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Shakedown and Creep Rupture Assessment of a Header Branch Pipe Using the Linear Matching Method

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Abstract

Many power plant components are subject to combined mechanical and thermal loading conditions during their operating lifetime. It is important that potential failure mechanisms of such components are extensively investigated in order to ensure sufficient confidence in their reliability. This paper presents shakedown and creep rupture analyses of a header branch pipe subjected to cyclic thermo-mechanical loading performed using the Linear Matching Method (LMM). The detailed investigation of failure mechanisms under the combined action of the internal pressure and the cyclic thermal load due to the temperature difference between the inner and outer pipe surfaces will be the primary focus of this paper. The header branch pipe considered here is composed of a single material with properties that are dependent upon both temperature and rupture life. A novel study investigating the effect that two geometric parameters – branch diameter and separation – have upon the failure mechanisms of the header branch pipe has also been carried out. The impact that these geometric parameters have upon the limit load, shakedown and creep rupture limits is one of the principal areas that is investigated in this work. In addition to this, an understanding of the dependency of the creep rupture limit upon the defined time to creep rupture is also studied. Verification of these results is then given by full elastic-plastic analyses performed within ABAQUS.

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Keywords: Linear Matching Method (LMM); Creep; Shakedown; Cyclic Plasticity.

1. Introduction

Engineering components such as pipe intersections and branch connections are employed within a diverse range of circumstances and can therefore be exposed to a wide variety of operating conditions, often including cyclic

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combinations of mechanical and thermal loads. In cases of pure mechanical loading it is possible for these components to experience phenomena such as strict shakedown, reverse plasticity or ratchetting. However, in the event that such components are also exposed to significant thermal loads there can be a significant impact upon the material properties, such as a reduction in the yield stress, and this can impact upon the shakedown limit for the component.

There are a number of ways in which the elastic-plastic behaviour of structures may be analysed; one such method is simulation through the use of incremental Finite Element Analysis (FEA). This is an effective method which enables the investigation of a wide range of load cycles; however there can be significant costs in relation to the computing time required to perform such analyses, especially with complex 3D models. One other drawback is an inability to determine the boundaries for the shakedown limits directly. It can only demonstrate what the failure mechanism is for specific loading conditions i.e. elastic shakedown, plastic shakedown or ratchetting. The R5 assessment procedure [1] that is used by specialists in the UK nuclear industry does allow for the operation of components outside of the classic elastic shakedown limit, provided that it is within the global shakedown region and is not experiencing ratcheting. However, due to the fact that the assessment procedures used in the code are simplified processes, it can often be difficult for the shakedown status of a component to be demonstrated. The Linear Matching Method (LMM) has therefore been developed and incorporated within ABAQUS, commercial FEA program, for calculating the limit boundary between shakedown and non-shakedown behaviours [2, 3, 4].

The component that has been considered in this paper has previously been investigated using the LMM plug-in [5] to determine the shakedown limit under specific operating conditions. This analysis was primarily performed in order to verify the LMM shakedown procedures for a complicated engineering problem. Included within this work there was a combination of pressure and thermal loads in addition to the application of a bending moment, with all of the geometric parameters kept constant throughout. The results obtained within this study through the use of the LMM software tool aligned with the elastic plastic analyses which were also performed, showing that the LMM is able to solve practical engineering problems with complex loading conditions.

Due to the complex nature of piping systems, many of the components used come in standardised geometries; however, in some cases it might be preferable for custom dimensions to be used in order to reduce costs or improve efficiency. One such study [6] used the Elastic Compensation Method to investigate how the geometry of a component, similar to the one considered within this paper, impacts upon the limit loads and shakedown limits. This was achieved through modification of pipe diameter, with the shell thickness also altered in order to ensure that thin walled theory was still applicable. In this case an Elastic-Perfectly Plastic (EPP) material model was used in order to perform the calculations necessary to determine the limit boundary, also known as the Bree Diagram [7], for each variation of geometry. From this study it is shown that there is a strong interaction between the applied loads, particularly when the branch diameter is significantly smaller than that of the main pipe.

This paper aims to utilise the LMM plugin to determine the impact that changing the geometry of a header that is commonly used in advanced gas-cooled reactor's (AGR's) has upon the shakedown limit of the component. Within the current work an overview of the most significant areas of the background is first given which is followed by a description of the model that is used. The initial analyses of the standard geometry, along with appropriate verification calculations and an investigation into the effect that creep rupture has on the allowable internal pressure load for various rupture times are then discussed in section 4. Also presented within this section of the paper is a parametric study of the impact that changing the distance between the two branch pipes has on the failure mechanism of the component. The final area of interest, a parametric study of the effect of changing branch pipe diameter, is also discussed within this section of the current work.

2. Problem background

The key focus of the current work is to investigate the impact of several key parameters of a header branch pipe that is used in nuclear power plants. It is therefore necessary for the manners in which a component structure may respond to its loading conditions to be understood in order to determine the significance of the results. In addition, the tools that can be employed to perform these analyses – such as the R5 assessment procedure and the LMM analysis tool – should be clearly understood in order to utilise them correctly

2.1. Summary of structural responses

Under monotonic loading conditions a structure is able to withstand a so-called “limit load” before a plastic hinge is formed within the structure. Beyond this it is not possible for the structure to sustain any additional loads. However, when cyclic loading acts on the component it can also experience failure at lower levels. Under such circumstances one of several structural responses may be seen depending upon the magnitude and nature of the loading cycle.

