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Towards an Empirical Test of Realism in Cognition

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Abstract. We review recent progress in designing an empirical test of (temporal) realism in cognition. Realism in this context is the property that cognitive variables always have well defined (if possibly unknown) values at all times. We focus most of our attention in this contribution on discussing the exact notion of realism that is to be tested, as we feel this issue has not received enough attention to date. We also give a brief outline of the empirical test, including some comments on an experimental realisation, and we discuss what we should conclude from any purported experimental 'disproof' of realism. This contribution is based on Yearsley and Pothos (2014).

1 Introduction

Our aim in this contribution is to give an overview of recent work that seeks to address the question of whether models of cognitive processes can be (temporally) realist. We will define exactly what we mean by realist below, but the key finding is that given a suitable definition this question can be empirically answered by simple experiments. This contribution is based on Yearsley and Pothos (2014), but instead of simply summarising this paper we will instead focus on two of the most important issues and discuss them in depth. The two issues we shall focus our attention on are firstly the exact notion of 'realism' which is to be empirically tested, and secondly the possibilities open to us should experiments rule out this particular notion of realism. We feel these are important topics to address because the empirical test we shall propose, which has been discussed before (Attmanspacher and Filk (2010)), is borrowed from the physics literature, and it is far from clear how this test is to be derived or interpreted in the context of cognitive models. We shall take advantage of the fact that this contribution is based on an existing paper to skip much of the technical detail; interested readers are invited to consult Yearsley and Pothos (2014).

The rest of this contribution is structured as follows; in Sect. 2 we discuss the notion of realism in cognitive models in a general way and in Sects. 2 and 3 we introduce the two smaller assumptions that together make up the assumption of realism proper. In Sect. 4 we make some very brief comments on the empirical test of realism we propose. In Sect. 5 we then discuss the options for cognitive modelling should our empirical test rule out realism. We conclude in Sect. 6.

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2 Realism in Cognitive Models

Every thought, feeling and memory that we have ultimately arises from, is processed by or is stored in the physical matter of our brains. Thus, in principle, if it were possible to know exactly the physical specification of a subject's brain at any moment of time, we should be able to know that subject's feelings and predict their judgments. Of course, such a scenario is the stuff of science fiction rather than current psychology, but the fundamental principle behind it, that the behaviour of cognitive variables such as feelings and judgments can be reduced to the physical specification of our neurophysical states does manifest itself in an important way in current cognitive models. In brief, most current cognitive models have a property that we might term 'realism,' that is, it is an implicit assumption of these models that all the cognitive variables whose values are described by a given model have definite values at all times (cf Raijmakers and Molenaar (2004)).

This assumption arises in a natural way when we consider the link between cognitive processes at the level of thoughts and feelings, and the underlying neurophysiology of the brain which is assumed to give rise to these thoughts and feelings. For the purposes of this contribution we assume the most fundamental processes in the brain relevant for cognition may be described by classical physics (the alternative hypothesis, that brain function at the neuro level is non-classical, is very controversial (beim Graben and Atmanspacher (2009).) It is a key feature of classical physics that the positions, electric charges, etc. of all classical particles are definite at all times, that is, whilst the values of these quantities may be *subjectively* uncertain (since we have only limited knowledge of them) they are nevertheless *objectively* certain. Thus, one might reasonably argue, if cognition is ultimately determined by brain neurophysiology, and if the most fundamental variables at the neurophysiological level have definite but unknown values, then presumably all cognitive variables must also have definite if unknown values. We will argue that this assumption is in fact highly questionable.

