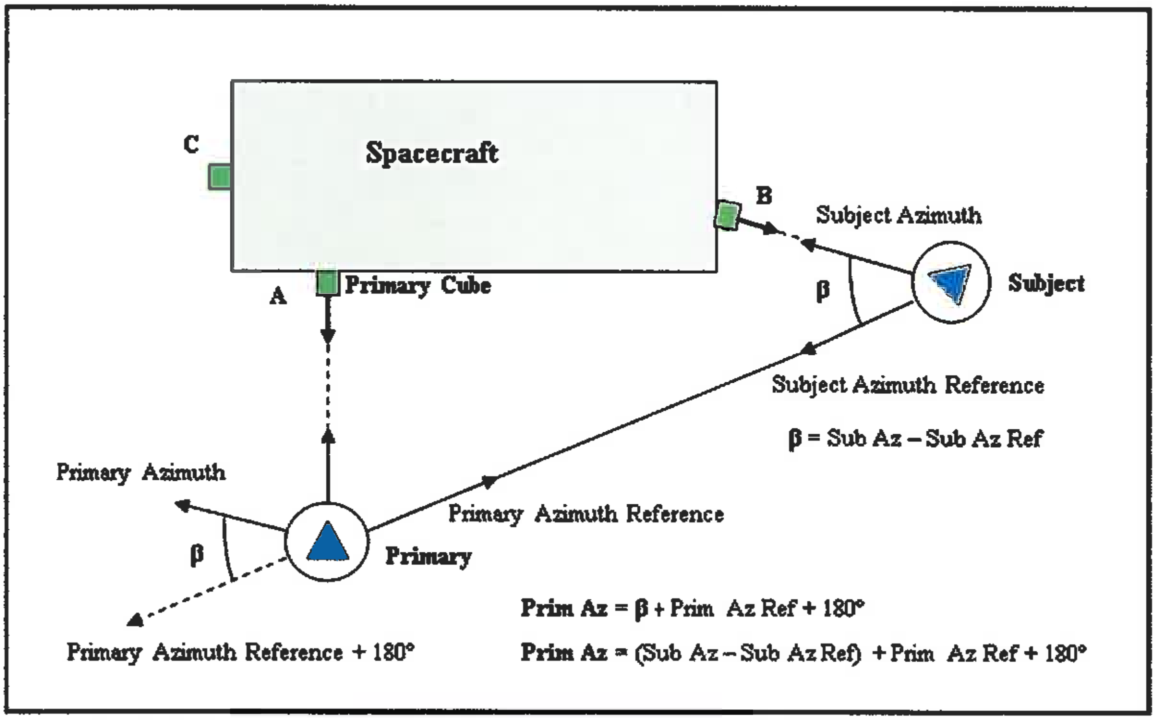

Figure 5. Calculation of subject theodolite azimuth in primary theodolite coordinates

Calculation of the roll with use of a relay theodolite (Subject to Secondary to Primary)

The last geometry of concern here is the case when the subject theodolite cannot or it is not practical to be referenced directly to the primary theodolite. In this case another theodolite can serve as an intermediate primary. Figure 6 shows a basic arrangement for this case.