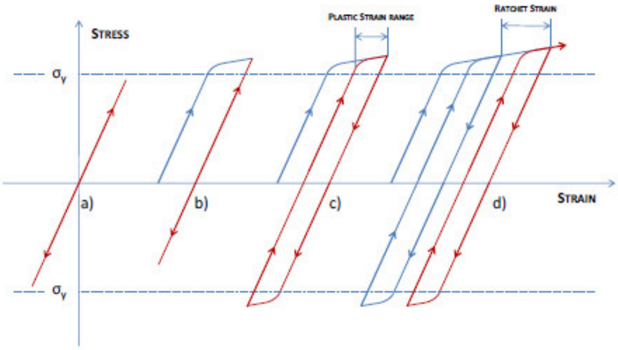


Fig. 1. Structural Responses to Cyclic Loading.

It is possible for the whole component to remain within the elastic limit throughout the complete loading cycle, as is shown in Fig. 1a, when load levels are sufficiently small. If the loading magnitude is increased beyond the elastic limit, then plastic strains begin to occur within the structure leading to one of three steady state responses:

- Strict shakedown: plastic strains accumulate in the initial load cycles producing residual stresses; response then becomes entirely elastic as shown in Fig. 1b.
- Global shakedown: plastic strains accumulate in all load cycles but is equal for both loading and unloading, as shown in Fig. 1c. A closed loop is formed and no net increase in the plastic strain occurs between cycles.
- Ratcheting: plastic strains accumulate in all load cycles, leading to an incremental plastic collapse of the entire structure, as shown in shown in Fig. 1d.

When the response is entirely elastic or within the strict shakedown region then such a condition is generally permissible. In fact, due to the initial plastic strains accumulated during strict shakedown, it is permissible for the elastic limit to be exceeded in some locations provided that a low cycle fatigue assessment is performed. The global shakedown of a component may also be permitted, so long as the presence of reverse plasticity is accounted for through a low cycle fatigue assessment. For the vast majority of cases it is not acceptable for the component to encounter ratcheting.

If a component is exposed to excessive temperatures then creep, a phenomenon involving the creation and movement of defects, may occur. The creep rupture limit then becomes another important design limit, as well as both the component shakedown and ratchet limits.

2.2. The Linear Matching Method

Having already been described extensively in various other works [8, 9], it is felt unnecessary for a full description of the LMM to be reported.

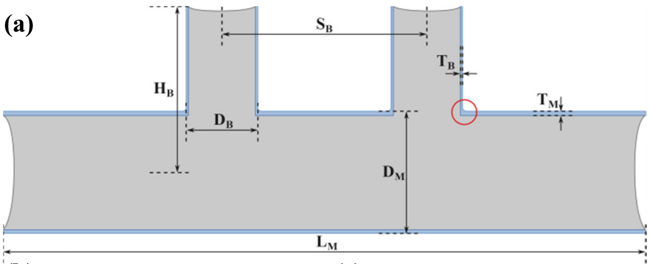
The LMM is based on the principle that a linear material model can be used within an iterative process to simulate nonlinear elastic-plastic behaviour of materials. This process involves the modulus throughout the model being reduced in regions where stress exceeds the material yield stress during sequential linear elastic analyses, therefore enabling the redistribution of stresses throughout the structure. This provides a very accurate depiction of the way such stresses are distributed in an elastic-plastic material. Each of the iterations may use a modified value of yield stress from the previous calculation, so that the temperature dependent yield stress and creep rupture stress could be easily adopted.

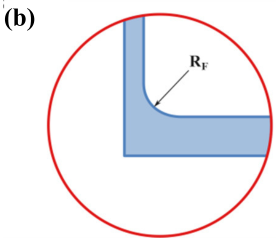
3. Finite Element Model

ABAQUS is one of the most widely used FEA software suites for analysing structural components, and for this reason the LMM subroutines have been implemented for it [4]. ABAQUS, together with the LMM subroutines, has been used within this study to perform the calculations necessary to determine the shakedown and creep rupture limit boundaries.

3.1. Header branch geometry

A full schematic of the header pipe from [5] is shown in Fig. 2a while Fig. 2b shows the intersection fillet in greater detail. The values for the dimensions highlighted are given in Fig. 2c. Branch pipe height is measured from the main pipe central axis, while separation measured from the central axes of the branch pipes. Outside diameter is used for both pipes, all other dimensions are self-explanatory.