To make the argument more concrete consider a simple example; the first author of this contribution enjoys crisps, and he also enjoys chocolate. At any given moment of time he will have a preference for either crisps over chocolate, or vice versa. Let us denote this cognitive variable by the function $C(t)$, which may take values between $+1$ (definitely prefer crisps) and -1 (definitely prefer chocolate.) If we desire we could measure this variable crudely by asking the author which snack he would prefer, or we could measure it more precisely by asking him to make some trade off between various quantities of crisps and chocolate. The key assumption of classical models of cognition is that this variable $C(t)$ always has a well defined value between ± 1 at all times. This may seem reasonable over the course of some short lab experiment, but does it really make sense over longer periods of time? What happens when the author is distracted by writing a conference contribution? Or what if he has just sated his appetite with a bag of jelly beans? Does this cognitive variable nevertheless still exist, tirelessly winding some intricate path between ± 1 which only the gods, or possibly the advertising executives, can trace?

The alternative to a realist account of cognitive variables is one wherein such variables do not possess values until they are measured, that is, measuring such variables is a *constructive* process. The idea that measuring the value of a cognitive variable can change that value has been considered before, and can be easily incorporated within classical models of cognition. However what we are suggesting here is slightly different, it is not a question of measurements changing the values of *existing* quantities, rather the process of measurement *creates* those values, where previously there were none. This idea may sound familiar to anyone who has come across quantum theory before (see e.g. Jammer (1966)), and we will have more to say on this connection below. For now note that in the example we gave above it is not so hard to imagine that the author's preference for crisps over chocolate simply isn't *defined* at time when he is not, consciously or unconsciously, thinking about eating.

Why should we care whether our cognitive theories are realist or not? Well there are two main reasons; the first is that there are certain types of behaviour that are possible in non-realist models that are impossible in realist ones. This means that there may be limits on the type of cognitive processes realist theories are able to describe. The second reason seems to the authors more important; the ultimate aim of constructing cognitive models is not simply to describe or even predict cognitive processes, but at some level to understand them. For this reason it is important to have some confidence that the structural features of our models match or map in some sense the way cognition happens in the brain. Although our understanding of the physiology of cognition is currently far too limited to be used to impose detailed constraints on cognitive models, there are nevertheless some basic constraints that we can impose that do limit the classes of cognitive models we should consider acceptable. One of these concerns the idea of 'bounded cognition'; we would argue a second one concerns realism (for some work in this direction see Jones and Love (2011).)

So how are we to tell whether cognition is realist or not? We hinted at the answer earlier; there are some types of behaviour that are impossible to reproduce within a realist cognitive model. Our task therefore is to produce a test which will allow us to determine whether a given set of judgments can be described by a realist cognitive model, and to suggest some possible cognitive variables which may fail this test. We will do this below, but first we need to be clearer on exactly what we mean by realism in cognitive models.

In the next two sections we will discus two reasonable assumptions which together we claim form the assumption of 'realism' in cognitive models. We will spend some time discussing these assumptions in depth, because they are really the most important part of this work. Obviously since our test of 'realism' is really a joint test of these assumptions its significance depends entirely on whether one believes these assumptions really capture the correct notion of realism in cognition. But as well as this because there are two separate assumptions any purported failure of 'realism' leaves us the option of retaining one of the assumptions, and if we want to be clear about which one (if either) we should retain we need first be clear on their meaning. Once we have done this our empirical test of realism follows by some elementary algebra, which we shall skip, and the task of choosing a likely cognitive variable is an exercise in experimental cunning rather than intellectual vigour.

3 Realism Part 1: Cognitive Realism

Let us set out our first assumption which, together with the assumption discussed in the next section, together define 'realism' in cognitive models.

Cognitive Realism: This is the assumption that the reason for any judgement at the cognitive level is ultimately (in principle, if not in practice) reducible to processes at the neurophysiological level.

