Consideration of moment redistribution (MR) in the design of continuous reinforced concrete (RC) beams results in an efficient and economical design. Adding fibre-reinforced polymer (FRP) materials to reinforced structures to enhance flexural capacity leads to a reduction in ductility, such that design standards severely limit use of the MR in their design. This has forced engineers to use elastic analyses for strengthening design, which can lead to FRP wastage. To overcome this, complicated or empirical solutions have been applied to solve the problem of MR in strengthened concrete members, with limited success. This paper presents a novel theoretical strategy for quantifying and tracking MR in such members by employing basic structural mechanics without any need for estimating rotation capacity or ductility. Fully non-linear flexural behaviour of continuous strengthened members can be predicted and any geometry, loading arrangement and strengthening technique or configuration can be considered. The numerical model is validated against existing experimental data from the literature. Good agreement is shown between the experimental and numerical data, with the significance of this work being that, potentially, for the first time MR could credibly and confidently be incorporated into design guides for FRP strengthening of RC structures.

**Notation**

- \( A_{frp} \) area of FRP
- \( A_s \) area of tension steel reinforcement
- \( A_{sc} \) area of compression steel reinforcement
- \( C \) total force in compression
- \( C_c \) compression in concrete
- \( C_s \) compression in steel reinforcement
- \( d \) effective depth to tension reinforcement
- \( d_c \) effective depth to compression reinforcement
- \( E_A \) tension stiffness of FRP
- \( E_{cm} \) Young’s modulus of concrete
- \( E_t \) tensile modulus of FRP
- \( EI \) flexural stiffness
- \( f_c' \) compressive strength of concrete
- \( f_{cm} \) mean compressive strength of concrete at 28 d
- \( f_{ck} \) characteristic cylinder strength of concrete
- \( f_{cm} \) characteristic tensile strength of concrete
- \( f_y \) yield strength of steel reinforcement
- \( h \) overall height of beam
- \( K \) curvature
- \( k_d \) neutral axis depth
- \( M \) bending moment (BM)
- \( M_{cr} \) cracking moment of concrete
- \( M_{elas} \) theoretical BM determined from elastic analysis
- \( M_{redis} \) redistributed BM
- \( M_u \) moment capacity
- \( P \) applied load
- \( P_u \) ultimate (failure) load
- \( T \) total force in tension
- \( T_c \) tension in concrete
There are various reasons why existing reinforced concrete (RC) structures may require strengthening or retrofitting, for example a need for greater strength, durability or even ductility. Adding fibre-reinforced polymer (FRP) materials to RC structures has been recognised as an effective technique to enhance the strength and durability of such structures (Hollaway and Leeming, 1999; Teng et al., 2001). However, research has demonstrated that FRP strengthening of flexural members reduces their original ductility prior to FRP debonding (Casadei et al., 2003; El-Refaie et al., 2003; Oehlerls and Seracino, 2004; Oehlerls et al., 2007). The elastic nature of the FRP generally leads to a more brittle failure of FRP-strengthened RC members.

Ductility is an intrinsic characteristic in many materials and allows them to deform plastically before failure. As discussed by Beeby (1997), one of the major advantages of ductility is that the bending moment (BM) can be redistributed automatically in a ductile continuous member from zones that are stressed plastically to zones that are not yet plastic. Having sufficient ductility helps to satisfy the lower bound theorem of plasticity in design, which in turn ensures that no undesired collapse mechanism occurs prior to the expected failure mode. In addition, the ability for redistribution of BMs in conventional statically indeterminate RC members allows for an efficient and economical design by reducing the cross-sectional area or internal reinforcement in the zones with maximum BM and congested reinforcement (Matlock, 1959; Scott and Whittle, 2005).

