A Novel Subspace-Averaging Direction of Arrival Estimation Technique

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***Abstract*—This paper presents a preliminary study of a novel subspace-averaging direction of arrival estimation technique (SADE), which jointly utilizes both the noise and signal subspaces to estimate the direction of arrival (DOAs) instead of just the former in a wide range of Signal to Noise (SNR) scenarios. The work was carried out by exploiting the common noise subspace properties with a modified covariance matrix. To further reduce the computational load, this work employs a simple polynomial root solving technique to determine the DOAs. From the simulation results of varying snapshot values and under Additive White Gaussian Noise (AWGN), the SADE technique manages to attain 99.84% of the Cramer Rao Bound (CRB) at >15dB SNR. In addition, from the simulation of varying antenna array elements, when the number of elements is less than 8, the SADE technique performs with a Root Mean Squared Error (RMSE) of 9.5% of the true direction compared to 15.6% and 16.7% of root-MUSIC and ESPRIT respectively with the potential application such as in a low-cost intelligent transportation localization system that requires high DOA estimation accuracy without the need for high hardware costs.**

***Index Terms*—Antenna array, DOA, Direction of Arrival, Snapshots**

I. Introduction

Direction-of-Arrival (DOA) estimation of impinging signals on a sensor array is a commonly occurring research problem with applications in the field of radar, sonar, and many other wireless communication systems [1]-[5]. With the rise of technologically advanced Vehicle-to-Vehicle (V2V) and Vehicle-to-Everything (V2X) wireless communication, DOA estimation will play a key role in realizing V2V and V2X that enables vehicles to intercommunicate and localize with other vehicles and the environment around them using short-range wireless signals [5]-[6].

Much effort has been made over the recent decades in developing high-resolution DOA estimators such as the Capon's Method, Multiple Signal Classification (MUSIC), Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) and its many improved derivatives [7]-[9]. To overcome this challenge, many improvements have been carried out to reduce the computational load and estimation accuracy such as the root-MUSIC and Unitary-ESPRIT [5]. The most recent state-of-the-art technique was the use of machine learning to mimic the performance of the aforementioned estimators. Machine learning-based techniques to solve for DOA estimation are known to be computationally demanding and time-consuming which can be deemed unfeasible for real-world active application [15]-[16]. Thus, these techniques present the pitfall in the lack of consideration of the signal subspace region that can be beneficial in determining a specific region for DOA estimation due to the robustness of the aforementioned subspace. Furthermore, it has been highlighted in past works of literature that the implementation of DOA algorithms is financially expensive due to high computational complexity where there is a need for a significantly high number of samples to accurately estimate the DOAs of interest.

The present paper aims at a preliminary attempt in developing a novel DOA estimation method for narrowband and short-range signal source environment called the Subspace-Averaging DOA Estimator (SADE). This is carried out by utilizing and averaging both the noise and signal subspaces concurrently to form a new covariance matrix which is a crucial data point for DOA estimation. The proposed DOA technique estimates exact DOA points at a range of SNR values (> 10 dB) while maintaining low computational complexity. The receiver is configured to process multiple DOA information impinging from different DOA position in space. The simulation results verify our proposed method and supersede existing DOA estimation techniques as an asymptotically unbiased estimator under static Additive White Gaussian Noise (AWGN) conditions.

This paper is outlined as follows. Section II presents the system model of a general DOA estimator. Section III introduces our proposed SADE technique. Section IV presents the simulation results under the environment of varying snapshots and antenna array elements. Finally, Section V concludes the paper with a conclusion and presents potential future works.

