High Resolution Image Reconstruction from Projection of Low Resolution Images Differing in Sub-pixel Shifts

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What is Super Resolution (SR)?

- A High Resolution (HR) image computed from several observed low-resolution (LR) images that differ in sub-pixel shifts and rotation is called Super Resolution (SR).

- SR Increases high frequency components and removing the degradations caused by the imaging process of the low resolution camera.

Why Super Resolution?

- Higher spatial resolution gives more details
- Spatial Resolutions cannot be increased indefinitely by hardware
- The limitations of spatial resolution come from
    - Diffraction of optical system
    - Signal to noise ratio of the sensor system
    - Spacecraft orbits in space-borne imaging

Approaches to Computing SR

- Frequency Domain Approach
- Spatial Domain Approaches
  - Non-Iterative approach: Interpolation and Restoration
  - Statistical Approaches:
    - Maximum A posterior (MAP)
    - Maximum Likelihood (MLE)
- Wavelet-Based construction
- Set Theoretic Methods

Spatial Domain Approaches to SR

- Iterative Back Projection (IBP)
- Interpolations
  - Nearest Neighbor (NN)
  - Inverse Distance Weighted (IDW) Interpolation
- Maximum Likelihood Estimation (MLE)
- Interpolation using Radial Basis Functions (RBF)

Imaging Model

- High Resolution (HR) Image
  - Let x be the HR image sought (in 1-D vector form scanned in lexicographical order)
- Low Resolution(LR) Images
  - Let y_1, y_2, .. y_k be LR images that differ in translation and rotation w.r.t image y1 (reference image)
  - Let Y be concatenation of y_1, y_2, .. y_k
- Each of the LR images (y_k) is given by
  - $y_k = D_k H_k F_k \, x + v_k$

    F_k : motion matrix

    H_k : Blurring matrix

    D_k: down sampling

    V_k : noise term

Imaging Model (2)

- The previous equation can be written as

  $\underline{y} = Mx + \underline{V}$

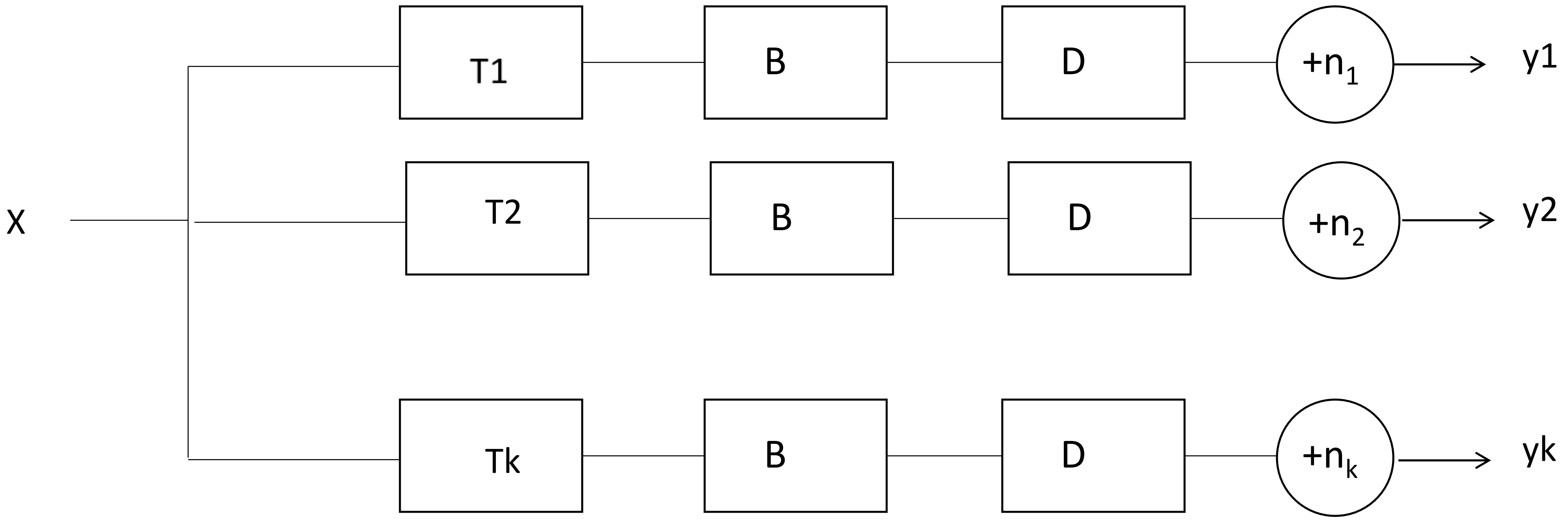
  - $M = [D_1H_1F_1\ D_2H_2F_2\ \ldots\ D_kH_kF_k]^t$
  - $\underline{y}$ is concatenation of $y_1, \ldots. y_k$
  - $\underline{V}$ concatenation of noise components $V_1 .. V_k$
  - Given y, to determine x; which is an ill-posed problem.
  - Also in imaging system the matrices used in the above equations are not known.

