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# Comparison of Models for Ball Bearing Dynamic Capacity and Life

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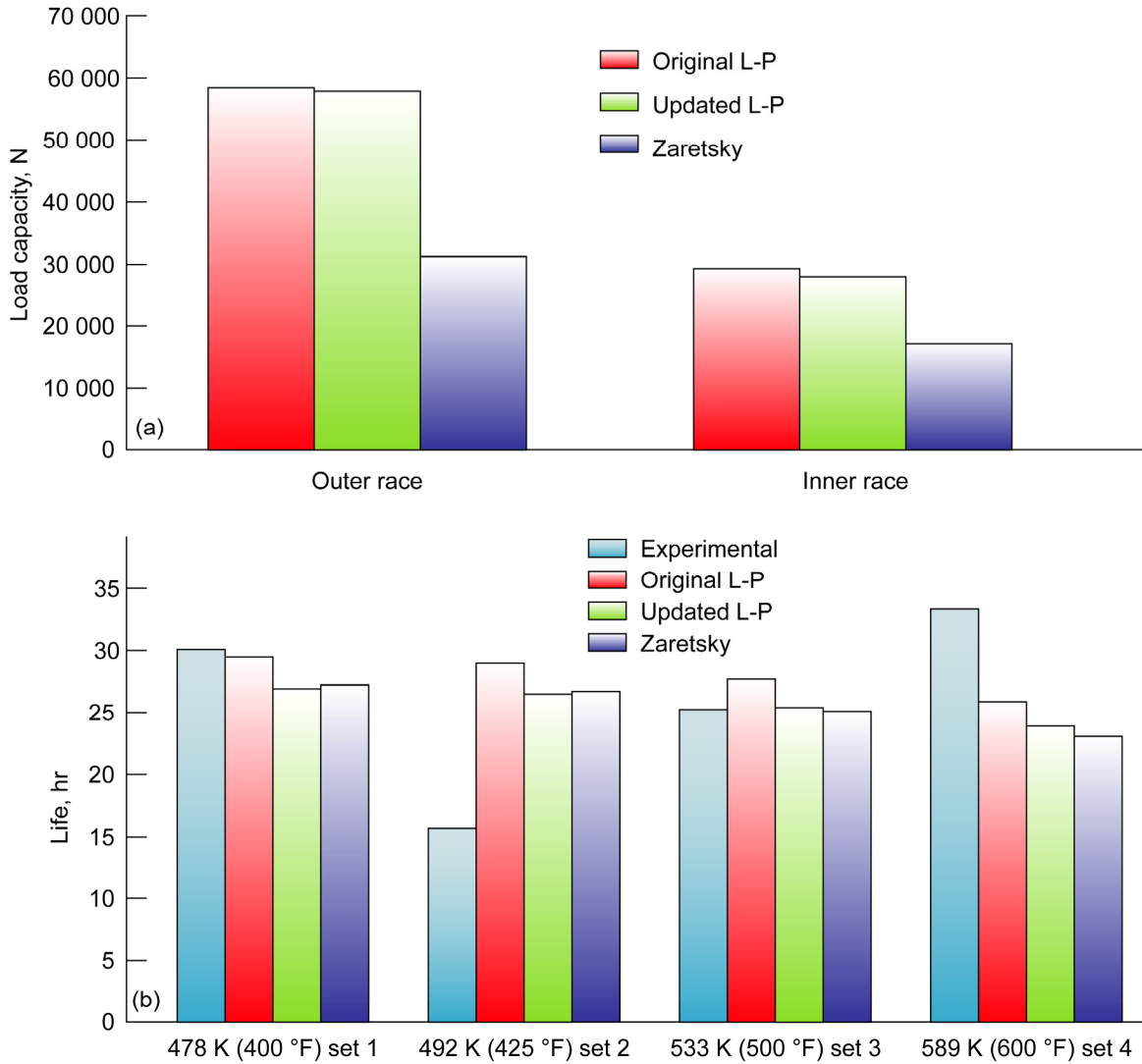
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Page 16, Figure 4(b) has been corrected.



Figure 4.—(a) Comparison of dynamic load capacities for the test bearing at 1.44 DN. (b) Comparison of basic $L_{10}$ bearing lives at 1.44 million DN. L-P is Lundberg-Palmgren.

# Comparison of Models for Ball Bearing
# Dynamic Capacity and Life

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## Abstract

Generalized formulations for dynamic capacity and life of ball bearings, based on the models introduced by Lundberg and Palmgren and Zaretsky, have been developed and implemented in the bearing dynamics computer code, ADORE. Unlike the original Lundberg-Palmgren dynamic capacity equation, where the elastic properties are part of the life constant, the generalized formulations permit variation of elastic properties of the interacting materials. The newly updated Lundberg-Palmgren model allows prediction of life as a function of elastic properties. For elastic properties similar to those of AISI 52100 bearing steel, both the original and updated Lundberg-Palmgren models provide identical results. A comparison between the Lundberg-Palmgren and the Zaretsky models shows that at relatively light loads the Zaretsky model predicts a much higher life than the Lundberg-Palmgren model. As the load increases, the Zaretsky model provides a much faster drop off in life. This is because the Zaretsky model is much more sensitive to load than the Lundberg-Palmgren model. The generalized implementation where all model parameters can be varied provides an effective tool for future model validation and enhancement in bearing life prediction capabilities.

## Introduction

For over a century, rolling-element fatigue has been the criterion for determining the life of rolling-element bearings. Classical rolling-element fatigue is stress- or load-cyclic dependent, beginning as a crack at a depth below the race or rolling-element surface. The crack propagates into a crack network that reaches the contacting surface to form a spall limited in area and depth of penetration (Ref. 1). This failure mode can be classified as high-cycle fatigue where most of the bearing life is related to crack initiation with a relatively short time related to crack propagation. Rolling-element fatigue is extremely variable but statistically predictable, depending on the steel type, steel processing, heat treatment, bearing manufacturing and type, lubricant used, and operating conditions (Ref. 1). Sadeghi, et al. (Ref. 2) provide an excellent review of this failure mode.

In 1924, Palmgren (Ref. 3) began and later together with Lundberg in 1947 and 1952 (Refs. 4 and 5), developed what is now referred to as the "Lundberg-Palmgren Model for Rolling Bearing Life Prediction." The Lundberg-Palmgren analysis is based on the Weibull fracture strength and life models (Refs. 6 to 10) and classical rolling-element fatigue (Ref. 11). In 1953, the Lundberg-Palmgren life model was adopted as the rolling bearing life prediction standard by American National Standards Institute (ANSI)/American Bearing Manufacturers Association (ABMA) (Refs. 12 and 13) and later by International Organization for Standardization (ISO) (Refs. 14 and 15). It is currently the basis for all bearing life prediction worldwide.

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Jones in 1952 (Ref. 16) was the first person to consider the effect of centrifugal loading on the life of angular-contact ball bearings. His analysis modified the bearing life analysis of Lundberg and Palmgren. Subsequently, Jones (Ref. 17) in 1959 published his method for determining the kinematics of a ball and sliding friction in a high-speed, angular-contact ball bearing. Following this analysis in 1960, Jones (Ref. 18) published a completely general solution for both ball and roller bearing kinematics, loading, and life. For the first time, a bearing analysis incorporated elastic deflection of the bearing shaft and supporting structure as well as centrifugal and gyroscopic loading of the rolling elements under combined loading and high-speed operation (Ref. 19). The solution, which was accomplished numerically by iterative techniques, was programmed by Jones for an IBM 704 digital computer. This was the first rolling-element bearing computer code and a major technical achievement.

The Lundberg-Palmgren (Refs. 4 and 5) bearing life analysis as well as that of Jones (Ref. 19) was benchmarked to a pre-1940 rolling bearing life database. With advancements in materials, steel processing, manufacturing techniques, and lubrication beginning in the early 1950s, rolling bearing fatigue life has significantly increased. Fatigue life predictions from the Lundberg-Palmgren model began to significantly underestimate bearing life (Ref. 20).

In 1987, Anderson (Ref. 21) discussed the limitations of the Lundberg-Palmgren life model. He stated that "in the decades since its development a number of shortcomings have become apparent. These shortcomings, which manifest themselves as discrepancies between predicted and actual bearing behavior, are partly due to limitations of the original model in accounting for all relevant phenomena and partly due to continuously advancing bearing technology." These limitations in the Lundberg-Palmgren life model have stimulated research on more comprehensive life models such as those of Ioannides and Harris (Ref. 22) and Zaretsky (Refs. 23 and 24). The limitations have also resulted in the introduction of life factors that attempt to correct for material and lubrication effects in the Lundberg-Palmgren life model (Refs. 25 to 27).

Ioannides and Harris (Ref. 22) modified the Lundberg-Palmgren model by introducing a shearing stress fatigue limit, below which the bearing life is assumed to be infinite. Thus, a limiting shear stress was subtracted from the base shear stress before computing the life (Ref. 11).

Zaretsky (Refs. 11, 23, and 24) made some fundamental changes to the Lundberg-Palmgren model. First, the Weibull equation was rewritten to make the shear-stress exponent independent of the Weibull slope. Second, the maximum shearing stress, rather than the orthogonal shearing stress, was chosen as the critical shearing stress. Third, the life dependence on the depth of the critical shear stress was eliminated. Awaiting further experimental validation of these fundamental modifications, the Lundberg-Palmgren model has continued to be the dominant life prediction model while life correction factors have been developed to reflect the behavior of improved materials in varied operating environments (Refs. 26 and 27).

As the modern bearing steels and manufacturing techniques have advanced, bearing life has increased several fold in comparison to that achievable in the 1940s. However, demands on operating environments in terms of speed, load, and temperatures have become significantly more adverse. This has resulted in renewed interest in fatigue life modeling. Currently, the three most referenced fatigue models are Lundberg-Palmgren (Refs. 4 and 5), Ioannides-Harris (Ref. 22) and Zaretsky (Refs. 11, 23, and 24). In the Lundberg-Palmgren model, as originally developed, certain material properties and survival statistics are embedded in the model constants. In addition, simplifying assumptions were made to implement the elastic contact solutions. Thus, the life factor approach has been the only viable approach to update the life prediction from this model, as it is currently used.

The shear-stress fatigue limit concept proposed by Ioannides and Harris (Ref. 22) has been a very controversial subject. Several researchers (Refs. 28 to 30) have demonstrated that a realistic fatigue limit on rolling bearing materials does not really exist. Thus, an input shear-stress fatigue limit in the Ioannides and Harris model (Ref. 22) may lack physical significance.

The Zaretsky model attempts to physically modify the fatigue process as a whole. Since the actual data used to develop the Lundberg-Palmgren model is not available in the open literature to revisit the model at a fundamental level, it is essential to reformulate the Lundberg-Palmgren life equation in a more

generalized fashion in order to establish the physical significance of each element of the model. Likewise, the Zaretsky model required development so that it can be applied to practical bearings.

Based upon the above discussion, the objectives of the investigation reported were to (a) develop generalized expressions for both the Lundberg-Palmgren and the Zaretsky models; (b) implement the models in a bearing performance simulation computer code; (c) carry out a parametric model evaluation to derive the model constants from available experimental data; and (d) compare the model predictions over the operating conditions as a function of key model elements.

