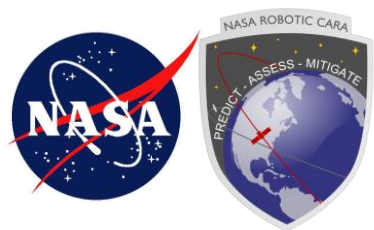


Conjunction Assessment Risk Analysis



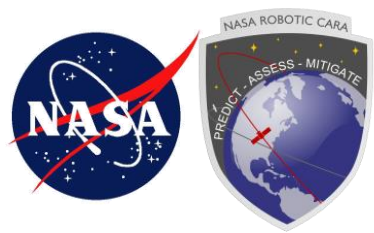
ASW Covariance Introduction and Formation

M.D. Hejduk | 13 APR 2016



OD Parameters Generated by ASW Solutions

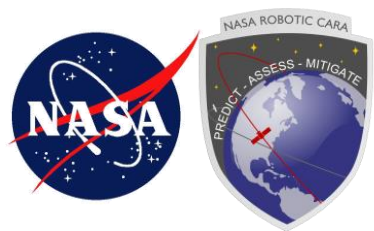
- **Solved for: State parameters**
 - Six parameters needed to determine 3-d state fully
 - Cartesian: three position and three velocity parameters in orthogonal system
 - Element: six orbital elements that describe the geometry of the orbit
- **Solved for: Non-conservative force parameters**
 - Ballistic coefficient ($C_D A/m$); describes vulnerability of spacecraft state to atmospheric drag
 - Solar radiation pressure (SRP) coefficient ($C_R A/m$); describes vulnerability of spacecraft state to visible light momentum from sun
- **Considered: ballistic coefficient and SRP consider parameter**
 - Not solved for but “considered” as part of the solution
 - Derived from information outside of the OD itself
- **Covariance matrix includes variances/covariances for all solved-for parameters, with potential alteration by consider parameters**



Covariance Matrix Construction: Symbolic Example

- Three estimated parameters (a, b, and c)
- Variances of each along diagonal
- Off-diagonal terms the product of two standard deviations and the correlation coefficient (ρ); matrix is symmetric

	a	b	c	...
a	σ_a^2	$\rho_{ab}\sigma_a\sigma_b$	$\rho_{ac}\sigma_a\sigma_c$...
b	$\rho_{ab}\sigma_a\sigma_b$	σ_b^2	$\rho_{bc}\sigma_b\sigma_c$...
c	$\rho_{ac}\sigma_a\sigma_c$	$\rho_{bc}\sigma_b\sigma_c$	σ_c^2	...
...



Example Covariance from CDM

- **8 x 8 matrix typical of most ASW updates**

- Some orbit regimes not suited to solution for both drag and SRP; these covariances 7 x 7

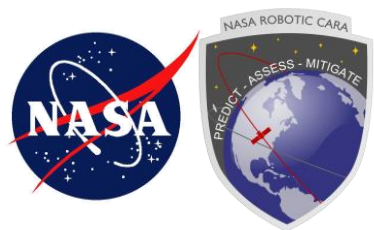
- **Mix of different units often creates poorly conditioned matrices**

- Condition number of matrix at right is $9.8E+11$ —terrible!

- **Often better numerically (and more intuitive) to separate matrix into sections**

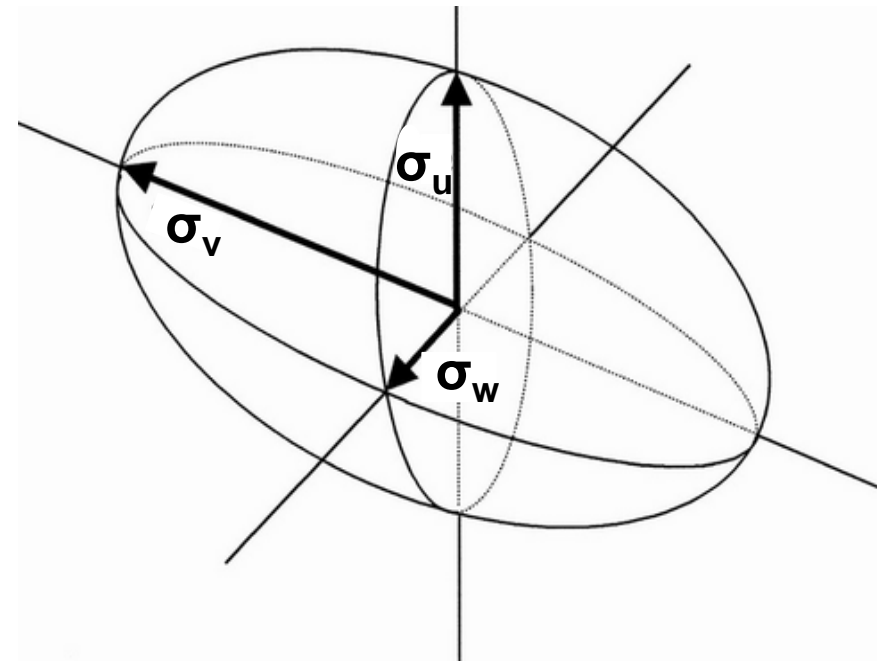
- **First 3 x 3 portion (amber) is *position* covariance—often considered separately**

