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Re-Thinking the Use of the OML Model in Electric-Sail Development

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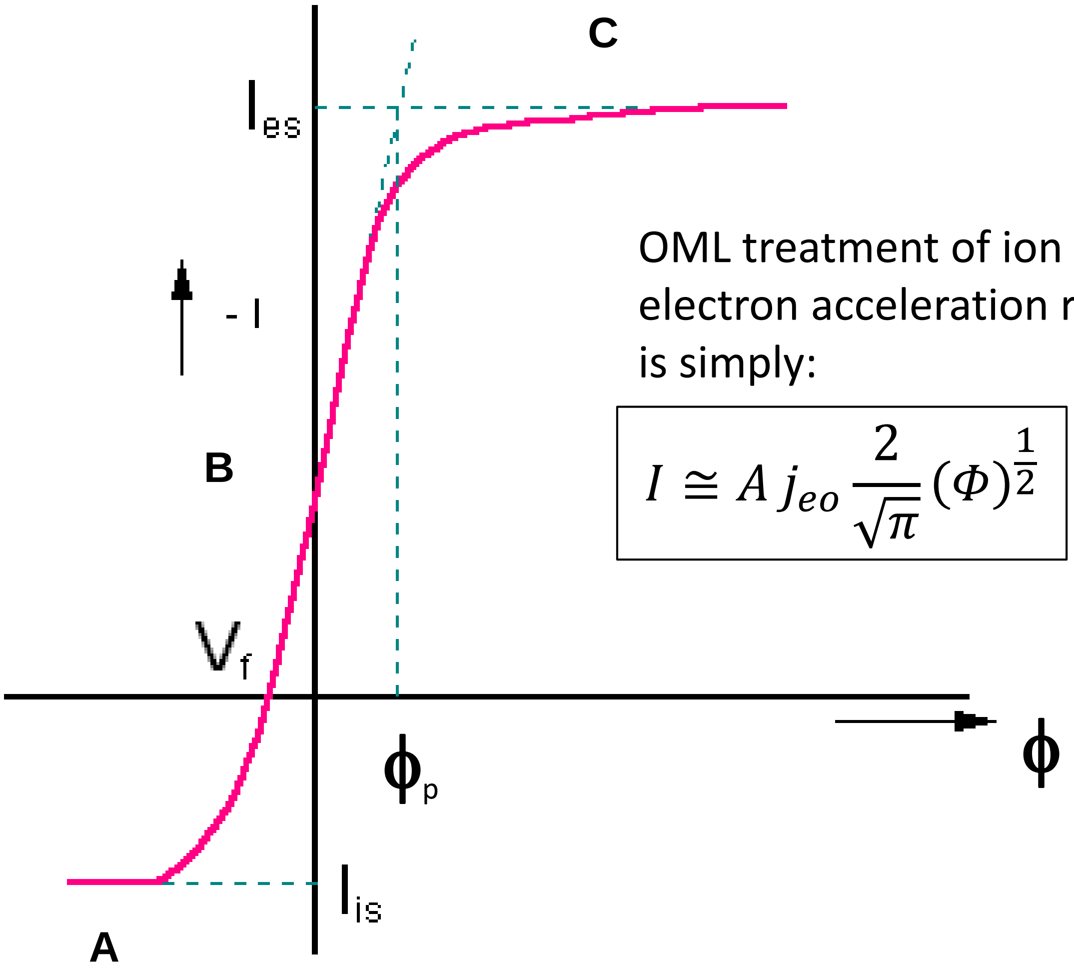
General Conditions on Application of the OML Model