II. SYSTEM MODEL

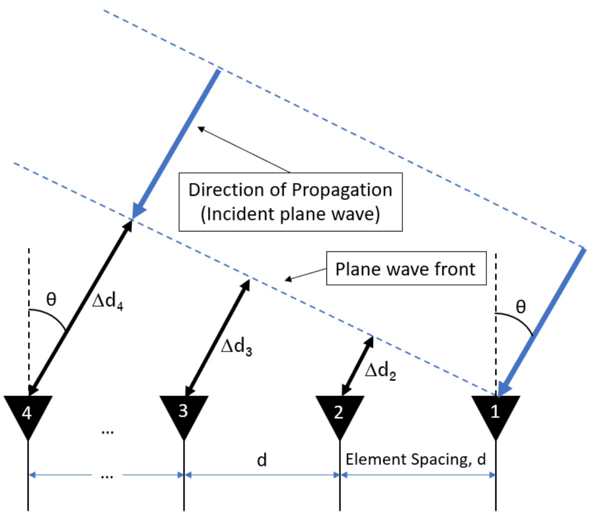

Figure 1: ULA Illustration

Fig. 1 shows a general data model illustration of a simple linear array which will be the basis of our algorithm design [4]. Assume that N far-field narrowband signals are impinging on a Uniform Linear Array (ULA) of $\mathrm{M}(>\mathrm{N})$ sensors with an inter-element spacing, d typically at $\frac{\lambda}{2}$ wavelength of its operating frequency. In this case, it is assumed that the signals are uncorrelated with noise. Under this assumption, the received signal at the array output for k^{th} snapshot is expressed as [4]:

$$\mathbf{x}(k) = \mathbf{A}s(k) + \mathbf{N}(k) \quad k = 1,2,\dots,\mathrm{K} \tag{1}$$

Where K is the total number of snapshots, $\boldsymbol{x}(k)$ is an $\mathrm{M}\times\mathrm{M}$ matrix of the received signal data consisting of signals and additive noise, $\mathbf{N}(k)$ while $\mathbf{A} \triangleq [a(\theta_1) \quad a(\theta_2) \quad \cdots \quad a(\theta_\mathrm{N})]$ is an $\mathrm{M}\times\mathrm{N}$ matrix containing the signal arrival vector information which consists of the relative phase shifts at the array elements. In addition, $a(\theta)$ in $\mathbf{A}$ is the steering vector which is defined as

$$\boldsymbol{a}(\theta) = \left[1 \quad e^{j2\pi\left(\frac{d}{\lambda}\right)\sin\theta} \quad \dots \quad e^{j2\pi(M-1)\left(\frac{d}{\lambda}\right)\sin\theta}\right]^T \tag{2}$$

$\boldsymbol{s}(k) \triangleq [s_1(k) \quad s_2(k) \quad \cdots \quad s_N(k)]^T$ is an $\mathrm{N}\times 1$ vector of N incident signal source values and n(k) is an $\mathrm{M}\times 1$ vector of sensor noise values [4]. Finally, $\theta = \{\theta_1, \quad \cdots \quad \theta_N\}$ are the parameters of interest that contain the DOA information which is required to be estimated. The sample covariance matrix is then obtained by [5]:

$$\widehat{\mathbf{R}}_{\mathrm{xx}} = \frac{1}{\mathrm{K}}\sum_{k=1}^{\mathrm{K}} \mathbf{X}\mathbf{X}^{\mathrm{H}} \tag{3}$$

Where the variable K is denoted as the number of snapshot samples as before. The next stage is the Eigenvalue Decomposition (EVD) to obtain the signal and noise subspaces. The first D highest eigenvalues of the covariance matrix represent the incoming signals. The $(\mathrm{M}-\mathrm{N})$ smallest eigenvalues represent noise. To that end, it is now possible to represent the signal, $\mathbf{R_s}$ and noise subspaces, $\mathbf{R_n}$ in vector form as:

$$\mathbf{R_s} = [W_1 \quad W_2 \quad \dots \quad W_D] \tag{4}$$

$$\mathbf{R_n} = [W_{D+1} \quad W_{D+2} \quad \dots \quad W_M] \tag{5}$$

Where W is the m^{th} eigenvector of the Eigen-decomposed covariance matrix of $\widehat{\mathbf{R}}_{\mathbf{xx}}$.

III. THE PROPOSED DOA ESTIMATION METHOD – SADE

The proposed method identifies and modifies the signal and noise subspaces within the received data. Then, the DOA is estimated using a polynomial solving technique to reduce computational complexity.