(c)

Dimension	Value [mm]
D_M	603.6
L_M	3153.3
T_M	20.0
D_B	351.6
H_B	820.0
T_B	10.7
S_B	1003.3
R_F	20.0

Fig. 2. (a) Header Geometry; (b) Fillet Geometry; (c) Header Dimensions

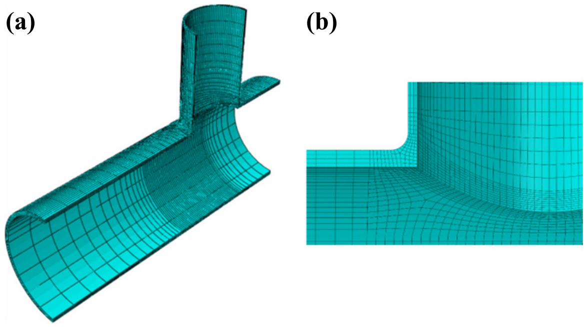


Fig. 3. (a) Header Pipe Mesh; (b) Intersection Mesh Detail

The FE model has been meshed using the quadratic brick element C3D20R within ABAQUS, as shown in Fig. 3a. Bias has been used in order to generate a denser mesh at the intersection between the branch and main pipe, resulting in a mesh that consists of 16,240 elements. Fig. 3b then shows how the component has been meshed around the intersection between the main and branch pipe in greater detail.

3.2. Material properties

The material considered in the current work is a stainless steel, with the key properties for this shown in Table 1. Other properties, such as material yield stress, are described where appropriate in later sections.

Table 1. Component Material Properties

Material Property	Value
Young's Modulus, E [GPa]	190.00
Poisson's Ratio, ν	0.30
Coefficient of Thermal Expansion, α [°C-1]	1.60×10-5
Thermal Conductivity, k [W/mm•K]	1.63×10-2

3.3. Boundary and loading conditions

In order to improve the computing efficiency a quarter-model with symmetry conditions was used as shown in Fig. 3 and Fig. 4. Fig. 4 shows which surfaces these symmetry conditions were applied to. An equation constraint was applied to the end surface of the branch pipe in order to simulate the plane condition. A multi-point constraint fixed to a central reference point was applied to the end surface of the main pipe. This allows the expansion/contraction of the pipe while also preventing the pipe from moving freely.

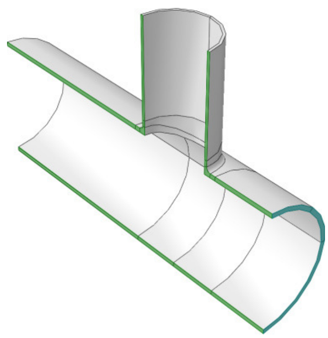


Fig. 4. Geometry Symmetry Conditions.

A reference pressure of 4.55 MPa is applied to all of the internal surfaces to simulate the operating conditions [5]. As both branch pipes are assumed to have closed ends, a force acts along the cylinder axis. This is calculated using thin-walled theory and is applied as a pressure to the end surfaces.

$$\sigma_Z = \frac{\Pr}{2t} \tag{1}$$

Each load cycle in the step-by-step analysis consists of three steps: uniform low temperature distribution, transient temperature distribution and uniform high temperature distribution, and so thermal analyses are performed to generate the output database files needed to apply these cyclic thermal loads as predefined fields. The DC3D20

ABAQUS element is used in the same mesh structure and suitable temperature boundary conditions are applied to the inner and outer surfaces.

4. Results and discussions

Throughout the current work analyses have been carried out using the LMM analysis tool so as to better understand the failure mechanisms of a practical engineering problem.

4.1. Shakedown and creep rupture analysis

In order to obtain the required results the LMM has been used to perform shakedown limit analyses; these are then verified using step-by-step analyses. The initial focus of the current work was also to investigate the impact of creep upon the header branch pipe considered within this paper.

Table 2. Temperature Dependent Yield Stress.

Temperature [°C]	Yield Stress [MPa]
26.85	206.85
126.85	167.49
226.85	143.38
326.85	128.64
426.85	121.36
526.85	117.62
626.85	112.52
726.85	102.99

4.1.1. Shakedown analysis and verification

The shakedown boundary is a theoretical line separating shakedown and non-shakedown behaviours, and is unique to a structure and the loads applied to it. The LMM plugin has been used in order to generate such a boundary for the header pipe component of interest within the current work. For this analysis temperature dependent yield stress has been used, with the values for this over a range of temperatures shown in Table 2 [10]. Fig. 5 shows the shakedown boundary for the header branch under the operating conditions that were applied. The thermal load has been normalised against a reference temperature of $\Delta\theta_0$=650 °C in order for creep conditions to be considered, while the mechanical load has been normalised against a reference pressure of P_0=4.55 MPa.

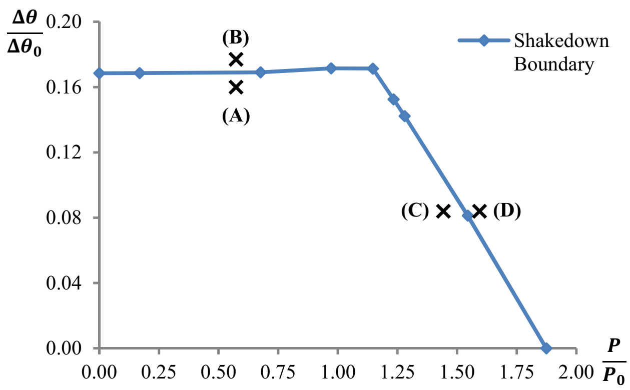


Fig. 5. Shakedown Limit Boundary.

From this analysis it can be seen that the multiplier for the temperature load at the reverse plasticity limit is very small, approximately 0.17, which means that such a header component would not be able to withstand temperatures that would be sufficiently high so as to cause creep. Generally cyclic thermal conditions such as these are a worst case scenario as the cyclic temperature range of a component is unlikely to be of this magnitude (0-650°C) under normal operating conditions, and therefore the actual thermal stresses produced are likely to be smaller in than those within the current model.