This assumption is perhaps what one might think of if one is asked to characterise realism. In fact it might seem like no further assumptions are needed, we will explain why this is not the case below. For now let us instead introduce some notation to help us make this assumption more precise, and to put it on the required mathematical footing needed for our empirical test. Consider again our example cognitive variable $C(t)$. Let us denote the complete neurophysiological state of a given subject as λ. Cognitive Realism means that there is a function which, given that the neurophysiological state of the subject is initially λ, will tell us the value of $C(t)$, let us denote this by $C(\lambda, t)$. This is what we meant in the introduction when we said that realism means that, in principle, were we to know the physical state of a subject's brain we would know all their feelings and be able to predict their judgments. However in practice of course we cannot know a subject's exact neurophysiological state, the best we can do is give some probability distribution based on the limited knowledge we do have. Let us denote the probability distribution representing our knowledge of a subject's λ as $\rho(\lambda)$. Then our best guess about the value of $C(t)$ given our knowledge of the neurophysiological state is,

$$\langle C(t) \rangle = \sum_{\lambda} C(\lambda, t)\rho(\lambda), \tag{1}$$

that is, the expected value of $C(t)$ is just the expectation value of $C(\lambda, t)$, given the probability distribution $\rho(\lambda)$.

Let us make few comments about this assumption, and its mathematical consequence Eq. (1).

- The observant reader may find the time dependence in Eq. (1) rather odd, in that is contained in the cognitive variable rather than in the distribution over neurophysiological states. This is purely notational, the current notation fits present purposes better.
- Let us stress that there is no expectation that we know the subject's λ, and also no requirement that we know the function $C(\lambda, t)$. Even if Cognitive Realism is true a subject's λ need not be knowable even in principle, but the λ's should be well defined and $C(\lambda, t)$ must exist.

– The assumption of cognitive realism may also be expressed in the following important way: for any set of judgements, and at all times, a subject has a definite opinion about all judgements.
– It is very difficult to see how this assumption could fail to be valid at some level. After all, if the values of cognitive variables are not determined by the brain, what are they determined by?
– The previous point notwithstanding, for the derivation of our empirical test to hold the λ need only be some variables which by assumption always have well defined values, it is not strictly necessary that they be neurophysiological. For more on this point see Yearsley and Pothos (2014).

Cognitive Realism may seem to totally capture the notion of realism we had in mind in the introduction. However there are two important missing ingredients in the discussion in this section. The first is a description of how measurement of a cognitive variable works. This may seem somewhat pedantic, but it is important to establish whether one can carry out reliable measurements of these variables, and to see how this fits in with realism. The second ingredient is some kind of assurance that we can take finite collections of cognitive variables and embed them into a cognitive model in a self-consistent way. In other words, Cognitive Realism is the assumption that the cognitive level can be connected to the neurophysiological level; what we also need is an assumption that the cognitive level can be *disconnected* from the neurophysiological level, and modelled on its own. That is the content of our second assumption.

4 Realism Part 2: Cognitive Completeness

Our second assumption is harder to state than our first. It concerns the cognitive state of subjects. This is defined to be the object that captures all the information needed to make predictions about a subject's judgments in the context of a particular cognitive model. It is therefore equivalent to an exhaustive set of probabilities for future measurement outcomes[1]. The exact form the cognitive state takes will depend on the model, and we want to state our assumption without reference to any particular form.

Cognitive Completeness: This is the assumption that the cognitive state of a person responding to such a set of judgements can be entirely determined by the probabilities for the judgement outcomes.

That is, observing participant behaviour can fully determine the underlying cognitive state, without the need to invoke neurophysiological variables. The reason this assumption is more vague than the first is that we have not defined exactly what the cognitive state is supposed to be. Generally this will depend on the model and we cannot assume, for example, that the cognitive state is a probability distribution over thoughts or judgments. However whatever the form

[1] The idea of defining the state of a system in this way occurs frequently in physics, see e.g. Hardy (2001).

of the cognitive state, if this is the object that allows us to predict judgment outcomes then it is important that it can be determined entirely in terms of them, otherwise it is not possible to establish this state empirically, making prediction impossible.

This assumption has an important consequence. Consider any measurement made on a group of participants that does not change the probabilities for the outcomes of any future judgement in the relevant cognitive model. Let us call such measurements non-disturbing. Whether or not a measurement is non-disturbing can be established empirically.