In order to achieve these objectives, the bearing dynamics computer code ADORE (Ref. 31), which is based on the bearing dynamic analysis of Jones (Ref. 19), was used as a baseline code. Published experimental data for a jet engine main-shaft bearings (Refs. 32 to 35) were used to derive the constants in the life equation. Model predictions were then compared with another set of high-speed turbine engine bearing life data (Ref. 36). The groundwork developed in this investigation is intended to provide a starting point for future model development and validation as experimental data on newer materials become available.

# Nomenclature

| | |
|---|---|
| $A$ | original Lundberg-Palmgren constant in load capacity equation, $N/m^{1.80}$ ($lbf/in.^{1.80}$) |
| $A_{LP}$ | updated Lundberg-Palmgren constant in load capacity equation, $N/m^{1.80}$ ($lbf/in.^{1.80}$) |
| $A_Z$ | Zaretsky constant in load capacity equation, $N/m^{1.332}$ ($lbf/in.^{1.332}$) |
| $a$ | semimajor width of Hertzian contact area perpendicular to direction of rolling, m (in.) |
| $b$ | semiminor width of Hertzian contact area in direction of rolling, m (in.) |
| $c$ | shear-stress-life exponent |
| $D$ | diameter of rolling element, m (in.) |
| $d$ | diameter of rolling-element running track, m (in.) |
| $d_m$ | pitch diameter, m (in.) |
| $f$ | raceway curvature factor: ratio of groove radius to ball diameter |
| $h$ | exponent for depth to critical shear stress |
| $K$ | empirical proportionality constant in fundamental Equation (6) |
| $L$ | individual contact life, millions of rotating-race revolutions |
| $L_S$ | life of stationary race, millions of rotating-race revolutions |
| $L_R$ | life of rotating race life, millions of rotating-race revolutions |
| $L_{10}$ | 10-percent bearing life: life at which 90 percent of a population survives, millions of rotating-race revolutions |
| $m$ | Weibull slope or modulus |
| $N$ | life, number of stress cycles |
| $n$ | number of rolling elements |
| $p_H$ | maximum Hertz stress, Pa ($lbf/in.^2$) |
| $Q$ | normal force (load) between rolling element and raceway, N (lbf) |
| $Q_c$ | dynamic radial load capacity, N (lb) |
| $S$ | probability of survival |
| $u$ | contact frequency of rolling element per revolution of bearing race |
| $V_o$ | stressed volume, $m^3$ ($in.^3$) |
| $z$ | distance below surface to critical shear stress, m (in.) |
| $\alpha$ | contact angle |

| | |
|---|---|
| $\gamma$ | $D \cos(\alpha)/d_m$ |
| $\zeta$ | ratio of critical shear stress to maximum Hertz stress, $\tau/p_H$ |
| $\dot{\theta}$ | rolling-element orbital velocity |
| $\lambda_E$ | elastic property ratio |
| $\xi$ | nondimensional depth to critical shearing stress, $z/b$ |
| $\tau$ | critical shearing stress, Pa (lbf/in.$^2$) |
| $\Omega$ | angular velocity of bearing race |

Subscripts:

| | |
|---|---|
| $i$ | designates race (1 = inner race, 2 = outer race) |
| $j$ | designates rolling-element number (from 1 to $n$) |
| $m$ | designates maximum sheer stress or depth of this stress |
| $o$ | designates orthogonal sheer stress or depth of this stress |

# Lundberg-Palmgren Model

Based on very simple observations, where mechanical failures were caused by some functions of applied stress and the stressed volume, Weibull (Refs. 6 and 7) introduced the Weibull distribution function for strength and life analysis, which is presently very commonly used in a wide range of applications. In very general terms the distribution has three parameters, a location parameter, a scaling factor, and a shape parameter. The location parameter defines a base time with respect to which life may be measured, the scaling factor is a reference life at a defined survival probability, relative to which reliability at any arbitrary survival probability may be measured, and the shape parameter essentially defines the shape of the distribution, more commonly known as Weibull slope. The statistical distribution of course does not define any functions of stress or the volume stressed. Following this fundamental development, Weibull (Refs. 8 and 9) applied this distribution to fatigue and other types of failures. Based on this work, Lundberg and Palmgren (Refs. 4 and 5) applied the Weibull distribution to rolling-bearing failures.

After a statistical analysis of a large amount of experimental bearing life data, it was postulated that in addition to subsurface shear stress and the stressed volume, as suggested by Weibull, rolling-element fatigue was also dependent on the depth at which the fracture is initiated. For this analysis the location parameter was set to zero and the characteristic life was determined with 90-percent survival probability, commonly known as $L_{10}$ life. Thus, based on a two-parameter Weibull distribution bearing life, $N$, in stress cycles, with a probability $S$, was written as a product of an empirical function of the maximum subsurface orthogonal shear stress, $\tau_o$, depth below the surface, $z_O$, at which the maximum orthogonal shear stress occurs, and the volume, $V_o$, of the material stressed:

$$\ln\frac{1}{S} \propto \tau_o^c N^m z_o^{-h} V_o \ln\frac{1}{0.90} \tag{1}$$

Here $c$, $m$ and $h$, are empirical exponents determined by fitting experimental data to the model. The factor $\ln\frac{1}{0.90}$ results from the fact that life at arbitrary survival probability, $S$, is normalized to 90 percent probability of survival.
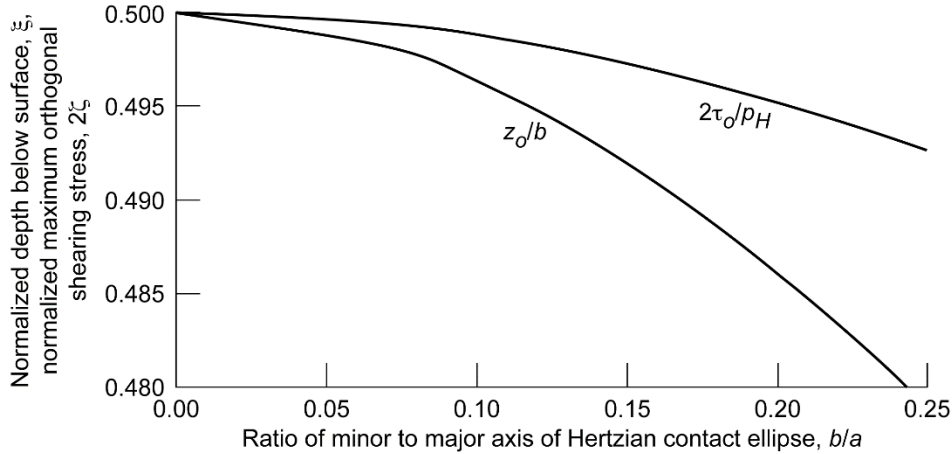
Figure 1.—Normalized orthogonal shearing stress and its depth below the Hertzian contacting surface for point and line contact.

Consider a concentrated (Hertzian) contact between two elastic bodies, with prescribed geometries and material properties, subjected to an applied load $Q$. For a generalized three-dimensional contact, Harris and Kotzalas (Ref. 37) have documented the elastic solutions in terms of contact half widths $a$ and $b$, normal to and along the rolling directions, respectively, the Hertzian contact stress, $p_H$, and the subsurface shear stress distribution as a function of depth below the surface. The relation between the maximum orthogonal shear stress, $\tau_o$ and the depth $z_o$, at which it occurs, is illustrated in Figure 1 as a function of the ratio of the contact half widths.

For most ball bearings, the ratio of the semiminor axis $b$ of the Hertzian contact ellipse to the semimajor axis $a$ is approximately 0.15 ($b/a \approx 0.15$) (Fig. 1). Thus,

$$\frac{2\tau_o}{p_H} \approx 0.497 \approx 0.5 \tag{2a}$$

$$\frac{z_o}{b} \approx 0.492 \approx 0.5 \tag{2b}$$

Although Lundberg and Palmgren (Refs. 1 and 4) used the orthogonal shear stress, $\tau_o$, it may be convenient to generalize the model in terms of any shear stress and the depth at which it occurs. Thus, in Equation (1), $\tau_o$ and $z_o$ may be simply replaced by $\tau$ and $z$. Equation (2) may be expressed in general terms as

$$\tau = \zeta p_H \quad \text{and} \quad z = \xi b \tag{3}$$

where the ratio of critical shear stress to maximum Hertz stress, $\zeta = 0.25$ and the nondimensional depth to the critical shearing stress, $\xi = 0.50$ for the Lundberg-Palmgren model.

Now, if the raceway life, $L$, is expressed in terms of millions of revolutions, while each contact exerts $u$ stress cycles per revolution, then

$$N = uL \times 10^6 \tag{4}$$

Also, if $d$ is the diameter of the rolling-element track on the raceway, then for point contact, the volume of material stressed is expressed as (Lundberg-Palmgren (Ref. 4))

$$V = az\pi d \tag{5}$$

where $a$ is the contact half width normal to the rolling direction, and $z$ is the depth at which the failure originates, as defined earlier.

Thus, after combining Equations (3) to (5) and introducing a proportionality constant, $K$, Equation (1) may be rearranged to express raceway life resulting from a single contact as

$$\frac{1}{L} = \left[ \frac{K(\zeta p_H)^c \pi \xi abd(\xi b)^{-h}}{\ln\frac{1}{S} \Big/ \ln\frac{1}{0.90}} \right]^{1/m} u \tag{6a}$$

where the constant $10^{6m}$ is absorbed in the proportionality constant $K$. Additionally, depending on computational preference, the constant term, $\ln(1/0.90)$, may either be absorbed in the constant, or consistent with ANSI/ABMA (Refs. 12 and 13) and ISO (Ref 14) standards, a reliability factor may be expressed as

$$a_1 = (\ln S / \ln 0.90)^{1/m} \tag{6b}$$

and Equation (6a) may be reduced to

$$\frac{1}{L} = \left[ K(\zeta p_H)^c \pi \xi abd (\xi b)^{-h} \right]^{1/m} \frac{u}{a_1} \tag{6c}$$

Equations (6a) or (6c) represents the generalized form of the Lundberg-Palmgren life equation, applicable to both point and line contacts. In the event the survival probability, $S = 0.90$, then of course Equations (6a) and (6c) will be identical and the reliability factor in Equation (6b) will be one. Note that the Hertzian contact stress, $p_H$ and the contact dimensions $a$ and $b$ are related to the applied contact load, geometry of the interacting surfaces, and the applicable material properties. Thus, specialized versions of Equations (6a) or (6c) for point and line contact configurations may be developed.