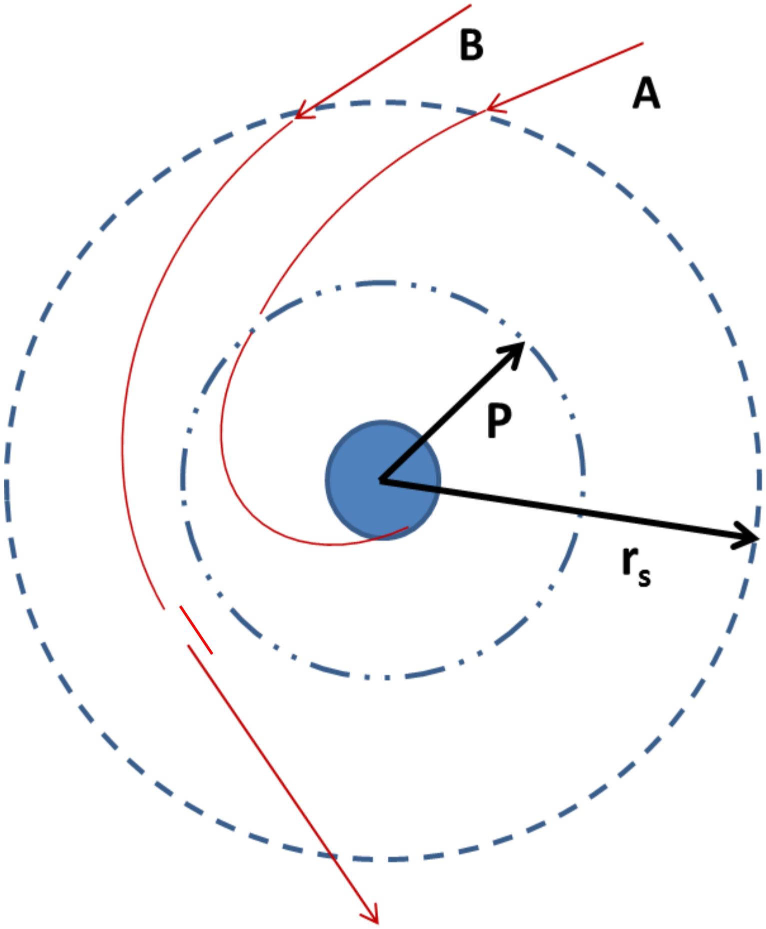


OML treatment of ion and electron acceleration regions is simply:

$$\boxed{I \cong A\, j_{eo} \frac{2}{\sqrt{\pi}} (\Phi)^{\frac{1}{2}}} \qquad (1)$$

Langmuir Probe i-v Characteristic

OML Assumed Electron Trajectories and Sheath E-Field Distribution



Assumptions:

(1) There exists an impact parameter, P, such that:

- All particles with trajectories for which $r_{min} > P$ are lost.
- All particles for which $r_{min} \geq P$ are collected.

(2) Electric field within the sheath is symmetrical and undistorted.

OML Derivation

$$f(u,v) = \left(\frac{m}{2\pi kT}\right) \exp\left(-\frac{m(u^2+v^2)}{2kT}\right)$$

$$j_{eo} = \frac{e\mathrm{n}}{4}\left(\frac{\mathrm{8kT}}{\mathrm{\pi m}}\right)^{\frac{1}{2}} .$$

$$P = (\frac{\mathrm{r}_s}{r_w}) erf\left(Z^{\frac{1}{2}}\right) e^{\Phi}\left[1 - \mathrm{erf}(\Phi + Z)^{\frac{1}{2}}\right]$$

where $\Phi = \left(\frac{e\phi_w}{kT}\right)$, and $z = \left(\frac{{r_s}^2}{{r_s}^2 - {r_w}^2}\right)\Phi$,

For $r_s >> r_w$ and $\Phi >> 1$,

$$\boxed{P \cong \frac{2}{\sqrt{\pi}}(\Phi + 1)^{\frac{1}{2}} \approx \frac{2}{\sqrt{\pi}}\Phi^{1/2}} \qquad (2)$$

OML Derivation (Cont'd)

$$I = A_w j_{eo} P$$

Inserting Eqn. (2) for P,

$$I \cong A_W \, j_{eo} \frac{2}{\sqrt{\pi}} (\Phi)^{\frac{1}{2}}$$

$$\boxed{I = \frac{en}{\pi} A_W \left(\frac{2\mathrm{e}\phi_w}{\mathrm{m}} \right)^{\frac{1}{2}}} \qquad (3)$$

where $A_w = 2\pi r_w L$

Eqn. (3) is identical to the standard OML representation given in Eqn. (1). Therefore, all assumptions and approximations used in arriving at Eqn. (3) apply to the OML model.

Assumptions and Approximations Made in the LMS Treatment

(1) Small body ($r_w << \lambda_D$).

(2) Quasi-static conditions ($V_{drift} << v_{th}$).

(3) Maxwellian distribution of the collected plasma component at the sheath boundary.

(4) Total absorption of the collected species that contact the electrode—particles are either collected or lost.

(5) Cylindrically symmetric E-field.