Cognitive completeness means that, as long as a measurement is non-disturbing, it can be assumed to have no effect on the neurophysiological state of a participant. This is because cognitive completeness tells us that the cognitive state of the participants may be fully determined by knowledge of the outcomes of all judgements in the relevant cognitive model. Thus, at most, a non-disturbing measurement may change the underlying neurophysiological state in a way that gives rise to the same cognitive state. However, any such change is undetectable by any measurement relevant to the cognitive model, and thus we can simply assume that no change in the neurophysiological state occurred.

It is useful to express this in a more mathematical way. Cognitive Completeness means that every cognitive model defines a set of similarity classes on the set of all probability distributions over the neuropsychological variables, with two distributions $\rho(\lambda)$ and $\rho'(\lambda)$ being similar, $\rho(\lambda) \sim \rho'(\lambda)$, if they lead to the same predictions for all judgements contained in the cognitive model. In general measurement of the cognitive variable $C(t_1)$ at t_1 will change the distribution of neurophysiological variables so that a subsequent measurement of, e.g. $C(t_2)$ with $t_2 > t_1$, will depend on whether or not the first measurement was made. Denote the new distribution over the λ after measurement at t_1 as $\rho(\lambda; t_1)$. Then joint measurement of $C(t_1)$ and $C(t_2)$ yields,

$$\langle C(t_1)C(t_2)\rangle = \sum_{\lambda} C(\lambda, t_1)C(\lambda, t_2)\rho(\lambda; t_1) \tag{2}$$

However if the measurement at t_1 was non-disturbing this is equal to

$$\langle C(t_1)C(t_2)\rangle = \sum_{\lambda} C(\lambda, t_1)C(\lambda, t_2)\rho(\lambda). \tag{3}$$

This is the mathematical result used in the derivation of our empirical test.

As a mathematical aside, we now sketch how Cognitive Realism and Cognitive Completeness may be used to derive the existence of a probability distribution over the cognitive variables[2]. We will assume that we are measuring the value of a cognitive variable $C(t)$ which may take a finite number of possible

[2] The existence of such a probability distribution in fact follows directly if our empirical test is satisfied, this is the cognitive analogue of Fine's theorem (Fine (1982), Halliwell (in press)). For a further discussion of the conditions under which such probability distributions may be defined, see Bruza et al. (2013).

values, which we will denote $\{c_i\}$[3]. We begin with the generalisation of Eq. (2) for a series of measurements of $C(t)$ at a number of times,

$$\langle C(t_1)C(t_2)...C(t_n)\rangle = \sum_{\lambda} C(\lambda, t_1)C(\lambda, t_2)...C(\lambda, t_n)\rho(\lambda; t_1, t_2, ...t_n) \quad (4)$$

For each measurement the set of possible λ may be split into subsets $\Lambda_i(t)$ according to the value of $C(\lambda, t)$. Denote the function which is 1 if $\lambda \in \Lambda_i(t)$ and 0 otherwise by $\chi_i(\lambda, t)$ This can be done for every measurement, so we can write,

$$\begin{aligned}\langle C(t_1)C(t_2)...C(t_n)\rangle &= \sum_{\lambda}\sum_{i} c_i\chi_i(\lambda, t_1)\sum_{j} c_j\chi_j(\lambda, t_2)...\sum_{k} c_k\chi(\lambda, t_n)\rho(\lambda; t_1, t_2, ...t_n)\\ &= \sum_{i,j...k} c_i, c_j...c_k P(i, t_1; j, t_2; ...k, t_3), \end{aligned} \quad (5)$$

where

$$P(i, t_1; j, t_2; ...k, t_3) = \sum_{\lambda} \chi_i(\lambda, t_1)\chi_j(\lambda, t_2)...\chi(\lambda, t_n)\rho(\lambda; t_1, t_2, ...t_n). \quad (6)$$

Because $P(i, t_1; j, t_2; ...k, t_n)$ is just a coarse-graining of the original $\rho(\lambda; t_1, t_2...t_n)$ it is guaranteed to be a probability distribution on the set of measurement outcomes. However because $\rho(\lambda; t_1, t_2...t_n)$ depends on whether or not the measurements are performed this probability does not obey the correct sum rules when one or more measurements are summed out. However if we use the assumption of Cognitive Completeness we can drop the dependence of ρ on the measurements provided they are non-disturbing, in which case $P(i, t_1, j, t_2...k, t_n)$ becomes independent of whether the measurements are performed or not and therefore obeys the correct sum rules. This is the way in which Cognitive Realism and Cognitive Completeness imply the existence of a probability distribution over the cognitive variables.