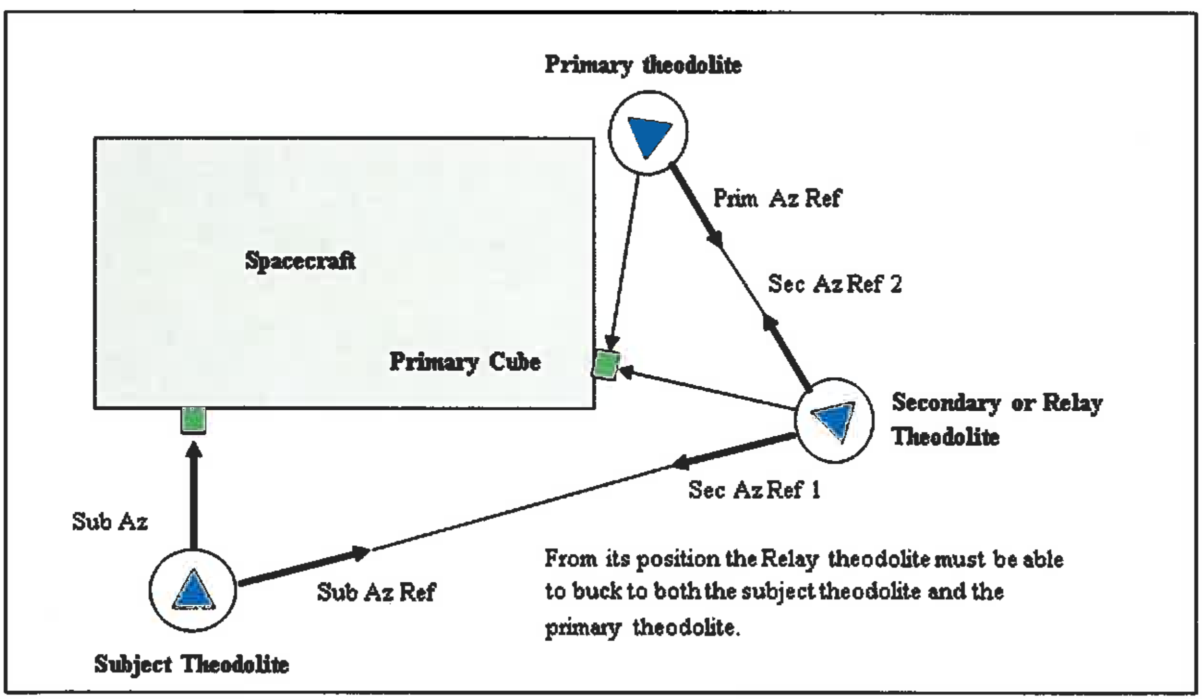

Figure 6: Typical setup for use of a relay theodolite

The secondary theodolite used is also termed a "relay" theodolite because it relays the reading of the subject theodolite to the primary theodolite. Thus, this relay theodolite must be positioned so that it can buck to both the subject and primary theodolite. The vector analysis of this arrangement is given in Figure 7. It shows how the measurement can be broken into two steps in which the "subject to primary" analysis, previously discussed, is applied to each step. First, the subject theodolite reading is transferred to the relay theodolite as if it were the primary theodolite. Second, the relay theodolite reading corresponding to the reading of the subject theodolite is transferred to the primary theodolite. The process is captured by the equations below:

$$\textbf{Sec Az} = (\text{Sub Az} - \text{Sub Az Ref}) + \text{Sec Az Ref } 1 + 180^\circ$$

$$\textbf{Prim Az} = (\text{Sec Az} - \text{Sec Az Ref } 2) + \text{Prim Az} + 180^\circ \qquad (5)$$

$$\textbf{Roll} = 360^\circ - \text{Prim Az}$$

Combining the results into the equation for roll leads to the equation below:

$$\textbf{Roll} = 360^\circ\text{-}\{([(\text{Sub Az-Sub Az Ref})+(\text{Sec Az Ref } 1 + 180^\circ)]\text{-Sec Az Ref } 2)+(\text{Prim Az Ref}+180^\circ)\} \qquad (6)$$