Shown in Fig. 6a is the plastic strain distribution for a pure thermal load case at the reverse plastic limit, while Fig. 6b shows this for the pure mechanical load case at the limit load. From this it can be seen that the failure mechanism of the component is highly dependent on the loading conditions. In the case of the purely thermal load cycle the critical location is found on the outer surface of the branch base and perpendicular to the central axis of the main pipe. In the pure mechanical load case, the critical location can be found at the base of the branch facing towards the other branch pipe.

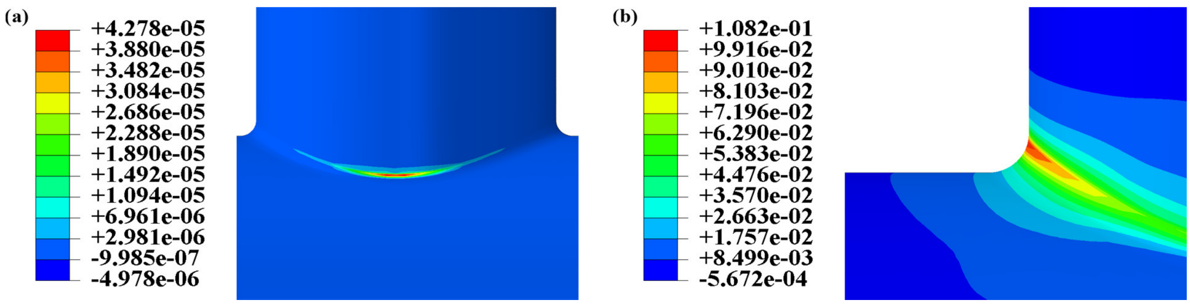


Fig. 6. Effective Plastic Strain under (a) Pure Thermal Loading; (b) Pure Mechanical Loading.

In order to ensure that the results obtained through the LMM plugin are accurate it is necessary for a series of step-by-step analyses to be performed for verification. These can be very time consuming as a result of requiring a significant number of loading cycles for a steady state response to be achieved. As a result only the four analyses have been performed, for the points highlighted in Fig. 5. Points' "A" and "B" are used to verify the reverse plasticity limit, while points; "C" and "D" are used to verify the ratchet imit. Each load cycle in the step-by-step analysis consists of three steps: uniform low temperature distribution, transient temperature distribution and uniform high temperature distribution, in addition to the proper mechanical load at each step.

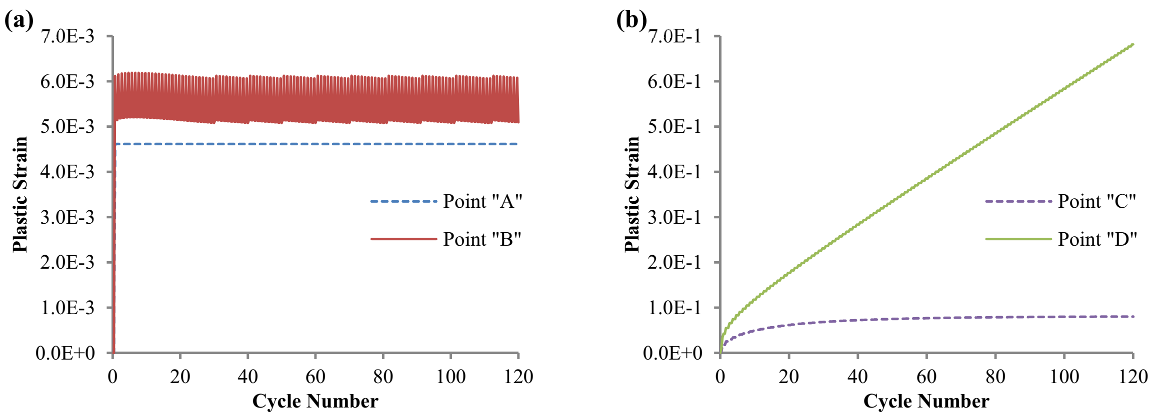


Fig. 7. (a) LMM Verification for Points' "A" & "B"; (b) LMM Verification for Points' "C" & "D".

Figure 7a provides confirmation of the reverse plasticity limit boundary that was determined using the LMM. From this it can be seen that the response at point “A”, below the limit, is strict shakedown. At point “B”, above the limit, it can be seen that the component is in global shakedown. Hence Fig. 7b confirms the calculated reverse plasticity limit for the component. The response of the component at point “C” requires a high number of cycles to reach steady state, at which point it can be seen that it is in the strict shakedown region. In contrast to this, point “D” reaches a steady state in a shorter time and can be seen to be ratcheting.

4.1.2. Creep rupture analysis

Creep is a phenomenon that is dependent upon both the operating temperature and the creep dwell time. To demonstrate how operating temperature and creep dwell time can impact on the maximum internal pressure the header can sustain, creep rupture limit analyses under uniform high temperature were performed. Unlike for the shakedown analysis, where the transient temperature distribution has a significant impact on the shakedown limit, the transient temperature distribution can be ignored in the creep rupture analysis, as the transient period is much shorter than the steady state operation period.