Based on the Hertzian elastic point contact solution (Harris and Kotzalas) (Ref. 37) as applicable to ball bearings, the relationships between the applied contact load, $Q$, and the contact parameters contained in Equation (6) may be written as

$$p_H = \frac{3Q}{2\pi ab} \tag{7a}$$

$$a = a^* \left( \frac{3}{2E' \sum \rho} \right)^{1/3} Q^{1/3} \text{ and } b = b^* \left( \frac{3}{2E' \sum \rho} \right)^{1/3} Q^{1/3} \tag{7b}$$

$$\frac{1}{E'} = \frac{1-\upsilon_1^2}{E_1} + \frac{1-\upsilon_2^2}{E_2} \tag{7c}$$

where $\Sigma\rho$ is summation of the principal curvatures of the two interacting surfaces, $E_1$, $\upsilon_1$, $E_2$, $\upsilon_2$ are, respectively, the elastic modulus and Poisson's ratio of the two surfaces, $a^*$ and $b^*$ are functions of elliptical integrals corresponding to the ratio of the contact half widths, $a$ and $b$.

Equations (7) may be substituted into Equation (6) to get a life-load relationship, which can be inverted to compute a load, which will sustain 1 million rotating ring revolutions of life. This load is defined as the dynamic load capacity of a bearing component. In the original Lundberg and Palmgren formulation Equations (4) and (5), $E'$ is considered as a constant, corresponding to elastic properties of AISI 52100 bearing steel, and it is included in the proportionality constant, $K$, in Equation (6). In order to permit elastic properties variation in the updated Lundberg-Palmgren relation, developed herein, it is convenient to define an elastic property ratio,

$$\lambda_E = \frac{1/E'}{1/E'_o} \qquad (8)$$

where $E'_o$ corresponds to the elastic properties of AISI 52100 steel.

Substitution of Equations (7a) to (7c) and (8) into Equation (6) provides a life equation in terms of bearing geometry, the applied loads, speeds, and applicable material properties. The proportionality constant, $K$, is determined by correlating the model predictions with experimental data. While Equation (6a) or (6c) can be readily used to compute bearing life, it is often customary to compute a load under which the bearing could sustain a life of one million revolutions with a given probability. This load is defined as the dynamic capacity of the bearing. Thus, after substituting Equations (7) and (8), Equation (6a) can be inverted to provide the following generalized expression for load capacity for ball bearings.

$$Q_{cLP} = A_{LP}\left[k_c k_a^{1-c} k_b^{1-h-c} d\right]^{\frac{-3}{c-h+2}} u^{\frac{-3m}{c-h+2}} \qquad (9)$$

where the various constants are

$$k_a = a^*\left(\frac{3\lambda_E}{2\sum\rho}\right)^{1/3} \qquad (10a)$$

$$k_b = b^*\left(\frac{3\lambda_E}{2\sum\rho}\right)^{1/3} \qquad (10b)$$

$$k_c = \left(\frac{3\zeta}{2\pi}\right)^c \left(\frac{2\pi\xi^{1-h}}{\ln\frac{1}{S}}\right) \qquad (10c)$$

$$A_{LP} = \left(\frac{K}{E_o\frac{2-h-2c}{3}}\right)^{-\frac{3}{2+c-h}} \qquad (10d)$$

Again, $k_c$ in above Equation (10c) is a constant, since the survival probability, $S$ for the $L_{10}$ life is 0.90, and $\zeta$ and $\xi$ corresponding to the magnitude and depth of maximum orthogonal subsurface shear stress are, respectively, 0.25 and 0.50 for the Lundberg-Palmgren model. Thus, $k_c$ can be included in the proportionality constant. However, $k_c$ is left here as a separate term in the generalized expression (Eq. 10c) in the event it is desired to change any of these parameters. Also, note that the constant $\ln\dfrac{1}{0.90}$ has been absorbed in the constant $K$ in Equation (10d). The constant $A_{LP}$ is the life constant, which has to be determined by fitting experimental data to the model.

With prescribed race angular velocities for outer and inner races as $\Omega_1$ and $\Omega_2$, respectively, if the rolling-element orbital velocity is $\dot{\theta}$, then the contact frequency per revolution of faster of the two races is

$$u_i = \frac{\left|\dot{\theta} - \Omega_i\right|}{\Omega_2} \quad i = 1,2 \text{ and } \Omega_2 > \Omega_1 \tag{11a}$$

$$u_i = \frac{\left|\dot{\theta} - \Omega_i\right|}{\Omega_1} \quad i = 1,2 \text{ and } \Omega_1 > \Omega_2 \tag{11b}$$

Note that in the entire generalized formulation above, life is modeled at each individual contact. For life of the entire bearing these lives are appropriately summed over the total number of rolling elements as discussed later in the paper. Thus, while the above contact frequency is for an individual contact, the total number of rolling elements does enter in the calculation while estimating life of the entire bearing. In commonly used simplified expressions where all rolling elements are assumed to operate with some effective load, the expression for contact frequency includes the number of rolling elements and the summation, as discussed later in the paper, is eliminated.

The contact parameters in Equation (9) are readily available in rolling-element bearing computer codes. As a result, implementation of Equation (9) is straight forward. The load capacity $Q_{cLP}$ is dimensional. The resultant units of the constant $A_{LP}$ will depend on the values of exponents $c$ and $h$. From Lundberg and Palmgren, with the values of 31/3 and 7/3 for $c$ and $h$, respectively, the units reduce to (Force/Length)$^{1.80}$.

While developing the original model in the 1940s, Lundberg and Palmgren (Refs. 1 and 4) made certain simplifications in relating the bearing geometry, material properties to compute the elastic contact solutions. Also, they assumed the materials to be air-melt AISI 52100 bearing steel and the values of the exponents, $c$, $h$, and $m$ were set to 31/3, 7/3, and 10/9, respectively. The result is the following dynamic capacity equation for a point contact in ball bearings with a survival probability of 90 percent (Ref. 38).

$$Q_c = A \left(\frac{2f}{2f-1}\right)^{0.41} (1 \mp \gamma)^{1.39} \left(\frac{D}{d_m}\right)^{0.30} D^{1.8} u^{-1/3} \tag{12a}$$

$$\gamma = \frac{D \cos \alpha}{d_m} \tag{12b}$$

where the upper sign refers to the inner race while the lower sign denotes outer race contact.

The variables, $D$, $d_m$, $\alpha$, $f$, and $u$ are, respectively, the ball diameter, pitch diameter, contact angle, race curvature factor, and the number of stress cycles exerted by a single rolling element on the raceway per revolution of the bearing. After correlating Equation (12a) with experimental data available at the time, Lundberg and Palmgren proposed a value of $2.464 \times 10^7$ N/m$^{1.8}$ (7450 lbf/in.$^{1.8}$) for the empirical constant $A$. However, the data used to perform the correlation is not available in the open literature.

Equation (12a) does not have any input for the elastic properties of the materials since they are included in the constant $A$. However, if the value of contact angle is set to that existing under the applied loads, the effect of changes in overall bearing deformation under the applied loads, speeds, and temperatures, will be accounted for.

Equation (12a) for its simplicity has been used extensively over many decades. With the advancing materials technology, the validity of this simple equation has increasingly been in question. Since computerized analysis of rolling contacts is readily available in modern models for rolling bearings, Equation (9) may be implemented as a more generalized and updated Lundberg-Palmgren model. The model constant, $A_{LP}$, has to be, of course, determined by correlating the model to experimental failure data. For the purpose of discussion in this paper, Equation (12a) is referred to as the "original" Lundberg-Palmgren model while the generalized Equation (9) is called the "updated" Lundberg-Palmgren model. Once the dynamic capacity is calculated from either Equation (9) or (12a), the raceway life due to a single contact subjected to a contact load, $Q$, which is an output from a detailed load distributed analysis under the applied operating conditions on the bearing, may be written as:

$$\frac{1}{L_j} = \left( \frac{Q_j}{Q_{cj}} \right)^{\frac{c-h+2}{3m}} \tag{13}$$

With the values of $c$, $h$ and $m$ as 31/3, 7/3, and 10/9, respectively, the value of the load exponent $\frac{c-h+2}{3m} = 3$, for the Lundberg-Palmgren model. Based upon a sensitivity study in Appendix A, the values of the Weibull modulus $m$ and $c$ chosen by Lundberg and Palmgren best reflects their database.

Now the life of the raceway will be calculated from an inverse summation of individual contact lives. For the stationary raceway each contact may be subject to a different load condition, thus, the summation over $n$ rolling elements will involve the Weibull dispersion slope, $m$,

$$\frac{1}{L_S} = \left( \sum_{j=1}^{n} \left( \frac{1}{L_j} \right)^m \right)^{1/m} = \left( \sum_{j=1}^{n} \left( \frac{Q_j}{Q_{cj}} \right)^{\frac{c-h+2}{3}} \right)^{1/m} \tag{14}$$

For the rotating raceway each rolling element applies an identical loading condition. Thus, the raceway life is computed by a simple summation over the $n$ rolling elements

$$\frac{1}{L_R} = \sum_{j=1}^{n} \frac{1}{L_j} = \sum_{j=1}^{n} \left( \frac{Q_j}{Q_{cj}} \right)^{\frac{c-h+2}{3m}} \tag{15}$$

Finally, the life of the entire bearing is computed by summation over the races

$$L_{10} = \left( L_S^{-m} + L_R^{-m} \right)^{-1/m} \tag{16}$$

It should be noted that by computing life individually for each contact in the bearing and then carrying out appropriate summation over the raceway eliminates the need for computing an "effective" or "equivalent" load as commonly done in simplified implementation of life models. Such a generalized implementation permits more precise modeling of load variation on the rolling elements.

## Zaretsky Model

Zaretsky (Ref. 23) proposed several modifications to the fundamental Weibull and Lundberg-Palmgren life equation, Equation (1):

1.  Zaretsky explained that in the Lundberg-Palmgren model the dependence of life on the depth, $z_o$, below the surface where the orthogonal shear stress is a maximum, implies that the life is dependent on the time it takes for the crack to propagate from the place of origination to the surface. However, since rolling bearing fatigue life can be categorized as "high-cycle fatigue," the crack propagation time is extremely small in comparison to the total running time of the bearing for a bearing made from modern vacuum-processed material. Thus, Zaretsky (Ref. 23) dispensed the term $z_o^{-h}$ in Equation (1). Subject to future evaluation and validation, the exponent, $c$, to the subsurface shear stress is kept as 31/3, as in the Lundberg-Palmgren model.
2.  The critical shear stress used in the Lundberg-Palmgren model is the maximum orthogonal shear stress, $\tau_o$. Zaretsky proposed to use the maximum shear stress instead, $\tau_m$, which in a ball bearing is 30 percent greater than the maximum orthogonal shear stress, $\tau_o$.
3.  The volume of material being fatigued in the Lundberg-Palmgren model is based on the depth, $z_o$, at which the orthogonal shear stress is a maximum. Zaretsky proposed that the volume should be based on the depth, $z_m$, at which the maximum shear stress occurs, which is 57 percent greater than the depth of maximum orthogonal shear stress.
4.  It is seen that when Equation (1) is inverted to express life as a function of shear stress, the shear stress exponent is dependent on scatter in the life data (the Weibull slope, $m$). Zaretsky demonstrated for most materials, the shear-stress-life exponent, $c$, is independent of scatter in the data. As a result, Zaretsky (Ref. 11) modified the shear-stress exponent to be equal to $cm$. This makes the shear-stress exponent independent of scatter in life data once Equation (1) is inverted to express life.