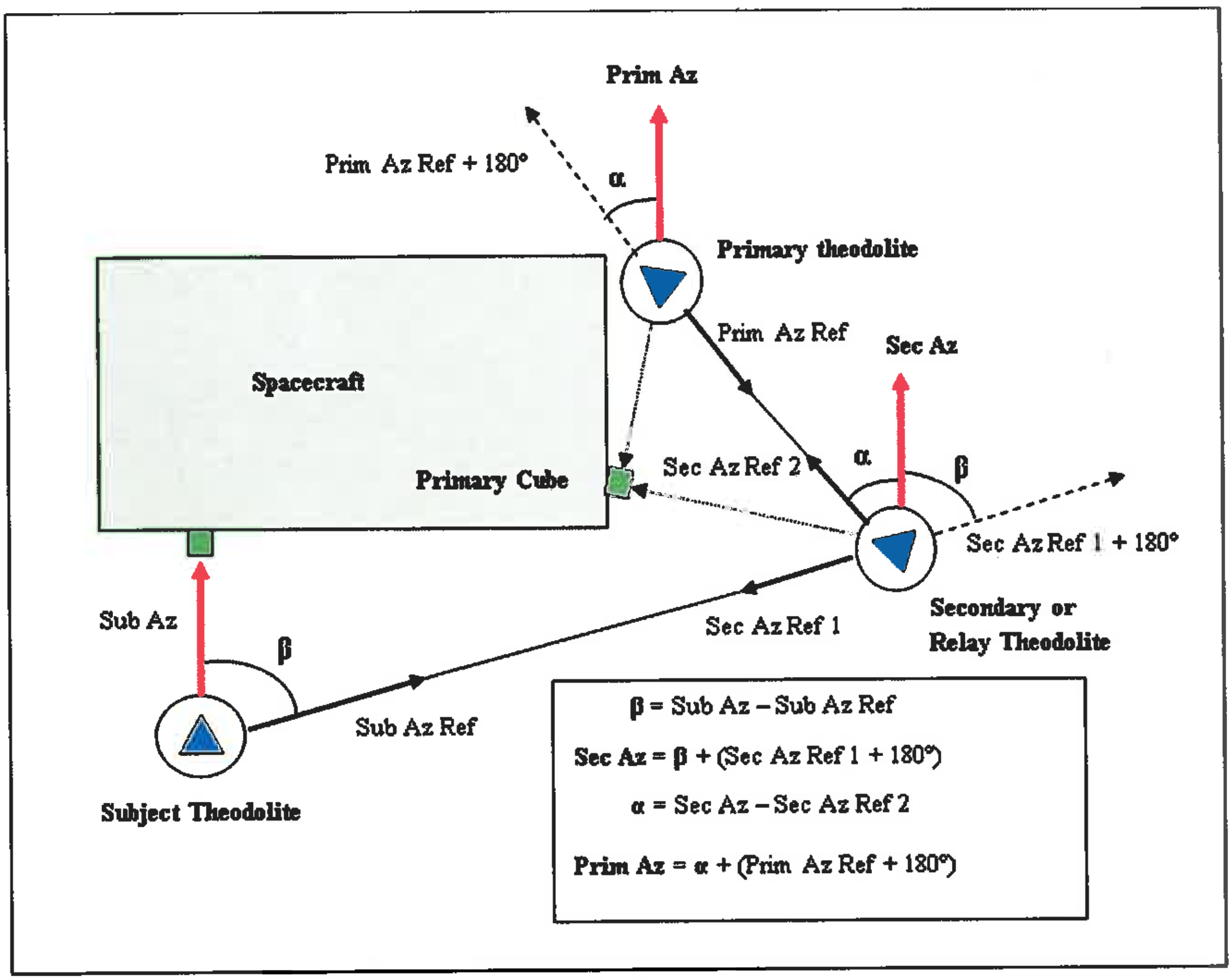

Figure 7: Vector analysis of relay theodolite geometry

4. DATA ANALYSIS

4.1 Direction cosines

It is convenient to analyze angular data obtained from theodolite autocollimation measurements using vector analysis. The pointing direction of any reflective surface can be represented as a unit vector with components that are direction cosines. Direction cosines are the projections of a unit vector along each coordinate axis. That is, for any unit vector **M**: **M** $= (M_X, M_Y, M_Z) = (1 \cdot \cos\alpha,\ 1 \cdot \cos\beta,\ 1 \cdot \cos\gamma)$.

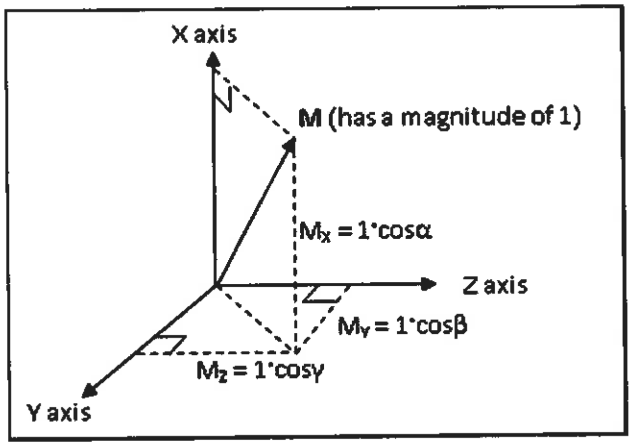

Figure 8 shows that for any vector **M**:

$M_X = M \cdot \cos(\text{zenith})$, $M_Y = M \cdot \sin(\text{zenith}) \cdot \sin(2\pi - \text{roll})$, and $M_Z = M \cdot \sin(\text{zenith}) \cdot \cos(2\pi - \text{roll})$.

$$\cos\alpha = M_X/M = \cos(\text{zenith});\ \cos\beta = M_Y/M = -\sin(\text{zenith}) \cdot \sin(\text{roll});\ \cos\gamma = M_Z/M = \sin(\text{zenith}) \cdot \cos(\text{roll}) \qquad (7)$$

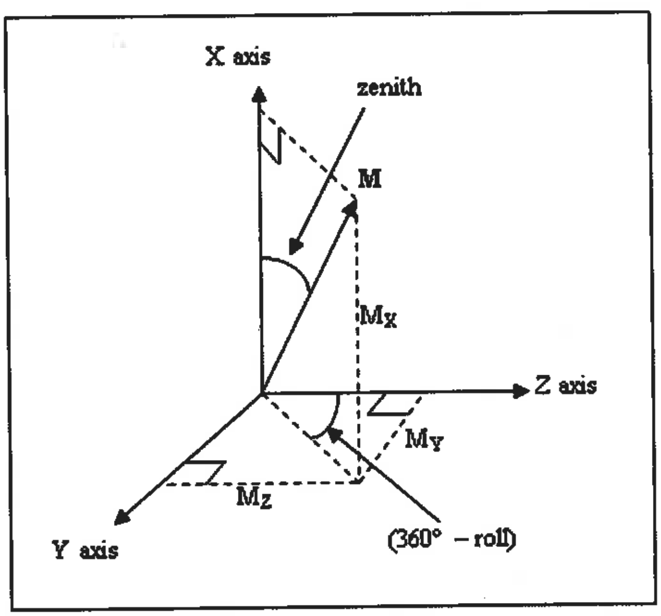

Figure 8. Relationship between the components of a vector and its roll and zenith

Any normalized vector can be represented as a set of direction cosines $(\cos\alpha, \cos\beta, \cos\gamma)$ and the vector direction of any reflective surface can be calculated from the roll and zenith of its normal.

4.2 Angular Projections in the Plane

If the direction cosines M_X, M_Y, and M_Z are known, the angular projections in the planes, roll (YZ plane), pitch (ZX plane), and yaw (XY plane), can be calculated using the definitions previously defined in Figure 1. For example, the pitch is defined as the counterclockwise rotation about the +Y axis relative to the +X axis (see Figure 9 below).

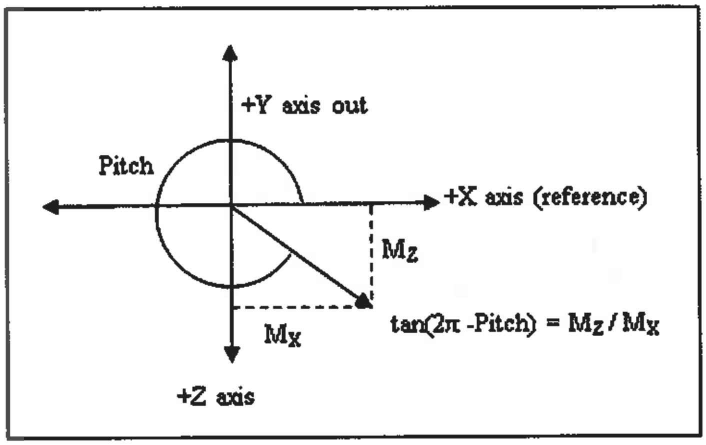

Figure 9. Calculation of the pitch using its definition from section 3.1

The geometry shows that $\tan(2\pi - \text{Pitch}) = M_Z/M_X$, therefore $\text{Pitch} = -\tan^{-1}(M_Z/M_X)$. In similar fashion, equations of the Yaw and Roll can also be found as given below:

$$\text{Pitch} = -\tan^{-1}(M_Z/M_X)$$
$$\text{Yaw} = -\tan^{-1}(M_X/M_Y) \qquad (8)$$
$$\text{Roll} = -\tan^{-1}(M_Y/M_Z)$$