Firstly, the modified received data signal, $\mathbf{Y}$ is defined as:

$$\mathbf{Y} = \mathbf{I}_{\mathrm{M}}\mathbf{X}^* \tag{6}$$

Where $\mathbf{X}^*$ is defined as the complex conjugate of the received data signal $\mathbf{X}$ and $\mathbf{I}_{\mathrm{M}}$ is an anti-diagonal identity matrix of size $\mathrm{M}\times\mathrm{M}$ represented as:

$$\mathbf{I_M} = \begin{bmatrix} 0 & \dots & 1 \\ \vdots & \ddots & \vdots \\ 1_{(\mathrm{MxM})} & \cdots & 0 \end{bmatrix} \tag{7}$$

Next, the reformulated secondary sample covariance matrix $\widehat{\mathbf{R}}_{\mathbf{yy}}$ is expressed as:

$$\widehat{\mathbf{R}}_{\mathbf{yy}} = \frac{1}{\mathrm{K}}\sum_{i=1}^{\mathrm{K}} \mathbf{Y}\mathbf{Y}^{\mathrm{H}} \tag{8}$$

To that end, the resultant sample covariance matrix $\mathbf{R}$ is obtained as follows:

$$\mathbf{R} = \frac{\left(\widehat{\mathbf{R}}_{\mathbf{xx}} + \widehat{\mathbf{R}}_{\mathbf{yy}}\right)}{2} \tag{9}$$

From the equation, we can observe the noise components of both covariance matrices have equal values. This can be proven using the Expectation Value formula represented as [12]:

$$\mathbf{R}_{\mathrm{yy}} = \mathrm{E}[\mathbf{Y}\mathbf{Y}^{\mathrm{H}}] \tag{10}$$

$$\mathbf{R}_{\mathrm{yy}} = \mathbf{I}_{\mathrm{M}}\mathbf{A}^*\mathbf{R_s}^*(\mathbf{A}^*)^{\mathrm{H}}\mathbf{I}_{\mathrm{M}} + \mathbf{R_n} \tag{11}$$

$$\mathbf{R}_{\mathrm{yy}} = \mathbf{I}_{\mathrm{M}}\mathbf{R_s}^*\mathbf{I}_{\mathrm{M}} \tag{12}$$

Thus, we utilize both the noise and signal subspaces to determine the DOA. Lastly, we employ a simple root polynomial technique to determine the DOAs with the purpose and benefit of lower computational complexity. This essentially means that scanning the entire span of possible DOA angles is not required – significantly reducing the costs. The poles of the pseudo spectrum are the corresponding roots that lie closest to the unit circle. For example, an M-element ULA covariance matrix is of dimension M × M and will have $2(\mathrm{M}-1)$ diagonals. Thus, each root can be written as [5][14]:

$$z_i = |z_i| e^{j\arg(z_i)} \quad i = 1,2,\dots,2(\mathrm{M}-1) \tag{13}$$

Where $z = e^{j\frac{2\pi}{\lambda}d\sin\theta_i}$ and $\arg(z_i)$ is the phase angle of z_i. By comparing $e^{j\arg(z_i)}$ and $e^{j\frac{2\pi}{\lambda}d\sin\theta_i}$, the p^{th} roots closest to the unit circle are mapped and converted into the estimated DOAs of interest by:

$$\theta_{i(p)} = sin^{-1}\left(\frac{\lambda}{2\pi d}\arg\left(z_{i(p)}\right)\right) \tag{14}$$

Where $\theta_{i(p)}$ are the estimated DOAs of interest. Note that the range of i values are dependent on the number of signal source. For example, if there are 2 signal sources, then the 2 roots closest to the unit circle are the estimated DOAs of interest and so forth. In this paper, as it is beyond the scope of this study, it is assumed that the number of signal sources is known. In summary, our proposed SADE algorithm is summed up in Table 1.

Table 1: Proposed SADE Algorithm

SADE Algorithm	
Step 1.	Obtain received signal data, $\mathbf{X}$
Step 2.	Obtain primary sample covariance matrix, $\hat{\mathbf{R}}_{\mathbf{xx}}$
Step 3.	Obtain secondary sample covariance matrix, $\hat{\mathbf{R}}_{\mathbf{yy}}$
Step 4.	Construct subspace-averaged covariance matrix, $\mathbf{R}$
Step 5.	Perform EVD to obtain signal and noise subspaces, $\mathbf{R_s}$ & $\mathbf{R_n}$
Step 6.	Perform polynomial rooting and determine DOAs, $\theta_{i(p)}$

IV. Simulation Results

The proposed SADE technique was implemented using MATLAB R2020b. It is assumed that the signal source is uncorrelated and only AWGN was considered for simplicity. The SADE algorithm is compared to ESPRIT and Root-MUSIC – a relatively similar but simple subspace DOA estimation technique for demonstration comparison and presentation of the key benefits of using a subspace-averaging based technique as the sample covariance matrix by leveraging on both the signal and noise subspaces instead of just the latter. The element spacing is half the operating frequency's wavelength. In this section, some key factors will be observed and discussed.