Table 3. Temperature Dependent Creep Rupture Stress.

Temperature [°C]	Rupture Stress [MPa] for considered Rupture Times [hrs]					
	1,000	3,000	10,000	30,000	100,000	300,000
500	118.56	118.56	118.56	118.56	118.56	118.56
550	116.18	116.18	116.18	116.18	116.18	116.18
600	113.80	113.80	113.80	112.00	94.00	79.00
650	107.76	107.00	88.00	70.00	57.00	46.00
700	89.00	72.00	57.00	45.00	34.00	27.00
750	63.00	50.00	38.00	29.00	21.00	16.00

The values for creep rupture stress used throughout this analysis are shown in Table 3 [10]. For certain temperatures and rupture times there is no difference between the creep rupture stress and the yield stress. It can be seen from Fig. 8 that when the component is operating at temperatures less than around 600°C there is no significant creep effect for the full range of rupture times that were considered. This is due to the fact that the creep rupture stress at these temperatures has a value which is no less than the yield stress of the material, meaning that the LMM does not replace yield stress with creep rupture stress.

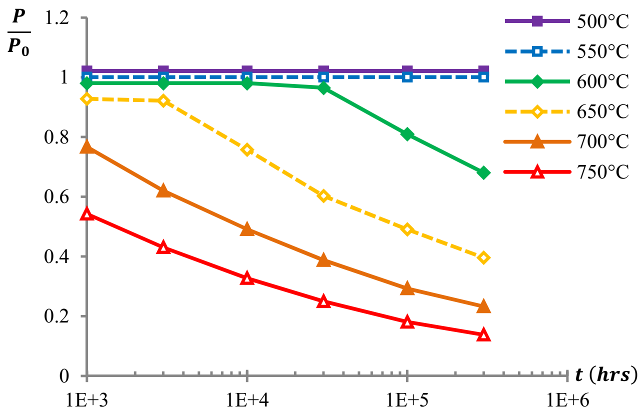


Fig. 8. Creep Rupture Limits with Different Temperatures and Allowable Creep Times.

However, as the operating temperature of the component is raised to 600°C it can be seen that for greater rupture times there begins to be a reduction in the creep rupture limit of the component. This is due to the fact that the creep rupture stress values for these temperature values have magnitudes which are less than that of the yield stress. In order for the component to withstand the accumulation of the creep strains for longer periods there must be a reduction in the applied pressure load, and this is shown in Fig. 8. As temperature is further increased it can be seen that the capacity of the component to withstand the internal pressure load is reduced to a greater extent than under non-creep conditions as a result of the rupture stress being significantly lower than the yield stress.

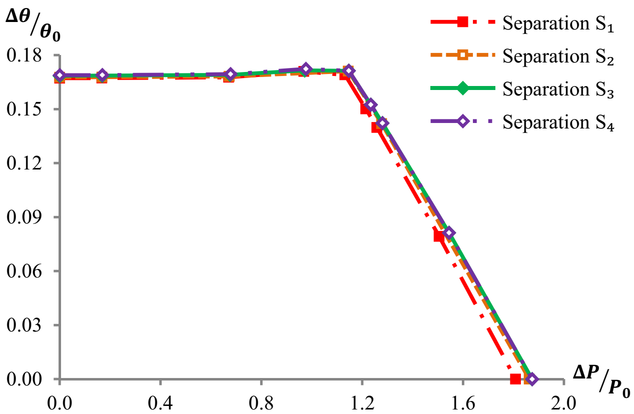


Fig. 9. Shakedown Limit Interaction Curves with Different Branch Separation.

Table 4. Variation of Branch Separation.

Geometry	SB [mm]
1	503.3
2	628.3
3	1003.3
4	1503.3

4.2. Effect of changing branch pipe separation

The Bree-like diagram for the varied separation analysis of the header branch pipe under constant internal pressure and the thermal cycle which have been chosen for this study, shown in Table 4, is shown in Fig. 9. As before, the constant internal pressure, P, has been normalised against the initial value, P_0, and the cyclic temperature load, $\Delta\theta$, has been normalised against the initial value, $\Delta\theta_0$. Values for temperature dependent yield stress from Table 2 were again used for these analyse.

These results demonstrate that there is no significant impact upon the reverse plasticity of the component as the separation is modified, however it can be seen that there is a slight decrease in the ratchet limit for a reduced separation. Fig. 10 shows the effective plastic strain of the component at the reverse plasticity limit under cyclic thermal loading for the various configurations. In the case of these large levels of cyclic loading the component failure mechanism predicted by the LMM is located on the outer surface of the branch pipe base where it intersects with the side of the main pipe. This is caused by the thermal stresses created by the non-uniform temperature distribution through the pipe wall thickness in the transitional stage.