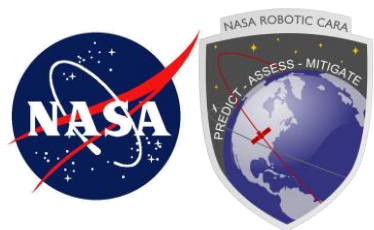
	U (m)	V (m)	W (m)	Udot (m/s)	Vdot (m/s)	Wdot (m/s)	B (m ² /kg)	AGOM (m ² /kg)
U	6.84E+01	-2.73E+02	6.38E+00	2.76E-01	-7.14E-02	8.75E-03	-3.83E-02	-3.83E-02
V	-2.73E+02	1.10E+05	3.23E+01	-1.17E+02	-8.99E-02	2.51E-02	-1.28E-01	-1.28E-01
W	6.38E+00	3.23E+01	4.47E+00	-3.26E-02	-6.83E-03	1.81E-03	-3.73E-03	-3.73E-03
Udot	2.76E-01	-1.17E+02	-3.26E-02	1.24E-01	1.10E-04	-2.47E-05	1.46E-04	1.46E-04
Vdot	-7.14E-02	-8.99E-02	-6.83E-03	1.10E-04	7.57E-05	-9.39E-06	4.10E-05	4.10E-05
Wdot	8.75E-03	2.51E-02	1.81E-03	-2.47E-05	-9.39E-06	2.06E-05	-4.39E-06	-4.39E-06
B	-5.07E-03	1.30E+00	4.34E-05	-1.38E-03	7.97E-07	7.26E-07	1.64E-05	-6.28E-07
AGOM	-3.83E-02	-1.28E-01	-3.73E-03	1.46E-04	4.10E-05	-4.39E-06	-6.28E-07	2.31E-05



Position Covariance Ellipse

- **Position covariance defines an “error ellipsoid”**
 - Placed at predicted satellite position
 - Square root of variance in each direction defines each semi-major axis (UVW system used here)
 - Off-diagonal terms rotate the ellipse from the nominal position shown
- **Ellipse of a certain “sigma” value contains a given percentage of the expected data points**
 - 1- σ : 19.9%
 - 2- σ : 73.9%
 - 3- σ : 97.1%
 - Note how much lower these are than the univariate normal percentage points





Batch Epoch Covariance Generation (1 of 2)

- **Batch least-squares update (ASW method) uses the following minimization equation**

- $dx = (A^TWA)^{-1}A^TWb$

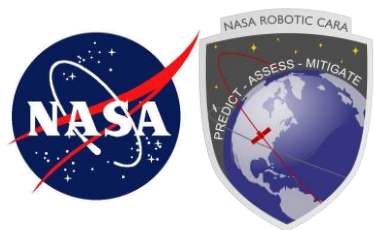
- dx is the vector of corrections to the state estimate
- A is the time-enabled partial derivative matrix, used to map the residuals into state-space
- W is the “weighting” matrix that provides relative weights of observation quality (usually $1/\sigma$, where σ is the standard deviation generated by the sensor calibration process)
- b is the vector of residuals (observations – predictions from existing state estimate)

- **Covariance is the collected term $(A^TWA)^{-1}$**

- A the product of two partial derivative matrices:

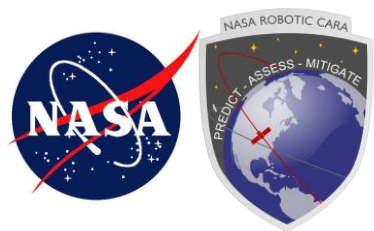
- $A = \frac{\partial(obs)}{\partial X_0} = \frac{\partial(obs)}{\partial X} \frac{\partial X}{\partial X_0}$

- First term: partial derivatives of observations with respect to state at obs time
- Second term: partial derivatives of state at obs time with respect to epoch state