The Generative Model of LR Images

$y_i = T_i BD(x) + n_{i,}$

T is sub-pixel translation, B is blurring, and D down sample operators

n_i is noise



Spatial Domain Approach to SR

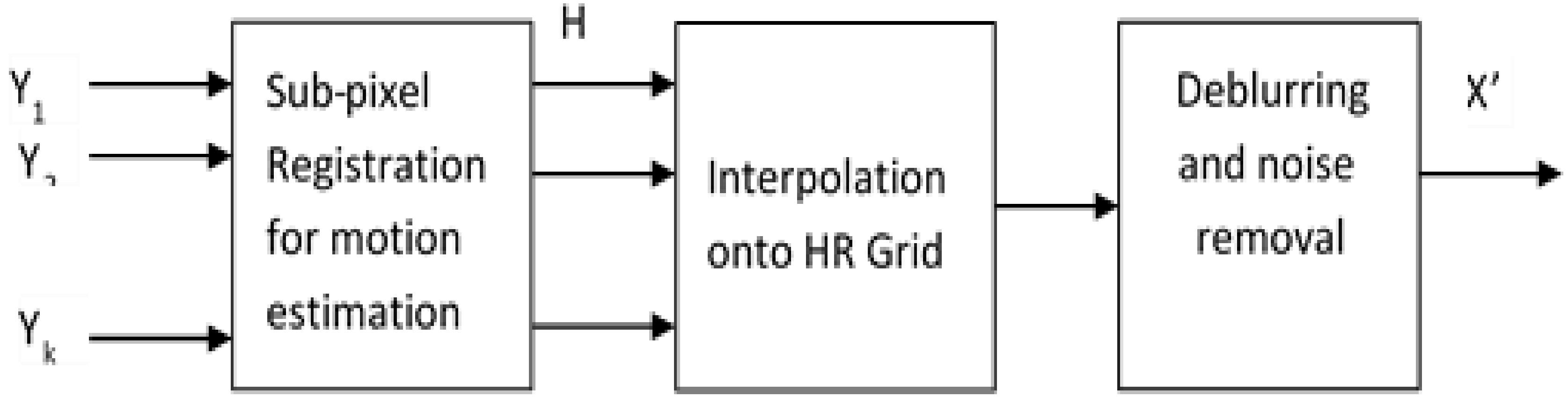


Figure 1. Block diagram reconstructing HR image from LR images

Sub-pixel Registration

- Uses Discrete Wavelet Transforms (DWT)
- Decomposes the Images into LR image and High Frequency Components in Horizontal and Vertical Directions (LL,LH,HL,HH) in multi-resolution fashion into a number of levels thus building a hierarchy low resolution images
- The decomposition is done for both (reference and input) images that are to be registered.
- At each level of decomposition features are extracted from low-frequency and high-frequency subbands for reference and input images and transformation function is computed using correlations.
- The transformation function is improved iteratively using images from the hierarchy.