If a structure is not ductile or if the original ductility is fully lost after strengthening, no advantage can be taken of moment redistribution (MR) in the structure. However, what level of ductility is required to allow some MR to occur? A lack of sufficient research looking at a link between the precise reduction in the ductility of RC members after FRP strengthening and any possible MR thereafter has resulted in uncertainty in this issue such that design standards worldwide have ignored (or overly conservatively limited) the exploitation of MR in FRP-strengthened RC flexural members (e.g. ACI-440-2R (ACI, 2008); TR55 (CS, 2012)). This means that RC members that need to be strengthened using FRP must be designed based on assumed elastic flexural behaviour up to failure, despite the fact that the original structure may have been designed with full consideration of ductility and MR. As Ibell and Silva (2004) described, this results in a very complex design condition because, after strengthening, the zones that were originally designed for a reduced BM must now be designed according to the original un-redistributed elastic BM plus any additional BM that is required for the strengthening requirement. Therefore, this can result in the need for great quantities of strengthening material. Consequently, it is very important that the profession knows exactly the level of MR that is likely, lest vast quantities of materials are wasted unnecessarily.

Quantifying MR in FRP-strengthened RC beams is potentially a complex problem. A few theoretical research studies have been conducted on this issue. Oehlerls et al. (2004) claimed that it is very hard to determine the adequacy of ductility in an FRP-strengthened RC beam. For quantifying MR, they proposed two different analytical approaches – the ‘flexural rigidity approach’ and the ‘plastic hinge approach’. In the first approach, stiffness variation is accommodated within zones with sagging and hogging BMs; in the second approach, it is assumed that flexural stiffness is constant along the entire beam except for the zones where plastic hinges are formed. Oehlerls et al. (2004) note that the hinge approach cannot be applied to FRP-strengthened beams because, usually, FRP debonding typically occurs prior to concrete crushing and the strengthened region usually behaves elastically prior to debonding. This means that no plastic hinge (i.e. a region of constant BM capacity with increase in curvature) can be formed in FRP-strengthened zones. However, using the rigidity approach, they indicate that the ductility of FRP-plated beams is lower than that of steel-plated beams. A simplified theoretical method was proposed by Ashour et al. (2004) to predict the load capacity of an FRP-strengthened beam. The method relies on equilibrium of forces and compatibility of deformations. This method can be used to calculate MR at failure, although it is assumed that the critical sections in the sagging and hogging zones reach their moment capacity at the time of failure.

Silva and Ibell (2008) applied a theoretical strategy to investigate ductility in such structures. They showed that an RC beam can still exhibit rotation capacity even after FRP strengthening, provided that the strengthened section has sufficient curvature ductility. They demonstrated that, although ductility is reduced in general, the BM can be redistributed out of an FRP-strengthened section by at least 7.5% provided the section has a curvature ductility capacity (defined as the ratio of the curvature at ultimate failure to the curvature at steel...
therefore, to be rather complex to implement in a general sense. A few studies have also been conducted more recently to predict or analyse MR in strengthened structures using a computer program based on finite-element modelling (Breveglieri et al., 2012; Dalfré and Barros, 2011). The results showed that the technique and configuration of strengthening significantly influences the degree of MR. Santos et al. (2013) and Lou et al. (2015) presented finite-element models to predict MR in FRP-reinforced RC beams. The models basically assume a specific damage model for concrete, elastic–plastic behaviour for steel, isotropic behaviour for the steel–concrete interface, linear elastic behaviour for the FRP and perfect bond for the FRP–concrete interface. The numerical simulations showed good agreement with the experimental findings.

There is a lack of sufficient research on defining clearly and relatively simply the extent to which MR can be relied on when an RC beam is strengthened using FRP. This paper presents a new numerical model that allows the redistribution of a BM in an FRP-strengthened RC beam to be quantified rigorously. To predict the flexural behaviour of the strengthened beam, the model applies a fundamental approach that is based on structural mechanics, not on empirical limits, and allows stiffness variations along the length of the beam to be found and updated during loading using an iterative approach. The degree of MR can be determined at any point along the beam length and at any applied load until failure. The new model was verified against experimental findings from the literature. It should be noted that the aim of this paper is only to present a model that can predict how MR occurs over the loading cycle, up to failure, based on assumed values for FRP debonding or rupture, and not to predict the actual failure mode. However, if required or desired, models for predicting failure modes (including concrete crushing, FRP debonding/rupture and even shear failure) can be accommodated in the numerical model presented.