5 Interlude: The Empirical Test and Some Experimental Considerations

Now that we have our two assumptions we can discuss the empirical test of realism that is the main achievement of this work. The test takes the form of a set of inequalities satisfied by realist systems but which may be violated by general systems. These inequalities may be easily derived from the mathematical expressions of Cognitive Realism and Cognitive Completeness, however rather than take up space in this contribution repeating algebra, we instead refer the interested reader to the appendix of Yearsley and Pothos (2014). We shall simply

[3] The extension to a cognitive variable with a continuous range of values is simple, but unenlightening.

quote the result, again in terms of our example variable $C(t)$ which recall takes values ± 1.

$$|\langle C(t_1)C(t_2)\rangle + \langle C(t_2)C(t_3)\rangle + \langle C(t_3)C(t_4)\rangle - \langle C(t_1)C(t_4)\rangle| \leq 2 \qquad (7)$$

Equation (7) is one of a collection of inequalities known as the temporal Bell[4], or Leggett-Garg inequalities, first derived as constraints on physical systems by Leggett and Garg (1985). Their significance in physics is much debated (see e.g. Ballentine (1987), Palacios-Laloy et al. (2010), Wilde (2012), George et al. (2013) and Yearsley (2013), and references therein), but note that the assumptions leading to their derivation in cognition are relatively uncontroversial. We will have nothing further to say about the use of these inequalities outside of cognition. AQ1

What would a concrete experimental set up to test these inequalities look like? Well we need four ingredients; the first is a cognitive variable which we are sure has two distinct values. There are many possible examples. The second is some way to manipulate the expected value of that variable, this could be through presentation of stimuli over which the experimenter does not have direct control, but which happen at regular time intervals, or it could be through presentation of stimuli over which the experimenter has direct control, in which case the frequency and order of presentation of the stimuli is not fixed in time, and the 't' variable in Eq. (7) is better thought of as a parameter rather than as a physical time. Again it is not hard to think of good examples.

The third ingredient is a reliably non-disturbing measurement process, in the sense outlined above. This might be hard to invent in general, but it is easy to establish whether a given measurement process is non-disturbing, so it presents no problem in principle. We mention in passing that it is not necessary that the measurement process be completely non-disturbing, being able to bound the disturbance to some low level is sufficient (Yearsley and Pothos (in preparation)). What might make a good non-disturbing measurement? In physics attention has focussed on so-called 'ideal negative measurements.' The idea is roughly that a particle which is not detected should not be disturbed by the detector (Leggett and Garg, 1985). One can therefore use an ideal negative measurement of whether a particle is in, say, $x < 0$ to establish that the particle is definitely in $x > 0$, but without causing any disturbance. It is not immediately clear whether these ideas can be translated to psychology.

However there may be another possibility available in psychology which is not available in physics. In psychology the extent to which a given judgment causes a change in the knowledge state of the subject can be influenced by details of the experimental design (e.g. White et al. (2014)). The psychological idea behind non-disturbing measurements would be to avoid a subject feeling as though they had made a strong commitment to a particular choice, and it is possible that this

[4] This term comes from the fact they these inequalities have a similar form to the standard Bell inequalities (see e.g. Bell (2004), for their significance in psychology see Conte et al. (2008) and Aerts et al. (2000). However beyond structural similarity the two sets of inequalities have little in common, and this terminology can sometimes lead to more confusion than clarity.

could be achieved through a sufficiently clever experimental design. Note that the question of whether or not this is possible is deeply connected to the issue of whether cognition is really 'quantum' (or possibly 'quantum-like'), so that fundamental limits such as the uncertainty principle always hold, or whether cognition merely has 'quantum features', in which case it may sometimes be possible to break what in physics are fundamental quantum limits.