(6) No collisional effects on particle trajectories. Electron trajectories through the sheath region are determined totally by their initial velocity at the sheath boundary and the sheath electric field.

Potential Effects of the Assumptions and Approximations

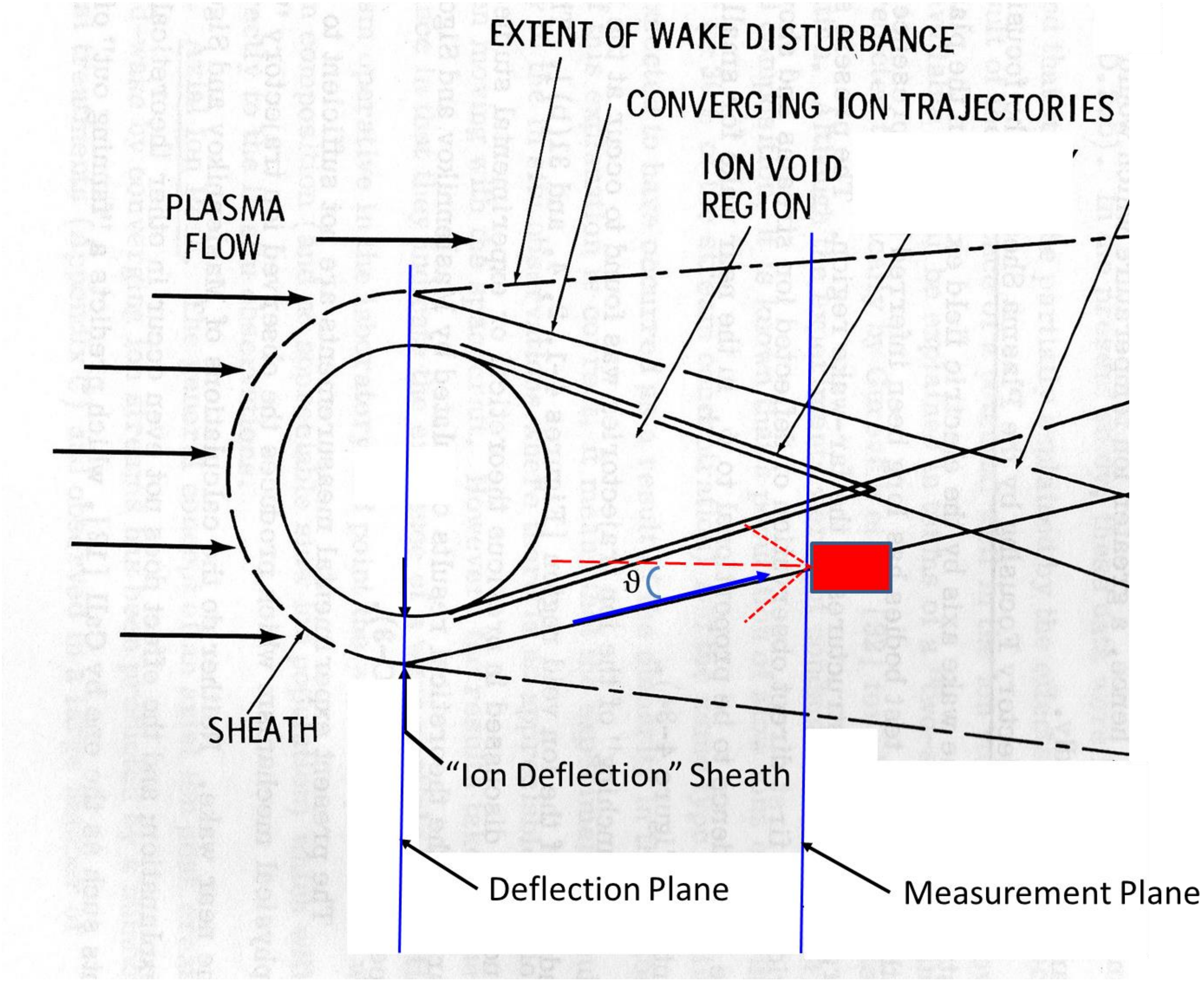
(1) Quasi-static conditions. (The Solar Wind has a very large drift velocity and on TSS-1R, Parker-Murphy erred by 200-300%—most probably because of orbital velocity effects.)

(2) Particles are either collected or. (Trapped particles occur in lab plasma sheaths and in the TSS-1R Satellite HV sheath.)