To evaluate the performance of our technique, we modelled a simple scenario where only a single far-field signal source is impinging onto the antenna array at 50° for ease of comparison. In our study, we focus on the performance of varying antenna array elements and snapshot values across SNR values.

A. Varying the Number of Snapshots

In this section, we observe our proposed SADE technique under varying snapshots against Root-MUSIC and ESPRIT. In this scenario, we assume that the number of the antenna array element is $M = 4$. The Cramer-Rao Bound (CRB) is also provided as an indicator of the statistical performance of our estimators. With reference from Fig. 2 to Fig. 4 there is clear indication and consistency across any value of snapshot that at low SNR value (< 15dB), the RMSE for SADE is approximately 23.01% higher than that of Root-MUSIC and ESPRIT respectively. However, as the SNR value approached >15dB, SADE presents a significantly lower RMSE when compared to the latter. At 30 dB SNR, SADE manages to attain closer to the CRB with an RMSE of approximately 99.84% accurate, when compared to Root-MUSIC and ESPRIT against the CRB value. Since the number of elements in this performance study is relatively small, the noise and signal subspace eigenvalues are inherently significant at high SNR. Furthermore, as SADE utilizes both the noise and subspace subspaces, the eigenvalues from both of these subspace components allows higher estimation resolution. This effect has an inversely proportional impact when the number of elements increases which are discussed in the next section.