Interpolation onto HR Grid

- Interpolation algorithms used to project the LR images onto HR
  - Nearest Neighbor (NN) Interpolation
  - Inverse Distance Weighted (IDW) Interpolation
  - Radial Basis Functions (RDF) Interpolation

Deblurring and Noise filter

- Simulation of LR images from Original Image includes a know Point Spread Function (PSF) and a noise component as shown in generative model LR images
- The reconstructed image deblurring is obtained by deconvolving using Weiner filter
- A low pass filter is used on the reconstructed SR image to filter out high frequency noise present in LR images

Statistical Methods in Computing SR image

- Maximum Likelihood Estimation (MLE)
  - In this method the SR image is estimated from a given set of LR images differing in sub-pixel shifts
  - $\mathrm{p}(\mathrm{x}|\mathrm{y1}, \mathrm{y2}..\mathrm{yk}) = \frac{p(y1, y2, ..yk|x)\, p(x)}{\int_x \mathrm{p}(\mathrm{y1}, \mathrm{y2}..\mathrm{yk}|\mathrm{x})\, p(x)}$
  - x is SR image, y1, y2, .. yk are LR images
  - Log Likelihood function L(x) = log($p(y1, y2, ..yk|x)\, p(x)$)
  - MLE assumes flat a prior p(x)
  - $x' = \underset{x}{argmax}\, L(x)$
  - Every pixel on HR grid is the one that maximizes the L(x)

Statistical Methods in Computing SR image (2)

- Maximum A Posterior (MAP) method
- $\mathrm{p(x|y1, y2..yk)} = \frac{p(y1, y2, ..yk|x)\, p(x)}{\int_x \mathrm{p(y1, y2..yk|x)}\, p(x)}$
- $x' = \underset{x}{argmax}\ \mathrm{p(y1, y2, ..yk|}x)p(x)$
- This method allows inducing priors in estimating x’
- The results of MLE and MAP are similar

Iterative Back Projection (IBP)

- Initial HR (x’)image is reconstructed from the reference LR image (y_1) by replicating pixels

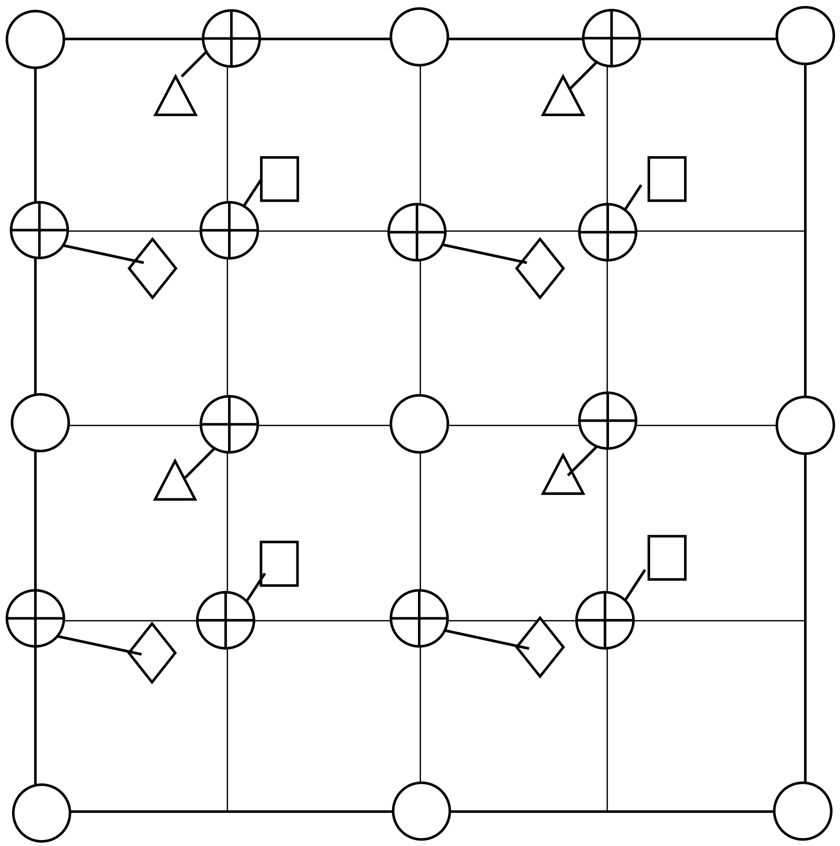
- From x’ the LR images are generated using the regenerative model (y'_1, y'_2, .. Y'_k)