(3) Symmetric E-field. (Proton deflection and/or particle collisions or trapping can distort the field.)

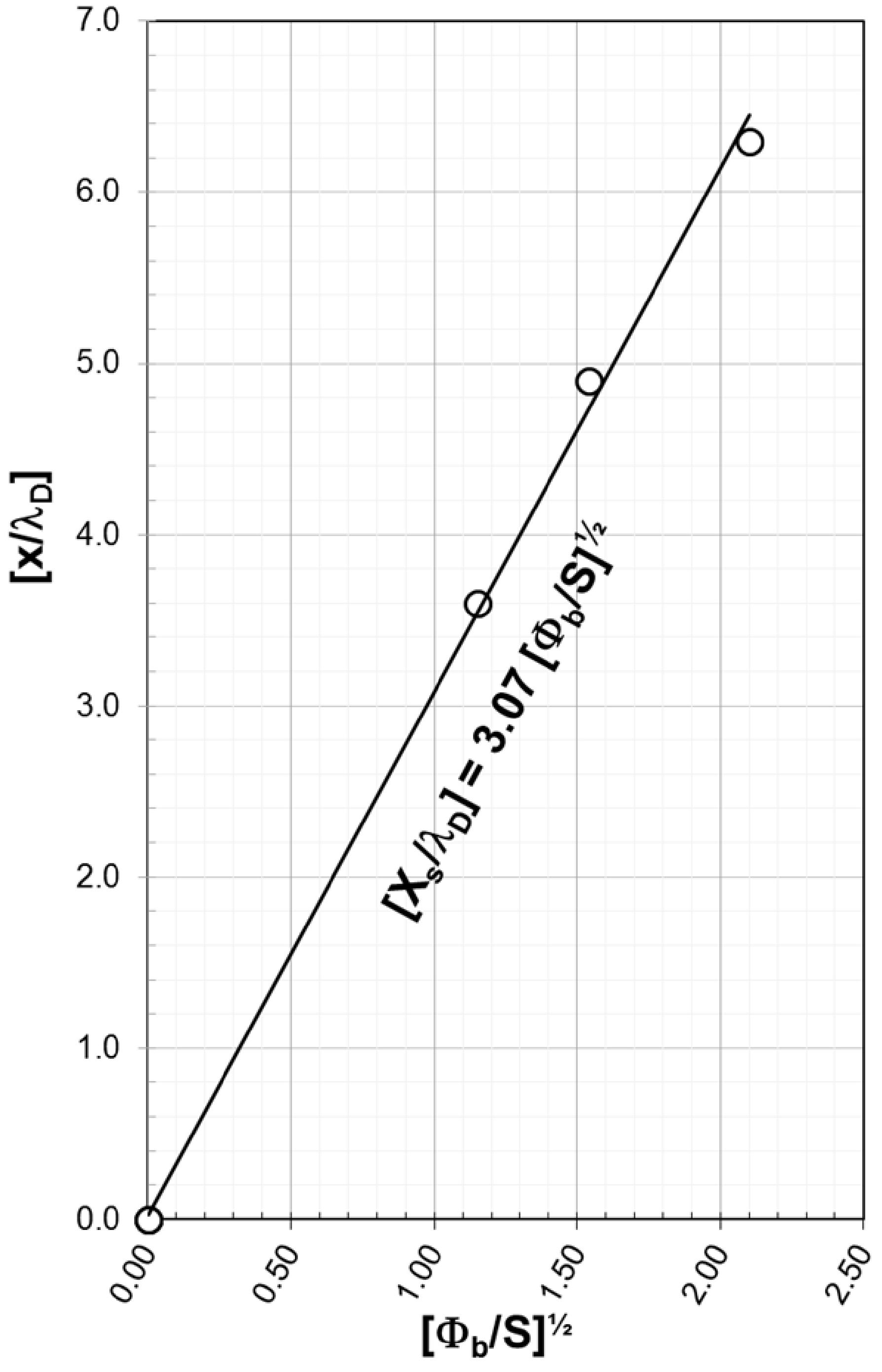
(4) No collisional effects on collected particle trajectories. This implies:
  - No photo-emission from surfaces.
  - No recombination of charged particles.
  - No charge exchange collisions.
  - No trapped particles.

Laboratory Observations of Body-Plasma Interactions



Regions of Disturbed Plasma Flow

Effective Sheath Width for Detectable Ion Deflection



Max Extent of Proton Deflection.