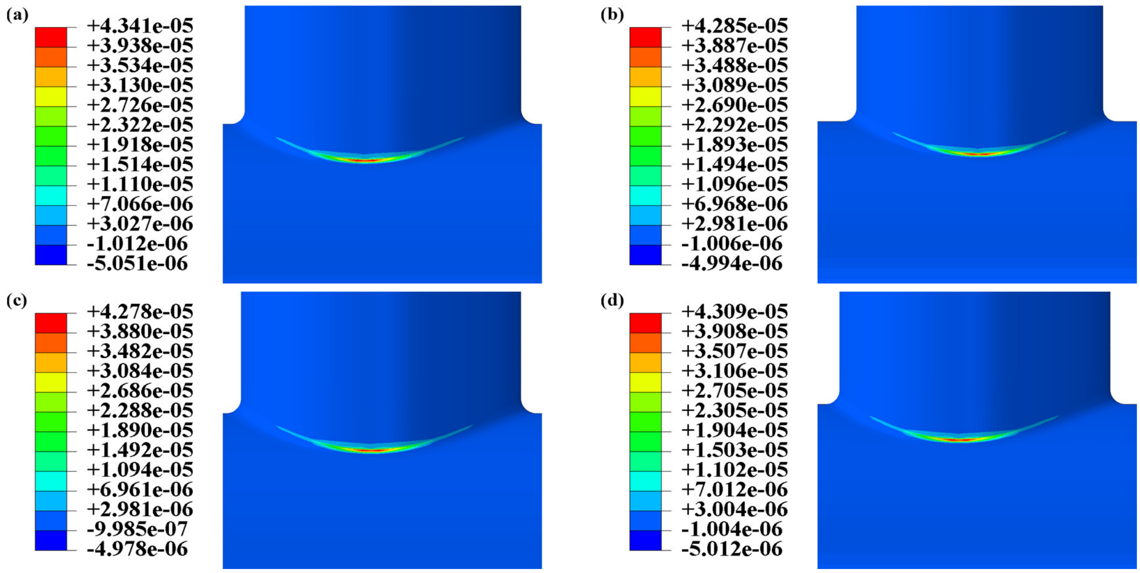


Fig. 10. Effective Plastic Strain under Pure Thermal Loading for (a) S1; (b) S2; (c) S3; (d) S4.

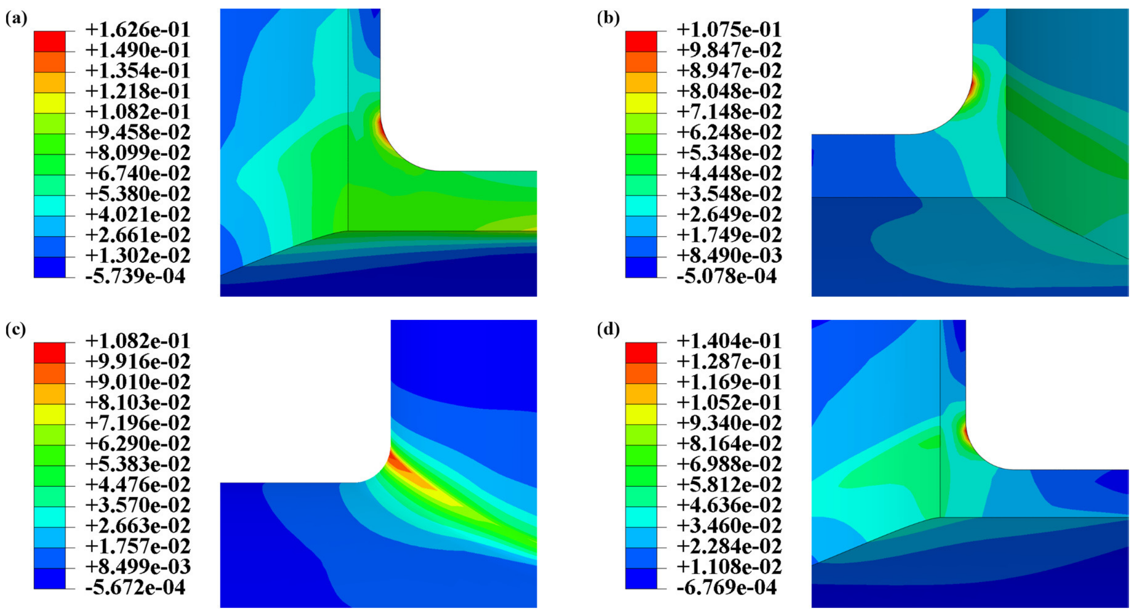


Fig. 11. Effective Plastic Strain under Pure Mechanical Loading for (a) S1; (b) S2; (c) S3; (d) S4.

When the component is exposed to pure mechanical loading the failure mechanism rotates around the base of the branch pipe and is located on the side that intersects with the top of the main pipe, as shown in Fig. 11. There is only a noticeable difference in the ratchet limit when separation of the branch pipes is sufficiently small, i.e. S_1=503.3mm. This phenomenon results from the interaction of the stress fields generated within the main pipe by the internal pressure on the branches. However, when the distance between the branches is large enough these do not overlap and therefore have no resulting impact upon the failure mechanism of the component.

4.3. Effect of changing branch pipe diameter

Values for temperature dependent yield stress from Table 2 were again used. Table 5 demonstrates how diameter has been changed for this study. Distance between pipes is kept constant by modifying the value for separation to maintain a constant distance between the outer surfaces of the branch pipes, while the wall thickness is kept at a constant ratio relative to branch diameter to ensure that only diameter is affecting the results.