- $x' = x' + \sum_{i_=2}^{k}(y_i - y'_i)$

Projection of LR images on HR Grid

- Assume the HR grid is 2n*2n where LR images are n*n
- Number of LR images differing in sub-pixel shift are 2^2
- HR grid kn*kn, requires at k^2 LR images for optimal reconstruction
- All LR images are assumed to differ in subpixel shifts that are different from each other.
- The subpixel shifts are global

Projection of LR images on HR Grid: NN Reconstruction



$x' = U^2(y1)$

U^2 is upsample operator by 2

$x' = U^2(y1)+\sum_{k=2}^{4} U^2(T_k(y_k)$

T_k Translation operator

Projection LR images on HR Grid using IDW

- Unlike in NN, all the LR images contribute to the estimation of every missing pixels of HR grid
- Pixel value depends on distance of the LR image pixels from the missing HR grid point by inverse distance
- $x' = U^2(y1) + \sum_{k=2}^{4} U^2(w_k\, y_k)$
- w_k is inverse distance of y_k from missing positions of x'
- $\sum_{k=2}^{4} w_k = 1$

Interpolation using Radial Basis Functions (RBF)

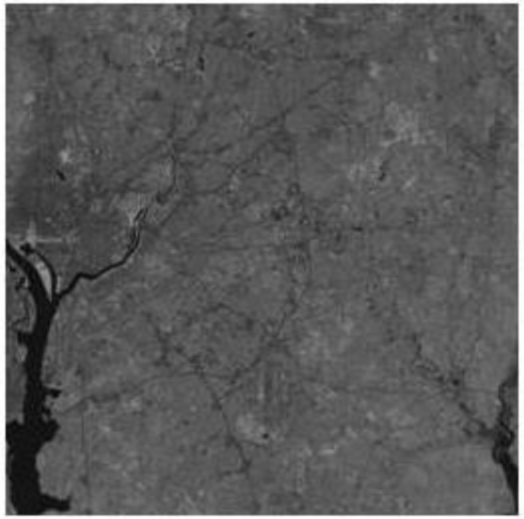
- A radial basis function (RBF) is a real valued function whose value depends on the distance from a given point x_i (x a variable not to be confused with image variable used for HR image).
- $\emptyset(x, x_i) = \emptyset(||x - x_i||)$
- When the distance function is Gaussian, the RBF is called Gaussian RBF
- $\emptyset(x, x_i) = e^{-(x-x_i)^2}$

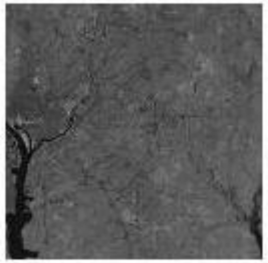
Interpolation using Radial Basis Functions (RBF)

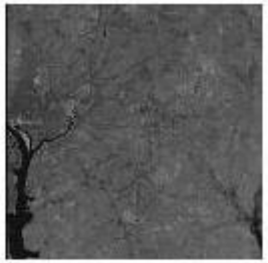
- The reference LR image(nxn) is used to fill up every other HR grid (2nx2n) pixels along the row and column.
- The remaining LR images are used to estimate the 3 missing cells of 2x2 HR grid as follows
- $x(i) = \sum_{k=2}^{4} \emptyset(i - T(k)) * y_k$
- $x(i) = \sum_{j=2}^{k} e^{-(||i-T(j)||)}\, y_j$
- In the above equation vector **i** is the position of the missing pixel from HR grid (0,1 for horizontal, 1,0 for vertical and 1,1 for diagonal)
- T(k) is the translation of LR image, y_k with respect to **i**.

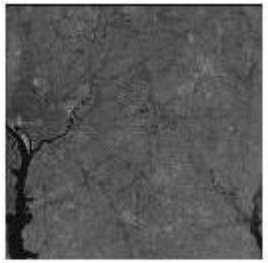
Experimental Results

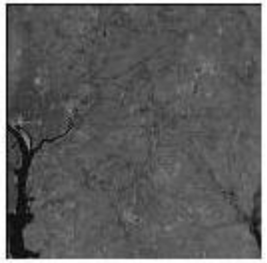
- Simulation of LR images from Chesapeake Bay Landsat Image