What of the final ingredient? Well this is simply the expectation that the cognitive variable in question does behave in a non-classical way. This is in some sense the most simple and the most difficult property to establish. It may be possible to use variables which have previously been shown to behave in non-classical ways (see e.g. Busemeyer et al. (2011), Pothos and Busemeyer (2009), Trueblood and Busemeyer (2011), Wang and Busemeyer (2013), Wang et al. (in press)), otherwise some experiential cunning will be needed to choose an appropriate set of judgments.

6 What Should We Conclude if 'Realism' Fails?

Suppose we find an appropriate experimental set up, conduct a test of realism in the way outlined above, and find a convincing violation of Eq. (7). What should we conclude? Assuming one agrees with the arguments which lead to Eq. (7) then the only conclusion is that one or both of our assumptions, Cognitive Realism and Cognitive Completeness, must be incorrect. But which one?

If one is committed to realism one might be tempted to drop Cognitive Completeness. The problem is that it is Cognitive Completeness that ensures that the cognitive state can be empirically determined, and since the cognitive state is the object which determines the probabilities for the outcomes of judgments, Cognitive Completeness ensures that any model has genuine predictive power.

Nevertheless one might argue that this problem can be circumvented. If we cannot fix the cognitive state in terms of the outcomes of judgments contained in our cognitive model, can we not simply add more judgments, the probabilities for which *would* be enough to fix the cognitive state? The answer is that we cannot. The full argument is given in Yearsley and Pothos (2014), but the essence is that adding any cognitive variables which can be measured in a non-disturbing way simply gives an extended cognitive model from which the original one can be recovered by coarse-graining, but since the original model isn't realist the extended one cannot be either. Adding in cognitive variables which cannot be measured in a non-disturbing way solves this problem, but having cognitive variables which cannot *in principle* be measured in a non-disturbing way means the new model still lacks predictive power. In summary, Cognitive Completeness is possibly even more central to cognitive modelling than realism.

So if we cannot drop Cognitive Completeness, can we drop Cognitive Realism? The answer is we can, we can model cognition with non-realist theories like quantum probability theory that include a constructive role for judgment. Quantum probability theory is often described as quantum theory without the physics (see e.g. Aerts and Aerts (1995), Atmanspacher et al. (2006)), and is potentially applicable in any situation where there is a need to quantify uncertainty (see e.g. beam Graben and Atmanspacher (2009)). Indeed there has been no small measure of success modelling some aspects of cognition in this way (e.g. Aerts and Gabora (2005), Busemeyer et al. (2011), Pothos and Busemeyer (2009), Trueblood and Busemeyer (2011), Wang and Busemeyer (2013), Wang et al. (in press), Bruza et al. (2009). For an overview see Busemeyer and Bruza (2011), Pothos and Busemeyer (2013).)

However we need to be cautious. Realism imposes a bound on the right hand side of Eq. (7) equal to 2, but quantum theory also implies a non-trivial bound on Eq. (7) of $2\sqrt{2}$ (Tsirelson (1980)). Since the logical bound is 4 we could well find that our experimental test of realism rules out not just realist theories of cognition but also quantum ones! Even if the evidence doesn't directly rule out quantum theory there are possibilities for non-realist theories other than quantum theory. In other words, our test of realism may rule out realist models of cognition, but it cannot 'rule in' quantum models. We need to search elsewhere for convincing evidence for the correctness of quantum approaches to cognition.

Finally we should mention that a failure of realism in cognition could have great significance for models of memory. If judgment is a constructive process then it is easy to imagine that memory retrieval may also be modelled constructively in a similar way (this has been suggested before, e.g. Howe and Courage (1997)). This could open up exciting new possibilities for modelling memories.