From Figure 8, the zenith can be calculated as follows:

$$M_X = \cos(\text{zenith}); \text{ therefore zenith} = \cos^{-1}(M_X) \qquad (9)$$

$$\text{Also, } \tan(\text{zenith}) = (M_Y^2 + M_Z^2)^{1/2}/\, M_X; \text{ therefore zenith} = \tan^{-1}((M_Y^2 + M_Z^2)^{1/2}/\, M_X) \qquad (10)$$

4.3 Roll and zenith from pitch and yaw

The Roll and zenith can also be calculated if the Pitch and Yaw are known. Following Redman[1], recall that

$\text{Roll} = -\tan^{-1}(M_Y/M_Z)$. Therefore:

$$\text{Roll} = -\tan^{-1}((M_Y/M_Z)\cdot(M_X/M_X))$$
$$= -\tan^{-1}((M_Y/M_X)\cdot(M_X/M_Z))$$
$$= -\tan^{-1}((M_Y/M_X)\cdot(M_X/M_Z))$$
$$= -\tan^{-1}[(1/\tan(-\text{Yaw}))\cdot(1/\tan(-\text{Pitch}))]$$
$$= -\tan^{-1}[1/(\tan(-\text{Yaw})\cdot\tan(-\text{Pitch}))] \qquad (11)$$

$$\text{Also, zenith} = -\tan^{-1}((M_Y^2 + M_Z^2)^{1/2}/M_X)$$
$$= -\tan^{-1}[(M_Y^2 + M_Z^2)/M_X^2]^{1/2}$$

$$= - \tan^{-1} [1/\tan^2(-\text{Yaw}) + \tan^2(-\text{Pitch})]^{1/2} \quad (12)$$

4.4 Coordinate transformations

The goal of an optical alignment set of measurements includes placement (the physical positioning of a component in angle and/or coordinates to a specified orientation), knowledge (determination of the position and/or orientation of a component), or stability (maintenance of a predetermined and bounded position and/or orientation of a component). Any one of these goals is accomplished more easily if the vectors and angles are represented in a coordinate system tied to the component, such as an alignment cube mounted on the component. Therefore, it is desirable to transform the raw data of a theodolite measurement system, initially represented in the coordinate system of the primary theodolite, to a coordinate system tied to the component, such as an alignment cube.

A coordinate system is described by a set of three basis vectors. Once the basis vectors of one coordinate system, O′, are found in another coordinate system O, transformations between the two systems are possible. The components of the "primed" coordinate axes of O′ expressed in terms of direction cosines in the O coordinate system form the rows of the rotation matrix that can transform any vector expressed in the O coordinate system into a corresponding vector in the O′ coordinate system. The transformation described above requires that the axes of O′ be mutually orthogonal[2], therefore, the rows of the transformation matrix must represent vectors that are mutually orthogonal.

In general, the faces of an alignment cube will not be orthogonal. The following procedure[3] can be used to construct the set of mutually orthogonal axes that form O′. Using the normal vectors of any two non-parallel surfaces calculated in system O, choose one to be the "primary" axis vector and one to be the "secondary" axis vector. The primary will be one of the axes (x′, y′ or z′) in the new system O′, while, in general, the secondary will only determine the plane of one of the other axis directions. Construct the axes of O′ as follows:

1. Let the primary and secondary axes be represented by unit vectors **p** and **s** in O. If we takes **p** to be the x axis of the new coordinate system O′ (for convenience of explanation), and let **s** lie in XY plane (**s** may point in a Y-like direction for the convenience of this explanation).
2. Find the cross product of the primary and secondary to calculate the tertiary vector—another axis direction in O′. In this case, $\mathbf{z}' = \alpha(\mathbf{p} \times \mathbf{s})$, is called the tertiary vector, in which "α" is the constant that makes **z′** a unit vector.
3. Take the cross product of the tertiary and the primary to obtain the third axis vector, $\mathbf{y}' = \mathbf{z}' \times \mathbf{p}$

In summary, the rows of the transformation matrix are:

$$\begin{aligned} \mathbf{x}' &= \mathbf{p} \\ \mathbf{y}' &= \mathbf{z}' \times \mathbf{p} \\ \mathbf{z}' &= \alpha\,(\mathbf{p} \times \mathbf{s}) \text{ where } \alpha = 1/|\mathbf{p} \times \mathbf{s}| \end{aligned} \quad (13)$$

Thus, the basis vectors of the new coordinate system O′ are determined in the O coordinate system, and the resulting transformation matrix can be used to transform any vector in O to a corresponding vector in O′. It is important to note that the matrix inverse of the resulting transformation matrix can be used to transform vectors from the O′ coordinate system into vectors in the O system.