2. Moment redistribution

In this section, the implication of MR is briefly presented through a particular (simple) example. An idealised elastic–plastic relationship between curvature (K) and bending moment (M) is considered in Figure 1 for all sections throughout the beam shown in Figure 2. A section reaches its moment capacity of at a curvature of when the steel reinforcement yields, and the section fails at an ultimate curvature of .

Figure 2 shows a statically indeterminate two-span conventional RC beam loaded symmetrically under a concentrated load at each mid-span. A constant flexural stiffness of EI is assumed along the entire length of the beam before loading.

Within the elastic range across the entire beam, the ratio of the hogging-zone BM to the sagging-zone BM remains constant. According to ‘elastic theory’, this ratio is 1:20 for this particular example. As long as this ratio is fixed, no redistribution of BM occurs in the beam. If the load increases further (load , as shown in Figure 2), the steel reinforcement will yield first in the hogging zone (over the central support), due to the loading arrangement adopted, and this zone will just reach its moment capacity of at a curvature of when the steel reinforcement yields. The solid line in Figure 2 (line B) shows the theoretical elastic BM diagram at the failure load, assuming that there had been no stiffness variation during loading to have led to MR. The primary reason that allows the increase in BM in the sagging zone (from (5/6) to ) to occur is the presence of curvature ductility of the hogging zone. It is seen that the ratio of hogging-zone BM to sagging-zone BM becomes 1:0 at ultimate failure, rather than the elastic ratio of 1:20. Hence, it can
be concluded that the BM has been redistributed from the hogging zone to the sagging zone, as shown in Figure 2.

This process becomes more complicated when FRP is added. As shown in Figure 3, there are various zones in an FRP-strengthened RC member that can be unstrengthened (such as zone A), or lightly strengthened (such as zone B) or heavily strengthened (such as zone C). When the member is loaded, these zones experience different rates of stiffness variation.

As illustrated in Figure 3, there is no horizontal plastic plateau in the $M-K$ relationship of the FRP-strengthened zones. This means that no plastic hinge is formed in the strengthened zones even if the steel reinforcement yields as the FRP withstands the applied load elastically until failure. In addition, various amounts of FRP can be added to the member in various configurations, affecting the mode of failure and the flexural behaviour of the strengthened member. These complexities indicate a need for a fundamental solution to this problem.

A novel numerical model, which can predict the flexural behaviour of FRP-strengthened RC members using a fundamental approach, is described in the next section. The model relies only on structural mechanics and tracks stiffness variations in the beam logically, whether strengthened or not.

3. The numerical model

The new model employs sectional analysis to determine stiffness variations in the beam over the loading cycle. A computer program has been written for the numerical calculations and analytical modelling. The given beam is initially subdivided into a large number of narrow vertical segments (e.g., slices of 10 mm thickness). The full $M-K$ relationship is found for each section along the length of the beam, whether strengthened with FRP or not. It is obvious that the more precise the relationship found between moment ($M$) and curvature ($K$) for each section, the required data for the numerical model include the geometry, specifications of the internal reinforcement and strengthening materials, and constitutive material models.