7 Conclusion

In this contribution we discussed some of the key issues involved in recent work on the question of realism in cognitive models. We have focussed on what we believe are the key conceptual issues, readers desiring the full technical details are again invited to consult Yearsley and Pothos (2014).

What can we conclude from this discussion? Well firstly we have argued that the standard notion of realism in cognition might be well motivated, but it is open to empirical challenge. The successes of the quantum cognition programme to date suggest, although do not prove, that realism may have to be abandoned as an assumption in models of cognition. The proposed empirical test of realism will hopefully settle the issue. This test is tricky to implement, but should be possible with the right choice of cognitive variable and measurement.

If our tests do rule out realism, this is not by itself reason to adopt quantum models of cognition. However such models can give valuable insight into what non-realist approaches may look like. In particular contextually and constructive judgments are central parts of quantum theory (Kitto (2008), Busemeyer and Bruza (2011), White et al. (2014)) and these will also be key features of any non-realist theory.

We wish to conclude by saying that an experimental realisation of this test is currently underway (Yearsley and Pothos (in preparation)). We await the results with considerable interest.

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References

Aerts, D., Aerts, S.: Applications of quantum statistics in psychological studies of decision processes. Found. Sci. **1**, 85–97 (1995)

Aerts, D., Aerts, S., Broekaert, J., Gabora, L.: The violation of Bell inequalities in the macroworld. Found. Phys. **30**(9), 1387–1414 (2000)

Aerts, D., Gabora, L.: A theory of concepts and their combinations II: a hilbert space representation. Kybernetes **34**, 192–221 (2005)

Atmanspacher, H., Romer, H., Wallach, H.: Weak quantum theory: formal framework and selected applications. Weak quantum theory: complementarity and entanglement in physics and beyond. Found. Phys. **32**, 379–406 (2006)

Atmanspacher, H., Filk, T.: A proposed test of temporal nonlocality in bistable perception. J. Math. Psychol. **54**, 314–321 (2010)

Bell, J.S.: Speakable and Unspeakable in Quantum Mechanics. Cambridge University Press, Cambridge (2004)

Bruza, P.D., Kitto, K., Nelson, D., McEvoy, C.L.: Is there something quantum-like about the human mental lexicon? J. Math. Psychol. **53**, 362–377 (2009)

Bruza, P.D., Kitto, K., Ramm, J.R., Sitbon, L.: A probabilistic framework for analysing the compositionality of conceptual combinations (2013). arXiv preprint arXiv:1305.5753

Busemeyer, J.R., Bruza, P.: Quantum Models of Cognition and Decision. Cambridge University Press, Cambridge (2011)

Busemeyer, J.R., Pothos, E.M., Franco, R., Trueblood, J.: A quantum theoretical explanation for probability judgment errors. Psychol. Rev. **118**, 193–218 (2011)

Conte, E., Khrennikov, A.Y., Todarello, O., De Robertis, R., Federici, A., Zbilut, J.P.: A preliminary experimental verification on the possibility of bell inequality violation in mental states. NeuroQuantology **6**(3), 214–221 (2008)

Fine, A.: Joint distributions, quantum correlations, and commuting observables. J. Math. Phys. **23**, 1306–1310 (1982)

George, R., Robledo, L., Maroney, O., Blok, M., Bernien, H., Markham, M., Twitchen, D., Morton, J., Briggs, A., Hanson, R.: Proc. Natl. Acad. Sci. **110**, 3777–3781 (2013)

Halliwell, J.J.: Two proofs of Fine's theorem (in press). Pre-print arXiv.org/abs/1403.7136

Graben, P.B., Atmanspacher, H.: Extending the philosophical significance of the idea of complementarity. In: Atmanspacher, H., Primas, H. (eds.) Recasting Reality. Wolfgang Pauli's Philosophical Ideas and Contemporary Science, pp. 99–113. Springer, Heidelberg (2009)