4.5 Theodolite measurements using a relay mirror

The use of a relay mirror may be indicated when the line-of-sight to a required cube face or reflective surface is blocked by structure, another component, etc. (see Figure 10 below). The relay mirror rotates the apparent direction of the subject mirror, as shown in Figure 11. The required measurements for its use are the pointing direction of the relay mirror itself and the direction of the resulting rotated vector (called the "through shot").

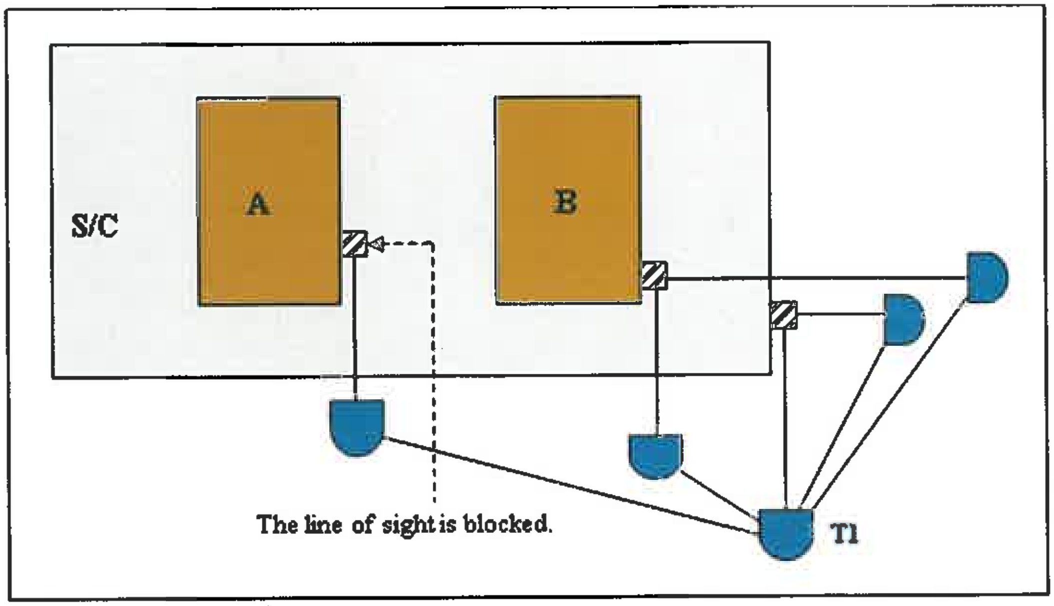

Figure 10. Example scenario of a blocked line-of-sight

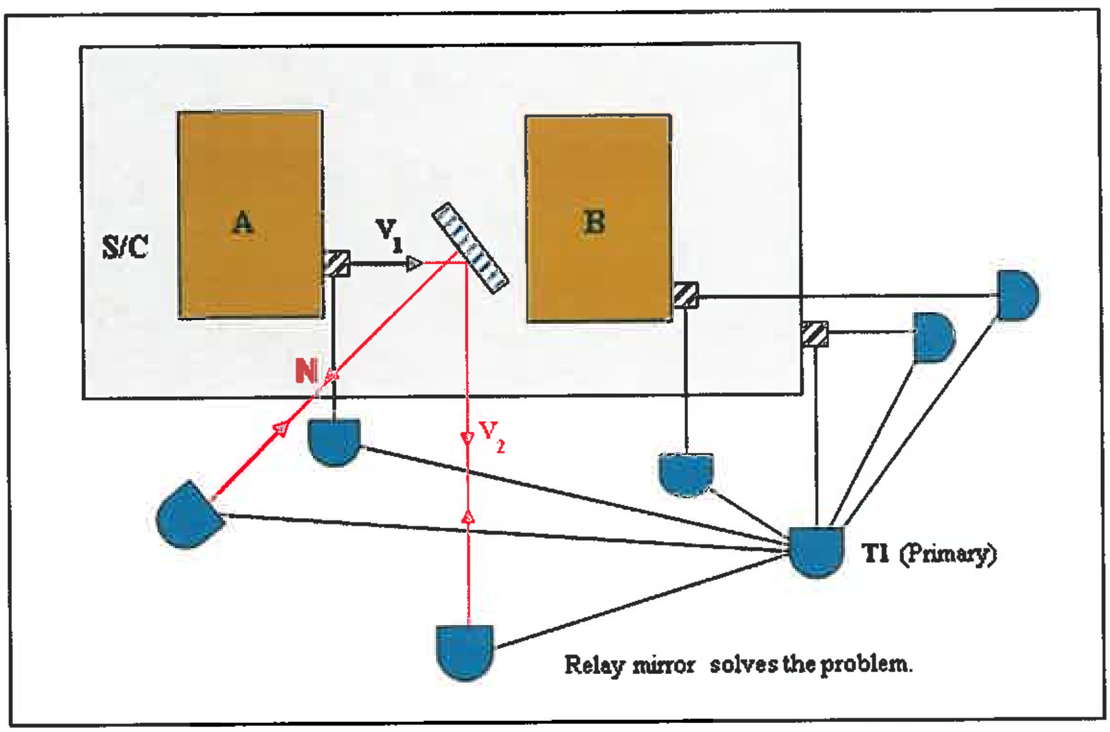

Figure 11. Use of a relay mirror in the scenario of a blocked line-of-sight

Measurements of the through shot $\mathbf{V}_2$ and the normal of the relay mirror **N,** where **N** = (L, M, N) are used to obtain the vector of interest $\mathbf{V}_1$ through application of Smith's Rotation Matrix[4]:

$$\begin{pmatrix} V_{1x} \\ V_{1y} \\ V_{1z} \end{pmatrix} = \begin{bmatrix} 1-2L^2 & -2LM & -2LN \\ -2ML & 1-2M^2 & -2MN \\ -2NL & -2NM & 1-2N^2 \end{bmatrix} \begin{pmatrix} V_{2x} \\ V_{2y} \\ V_{2z} \end{pmatrix} \tag{14}$$