Once the Lundberg-Palmgren model is implemented in generalized form (Eq. (9)), the implementation of the Zaretsky model is quite straightforward. Modification (1) is implemented by simply setting the depth exponent, $h$, in Equation (1) to zero; modifications (2) and (3) are simply accomplished by setting the values of $\zeta$ and $\xi$ to 0.30 and 0.786, respectively; finally, changing the shear-stress exponent in Equation (1) from $c$ to $cm$ satisfies modification (4). Thus, for the Zaretsky model the load capacity Equation (9) becomes

$$Q_{cZ} = A_Z \left[ k_{cZ} \left( k_a k_b \right)^{1-c} d \right]^{\frac{-3}{cm+2}} u^{\frac{-3m}{cm+2}} \tag{17}$$

where $k_a$ and $k_b$ are the same as defined in Equations (10a) and (10b), while the constants $k_{cZ}$ and $A_Z$ are modified as

$$k_{cZ} = \frac{2\pi\xi}{\ln\frac{1}{S}} \left( \frac{3\zeta}{2\pi} \right)^m \quad \text{and} \quad A_Z = \left( \frac{K}{E_o^{\frac{2(1-c)}{3}}} \right)^{-\frac{3}{2+c}} \tag{18}$$

Single contact, stationary and rotating raceway, and bearing lives are written similar to Equations (13) to (14) as

$$\frac{1}{L_j} = \left(\frac{Q_j}{Q_{cj}}\right)^{\frac{cm+2}{3m}} \tag{19}$$

$$\frac{1}{L_S} = \left[\sum_{j=1}^{n}\left(\frac{1}{L_j}\right)^m\right]^{1/m} = \left[\sum_{j=1}^{n}\left(\frac{Q_j}{Q_{cj}}\right)^{\frac{cm+2}{3}}\right]^{1/m} \tag{20}$$

$$\frac{1}{L_R} = \sum_{j=1}^{n}\frac{1}{L_j} = \sum_{j=1}^{n}\left(\frac{Q_j}{Q_{cj}}\right)^{\frac{c-h+2}{3m}} \tag{21}$$

and the equation of life of the bearing is identical to Equation (16). Note that we used the same Lundberg-Palmgren values for the shear-stress-life exponent $c$ and the Weibull modules $m$ of 31/3 and 10/9, respectively. The load-life exponent $p$ in Equation (19) comes out to be approximately 4.0 for the Zaretsky model as compared to $p = 3.0$ for the Lundberg-Palmgren model. Thus, as the load increases, the Zaretsky model shows a much faster dropoff in life. A schematic comparison of the two models is shown in Figure 2.

Appendix A contains a sensitivity study varying the values for $c$ and $m$ on $p$ and $n$. For the Zaretsky model, the values for $p$ and $n$ are less sensitive to variations in the Weibull modulus $m$ than the Lundberg-Palmgren model.
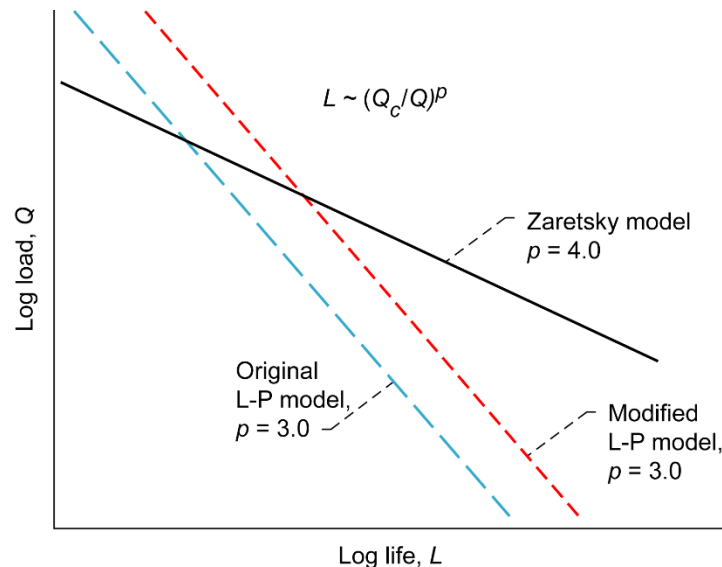


Figure 2.—Comparison of theoretical load-life relationships for the Lundberg-Palmgren (L-P) and Zaretsky models for ball (point contact) bearings.

# Model Implementation

The fatigue life module in the bearing dynamics computer code ADORE (Ref. 31) was rewritten to program both the Lundberg-Palmgren and the Zaretsky models in the generalized forms, as presented above. In addition, the existing original Lundberg-Palmgren model was preserved for comparison purposes. All model coefficients such as shear-stress exponent, $c$, depth exponent, $h$, Weibull slope, $m$, and survival probability, $S$, are kept as variable inputs. Such an implementation eases the development of new values for the model coefficients when correlating model predictions with experimental data. The coefficients in the original Lundberg-Palmgren model, Equation (12), of course, remain constant. Along with applied loads, ADORE models centrifugal and thermal expansion of all bearing elements. Thus, the change in internal clearance and contact stresses are taken into consideration when solving for the elastic contact solutions.

# Experimental Data

As stated earlier, model coefficients in the original Lundberg-Palmgren model are based on a large amount of unpublished pre-1940 experimental data. For this paper four sets of experimental data reported in References (32) and (33) were selected for computing the model coefficients. All the reported data was obtained with sets of ABEC-5 grade, split inner race, 120-mm bore angular-contact ball bearings, operating with a thrust load of 25,800 N (5800 lbf), at an operating speed of 12 000 rpm (1.44 DN). The surface finishes were approximately 0.05 to 0.75 µm (2 to 3 µin.) rms on the races and 0.025 to 0.05 µm (1 to 2 µin.) rms on the balls. Bearing geometry as input into the bearing dynamics code (Ref. 31), is documented in Table 1.

All the four sets of test bearings were lubricated with polyalphaolefin (PAO) lubricant. Properties of the lubricant are tabulated in Table 2. Note that dataset number 2 used less viscous (thinner) oil than used in sets 1, 3, and 4. The operating temperatures of the four datasets were respectively, 478 K (400 °F), 492 K (425 °F), 533 K (500 °F), and 589 K (600 °F). The lubricant film thickness to composite roughness ratios for the four sets were 5 or above. Standard Weibull analysis (Refs. 20 and 21) of the experimental data was carried out to compute the $L_{10}$ bearing life along with a Weibull slope defined by a least-squared regression line through the data points.

Due to the limited amount of experimental data the statistical variation in both the expected life and Weibull slope is significant. Thus, the data is inadequate for computation of expected Weibull slope and the expected $L_{10}$ life. Therefore, the Weibayes method (Ref. 38) is used to compute the expected experimental $L_{10}$ life with the Weibull slope held fixed at a value of 1.11, as recommended by Lundberg-Palmgren. This results in four data points, which were fitted to the life models to derive the model constant by using a least-squared deviation fit analysis.

TABLE 1.—ANGULAR-CONTACT BALL BEARING DETAILS

| | |
|---|---|
| Bearing bore | 120 mm |
| Bearing outer diameter (OD) | 190 mm |
| Number of balls | 15 |
| Ball diameter | 20.6375 mm |
| Pitch diameter | 155 mm |
| Contact angle | 20° |
| Outer race curvature factor | 0.52 |
| Inner race curvature factor | 0.54 |
| Bearing width | 35 mm |
| Outer race shoulder diameter | 165 mm |
| Inner race shoulder diameter | 145 mm |
| Bearing material (balls and races) | CEVM AISI M−50 Steel |
| Lubricant | Polyalphaolefin |

TABLE 2.—TEST LUBRICANT PROPERTIES

| PAO oil lubricant | Test sets 1,3,4 (Ref. 32) | Test set 2 (Ref. 33) |
|---|---|---|
| Additives | Antiwear<br>Antifoam | Antiwear<br>Oxidation inhibitor |
| Kinematic viscosity, cSt at<br>311 K (100 °F)<br>372 K (210 °F)<br>478 K (400 °F) | 443.3<br>39.7<br>5.8 | 60<br>8.9<br>1.9 |
| Pour point K (°F) | 236 (−35) | 225 (−55) |
| Specific heat | 2910 J/(kg K) at 533 K<br>(0.695 BTU/(lb °F) at 500 °F) | 2810 J/(kg K) at 477 K<br>(0.671 BTU/(lb °F) at 400 °F) |
| Thermal conductivity<br>J/(kg K) (BTU/(hr ft °F)) | 0.12 at 533 K<br>(0.070 at 500 °F) | 0.12 at 477 K<br>(0.073 at 400 °F) |
| Specific gravity | 0.71 at 533 K (500 °F) | 0.85 at 477 K (400 °F) |

TABLE 3.—EXPECTED $L_{10}$ BEARING LIVES FOR THE EXPERIMENTAL DATASETS

| Dataset | Failure index | Weibull least-squared fit analysis | | Expected $L_{10}$ life, hr with a<br>Weibull slope of 1.11 |
|---|---|---|---|---|
| | | Weibull slope | $L_{10}$ life, hr | |
| 1 | 10/27 | 1.724 | 311.3 | 195.9 |
| 2 | 14/27 | 1.906 | 195.7 | 101.5 |
| 3 | 11/26 | 3.560 | 432.2 | 161.7 |
| 4 | 6/26 | 1.920 | 260.9 | 205.0 |
| 5 | 6/30 | 1.873 | 1640 | 1535 |

Once the model constants were derived, model predictions were tested against another experimental dataset; this one obtained by Bamberger, et al. (Ref. 34) for a set of 30 each 120-mm bore angular-contact ball bearings made from VIM−VAR AISI M−50 steel. The geometry of these bearings was identical to those outlined in Table 1, except that the free contact angle of these bearings was increased to 24°. The bearings operated with a thrust load of 22 240 N (5000 lbf) at an operating speed of 25 000 rpm, which results in a DN value of three million. The operating temperature was 492 K (425 °F) and the bearing was lubricated with the MIL−L−23699 lubricant. Lubricant properties are documented in Reference 37.

Just for the purpose of comparison, the computed Weibull slope, associated $L_{10}$ life as computed from the Weibull least-squared regression along with the estimated lives at a Weibull slope of 1.11 using the Weibayes method, are summarized in Table 3. The failure index represents the number of bearings failed out of the total number in the set. Datasets 1 to 4 represent the four datasets used to compute the model constants, while set 5 represents the high-speed bearing case used to perform the final model validation.

Typical Weibull plots of the experimental data are shown in Figures 3(a) and (b). Clearly, the life estimated by Weibayes method with the Lundberg-Palmgren value of Weibull slope of 1.11 is different from the least-squared fit to the experimental data. However, as discussed above, due to the limited sample size in the present investigation and large expected variability in Weibull slope, the $L_{10}$ life estimated by the Weibayes method is used to correlate the model predictions and derive the model constants. For a valid correlation of experimental life with analytical model predictions in all of the five datasets, it is essential to apply life modification factors to determine the corresponding basic subsurface life since the experimental data represents life under actual operating conditions. The commonly used Society of Tribologists and Lubrication Engineers (STLE) publication (Ref. 26) provides simple life modification factors for improved materials and lubrication conditions; the factors are applied as simple multipliers on the basic subsurface (Lundberg-Palmgren) life.