The Bree diagram for the chosen values of diameter of the header branch pipe is shown in Fig. 12. As before, the constant internal pressure, P, has been normalised against the initial value, P_0, and the cyclic temperature load, $\Delta\theta$, has been normalised against the initial value, $\Delta\theta_0$. These results demonstrate that both the reverse plasticity and ratchet limits of the component are significantly affected by changes in the diameter of the branch pipes. The increase of the diameter of the branch pipes leads to a significant increase of the reverse plastic limit, but a decreased ratchet limit and limit load.

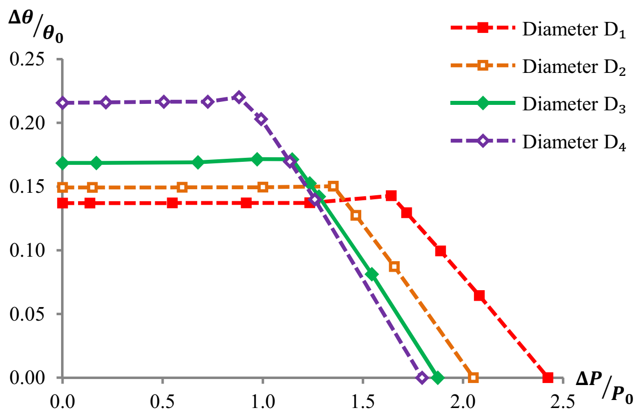


Fig. 12. Shakedown Limit Interaction Curves with Different Branch Diameters.

In Fig. 13 the effective plastic strain of the component under the cyclic thermal loading condition only at the reverse plastic limit is shown. The LMM predicts that the failure mechanism under such loading will occur at the base of the branch pipes. When the diameter is reduced to a small enough value this failure occurs around the complete circumference of the branch base. Whereas it can be seen for medium sized diameters that failure is localised to the outer surface of the branch pipe base, where it intersects with the side of the main pipe. In the case of a very large diameter, the failure mechanism is located in the same region of the branch pipe base but on the internal surface. This increased resistance to thermal loading is partly due to the increased wall thickness, which reduces thermal stresses in the pipe.

Table 5. Variation of Branch Diameter.

Geometry	D_B [mm]	D_B/ D_M	S_B [mm]	T_B [mm]
1	176.6	0.293	817.64	5.37
2	264.1	0.438	910.48	8.04
3	351.6	0.583	1003.30	10.70
4	526.6	0.872	1188.96	16.03

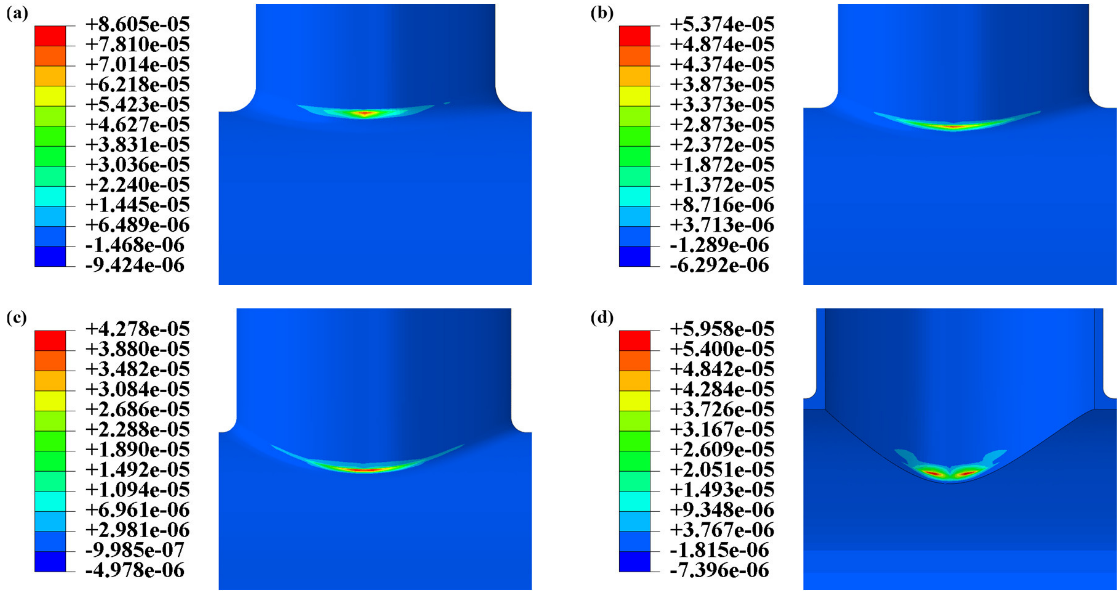


Fig. 13. Effective Plastic Strain under Pure Thermal Loading for (a) D_1; (b) D_2; (c) D_3; (d) D_4.

When the component is exposed to pure mechanical loads, shown in Fig. 14, the failure mechanism is similar to the original header geometry, with increasing magnitude of plastic strains as diameter is reduced. However, for larger diameter branch pipes the mechanism of failure is instead at the base of the branch pipe where it intersects with the side of the main pipe.