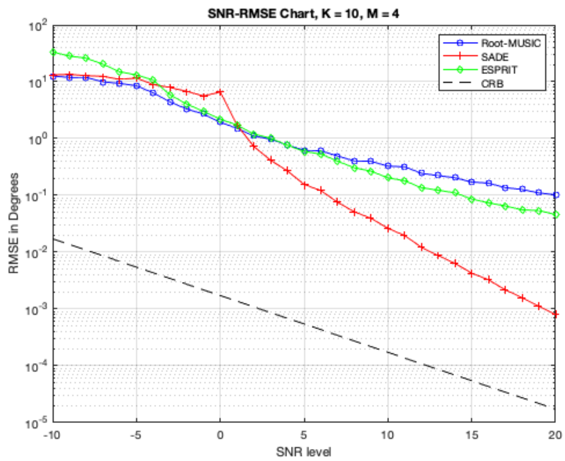

Figure 2: SNR-RMSE for K Number of Snapshots = 10

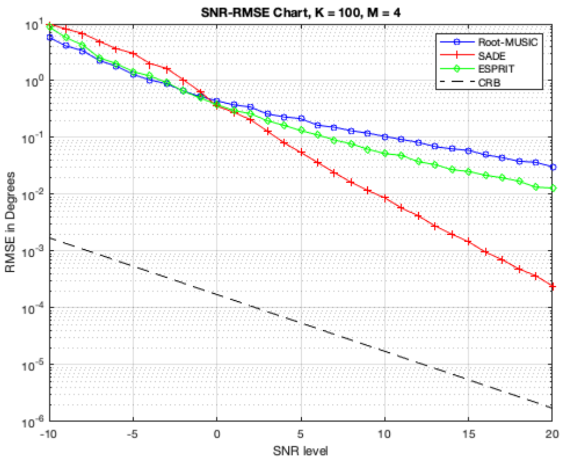

Figure 3: SNR-RMSE for K Number of Snapshots = 100

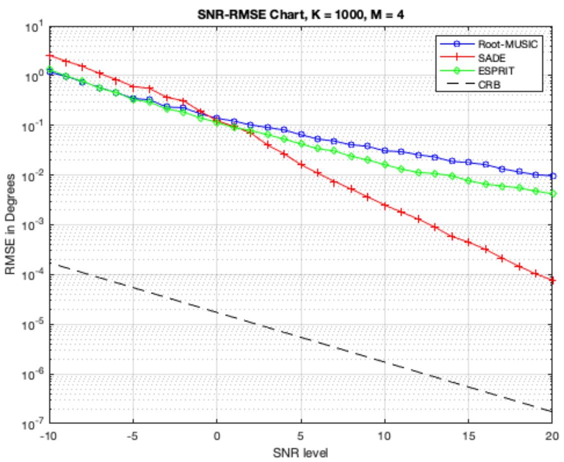

Figure 4: SNR-RMSE for K Number of Snapshots = 1000

B. *Varying the Number of Array Elements*

In this section, we observed the performance of our SADE algorithm under varying antenna array elements. For simplicity, we omit the CRB and observe closely and specifically on the SNR-RMSE performance among the 3 estimators. We also set a fixed snapshot value $K = 1000$ for consistent comparison. In the case where $M = 4$ is demonstrated in Fig. 4, we observe that the RMSE of SADE is significantly lower when compared to the other DOA techniques across the wide range of SNR values. Fig. 4 shows that RMSE for SADE obtains lower RMSE when compared to root-MUSIC and ESPRIT as the SNR gets higher. However, as the number of elements increase, the performance of SADE decays when compared to root-MUSIC. This is because the signal subspace E_s as the number of antenna array increases, the noise subspace, E_n are significantly higher in value when compared to the signal subspace $(\mathbf{R_n} \gg \mathbf{R_s})$. Therefore, the averaging technique loses estimation performance as the eigenvalues in the signal subspace approaches insignificant values. Thus, conducting a subspace-averaging technique would result in poorer performance. This trend is observed from Fig. 5 and Fig. 6 where SADE performs slightly worse than Root-MUSIC but higher accuracy when compared to ESPRIT. When the number of elements is less than 8, the SADE has an average RMSE of 9.5% when compared to root-MUSIC and ESPRIT at 15.6% and 16.7% respectively when compared to the true signal source DOA.

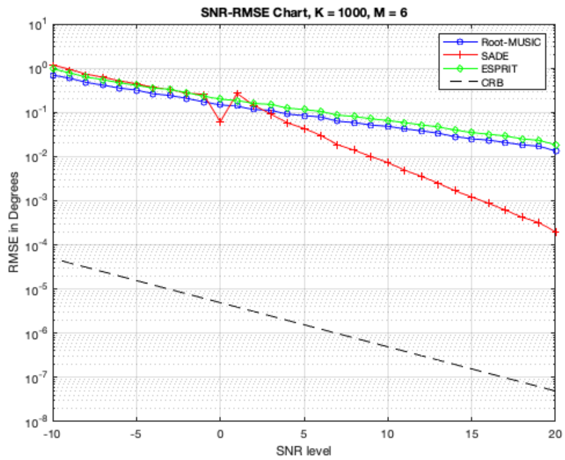

Figure 5: SNR-RMSE for Antenna Elements M = 6

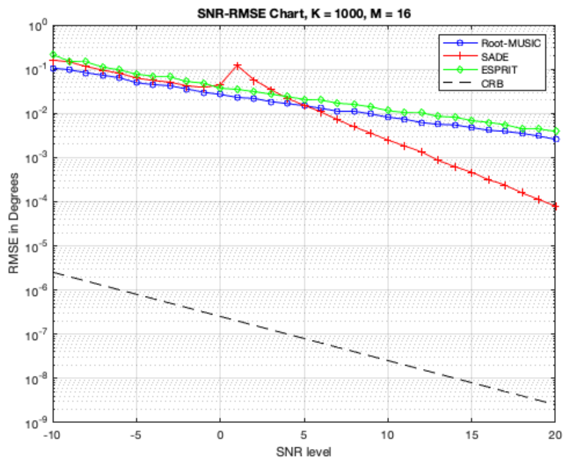

Figure 6: SNR-RMSE for Antenna Elements M = 16

V. Conclusion

The proposed subspace-averaging DOA estimation technique, SADE was presented to significantly improve the performance of an estimator without the expense of computational costs and improve estimation accuracy based on the RMSE results. From the simulation results of varying snapshot values and under Additive White Gaussian Noise (AWGN), the SADE technique manages to attain 99.84% of the Cramer Rao Bound (CRB) at >15dB SNR. In addition, from the simulation of varying antenna array elements, when the number of elements is less than 8, the SADE technique performs with a Root Mean Squared Error (RMSE) of 9.5% of the true direction compared to 15.6% and 16.7% of root-MUSIC and ESPRIT. A key potential application for the SADE algorithm would be in a Wi-Fi-based wireless communication system such as in an intelligent transportation environment or roadside to vehicular wireless links like in V2I or V2X applications where small antenna size is key to its mobility [13]. Another potential application would be in an Unmanned Aerial Vehicle (UAV) localization where the antenna size must be small to accommodate weight and flight optimization without the loss of localization accuracy.

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