The effect of this matrix multiplication is to correct the measured direction cosines of $\mathbf{V}_2$ into the direction cosines of $\mathbf{V}_1$ by transforming or rotating $\mathbf{V}_2$ into $\mathbf{V}_1$.

5. THE ALIGNMENT SEQUENCE

The spacecraft optical alignment sequence generally begins with the establishment of a mechanical reference frame (MRF) and its measurement with respect to an optical reference cube, the master reference cube (MRC). A common practice at GSFC is to have reference points or surfaces that can be easily measured designed into the structure so that spacecraft coordinate axes can be located with respect to the MRC. The coordinate system constructed using two adjacent faces of the MRC is termed the MRC Frame or MRCF. If the coordinate axes of the MRF and two adjacent faces of the MRC are measured with respect to a common reference frame, usually the primary theodolite frame, then transformation matrices between the MRCF and MRF can be calculated. Thus, any pointing direction that is known in the MRCF can be transformed into the MRF.

The basic sequence for aligning a component in the MRF if its optic or science axis direction is known with respect to its alignment cube (that is, direction cosines of the science axis are known with respect to a cube mounted to its structure) is as follows:

1. Measure the pointing directions of two adjacent faces of the component's optical reference cube and two adjacent faces of the MRC using theodolites.
2. Calculate measured directions in the primary theodolite frame.
3. Transform all measured directions in the primary theodolite frame to the MRCF.
4. Calculate the matrix transformation between the component's cube frame and the MRCF.
5. Transform the component's science axis direction to a vector in the MRCF.
6. Use the matrix transformation between the MRCF and MRF to transform the component's science axis direction to a vector in the MRF.
7. Adjust the orientation of the component as required for aligning its science axis in the MRF.
8. Repeat the measurement of the two adjacent faces of the component's optical reference cube.
9. Repeat the calculations described in steps 2 through 6 to verify the success of the alignment.
10. Repeat steps 7 through 9 until satisfied with the alignment.

6. ALTERNATIVE METHOD

The background necessary to complete the steps of the "Alignment Sequence" has been established. The discussion that follows is specifically relevant to the alignment of the GPM star trackers. Consider the use of a relay mirror discussed earlier. The basic set up is shown in the schematic below (Figure 12).

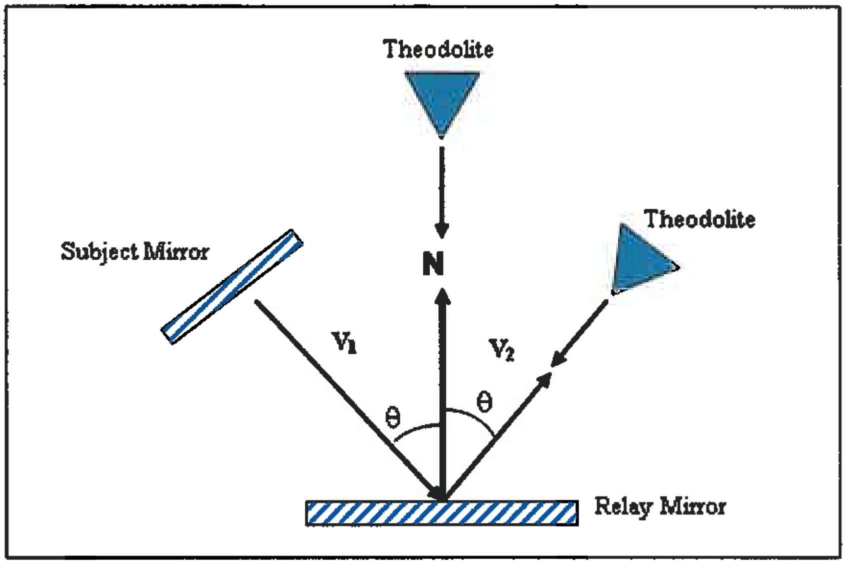

Figure 12. Use of a Relay Mirror

Consider the addition of the unit vectors $-\mathbf{V}_1$ and $\mathbf{V}_2$ as shown below in Figure 13, where $-\mathbf{V}_1$ represents the reflected light from the theodolite and $\mathbf{V}_2$ represents the light returning to the theodolite (definition of autocollimation).