Tallian (Ref. 27) presented a more comprehensive formulation of experimentally validated life modification factors; these factors are applied to each individual contact in the bearing and thus, the implementation is significantly more complex. In the present investigation since life is computed at each contact before summation over the races, no additional work was involved in implementing the Tallian life factors. Simply for the purpose of comparison, life factors based on both methods were computed for the five experimental datasets. The results are summarized Table 4 along with a summary of bearing materials and operating conditions. It is interesting to note that while the Tallian life factors are somewhat more conservative, the overall magnitudes of the two sets of life factors are not greatly different.
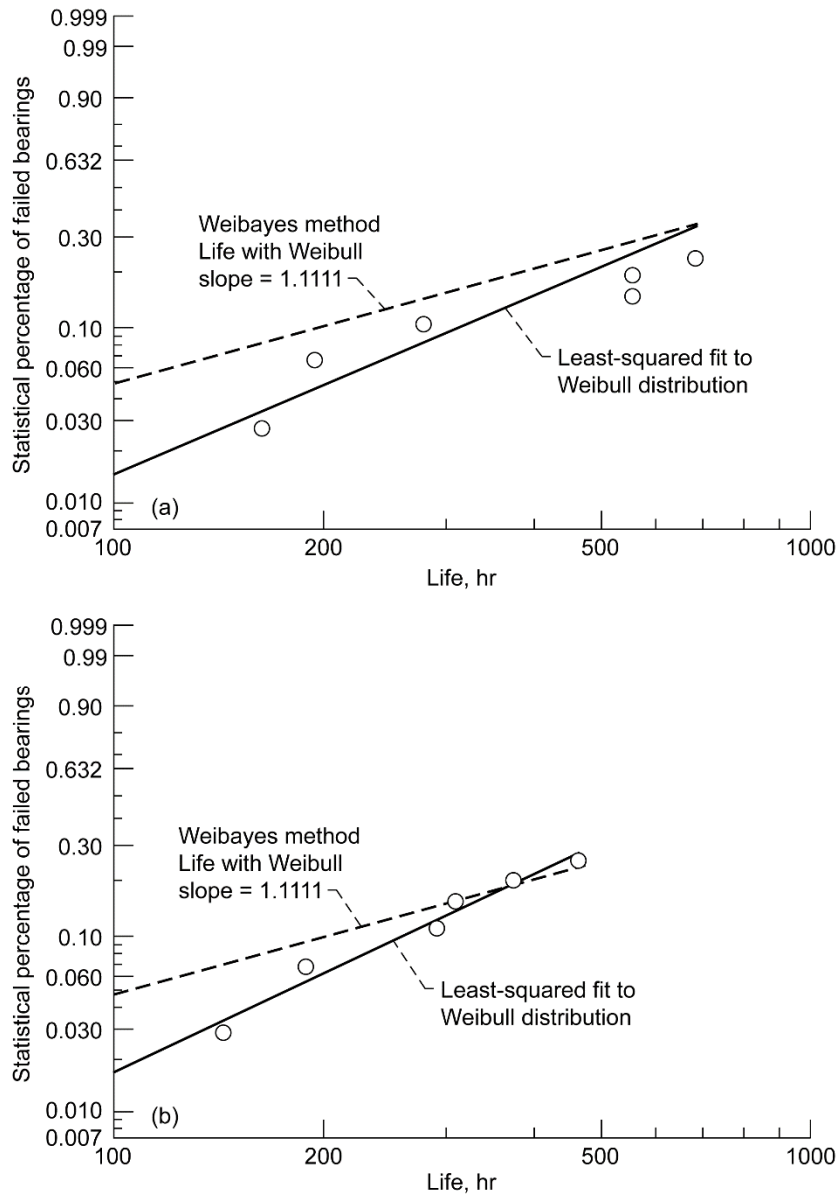
Figure 3.—Weibull plots for endurance characteristics of two data sets of 120-mm bore angular-contact ball bearings. Thrust load, 22.24 kN (5000 lbf), inner race, forged; material, CEVM AISI M–50 steel, material hardness at room temperature, 63 RC; contact angle, 20°, speed, 12 000 rpm. (a) Experimental data set 1, temperature 478 K (400 °F), failure index 10 out of 23 bearings tested. (b) Experimental data set 4, temperature 589 K (600 °F), failure index 6 out of 26 bearings tested.

TABLE 4.—SUMMARY OF EXPERIMENTAL DATASETS AND APPLICABLE LIFE MODIFICATION FACTORS

| Dataset | Bearing material | Lubricant | Operating temperature, K | Thrust load, kN | Inner race speed, rpm | STLE life factor (Ref. 26) | Tallian life factor (Ref. 27) |
|---|---|---|---|---|---|---|---|
| 1 | CVM AISI M−50 | PAO | 478 (400 °F) | 25.80 | 12 000 | 8.88 | 6.51 |
| 2 | CVM AISI M−50 | PAO | 492 (425 °F) | 25.80 | 12 000 | 8.25 | 6.48 |
| 3 | CVM AISI M−50 | PAO | 533 (500 °F) | 25.80 | 12 000 | 8.53 | 6.39 |
| 4 | CVM AISI M−50 | PAO | 589 (600 °F) | 25.80 | 12 000 | 8.10 | 6.15 |
| 5 | VIM−VAR AISI M−50 | MIL−L−23699 | 492 (425 °F) | 22.24 | 25 000 | 32.19 | 30.46 |

Therefore, simply due to their more comprehensive and conservative nature, the Tallian life factors were used in the present investigation to derive the basic subsurface fatigue life, corresponding to the experimental life, for validating the life models. Note that life modification factors are only used to make a valid comparison of analytical predictions with actual experimental data. In all other parametric studies only the basic subsurface lives free of any life modification factors are presented since the principal objective of the present investigation is to compare the various subsurface fatigue life models.

Based on the experimental data discussed above, the model constants, $A_{LP}$ and $A_Z$ in Equations (9) and (16), were computed by least-squared fit analysis of the predicted lives against the experimental lives for datasets 1 to 4. Once the model constants were established, the experimental life with dataset 5 was then compared with model prediction to further establish reliability of model predictions.

Since the generalized formulations presented herein show sensitivity of life with elastic modulus, the physical properties used for the AISI M−50 steel in comparison to those of AISI 52100 steel are summarized below in Table 5.

For VIM−VAR AISI M−50 steel a significant drop in elastic modulus with increased temperature has been reported in the literature (Ref. 39). At the operating temperature of 492 K for dataset 5, the elastic modulus drops to 166 GPa. No such data is presently available for the CVM AISI M−50 steel. Thus, a constant value of elastic modulus for datasets 1 to 4 was assumed.

It should be pointed out that in the present investigation no thermal analysis was undertaken. The available experimental data simply reports one operating temperature. This temperature is used for all both races and all the rolling elements. In reality the bearing will have a temperature field, which will vary with applied load and operating speeds. So a more advanced modeling of life will require a close integration of life equations and thermal interactions. Unlike the original Lundberg-Palmgren model, where the empirical life constant includes the elastic properties of AISI 52100 bearing steel, the newly developed updated Lundberg-Palmgren and Zaretsky models introduce a new elastic property parameter and the empirical life constants are free of any elastic properties. However, the change in contact load and geometry resulting from the change in elastic properties and thermal expansion of the bearing elements is accounted for in all models. While the operating temperature results in thermal distortion of bearing elements, the change in elastic properties affect the centrifugal expansion of the rotating race. Thus, both the operating temperature and speed affect the operating internal clearance and hence, the load distribution in the bearing. The room temperature and operating internal clearances for the five experimental datasets are summarized in Table 6.

TABLE 5.—PHYSICAL PROPERTIES OF BEARING MATERIALS AT ROOM TEMPERATURE

| Property | CVM AISI M−50 | VIM−VAR AISI M−50 | CVM AISI 52100 |
|---|---|---|---|
| Density (kg/m$^3$) | 7830 | 8027 | 7827 |
| Elastic Modulus (GPa) | 203 | 203 | 201 |
| Poisson's Ratio | 0.28 | 0.28 | 0.277 |
| Thermal Coefficient of Expansion (m/m/K) | $1.298 \times 10^{-5}$ | $1.006 \times 10^{-5}$ | $1.15 \times 10^{-5}$ |

TABLE 6.—ROOM TEMPERATURE AND OPERATING INTERNAL CLEARANCES
UNDER EXPERIMENTAL CONDITIONS

| Dataset | Room temperature clearance, mm | Operating temperature, K | Operating speed, rpm | Operating clearance, mm |
|---|---|---|---|---|
| 1 | 0.1493 | 478 (400 °F) | 12 000 | 0.1051 |
| 2 | 0.1493 | 492 (425 °F) | 12 000 | 0.1022 |
| 3 | 0.1493 | 533 (500 °F) | 12 000 | 0.09374 |
| 4 | 0.1493 | 589 (600 °F) | 12 000 | 0.08218 |
| 5 | 0.2141 | 492 (425 °F) | 25 000 | 0.04734 |

# Model Correlation and Validation

A least-squared deviation analysis between the predicted and experimental lives, for experimental datasets 1 to 4, yields the values of model constants, $A_{LP}$ and $A_Z$, as $3.8540 \times 10^6$ N/m$^{1.80}$ and $3.6635 \times 10^5$ N/m$^{1.332}$, respectively, for the updated Lundberg-Palmgren and Zaretsky models. The constant $A$, in the original Lundberg-Palmgren model is, of course, $2.4640 \times 10^7$ N/m$^{1.80}$ as stated earlier.

Although, as expected, units of the model constant in the updated and original Lundberg-Palmgren models are identical, the difference in the values corresponds to the different variable constituents of Equations (9) and (12a). The difference in units of the model constants between the Lundberg-Palmgren and Zaretsky models corresponds to elimination of the shear stress depth, $h$, term and replacement of exponent $c$ with $cm$ in the fundamental life Equation (1) for the Zaretsky model.

