Attenuation of plane and high order modes in a circular and annular lined duct

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ATTENUATION OF PLANE AND HIGH ORDER MODES IN A CIRCULAR AND ANNULAR LINED DUCT

by

D. J. SNOW

A dissertation submitted as a requirement for the degree of M.Sc (by research).

Loughborough University 1971.
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I Derivation of the Wave Equation
II Solution of the Boundary Condition Equation
III Evaluation of the Complex Bessel Function
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SUMMARY

An experimental and theoretical programme was undertaken to measure and predict the attenuation of plane and spiral modes within a cylindrical and annular duct. The duct was lined with a partitioned absorber designed to act as a locally reacting surface. A detailed duct modal theory was evolved for the cylindrical duct and used to compare with the measured results. A thin annulus theory was adopted for the annular duct which made use of an existing computer programme originally written for the rectangular duct problem. (Ref 1-3).

The experimental work was conducted using a siren rig and also, in order to obtain greater detail and reliability, a loudspeaker rig was built and used extensively for the $m = 0$ and $m = 1$ modes. These results confirm, within the limits of experimental accuracy, that the theoretical approach used is a valid one at least under the prevailing laboratory conditions of zero mean air flow and low sound pressure levels. Excellent agreement was obtained between theory and experiment for the cylindrical duct. In the case of the annular duct the comparison was less satisfactory but provided at least qualitative agreement.

The principle observed effects are the increase of attenuation rate with increasing mode number and decreasing cut-off frequency ratio.

The thesis is written with a bias towards the problems of the aero-engine industry and includes a brief account of present day absorption technology in this field.
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NOMENCLATURE

\( P = \) duct perimeter
\( A = \) duct X-sect area
\( \bar{\alpha} = \) random incidence absorption coefficient
\( z' = \) Acoustic impedance
\( P = \) Acoustic pressure
\( Y = \) Acoustic velocity
\( \bar{z} = \) Normalised acoustic impedance
\( \rho / \rho_0 = \) density of medium
\( C = \) velocity of sound in medium
\( \phi = \) velocity potential
\( K = \) free space wave number \( \left( \frac{2\pi}{\lambda} \right) \)
\( k_x,k_y,k_z = \) coordinate wavenumbers
\( \alpha,\gamma,\beta = \) cartesian coordinates
\( \bar{\lambda} = \) duct width
\( \beta = \) Acoustic admittance
\( g = \) \( (\xi+i\mu) \) Morse's distribution parameter
\( \gamma = \) Duct to wavelength ratio, \( \xi/\lambda \)
\( L = \) lining depth
\( \Gamma = \) complex propagation constant through the airspace
\( h = \) " " " " " " absorber
\( \rho'c' = \) characteristic impedance of the absorber
\( h = \) \( |\beta| \xi/\lambda \)
\( \phi = \) \( \tan^{-1} (\alpha/\rho) \) Morse parameters
\( \text{Zopt} = \) optimum impedance
\( E = \) Acoustic energy
\( I = \) Acoustic intensity
\( I.L. = \) Insertion Loss
\( T.L. = \) Transmission Loss
\( \text{Re( )} = \) Real part of ( )
\( \text{Im( )} = \) Imaginary part of ( )
\( P_m = \) Pressure amplitude coefficient of 'm'th mode
\( U_m = \) Velocity " " " " " "
\( J_m = \) Bessel function of the 'm'th order
\( \text{HTR} = \) HUB-TIP Ratio
INTRODUCTION

The rapid growth in volume of air traffic over the last decade has brought about the present state of public awareness of aircraft noise. This is well illustrated, for example, by the degree of hostility engendered over the question of new airport sittings. As a consequence of this hostility the airframe and engine manufacturers have been forced via their governments' and civil authorities to spend increasing sums on noise reduction schemes.

For the present generation of engines the only methods of directly reducing engine noise are the use of sound absorbing materials for forward and rear arc noise and the conventional lobed or multiple nozzle silencer for the rear arc jet noise. The familiar powerful whine of these engines, produced by the closely spaced blade rows, can be greatly reduced by acoustic materials. However, this benefit is limited in its effectiveness by the high jet noise levels so that in the worst case, at take off in the rear arc, almost no relief is afforded. In retrospect it can be seen that this generation of engines have a basic design of which many features are in direct conflict with a requirement for low noise.