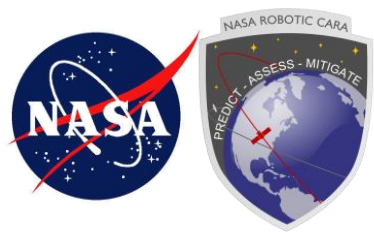
Batch Epoch Covariance Generation (2 of 2)

- **Formulated this way, this covariance matrix is called an *a priori* covariance**
 - A does not contain actual residuals, only transformational partial derivatives
 - So $(A^TWA)^{-1}$ is a function only of the amount of tracking, times of tracks, and sensor calibration relative weights among those tracks
 - Not a function of the actual residuals from the correction
- **Limitations of *a priori* covariance**
 - Does not account well for unmodeled errors, such as transient atmospheric density prediction errors
 - Because not examining actual fit residuals
 - W-matrix only as good as sensor calibration process
 - Principal weakness of present process, but expected to be improved eventually with JSpOC Mission System (JMS) upgrades



ASW Covariance Propagation

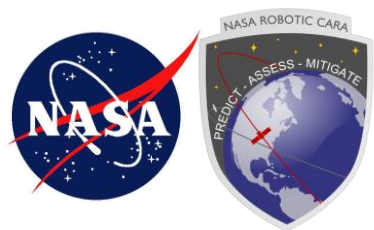
- **Covariance in VCM is virginal (unaltered) covariance**
- **When propagating VCM covariance, the propagator**
 - Scales the covariance by the weighted RMS if it is greater than unity
 - $C^* = C * WRMS^2$
 - Was an early attempt to improve covariance realism; not clear this is still a good idea
 - Applies the consider parameter to the ballistic coefficient variance
 - $C^*(7,7) = C^*(7,7) + Cpd^2$
 - More later on how this value is determined
 - May apply a consider parameter to the solar radiation pressure variance
 - $C^*(9,9) = C^*(9,9) + Cps^2$
 - Presently not used (Cps set to 0)
 - Propagates the altered covariance using linearized dynamics
 - $\Phi * C^{**} * \Phi^T$
 - Converts propagated matrix from equinoctial to Cartesian coordinates



Altered Covariance Positions

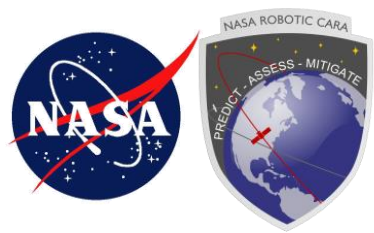
- Ballistic coefficient consider parameter (DCP) applied to ballistic coefficient variance (orange)
- If used, solar radiation pressure consider parameter applied to solar radiation pressure variance (purple)

	U (m)	V (m)	W (m)	Udot (m/s)	Vdot (m/s)	Wdot (m/s)	B (m ² /kg)	AGOM (m ² /kg)
U	6.84E+01	-2.73E+02	6.38E+00	2.76E-01	-7.14E-02	8.75E-03	-3.83E-02	-3.83E-02
V	-2.73E+02	1.10E+05	3.23E+01	-1.17E+02	-8.99E-02	2.51E-02	-1.28E-01	-1.28E-01
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Wdot	8.75E-03	2.51E-02	1.81E-03	-2.47E-05	-9.39E-06	2.06E-05	-4.39E-06	-4.39E-06
B	-5.07E-03	1.30E+00	4.34E-05	-1.38E-03	7.97E-07	7.26E-07	1.64E-05	-6.28E-07
AGOM	-3.83E-02	-1.28E-01	-3.73E-03	1.46E-04	4.10E-05	-4.39E-06	-6.28E-07	2.31E-05