With the above model constants, the computed dynamic load capacities for the test bearing are compared in Figure 4(a). As expected, the load capacities between the original and updated Lundberg-Palmgren models are almost identical. The observed very small difference is attributed to a small difference in elastic properties between the AISI M−50 and AISI 52100 steels. The Zaretsky model provides a significantly lower load capacity. This is primarily due to a higher load-life exponent in the Zaretsky model because the life at 1 million revolutions involves a very high load.
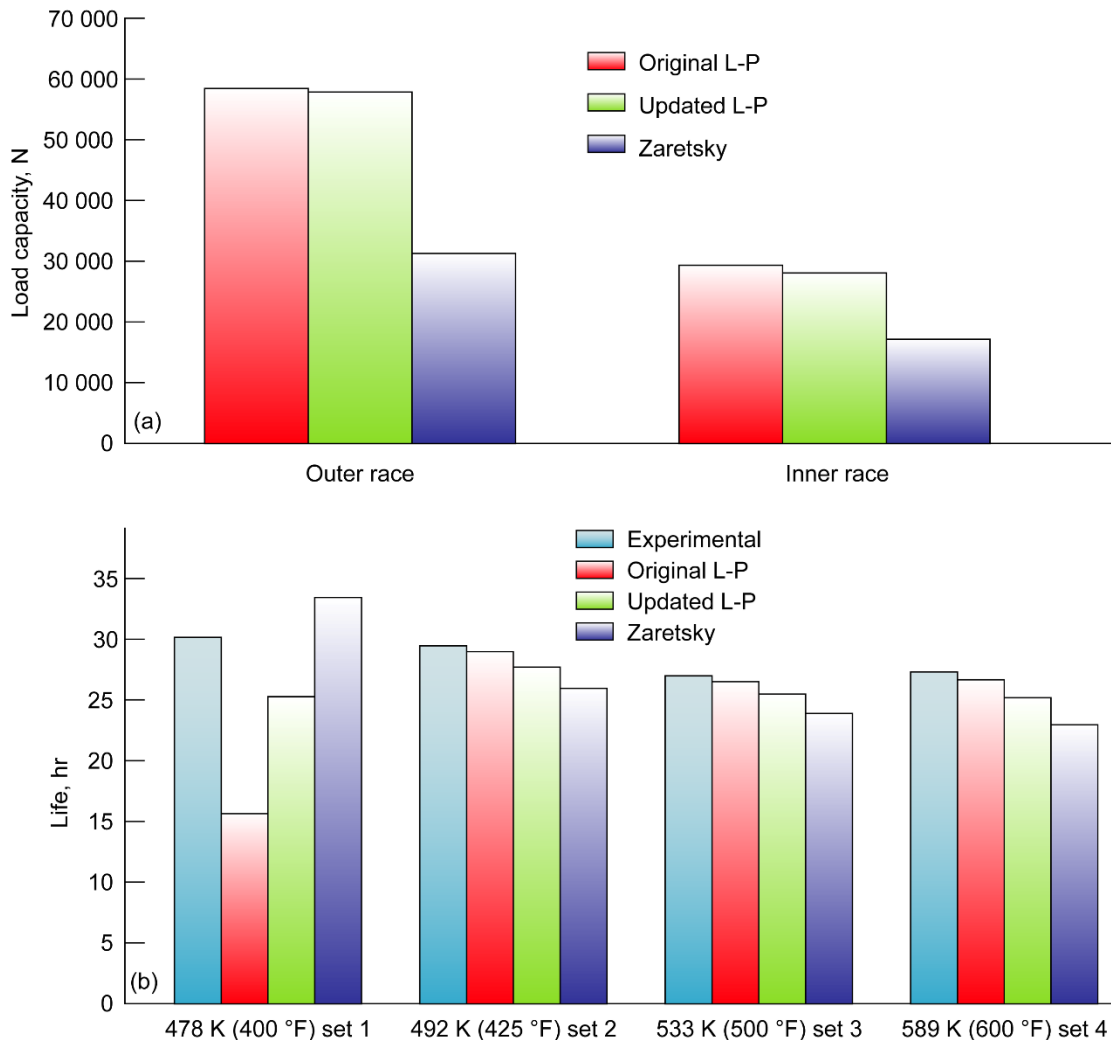


Figure 4.—(a) Comparison of dynamic load capacities for the test bearing at 1.44 DN. (b) Comparison of basic $L_{10}$ bearing lives at 1.44 million DN. L-P is Lundberg-Palmgren.

Comparison of predicted $L_{10}$ lives against the observed experimental life for the four test cases is shown in Figure 4(b). Again the small difference in life between the original and updated Lundberg-Palmgren lives is attributed to a small difference in elastic properties. Both the load capacities and predicted lives with the original and updated Lundberg-Palmgren models are identical if the elastic properties are set to those of AISI 52100 steel. This established analytically sound implementation of the models in the computer code. Predictions of the Zaretsky model under the test operating conditions are almost identical to those of the updated Lundberg-Palmgren model. Except for the scatter in the experimental data, the lives for the four test conditions are almost the same. Statistical analysis of the variance of experimental data shows that the differences in lives of the four datasets are statistically indistinguishable.

Once the model constants are established by regression analysis of the above four experimental datasets, the model predictions are compared with another experimental dataset obtained with another 120-mm turbine engine angular-contact ball bearing operating at 3 million DN. The material for this bearing is VIM−VAR AISI M−50 steel. At an operating temperature of 492 K (425 °F), it has been reported that the elastic modulus reduces to $1.66 \times 10^{11}$ N/m$^2$ from a room temperature value of $2.03 \times 10^{11}$ N/m$^2$ (Ref. 39). With such a reduction in elastic modulus (about 18 percent) the $L_{10}$ lives predicted by both the updated Lundberg-Palmgren and Zaretsky models are more than three times higher than those calculated by the original Lundberg-Palmgren formulation.

Figure 5(a) shows a comparison of load capacities, while Figure 5(b) compares the predicted lives against the experimental data. Note that the lives predicted by both the updated Lundberg-Palmgren and Zaretsky models are very close to the life observed experimentally. Life predicted with the original Lundberg-Palmgren model, where the elastic properties are fixed to those of AISI 52100 steel at room temperature, is significantly lower. Also shown in Figure 5(b) are life predictions at room temperature, which are very similar and comparable between the various models.

It is interesting to note while the predicted lives at room temperature are nearly identical between the original and updated Lundberg-Palmgren models, the original model shows a slight drop in life at the higher operating temperature where the elastic modulus is significantly lower. This is primarily attributed to increased centrifugal expansion of the rotating inner race with the lower elastic modulus at the high operating temperature. While the original Lundberg-Palmgren life equation does not provide for variation of elastic properties, the increase in contact load resulting from reduced internal clearance due to increased centrifugal expansion of the inner race is accounted for when the model is implemented in the computer code. As a result, there is a small drop in life at the higher temperature as shown in Figure 5(b).

Although the Lundberg-Palmgren and Zaretsky models show similar results in the comparisons discussed above, the Zaretsky model, due to a higher load-life exponent, predicted a significantly higher life at light loads and a much faster drop off in life at the load increases. This is seen in Figure 6(a), where the predicted lives are plotted as a function of speed with the VIM−VAR AISI M−50 bearing operating at 492 K (425 °F). As the speed increases, the centrifugal loading increases, which results in a reduction of life. Due to the faster drop off in life with increasing load with the Zaretsky model, the predicted life is actually somewhat lower than that predicted by the updated Lundberg-Palmgren model. The significantly lower life prediction by the original Lundberg-Palmgren model is directly related to the fixed and significantly higher value of elastic modulus.

Just for comparison, the results of Figure 6(a) are re-plotted in Figure 6(b) with the elastic modulus set to the room temperature value. Now both the original and updated Lundberg-Palmgren models show nearly identical lives while the Zaretsky model predicts a higher life at low speeds.