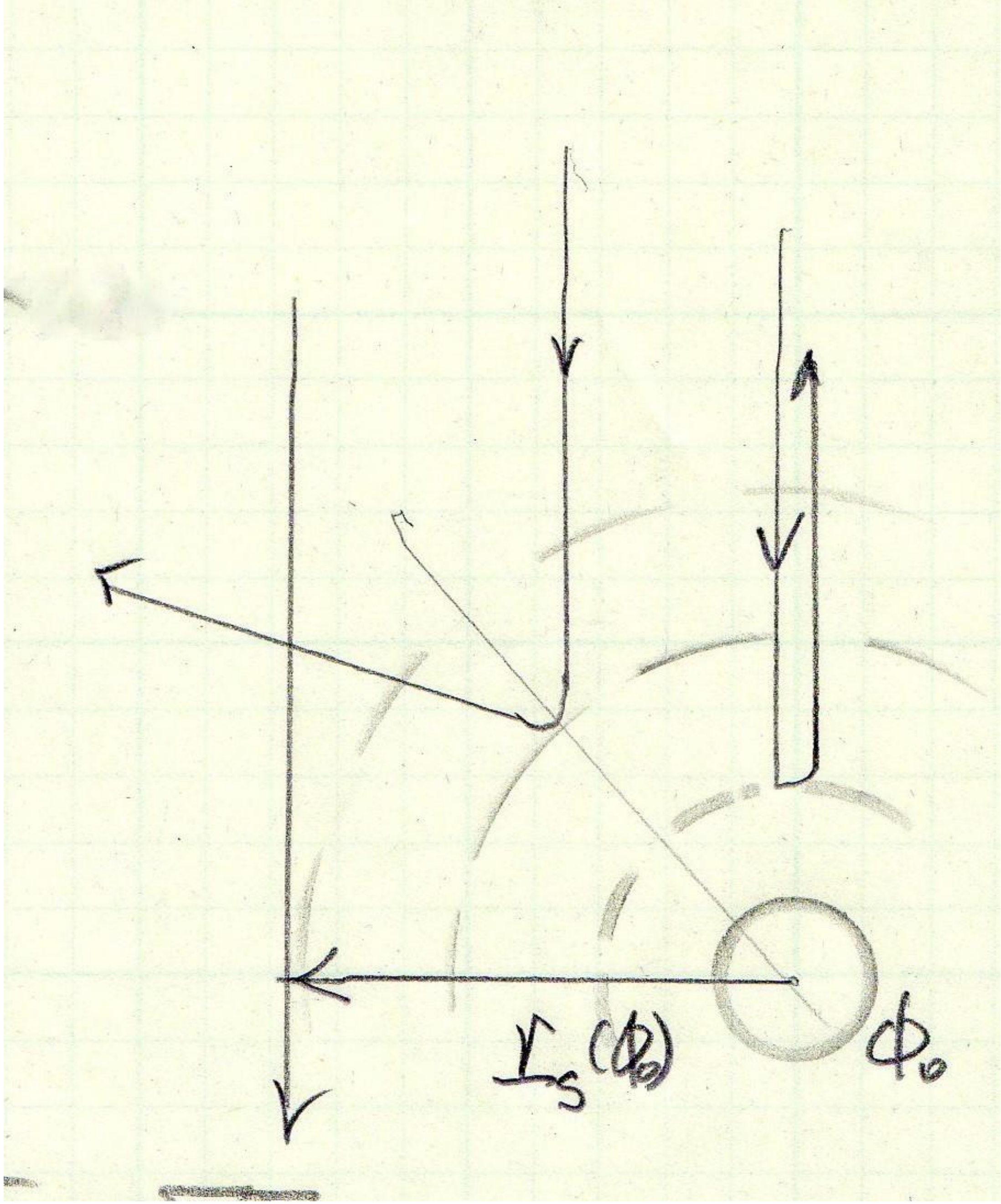