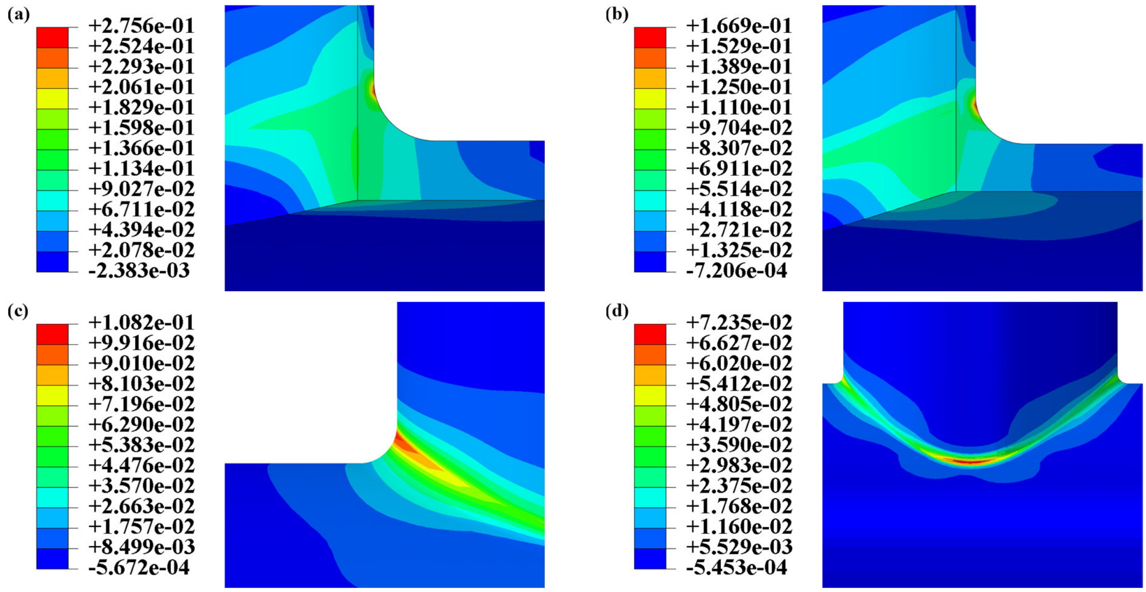


Fig. 14. Effective Plastic Strain under Purely Mechanical Loading for (a) D1; (b) D2; (c) D3; (d) D4.

Shown in Fig. 14 is the impact that branch diameter has upon the limit load and reverse plastic limit for both the internal pressure and temperature respectively. This plot again shows that there is an inverse correlation between the diameter of the branch pipes and the maximum internal pressure that the component can withstand. In contrast to this, there is a positive correlation between the diameter of the branch pipe and the maximum internal temperature which the component can be exposed to safely.

When the internal temperature is sufficiently high then there are also large levels of local stress, which result in the component experiencing reverse plasticity. Internal pressure generates a higher overall stress within the component when it is elevated, and this is what causes global failure in this case.

5. Conclusions

Presented within this paper is work investigating how significant an effect the operating temperature has upon the internal pressure a header pipe is able to withstand prior to creep rupture for various rupture times. In addition to this a study of the effects of two key geometric parameters of the header, branch separation and diameter, on the shakedown limit boundary has also been presented. The header pipe has been modelled using symmetry conditions to simplify the model and allow for the computing time to be minimised.

A study of the structural response of the component to different cyclic load points has been performed using the LMM plug-in tool. The cyclic thermal load is found to have a more significant impact upon the shakedown boundary than the mechanical load. It has also been shown how creep can impact upon component life and therefore be a significant factor in determining the maximum pressure on the internal surfaces that is permissible before creep rupture will occur.

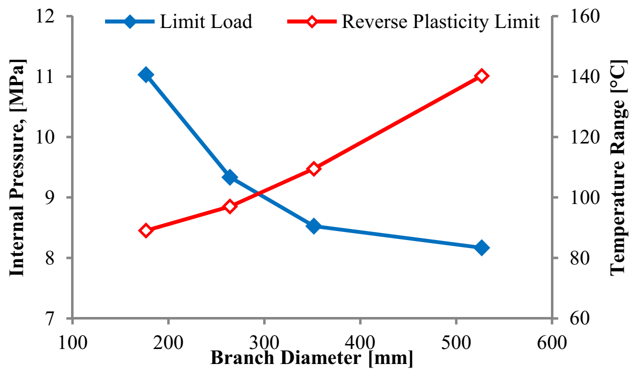


Fig. 15. Impact of Diameter on Shakedown Limits.

In addition to these results, an investigation into various geometric parameters has been conducted to determine how significantly they impact upon the shakedown limit boundary of the component. This study first looked at the separation of the branch pipes and the manner in which the limit boundary and failure mechanisms change as this distance both reduces and increases from the original value. From the results it can be seen that the separation does not have a significant impact. There is only a slight decrease in both the reverse plasticity limit and the ratchet limit when there is a very small distance between the branch pipes. However, when the separation was increased there was no impact on the limit.

The second area that was investigated within the current work was the significance of branch pipe diameter and how it affects the failure mechanism when modified. So as to ensure that only the diameter of the pipe was affecting the results, the separation between the pipes was kept constant. It can be seen that the branch pipe diameter has a more substantial impact upon the failure of the component than separation. As the diameter is reduced there are two outcomes: the limit load and the ratchet limit of the component are increased, while the reverse plasticity limit is reduced.

This investigation into both the creep behaviour and the importance of certain geometric parameters on the failure of a header branch pipe has shown how the LMM subroutines can be used to analyse practical engineering problems.

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