Load-life dependence with the various models is somewhat better seen in Figure 7, where the predicted lives are plotted as a function of applied load at the lower speed (1.44 million DN) with AISI M-50 elastic modulus at room temperature. Now the predicted lives with the original and updated Lundberg-Palmgren models are closely identical and the Zaretsky model predicts a much higher life at lighter loads. As the load increases the difference in predicted lives reduces. At the highest loads the life predicted by the Zaretsky model is actually lower than that estimated with Lundberg-Palmgren model.
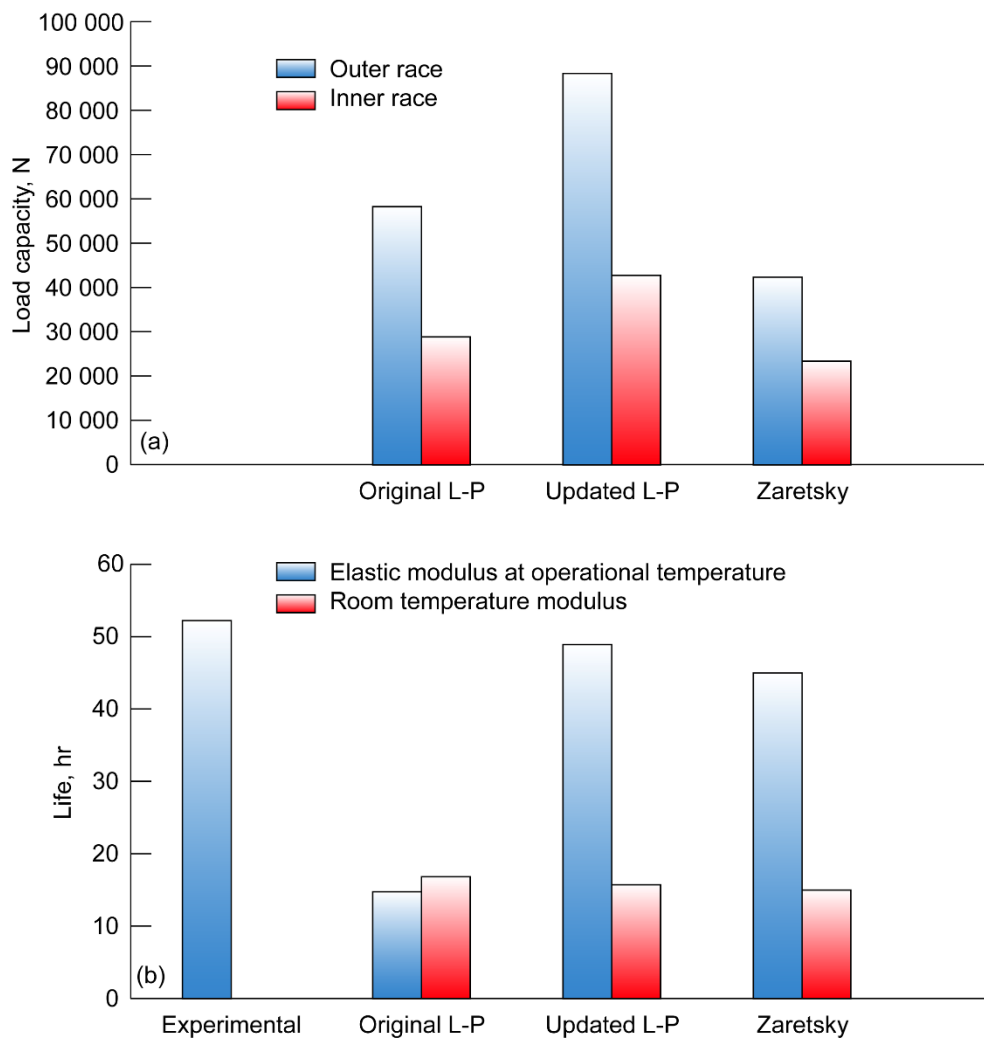
Figure 5.—(a) Comparison of load capacities at 3 million DN with VIM–VAR AISI M–50 properties (Experimental data set 5). (b) Comparison of basic $L_{10}$ bearing lives at 3 million DN with VIM–VAR AISI M–50 steel at 492 K (425 °F) and room temperature (experimental data set 5). L-P is Lundberg-Palmgren.
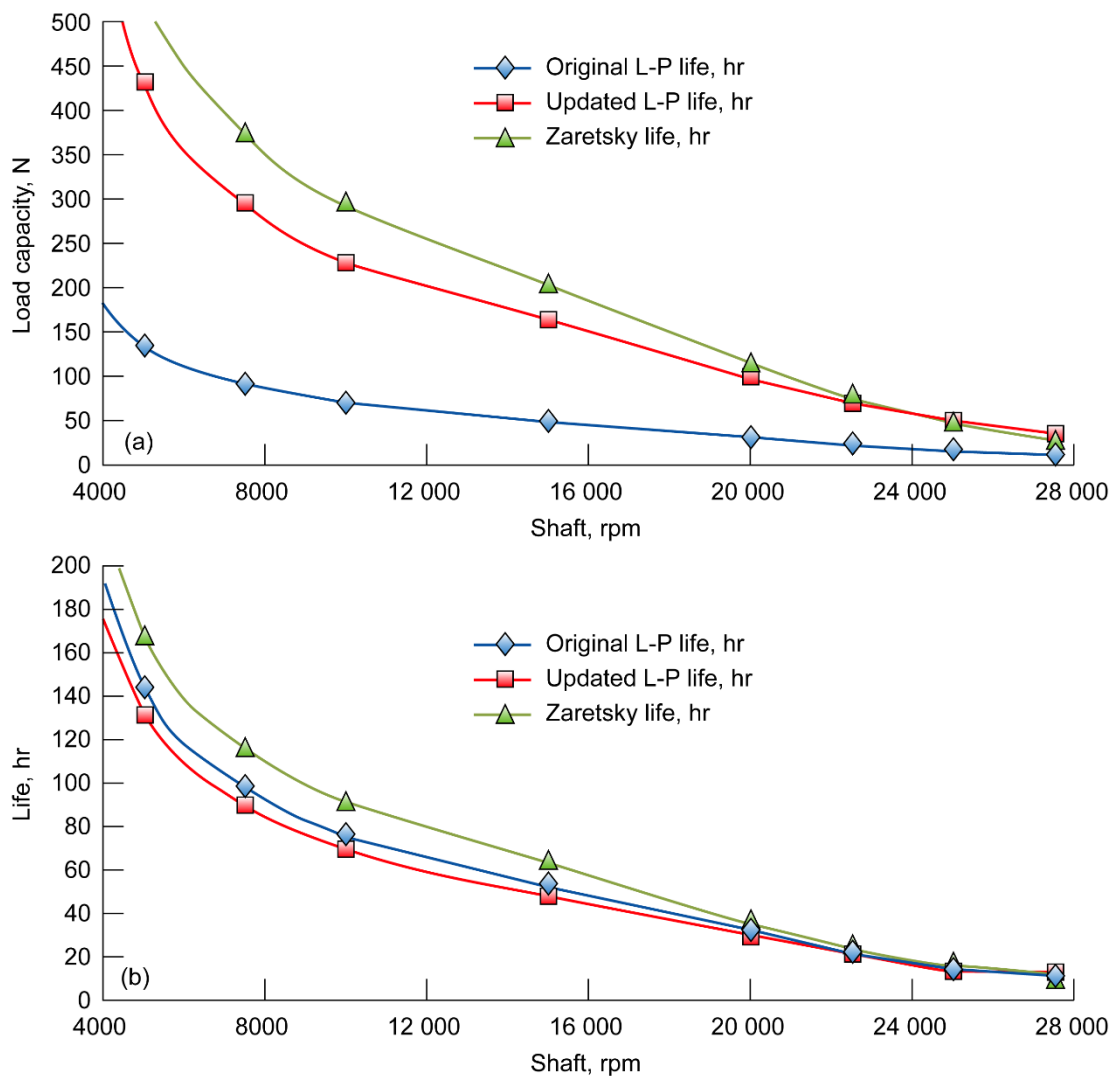
Figure 6.—(a) Life as a function of speed with AISI M–50 elastic modulus at operating temperature of 492 K (400 °F). (b) Life as a function of speed with AISI M–50 elastic modulus at room temperature. L-P is Lundberg-Palmgren.
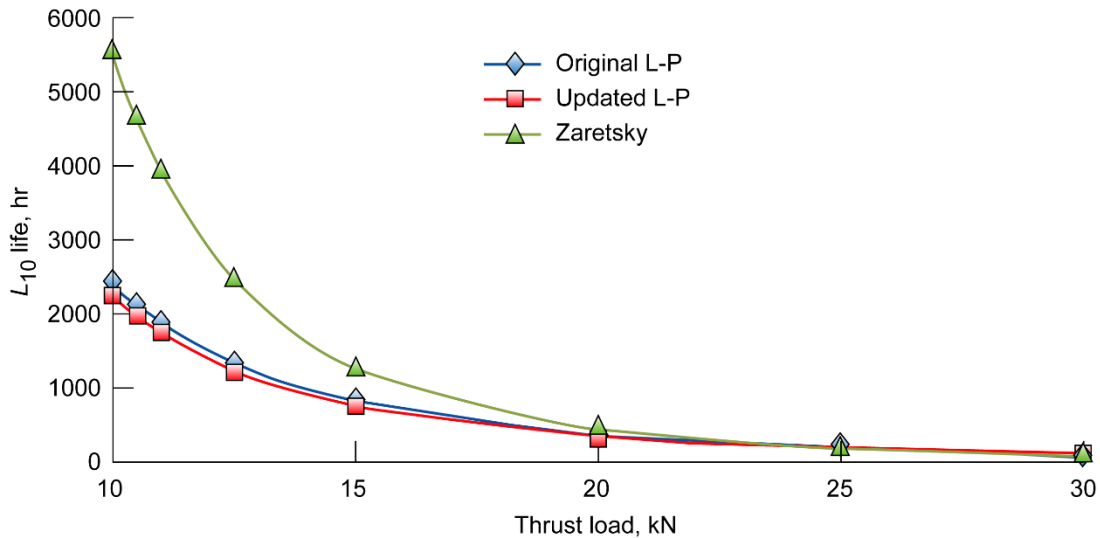
Figure 7.—Life as a function of thrust load with the CVM AISI M–50 120-mm bore angular-contact ball bearing, operating at 12 000 rpm with PAO lubricant at a temperature of 478 K (400 °F). L-P is Lundberg-Palmgren.

The key contribution of the generalized expressions developed in the present investigation is improved modeling of variation in elastic properties of the bearing materials. Although the changes in contact geometry and the resulting effect on volume of the material stressed is accounted for in the original Lundberg-Palmgren life equation, the empirical life constant is based on constant elastic properties corresponding to the common AISI 52100 bearing steel and there is no provision for variation of elastic properties in the life equation. The empirical constants in both the updated Lundberg-Palmgren and Zaretsky models are free of elastic properties of the bearing materials and the life equations include a new elastic property ratio term, which defines the elastic properties variation in terms of a ratio of properties of the current material to those of room temperature AISI 52100 bearing steel.

To more precisely demonstrate the significance of elastic property variation, Figure 8 plots life variation as a function of elastic modulus of the bearing material. The test ball bearing of experimental datasets 1 to 4 is used for these parametric runs but the elastic modulus of bearing material is varied arbitrarily. All runs are made at room temperature with an applied thrust load of 25 000 N and at an operating speed of 12 000 rpm. Although the elastic property term in the newly developed equations permits independent variation of elastic properties of the rolling elements and both races, just for simplicity the same modulus values for the rolling elements and the races are used in these parametric runs.

As the modulus drops, lives computed with the updated Lundeberg-Palmgren models sharply increase; lives with the Zaretsky model are slightly greater in comparison to the updated Lundberg-Palmgren model but the differences are quite small under the operating conditions for these parametric runs. Life variation as a function of the modulus is insignificant with the original Lundberg-Palmgren model. A very small drop in life predicted by the original Lundberg-Palmgren model with increasing modulus, not clearly seen in Figure 8, is a result of variation of contact geometry and centrifugal expansion of the rotating race with elastic modulus. This proves that variations in life due to these effects are insignificant in comparison to those contributed by elimination of elastic properties from the empirical life constant. It may also be noted that at a modulus value of about 200 GPa, which is the value for AISI 52100 bearing steel at room temperature, life computed by all the models is about the same. In fact, the lives computed by the original and updated Lundberg-Palmgren models are identical as expected.

As the modulus increases, the internal clearance in the bearing increases due to reducing centrifugal expansion of the rotating inner race, and the contact stress increases primarily due to reduced contact size for given load on the bearing. These effects are taken into account in all models. Figure 9 shows the variation in internal clearance and contact stress with modulus for the parametric runs.
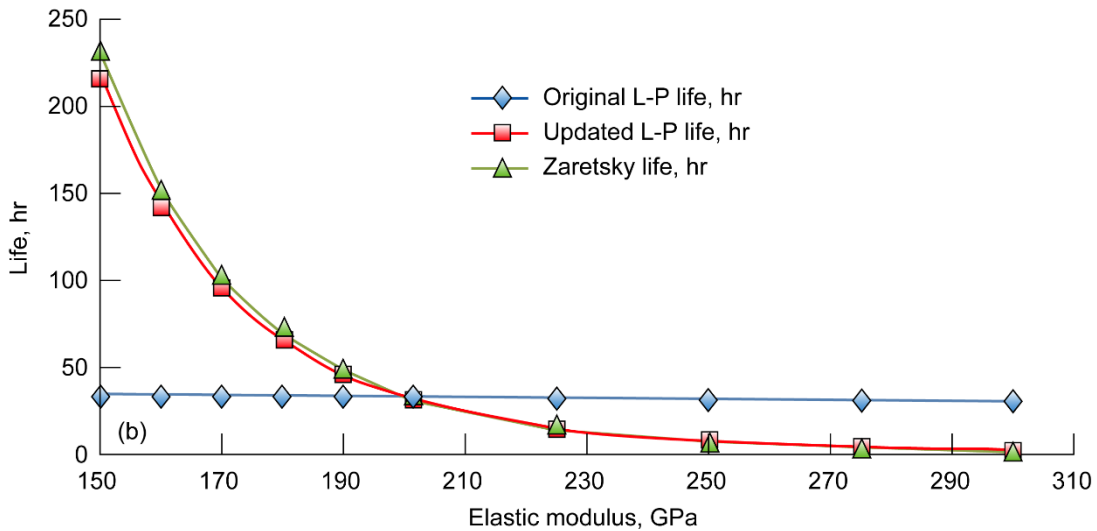
Figure 8.—Life as a function of elastic modulus for the test ball bearing at room temperature with a thrust load of 25 000 N and operating speed of 12 000 rpm. L-P is Lundberg-Palmgren.
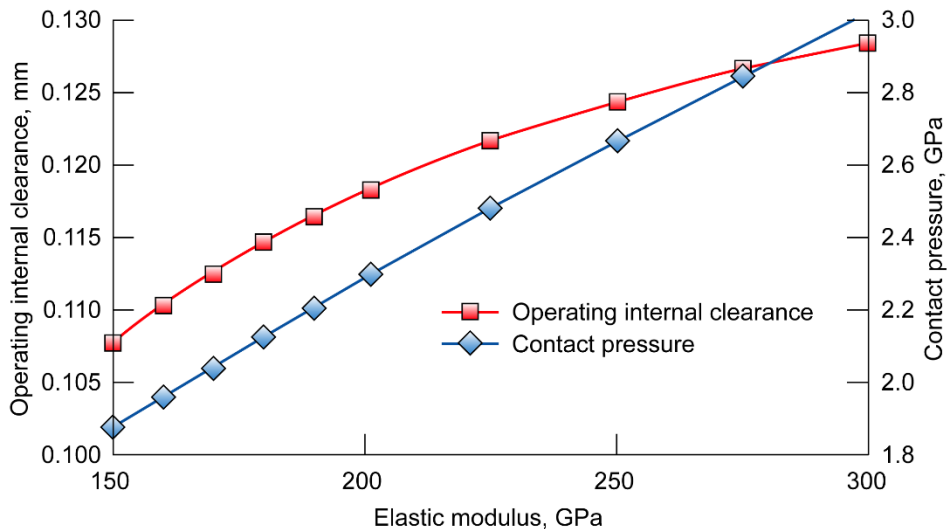


Figure 9.—Variations in operating internal clearance and maximum contact stress in the bearing as a function of elastic modulus for the test ball bearing at room temperature with a thrust load of 25 000 N and operating speed of 12 000 rpm.