Figure 4 illustrates the material models adopted for concrete, steel and FRP in this numerical technique. A parabolic curve has been adopted for the stress–strain relationship of the concrete in compression, according to BS EN 1992-1-1: 2004 (BSI, 2004) (Figure 4(a)). $\epsilon_{c1}$ is the strain at peak stress, and is equal to $0.7f_{cm}^{0.31}$, where $f_{cm}$ is the mean compressive strength of concrete at 28 d. $\epsilon_{cu1}$ is the ultimate compressive strain in concrete, which is considered to be 0.35% in this parabolic model. A linear relationship between stress and strain has been adopted for concrete in tension, according to BS EN 1992-1-1: 2004 (BSI, 2004) (Figure 4(b)). The concrete tensile strength

3.1 Determination of the $M-K$ relationship

To find a precise relationship between moment ($M$) and curvature ($K$) for each section, the required data for the numerical model include the geometry, specifications of the internal reinforcement and strengthening materials, and constitutive material models.

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(f_{ck}) is equal to 0.3f_{ck}^{2/3} (in MPa), where f_{ck} is the characteristic cylinder strength of concrete (f_{ck} = f_{cm} – 8 (MPa)). In addition, the tensile strain \( \varepsilon_t = f_{ctm}/E_{cm} \), where \( E_{cm} \) is the modulus of elasticity of concrete (GPa), equal to 22(\( f_{cm} \))/10^3. Softening of the concrete under tension is ignored in the numerical model as it does not play any role in the degree of MR quantified at failure. The behaviour of the steel reinforcement is represented by a bilinear model (Figure 4(c)) with a linear elastic branch ending at the yield stress (f_y) and a linear inclined plastic branch that shows strain hardening in the steel reinforcement after yielding, ending at ultimate fracture (f_u). The relationship between stress and strain for FRP is considered linear-elastic up to rupture (Figure 4(d)).

The M-K relationship for each section along the beam is found according to standard procedures, which are outlined here for completeness. Each cross-section along the beam is divided into horizontal segments of 1 mm thickness. For varying curvatures starting from zero, strains in each segment are found using an initial estimate for the neutral axis depth (k_d) by assuming that there is a perfect bond between the concrete and steel reinforcement, and between the concrete and FRP, and also that plane sections remain plane. Using the adopted material models, the stresses and forces are calculated separately for the tension and compression zones of the section, by knowing the corresponding strains in each constitutive material. As shown in Figure 5, the overall tension force (T) includes tension in the steel reinforcement (T_s), concrete (T_c) and FRP (T_{frp}), and the overall compression force (C) includes compression in the concrete (C_c) and compression steel (C_s). If the overall tension force is not in equilibrium with the overall compression force (i.e. T ≠ C), the neutral axis position is adjusted and the forces are recalculated while maintaining the same curvature. This calculation is performed iteratively until equilibrium is achieved and a precise position for the neutral axis is found. Note that it is a simple matter to assume T = 0 if this is thought sensible.

Finally, the corresponding moment of resistance (M) is determined from the calculated k_d for the adopted level of curvature by taking moments for the tension and compression forces about the neutral axis

\[
M = (C_c \times \bar{y}) + [C_s(k_d - d_s)] + (T_c \times \bar{y}') + [T_s(d - k_d)] + [T_{frp} \times (h - k_d)]
\]

where \( \bar{y} \) represents the distance between the neutral axis and the centroid of the concrete's compression zone, given by

\[
\bar{y} = \frac{\Sigma (A_i \times C_c \times y_i)}{\Sigma (A_i \times C_s)}
\]

where \( A_i \) is the area of horizontal layer \( i \), \( C_c \) is the compression force in layer \( i \) and \( y_i \) is the depth from the centroid of layer \( i \) to the neutral axis. Similarly, \( f' \) is the distance between the neutral axis and the centroid of the concrete’s tension zone.

A complete M-K relationship can be found for all cross-sections along the beam by repeating these calculations for different curvature values, until failure. Ultimate failure is simply controlled through specifying limiting values for strains in the concrete and FRP. In this study, a typical strain value of 0.35% is adopted for crushing of concrete in compression, and values of 0.8% and 1.5% are assumed for failure of the FRP through debonding (usual) and rupture (if the FRP is fully anchored) respectively. These values are based on what has been observed in the literature but are not definitive. If required, these assumed values can be refined appropriately.