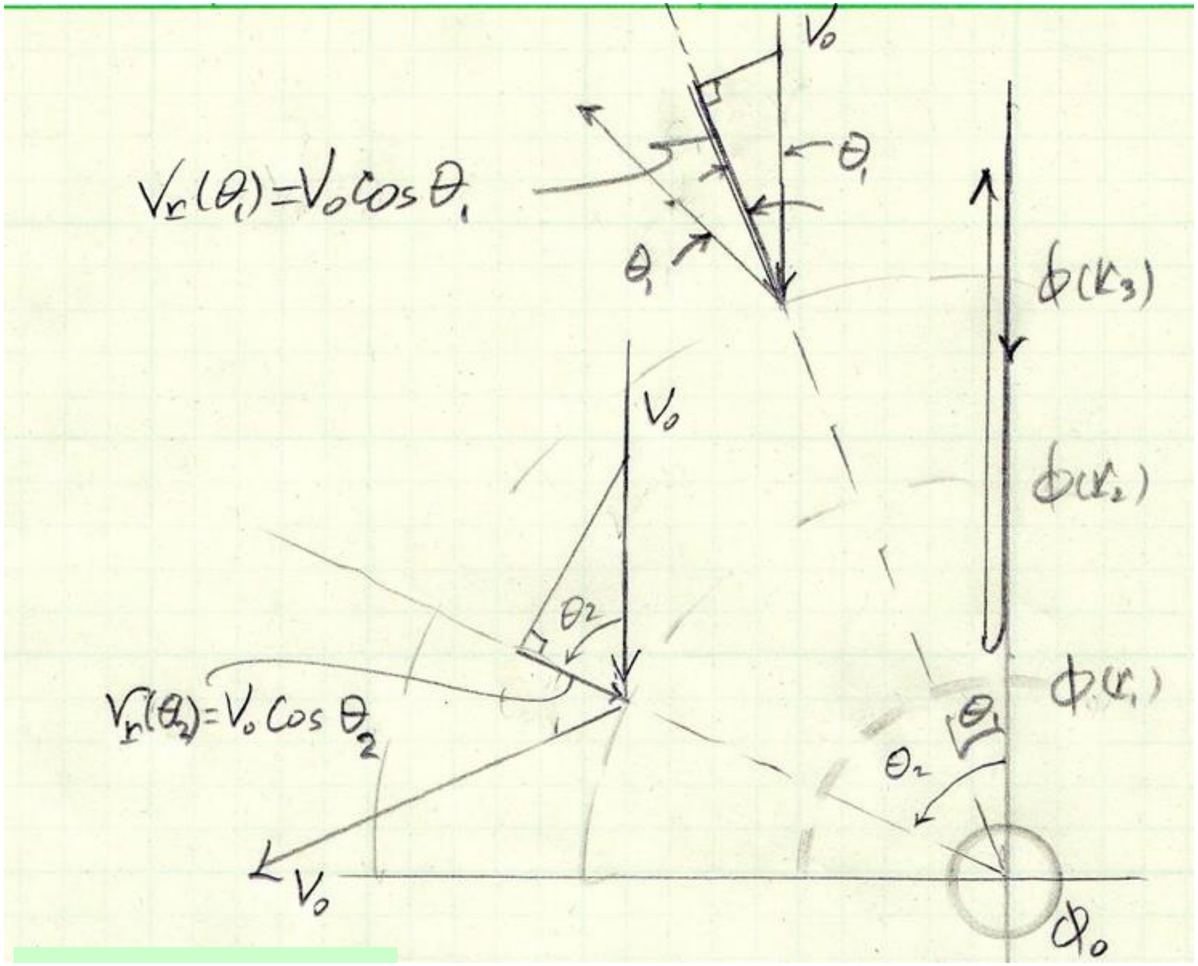