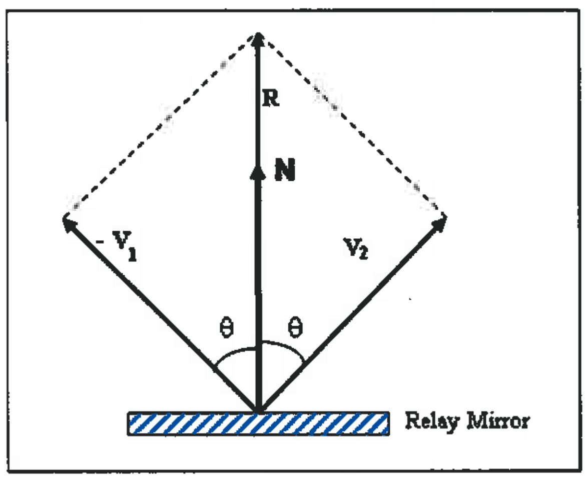

Figure 13. Graphical Addition of $-\mathbf{V}_1$ and $\mathbf{V}_2$

The Law of Reflection guarantees that vectors $\mathbf{V}_1$, $\mathbf{V}_2$, and $\mathbf{N}$ all exist in the same plane. Analysis of Figure 13 then leads to the following results:

$\mathbf{R} = -\mathbf{V}_1 + \mathbf{V}_2$ (vector addition);

$\mathbf{N} = \mathbf{R}/R$ (By symmetry $\mathbf{R}$ and $\mathbf{N}$ point in the same direction); then $\mathbf{R} = R\mathbf{N}$ where R is the magnitude of the vector $\mathbf{R}$ and from the vector diagram:

$R = 2\ V_1\cos\theta = 2\ V_2\cos\theta = 2\cos\theta = 2\mathbf{N}\cdot\mathbf{V}_2$ ($\mathbf{V}_1$ and $\mathbf{V}_2$ both have unit magnitude.)

Thus $\mathbf{R} = -\mathbf{V}_1 + \mathbf{V}_2 = (2\mathbf{N}\cdot\mathbf{V}_2)\mathbf{N}$; therefore $\mathbf{V}_1 = \mathbf{V}_2 - (2\mathbf{N}\cdot\mathbf{V}_2)\mathbf{N}$, and if $\mathbf{N} = (L, M, N)$, then in component form:

$$\mathbf{V}_1 = \{V_{2x} - (2\mathbf{N}\cdot\mathbf{V}_2)L,\ \ V_{2y} - (2\mathbf{N}\cdot\mathbf{V}_2)M,\ \ V_{2z} - (2\mathbf{N}\cdot\mathbf{V}_2)N\} \tag{15}$$

where $\mathbf{N}\cdot\mathbf{V}_2 = LV_{2x} + MV_{2y} + NV_{2z}$.

Writing the result as a matrix equation yields the following:

$$\begin{pmatrix} V_{1x} \\ V_{1y} \\ V_{1z} \end{pmatrix} = \begin{bmatrix} 1-2L^2 & -2LM & -2LN \\ -2ML & 1-2M^2 & -2MN \\ -2NL & -2NM & 1-2N^2 \end{bmatrix} \begin{pmatrix} V_{2x} \\ V_{2y} \\ V_{2z} \end{pmatrix}$$

The result is equation (14), that is $\mathbf{V}_1$ and $\mathbf{V}_2$ are related through a rotation described by Smith's Rotation Matrix.

Consider the set up shown in Figure 14, which is similar to Figure 12, except that the subject mirror has been replaced by a theodolite and the relay mirror has been replaced by the subject mirror. In this case $\mathbf{V}_1$ is the collimated light from Theodolite 2 and $\mathbf{V}_2$ is the collimated light from Theodolite 1.

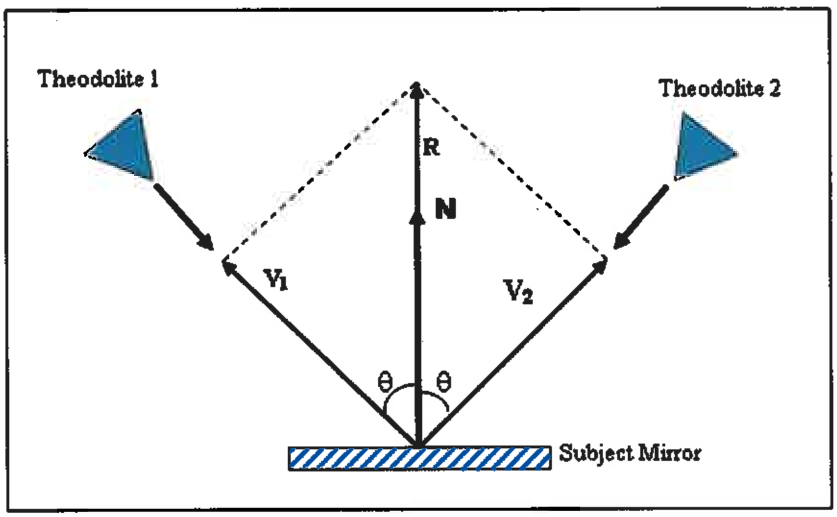

Figure 14. Graphical Addition of Vectors $\mathbf{V}_1$ and $\mathbf{V}_2$

The analysis of Figure 14 yields:

$$\mathbf{R} = \mathbf{V}_1 + \mathbf{V}_2$$

Where $\mathbf{V}_1 = (V_{1x}, V_{1y}, V_{1z})$ and $\mathbf{V}_2 = (V_{2x}, V_{2y}, V_{2z})$ are the arrays of direction cosines resulting from the measurements from theodolite 1 and theodolite 2, respectively. Therefore, the resultant is given:

$$\mathbf{R} = (V_{1x} + V_{2x}, V_{1y} + V_{2y}, V_{1z} + V_{2z}) \text{ and}$$

$$\mathbf{N} = \mathbf{R}/R \tag{16}$$

where R is the magnitude of the vector **R**. Thus, the mirror normal **N** is just the normalized vector sum of $\mathbf{V}_1$ and $\mathbf{V}_2$.