3.2 Determination of the real BM distribution

The real distribution of BMs along the beam length is now determined for each applied load using the M-K relationships found in the previous section. For a load increment starting from zero, the elastic BM is determined for all sections along the beam using, for example, the virtual work method and using the baseline uncracked flexural stiffness for each section. Knowing the BM at each section and using the corresponding M-K relationship, the curvature of each section is

\[\text{Figure 5. Calculation of tension and compression forces in an FRP-strengthened RC section}\]
From ‘elasticity theory’, the actual effective stiffness, $\left( \frac{EI}{\text{effective}} \right)$, can then be found for each section according to

$$3. \quad \left( \frac{EI}{\text{effective}} \right) = \frac{M}{K}$$

where $M$ is the bending moment and $K$ is the curvature of each section. Now, a new distribution of BMs is found according to

Figure 6. Schematic illustration of how iterations are conducted using the new numerical model.
It should be noted, however, that the proposed model produces less accurate results when the zone that is controlling MR is unstrengthened. This is because, in this specific case, the plastic plateau of the $M-K$ relationship related to the critical zone is almost a horizontal line (line A in Figure 3(b)), making it difficult or impossible to define a unique and accurate curvature for a given BM after steel yield. Hence, the numerical model requires a non-horizontal plastic plateau to be able to complete the computational iteration required for the calculation of BMs described earlier. To overcome this problem, an alternative approach based on equilibrium of BMs in the sagging and hogging zones was developed (Tajaddini, 2015). This is not required for the cases presented in this paper.

5. Verification of the new model
The numerical model was validated against existing experimental data reported in the literature. Figure 7 illustrates a schematic image of the geometry and loading arrangement of the experiments conducted by El-Refaie et al. (2003), Oehlers et al. (2004) and Aiello and Ombres (2007). All specimens were two-span rectangular RC beams loaded under concentrated loads at each mid-span, symmetrically. The experiments were carried out to investigate MR arising after flexural strengthening of continuous RC flexural members. Various strengthening techniques were used for the specimens in the different test series. Details of the test specimens, specifications of the test layouts and configurations of FRP strengthening are summarised in Table 1.

Figure 8 compares the experimental data and the numerical results obtained from the new model for the failure load and hogging-zone BM at failure in the specimens. It should be noted that the numerical results were obtained assuming similar failure strains for FRP debonding or rupture to those recorded experimentally. Except for beam SF4, reasonable agreement can be seen between the experimental and numerical results, indicating the ability of the numerical model to predict the flexural behaviour of continuous RC members strengthened using FRP.

The numerical model generally predicts correctly the flexural softening and mode of failure in the critical zones of the tests reported in the literature. Using the proposed model, progression in flexural softening can be tracked and monitored logically. Table 2 provides a comparison of the experimental data and corresponding numerical results. The table summarises the modes of failure, the load values at which first cracking