Hardy, L.: Quantum theory from five reasonable axioms (2001). arXiv preprint arXiv:quant-ph/0101012

Howe, M.L., Courage, M.L.: The emergence and early development of autobiographical memory. Psychol. Rev. **104**, 499–523 (1997)

Jammer, M.: The Conceptual Development of Quantum Mechanics. McGraw Hill, New York (1966)

Jones, M., Love, B.C.: Bayesian fundamentalism or enlightenment? On the explanatory status and theoretical contributions of Bayesian models of cognition. Behav. Brain Sci. **34**(169), 231 (2011)

Kitto, K.: Why quantum theory? In: Proceedings of the Second Quantum Interaction Symposium, pp. 11–18. College Publications (2008)

Leggett, A.J., Garg, A.: Quantum mechanics versus macroscopic realism: is the flux there when nobody looks? Phys. Rev. Lett. **54**, 857–860 (1985)

Palacios-Laloy, A., Mallet, F., Nugyen, F., Bernet, P., Vion, D., Esteve, D., Korotkov, A.N.: Experimental violation of a Bell's inequality in time with weak measurement. Nat. Phys. **6**, 442 (2010)

Pothos, E.M., Busemeyer, J.R.: A quantum probability explanation for violations of 'rational' decision theory. Proc. R. Soc. B **276**, 2171–2178 (2009)

Pothos, E.M., Busemeyer, J.R.: Can quantum probability provide a new direction for cognitive modelling? Behav. Brain Sci. **36**, 255–327 (2013)

Raaijmakers, J.G., Shiffrin, R.M.: Models for recall and recognition. Annu. Rev. Psychol. **43**, 205–234 (1992) AQ2

Raijmakers, M.E.J., Molenaar, P.C.M.: Modelling developmental transitions in adaptive resonance theory. Dev. Sci. **7**, 149–157 (2004)

Shafir, E., Tversky, A.: Thinking through uncertainty: nonconsequential reasoning and choice. Cogn. Psychol. **24**, 449–474 (1992)

Shiffrin, R.M.: Forgetting, trace erosion or retrieval failure? Science **168**, 1601–1603 (1970)

Tenenbaum, J.B., Kemp, C., Griffiths, T.L., Goodman, N.: How to grow a mind: statistics, structure, and abstraction. Science **331**, 1279–1285 (2011)

Trueblood, J.S., Busemeyer, J.R.: A comparison of the belief-adjustment model and the quantum inference model as explanations of order effects in human inference. Cogn. Sci. **35**, 1518–1552 (2011)

Trueblood, J.S., Brown, S.D., Heathcote, A., Busemeyer, J.R.: Not just for consumers: context effects are fundamental to decision-making. Psych. Sci. (in press) AQ3

Tsirelson, B.S.: Quantum generalizations of Bell's inequality. Lett. Math. Phys. **4**, 93 (1980)

Wang, Z., Busemeyer, J.R.: A quantum question order model supported by empirical tests of an a priori and precise prediction. TICS **5**, 689–710 (2013)

Wang, Z., Solloway, T., Shiffrin, R.M., Busemeyer, J.: Context effects produced by question orders reveal quantum nature of human judgments. PNAS (in press)

White, L.C., Pothos, E.M., Busemeyer, J.R.: Sometimes it does hurt to ask: the constructive role of articulating impressions. Cognition **133**(1), 48–64 (2014) AQ4

Wilde, M.M., Mizel, A.: Addressing the clumsiness loophole in a Leggett-Garg test of macrorealism. Found. Phys. **42**, 256–265 (2012)

Yearsley, J.M.: The Leggett-Garg inequalities and non-invasive measurability (2013). Pre-print arXiv.org/abs/1310.2149

Yearsley, J.M., Pothos, E.M.: Challenging the classical notion of time in cognition: a quantum perspective. Proc. R. Soc. B **281**(1781), 20133056 (2014)

Yearsley, J.M., Pothos, E.M.: An Empirical Test of Temporal Realism in Cognition (in preparation)

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Chapter 21

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