From this rather straight-forward analysis, it can be concluded that an alternative method for measuring a mirror normal is to buck one theodolite to another using the subject mirror as if it were a relay mirror. This alternative method of measurement, which we call the "Bounce Shot," was used to effect the measurement of critical GPM star tracker alignment cube faces in which the line-of-sight to the cube face normals were not available for direct theodolite autocollimation measurements (See Figure 15). The "bounce shot" relies on the symmetry of the Law of Reflection. In practice, one of the theodolites is designated the source theodolite and must be maintained in a set orientation during the measurements. One important advantage of the "bounce shot" over a measurement employing a relay mirror is the concern for the stability of the relay mirror. The calculation of the mirror normal requires only the direction cosines from the readings of each theodolite in a common reference frame and results in half the probable uncertainty of a relay set of measurements. Using this method, the theodolite operator can measure around obstacles that prevent direct line-of-sight measurement of a mirror surface normal, which can be especially useful when it is not possible or practical to use a relay mirror.

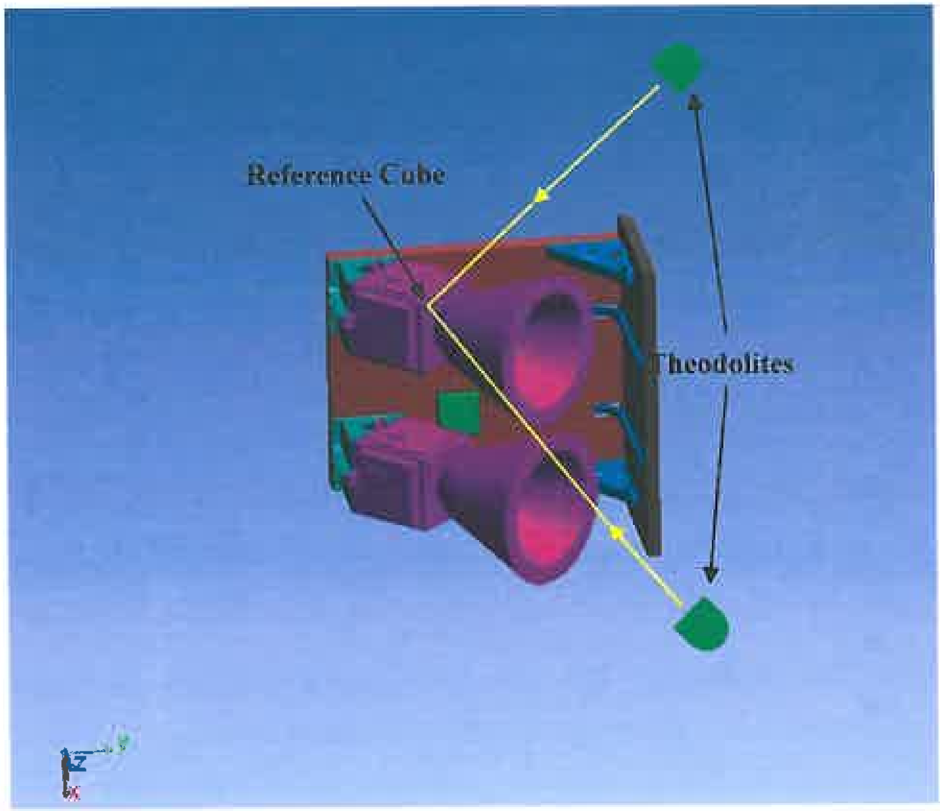

Figure 15. Measuring the GPM Star Tracker Cubes Using the Alternative Method

7. DATA ANALYSIS PROGRAM

GSFC theodolite autocollimation data analysis was generally performed using Microsoft Excel spreadsheets powered by a data analysis package written in Excel macro language. It was developed in house[5] and given the name "OAFDAMs" (Optical Alignment Facility Data Analysis Macros) more than twenty years ago. Recently, "OAFDAMs" was re-written in Visual Basic to be compatible with recent versions of Microsoft Excel. It was also expanded to include a vast number of related functions and two and three dimensional geometrical fitting.

"OAFDAMs" is used in the form of a spreadsheet template with a standard order of the basic data analysis as discussed here. For example, to determine the direction cosines of a measured cube face in the primary theodolite coordinate system, the data is inserted in a specified order: Cube name, direction, subject azimuth, subject elevation, subject azimuth reference, primary azimuth reference, subject theodolite number, and primary theodolite number. The spreadsheet instantly calculates in the order of calibrated elevation, zenith, roll, and direction cosines.

8. CONCLUSIONS

We have described an innovative technique in theodolite metrology that was used in the alignment of the GPM star trackers. This method can be used to measure around obstacles that prevent direct line-of-sight autocollimation measurements of a mirror surface normal, which is especially useful when it is not possible or practical to use the relay mirror technique. In order to provide a clear understanding of the spacecraft component alignment sequence as well as to provide a preface to this technique, we have presented a detailed account of the theory of theodolite autocollimation measurements and data analysis as practiced at GSFC.

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