| Beam | Width × depth: mm | $L$: mm | $L_1$: mm | Top steel bars | Bottom steel bars | FRP position | FRP $E_A$ value: kN | Steel yield strength: MPa | Concrete strength: MPa
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>H2\textsuperscript{a}</td>
<td>150 × 250</td>
<td>3830</td>
<td>1915</td>
<td>2T8</td>
<td>2T20</td>
<td>Hogging</td>
<td>6180</td>
<td>505–510</td>
<td>43·5 cu</td>
</tr>
<tr>
<td>H3\textsuperscript{a}</td>
<td>150 × 250</td>
<td>3830</td>
<td>1915</td>
<td>2T8</td>
<td>2T20</td>
<td>Hogging</td>
<td>18 500</td>
<td>505–510</td>
<td>33·2 cu</td>
</tr>
<tr>
<td>H4\textsuperscript{a}</td>
<td>150 × 250</td>
<td>3830</td>
<td>1915</td>
<td>2T8</td>
<td>2T20</td>
<td>Hogging</td>
<td>30 900</td>
<td>505–510</td>
<td>46·0 cu</td>
</tr>
<tr>
<td>H5\textsuperscript{a}</td>
<td>150 × 250</td>
<td>3830</td>
<td>1915</td>
<td>2T8</td>
<td>2T20</td>
<td>Hogging</td>
<td>18 500</td>
<td>505–510</td>
<td>44·0 cu</td>
</tr>
<tr>
<td>H6\textsuperscript{a}</td>
<td>150 × 250</td>
<td>3830</td>
<td>1915</td>
<td>2T8</td>
<td>2T20</td>
<td>Hogging and sagging</td>
<td>6180\textsuperscript{b}</td>
<td>505–510</td>
<td></td>
</tr>
<tr>
<td>SF2\textsuperscript{c}</td>
<td>375 × 120</td>
<td>2400</td>
<td>1200</td>
<td>2T12</td>
<td>4T16</td>
<td>Hogging</td>
<td>8640</td>
<td>601–540</td>
<td>39·0 cu</td>
</tr>
<tr>
<td>SF3\textsuperscript{c}</td>
<td>375 × 120</td>
<td>2400</td>
<td>1200</td>
<td>2T12</td>
<td>4T16</td>
<td>Hogging</td>
<td>13 800</td>
<td>601–540</td>
<td>39·0 cu</td>
</tr>
<tr>
<td>SF4\textsuperscript{c}</td>
<td>375 × 120</td>
<td>2400</td>
<td>1200</td>
<td>2T12</td>
<td>4T16</td>
<td>Hogging</td>
<td>10 500</td>
<td>601–540</td>
<td>48·0 cu</td>
</tr>
<tr>
<td>SO-1\textsuperscript{d}</td>
<td>150 × 200</td>
<td>1750</td>
<td>800</td>
<td>2T12</td>
<td>2T12</td>
<td>Hogg &amp; Sag</td>
<td>5700\textsuperscript{b}</td>
<td>557</td>
<td>21·1 cy</td>
</tr>
<tr>
<td>S1-1\textsuperscript{d}</td>
<td>150 × 200</td>
<td>1750</td>
<td>800</td>
<td>2T12</td>
<td>2T12</td>
<td>Hogg &amp; Sag</td>
<td>5700\textsuperscript{b}</td>
<td>557</td>
<td>21·1 cy</td>
</tr>
</tbody>
</table>

\textsuperscript{a}Beams tested by El-Refaie et al. (2003)
\textsuperscript{b}EA value at each of the sagging and hogging zones
\textsuperscript{c}Beams tested by Oehlers et al. (2004)
\textsuperscript{d}Beams tested by Aiello and Ombres (2007)
\textsuperscript{e}cu, cube strength; cy, cylinder strength

Table 1. Details of experiments reported in the literature
occurred, the load values at which first steel yield occurred and the values of experimentally recorded strain in the FRP at failure. All the numerical predictions are based on the recorded strains. As seen in Table 2, the correlation between the experimental and numerical data, over the full extent of loading, is reasonably good.

The MR was quantified for the beams tested, using the numerical model and applying Equation 4, and then compared with the experimental data reported in the corresponding literature. A reasonable correlation was found between the experimental and numerical results at failure, as illustrated in Figure 9, indicating that the new model can reasonably predict the degree of MR in continuous FRP-strengthened RC beams. In addition, as seen in Figure 9 and reported in the corresponding literature, MR can occur in FRP-strengthened RC beams to a reasonable extent, even up to 35% in the present study, although increasing the amount of FRP in the zone from which the BM is redistributed reduces the level of redistribution, as observed in beam H4. Also, this may cause the BM to be redistributed conversely from the sagging zone to the hogging zone, as observed in beam S0-1. It should be noted that predictions for MR, assuming a strain of 0.8% (where the FRP debonded in the test) or 1.5% (where the FRP actually ruptured in the test), are also provided in Figure 9, such that the predicted results are consistent. Generally, such predictions are adequate across the full range.