# Summary of Results

The bearing dynamics computer code ADORE (Ref. 31), which is based on the bearing dynamic analysis of Jones (Ref. 19), was used to develop generalized expressions for both the Lundberg-Palmgren and the Zaretsky rolling-element fatigue life models. Published experimental data for a jet engine bearing (Refs. 32 to 34) were used to derive the constants in the life equation. Model predictions were then compared with another set of high-speed turbine engine bearing life data (Ref. 36). Subsurface fatigue life models for ball bearings were generalized to permit parametric elastic property variation on subsurface shearing stress. Model constants were derived by least-squared regression analysis of available experimental life data.

Generalized life equations for both the Lundberg-Palmgren and the Zaretsky models in which, unlike the Lundberg-Palmgren model, the shearing stress-life exponent in the fundamental life equation is independent of scatter in life data and life dependence on depth of critical failure stress is eliminated, were developed and incorporated into the commercially available bearing dynamics code, ADORE. The following results were obtained:

1. For AISI 52100 steel the updated Lundberg-Palmgren model shows identical results when compared with the original Lundberg-Palmgren formulation. As the elastic properties deviate from those of AISI 52100 the predicted lives between the original and updated Lundberg-Palmgren models begin to deviate.

2. For the VIM−VAR AISI M−50 bearing steel at a temperature of 492 K (425 °F), where a reduction in elastic modulus of as much as 18 percent has been reported, the predicted $L_{10}$ life of a typical 3 million DN gas turbine engine bearing with both the updated Lundberg-Palmgren and the Zaretsky models is in complete agreement with experimental data. The life calculated by the original Lundberg-Palmgren model is about three times lower.

3. Parametric evaluation of the Lundberg-Palmgren and Zaretsky models demonstrates that at light loads the Lundberg-Palmgren models significantly underestimates the bearing life. As the applied load increases, the Zaretsky model shows a much faster drop off in life in comparison to the Lundberg-Palmgren model. Such a behavior is a direct result of the ball bearing load-life exponent of 4 in the Zaretsky model in comparison to 3 in the Lundberg-Palmgren model.

# Appendix A.—Effect of Varying the Weibull Modulus *m* and Shear-Stress Exponent *c* on the Load-Life Exponent *p* and the Hertz Stress Life Exponent *n*—A Sensitivity Study

## Lundberg-Palmgren Model

In 1947, Lundberg and Palmgren (Ref. 4) applied the Weibull analysis to the prediction of rolling-element bearing fatigue life. In order to better match the values of the Hertz stress-life exponent *n* and the load-life exponent *p* with experimentally determined values from pre-1940 tests on air-melt steel bearings, they introduced another variable, the depth to the critical shearing stress *z* to the *h* power where $f(N)$ in Equation (1) of the text can be expressed using the product law of reliabilities as

$$\ln\frac{1}{S} = \int_V f(N)dV \tag{A1}$$

and where

$$f(N) \propto \frac{\tau^c N^m}{z^h}\ln\frac{1}{0.90} \tag{A2}$$

The rationale for introducing $z^h$ was that it took a finite time period for a crack to propagate at a distance from the depth of the critical shearing to the rolling surface. Lundberg and Palmgren assumed that the time for crack propagation was a function of $z^h$.

Applying Equation (A2) to (A1),

$$N \propto \left(\frac{1}{\tau}\right)^{c/m}\left(\frac{1}{V}\right)^{1/m}(z)^{h/m}\left(\ln\frac{1}{S}\Big/\ln\frac{1}{0.90}\right)^{1/m} \tag{A3}$$

where *N* is the life in stress cycles for an arbitrary reliability *S*.

For their critical shearing stress, Lundberg and Palmgren chose the orthogonal shearing stress $\tau_o$. From Hertz theory (Jones) (Ref. 40),

$$z \propto p_H \tag{A4a}$$

$$\tau \propto p_H \tag{A4b}$$

$$V \propto d \times p_H^2 \tag{A4c}$$

For point contact, substituting Equations (A4a), (A4b) and (A4c) in Equation (A3), and denoting life as *L* (millions of revolutions) instead on *N* (number of stress cycles), equation (A3) may be written as

$$L \propto \left(\frac{1}{p_H}\right)^{c/m}\left(\frac{1}{p_H^2}\right)^{1/m}(p_H)^{h/m} \propto \frac{1}{p_H^n} \tag{A5}$$

From Zaretsky, et al. (Ref. 41), solving for the value of the exponent $n$ for point contact (ball on a raceway) from Equation (A5) gives

$$n = \frac{c + 2 - h}{m} \qquad \text{(A6)}$$

Lundberg and Palmgren (Ref. 4), using values of 1.11 for $m$, $c = 10.33$, and $h = 2.33$, from Equation (A6) for point contact

$$n = \frac{10.33 + 2 - 2.33}{1.11} = 9 \qquad \text{(A7)}$$

From Hertz theory (Ref. 40) for point contact

$$p_H \propto Q^{1/3} \qquad \text{(A8)}$$

Equation (A5) becomes

$$L \propto \frac{1}{p_H^n} \propto \frac{1}{Q^p} \qquad \text{(A9)}$$

From Equations (A8) and (A9) for point contact, where $n = 9$,

$$p = \frac{n}{3} = \frac{9}{3} = 3 \qquad \text{(A10)}$$

These values of $n$ and $p$ for point contact correlated to the then-existing rolling-element bearing database, which can be assumed were generated in Sweden with air-melt-processed AISI 52100 steel prior to the Second World War. In their 1947 paper (Ref. 4), Lundberg and Palmgren stated that their database reflected a variation in the Weibull modulus $m$ between 1.1 and 2.1 and a load-life exponent $p$ equal 3. It is not intuitively obvious how they selected values for the shear-stress-life exponent $c$ and the exponent $h$ related to the depth to critical shearing stress $z$. It was assumed by us that these values were made to fit their existing values by trial and error.

To determine the sensitivity of the Weibull modulus $m$ on the load-life exponent $p$ and the Hertz stress-life exponent $n$, the Weibull modulus $m$ in Equation (A6) was assumed to be 1, 1.5, and 2 with $c = 10.33$, and $h = 2.33$. The resultant values for the load-life exponent $p$ were 3.33, 2.22, and 1.66, respectively. For values the Hertz stress-life exponent $n$ was 10, 6.67, and 5, respectively. Clearly, a Weibull modulus of approximately 1 best reflects the Lundberg-Palmgren database.

The shear-stress-life exponent $c$ in Equation (A7) was varied and assumed to be 9, 10, and 11 for a Weibull modulus $m$ equal to 1. The resultant values for the load-life exponent $p$ were 2.89, 3.22, and 3.56. For these values, the Hertz stress-life exponent $n$ was 8.67, 9.67, and 10.67. Clearly, for the Lundberg-Palmgren model, life can be more sensitive to variations in the Weibull modulus $m$ than to variations in the shear-stress-life exponent $c$.

# Zaretsky Model

Applying the Weibull distribution function with reliability $S_1$, the function $f(N)$ expressed by an arbitrary life $N$ can be developed by Equation (A11).

$$f(N) = \ln\frac{1}{S_1} = \left(\frac{N}{N_{63}}\right)^m = \left(\frac{N}{N_{10}}\right)^m \ln\frac{1}{0.9} \qquad (A11)$$

Where $N_{63}$ is the characteristic life at which 63.2 percent of the bearings are expected to fail. The conventional $f(N)$ functions are almost always lacking in the $(\ln 1/0.9)$ term.

Further, introducing the stress-life relation $N_{10} \propto \tau^{-c}$ into Equation (A11) and then substituting this into Equation (A1), the life $N$ in stress cycles is given by Equation (A12). Here $a_1$ denotes a reliability factor.

$$N \propto a_1\left(\frac{1}{\tau}\right)^c\left(\frac{1}{V}\right)^{1/m}, \quad a_1 = \left(\frac{\ln S}{\ln 0.9}\right)^{1/m} \qquad (A12)$$

For critical shearing stress $\tau$, Zaretsky (Ref. 23) chose the maximum shearing stress $\tau_m$.

In the case where $S = 0.9$, $a_1 = 1$ and $L = N/u$ denotes life in units of $10^6$ rev unit for one rotating ring, then Equation (A12) can be written as

$$L \propto \left(\frac{1}{\tau}\right)^c\left(\frac{1}{V}\right)^{1/m} \propto \frac{1}{p_H^n} \qquad (A13)$$

From Zaretsky, et al. (Ref. 41), solving for the value of the Hertz stress-life exponent $n$, for point contact from Equation (A13) gives

$$n = c + \frac{2}{m} \qquad (A14a)$$

and from Equation (A10)

$$p = \frac{n}{3} \qquad (A14b)$$

If Lundberg-Palmgren values are assumed, where $c = 10.33$ and $m = 1.11$, $p = 4.04$ and $n = 12.13$ for point contact.

To determine the sensitivity of the Weibull modulus $m$ on the load-life exponent $p$ and the Hertz stress-life exponent $n$, the Weibull modulus $m$ in Equation (A14) was assumed to be 1, 1.5, and 2 with $c = 10.33$. The resultant values for the load-life exponent $p$ were 4.11, 3.89, and 3.78, respectively. The values for the Hertz stress-life exponent $n$ were 12.33, 11.66, and 11.33, respectively. The values for $p$ and $n$ are less sensitive to variations in the Weibull modulus $m$ for the Zaretsky model than for the Lundberg-Palmgren model.

The shear-stress-life exponent $c$ in Equation (A14a) was varied and assumed to be 9, 10, and 11 for a Weibull modulus $m$ equal to 1. The resultant values for the load-life exponent $p$ were 3.67, 4, and 4.33. For values for the Hertz stress-life exponent $n$, the values were 11, 12, and 13. As with the Weibull modulus $m$, the variation in the values for $p$ and $n$ are within those of the existing database (Ref. 42).

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