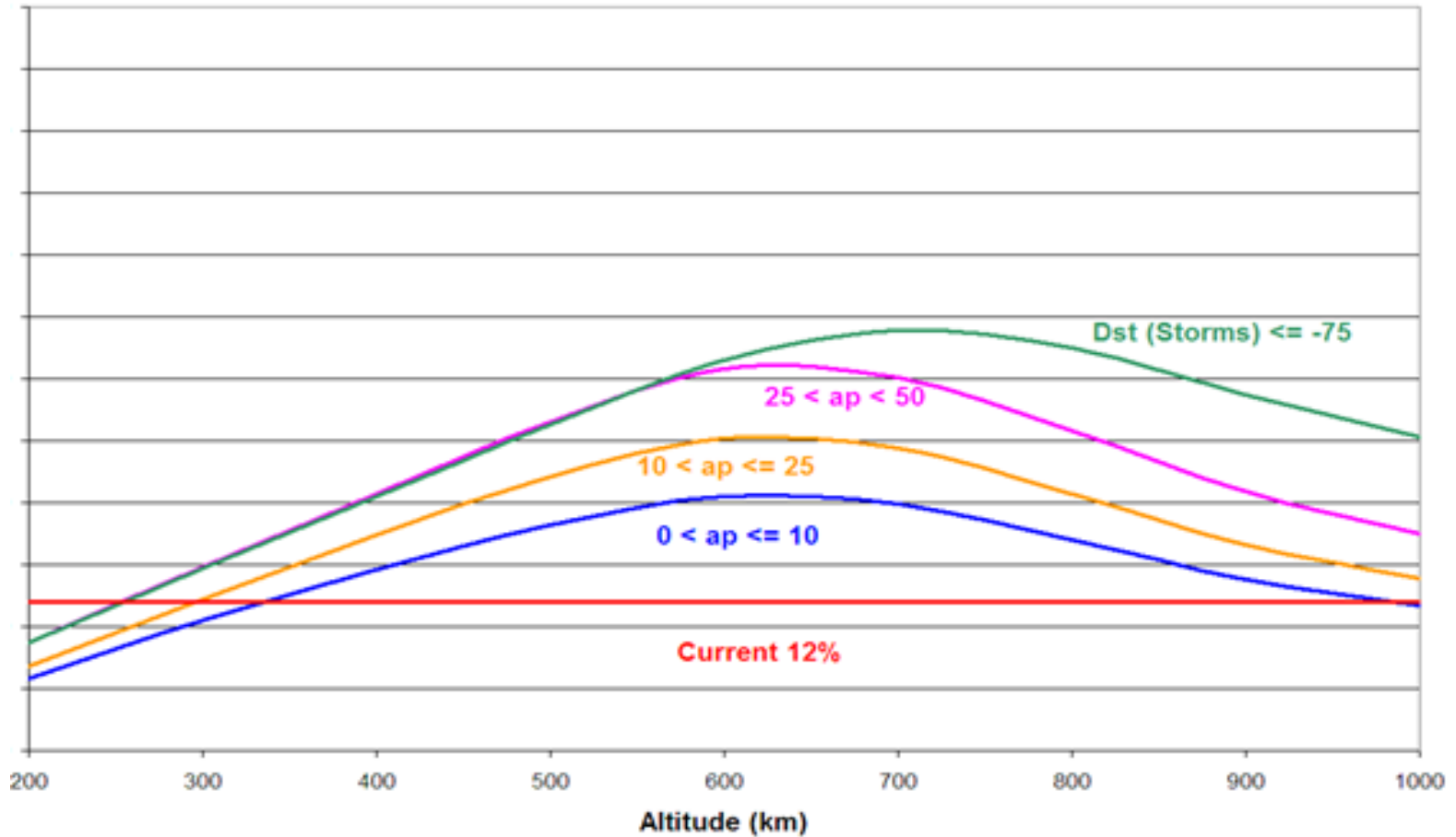
Dynamic Consider Parameter (DCP)

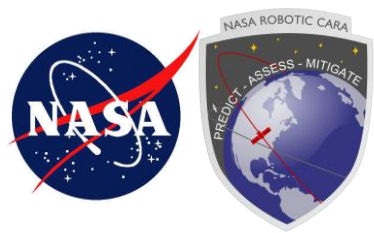
- **Specifies global error in the atmospheric density forecast**
 - Parameterizes percent RMS error in terms of
 - Satellite height (perigee altitude)
 - Geomagnetic activity (a_p and Dst)
 - Density forecast error combination of solar/geomagnetic indices and DCA
 - Directly compared numerous forecast densities to actual density
 - Discretized heights from 200 km to 1000 km averaged over lat / lon
 - Sample high, low, and medium solar cycles (2001, 2005, 2013)
 - Found most variation parameterizable via a_p conditions (versus F_{10})
 - Functions optimized for 3-day predictions—this is the tuning point!
- **Determines satellite-specific frontal area variation in prediction**
 - Quantifies ballistic coefficient RMS error through satellite histories
 - Looks back in time up to a year in most cases (upfront preprocessing)
 - Ascertains error / target for 3-day predictions (accounting for time-lags)
- **Combine the two uncertainty components to obtain DCP value**
 - Additive in variance sense as the root sum of squares



B Consider Parameter Values

dRho STD Predict ap & Dst





DCP Components RMS Uncertainty