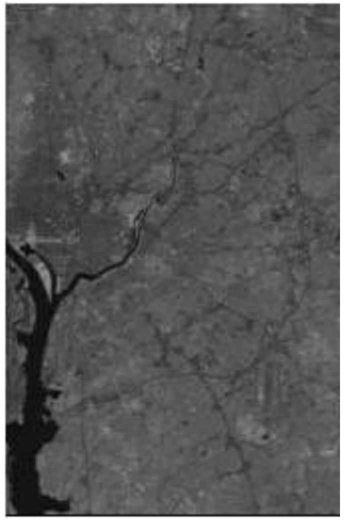




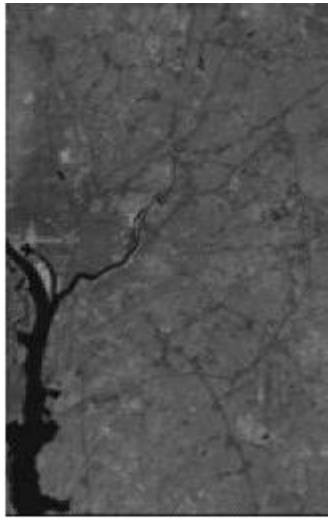




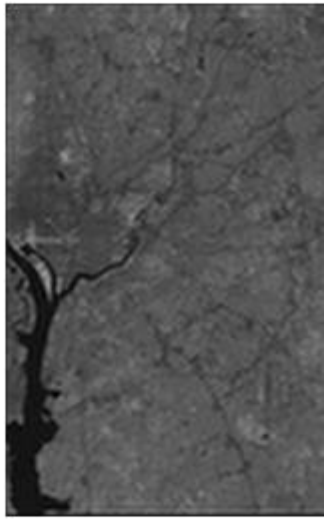
Results



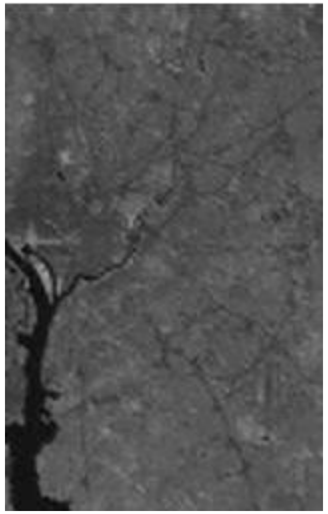
NN



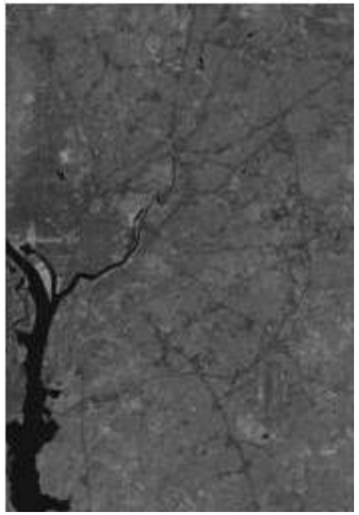
IDW



MLE



IBP



RBF

Results (SSE)

y2 : tr2 = 0.9, tc2 = 0.24

y3: tr3 = 0.12, tc3 = 0.82

y4: tr4 = 0.9, tc4 = 0.55

Algorithm	SSE
NN(Nearest Neighbor) interpolation	87799
IDW (inverse Distance Weighted)	128624
MLE (Maximum Likelihood)	167781
IBP (Iterative Back Projection)	220468
RBF (Radial Basis Function) Interpolation	77732

Results (SSE)

y2: tr2 = 082, tc2 = 0.08

y3: tr3 = 0.17, tc3 = 0.84

y4: tr4 = 0..64, tc4 = 0.56

Algorithm	SSE
NN(Nearest Neighbor) interpolation	886717
IDW (inverse Distance Weighted)	120953
MLE (Maximum Likelihood)	167938
IBP (Iterative Back Projection)	226565
RBF (Radial Basis Function) Interpolation	71946

Concluding Remarks

- Various algorithms are implemented to compare their performance in reconstructing HR image from a set of LR images differing subpixel shifts
- Interpolation using RBFs performed the best in most cases.
- HR reconstruction accuracy depends on the subpixel shifts and number of LR images
- We have experimented with four LR images to increase the spatial resolution by a factor of 2
- Further research is required to improve the spatial resolution by an arbitrary number with a given number of LR images