It is worth noting that, as shown in Figure 9, the initial condition and design of the specimens influence their capacity for MR such that beams S0-1 and S1-1 exhibited a lower degree of MR at failure compared to the other beams. This is

<table>
<thead>
<tr>
<th>Beam</th>
<th>Failure mode</th>
<th>Cracking load: kN</th>
<th>Yield load: kN</th>
<th>FRP failure strain: %</th>
</tr>
</thead>
<tbody>
<tr>
<td>H2</td>
<td>FRP rupture</td>
<td>FRP rupture</td>
<td>19.5</td>
<td>20.1</td>
</tr>
<tr>
<td>H3</td>
<td>FRP debonding</td>
<td>FRP debonding</td>
<td>20</td>
<td>20.3</td>
</tr>
<tr>
<td>H4</td>
<td>FRP debonding</td>
<td>FRP debonding</td>
<td>20.5</td>
<td>21</td>
</tr>
<tr>
<td>H5</td>
<td>FRP debonding</td>
<td>FRP debonding</td>
<td>20</td>
<td>20.2</td>
</tr>
<tr>
<td>H6</td>
<td>FRP rupture</td>
<td>FRP rupture</td>
<td>19.5</td>
<td>19.8</td>
</tr>
<tr>
<td>SF2</td>
<td>FRP debonding</td>
<td>FRP debonding</td>
<td>20.1</td>
<td>21.5</td>
</tr>
<tr>
<td>SF3</td>
<td>FRP debonding</td>
<td>FRP debonding</td>
<td>33.6</td>
<td>34</td>
</tr>
<tr>
<td>SF4</td>
<td>FRP debonding</td>
<td>FRP debonding</td>
<td>36.7</td>
<td>34.8</td>
</tr>
<tr>
<td>S0-1</td>
<td>CC, followed</td>
<td>CC, followed by FRP rupture</td>
<td>22.7</td>
<td>19.8</td>
</tr>
<tr>
<td></td>
<td>by FRP rupture</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>S1-1</td>
<td>CC, followed</td>
<td>CC, followed by FRP rupture</td>
<td>21.2</td>
<td>19.1</td>
</tr>
</tbody>
</table>

aCC = concrete crushing

Table 2. Experimental data versus numerical predictions over the loading cycle
due to the fact that the arrangement of internal reinforcement in beams S0-1 and S1-1 reduced their overall capacity for MR, while the other beams had higher capacities due to the difference between the proportion of steel reinforcement in the top and bottom of the cross-section. It should also be noted that the reason for beam H4 exhibiting low capacity for MR is the quantity of the FRP used for strengthening.

6. Conclusions
A new numerical approach to model the flexural behaviour of RC continuous members strengthened using FRP materials has been proposed. The model applies basic structural mechanics and can quantify the redistribution of bending moment (BM) over the full loading cycle. The numerical model was validated against experimental data from the literature. The following conclusions are drawn based upon the study conducted.

- Various beam geometries, loading arrangements, strengthening techniques or configurations can be adopted in the numerical model.
- Good agreement was observed between the numerical results obtained from the model and the test findings and observations in terms of predicting the flexural behaviour of continuous FRP-strengthened RC members over loading and also in terms of failure mode, failure load and BM at failure.
- Reasonable agreement was found between the numerical predictions and experimental results for the degree of moment redistribution (MR) that occurred in the test specimens, assuming that the debonding strain in the FRP remained constant. However, the failure mode could potentially be predicted in future work by adopting a reliable debonding/failure model.
- This work opens the possibility for MR to be quantified and included explicitly in FRP-strengthening design guidelines.

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