Figure 4.
Specular Proton Deflection Calculation



At any deflection point, $r^*(\vartheta,\phi)$,

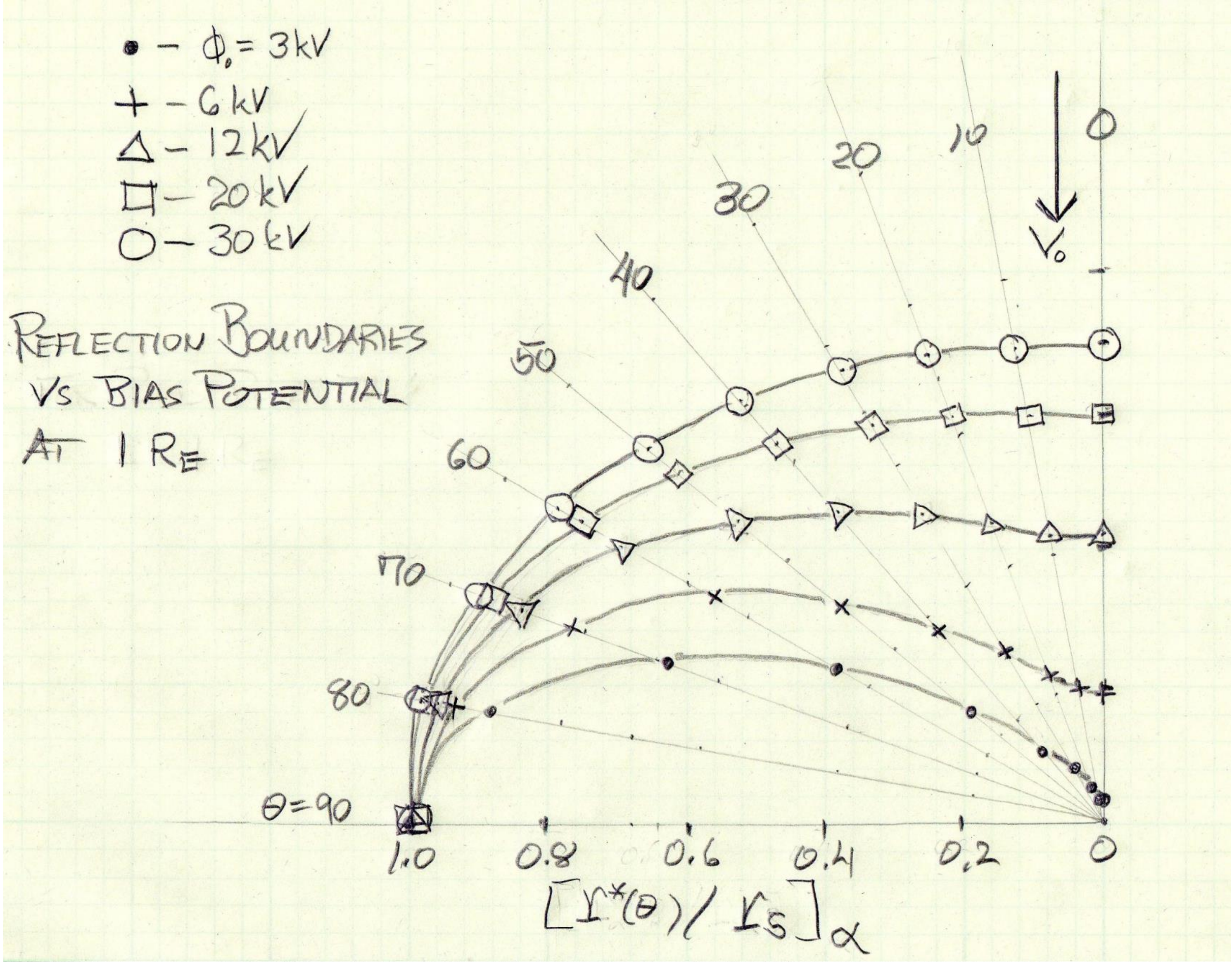
$e\phi(\mathrm{r}) = ½ m_p V_o^2$
$= ½ m_p V_o \cos\vartheta)^2$

and

$\phi(\mathrm{r}) = \phi_o \ln(r_s/r)/\ln(r_s/r_w)$, where $\phi(r=0) = 0$

Then

$$\boxed{r^*(\vartheta) = r_s(r_s/r_w)^{\beta\cos^2\vartheta}} \qquad (4)$$



Reflection Boundaries Within Sheath—at $r^*(\vartheta,\phi)$ given by Eqn. (4)—for Various Proton Drift Energies

Thrust Calculations
Based On Laboratory Experimental Results

Calculation of Momentum Exchange

$$f = n_o v_o (M_{in} - M_{out})$$

$$M_{in} = (m_p v_o)$$

$$M_{out} = m_p v_o \int_0^{\pi/2} \cos(2\vartheta)\, d\vartheta = 0$$

$$F = 2r_s(\phi_b)f = 2r_s n_o m_p v_o^2$$

$$\boxed{F = 0.87\ \mu\text{N/m}}$$

F is thrust generated per m of wire for nominal solar wind (V_o = 400 km/s, n_o = $7\text{x}10^6$ m^{-3}, and T_e = $1.5\text{x}10^5$ ^{o}k).

Schematic of the complex array of physical effects *observed* in the near plasma environment of the TSS satellite.