The change in design philosophy to high by-pass ratio engines was not a priori a noise reduction scheme but was mainly intended as a means of improving overall economic performance. However, an additional benefit of this design change was to lower the level of jet exhaust noise and to swing the emphasis to internally generated components of engine noise. It is worth noting that these components are just those that can be attacked via the use of acoustic linings. Also incorporated in the new engine designs were lessons learnt from testing of the older generation engines i.e. closely spaced blade rows were avoided wherever possible so that the annoying discrete tone noise
would be minimised. In order to further reduce both forward and rearward noise acoustic linings were specified and fitted as an integral part of the engine design. Thus the new engine concept has produced a step improvement both in noise emission and in the scope available within the engine design for further noise reduction as engine development continues. The latter operation is aided, in contrast to the previous generation of engines, by the much lower jet noise floor levels.

To obtain the best possible attenuation from the provision of expensive acoustic linings it is necessary to match the lining material to the particular acoustic field existing within the engine duct. Clearly this is a complex task and implies a quantitative understanding of many acoustic and aerodynamic processes and interactions. As a step in this direction it was discovered by Tyler & Sofrin (Ref 8) that the noise field excited by rotating machinery generally consists of high order circumferential modes, often called spinning modes by reference to their apparent direction of travel. However, available theoretical models of the attenuation process generally in use take no account of this unusual modal distribution. Thus this project was initiated in order to carry out an experimental and theoretical appraisal of the attenuating behaviour of these high order modes with the intention of thereby improving the design efficiency of acoustic materials.

Concurrently the same problem has been investigated in varying degrees of complexity by: Benzakein (Ref: 13)
Rice (Refs: 7, 33)
Mungur (Refs: 15, 16)
Chapter 1  An Introduction and Outline of Previous Work

The stimulus which motivated most of the early engineering work on duct attenuation was provided by the problems of silencing ventilation ducts. Various empirical and semi-empirical formulae were constructed usually based on the use of an absorption coefficient at random incidence. Some of the earliest work was carried out by Parkinson (Ref 46), in 1937, who proposed the formulae.

\[
\text{dB/ft} = -2.84 \frac{P}{A} \log_{10} (1 - \alpha)
\]

where

- \( P \) = lined duct perimeter (feet)
- \( A \) = duct cross-sectional area (ft\(^2\))
- \( \alpha \) = random incidence absorption coefficient

In the same year Sivian (Ref 25) analysed the problem in terms of the complex acoustic impedance of the lining material. The attenuation was now considered to be a linear function of \( P/A \) and reasonable agreement with measured results was found, at least at low frequencies. Sabine (Ref. 26), 1940, produced a formula which fitted his own experimental results and which was an even simpler expression than that of Parkinson above. Sabine's well known equation is:

\[
\text{dB/ft} = 1.05 \left( \frac{P}{A} \right) (\alpha)^{1.4}
\]

Despite its limitations this formula became generally accepted for most engineering applications. King (1944 Ref. 47), Rogers (1940 Ref. 28) and Bruel (1948 Ref. 48) have also produced simplified duct attenuation formulae.

All the work so far discussed suffers from similar limitations.

The equations produced are, by their nature, incapable of covering all parameter ranges and fit reasonably well only a particular set
of data or simply the data produced by that investigator.

A more rigorous treatment of the problem, suitable for any duct configuration, was carried out by Morse (Ref. 14, 1939) who showed how an exact solution could be obtained for ducts of any size and wall impedance.

A locally reacting boundary condition was assumed by Morse and, although not all materials satisfy this condition, the theory is otherwise generally applicable. Beranek (Ref. 27, 1940) re-examined Sabine's data using Morse's theory and concluded that this approach gave an improved fit to that data. Jones (Ref. 49, 1955) compared several prediction methods with his experiments and also decided that Morse's more complex solution was the more accurate. Kerka (Ref. 24, 1963) carried out a similar exercise and his results are plotted in Fig. 1. Although the measured results are not completely predicted by any one method, Morse's solution is the only one to show their sharply resonant character. Kerka attributes the discrepancy to the "telegraphing" of sound along the duct walls i.e. the non-locally reacting boundary (see Chapter 2.) Models based on the use of a generally fail for duct widths greater than half a wavelength, when more than one propagating mode can exist. The complexity of the complete Morse theory unfortunately prevents its ready usage, so that the problem is beset either by inaccuracy or complexity. Tester (Refs. 1-3, 1969) and others have programmed the Morse theory and so, this computer programme being available, comparisons with other experimental results can be much simplified. Figure 2 is a sample of the kind of agreement obtained with the results published by Ref. 50. The wall lining material used in Ref. 50 was a partitioned fibre material covered with a perforated metal skin. The attenuation curves obtained are closely predicted by Morse's theory so that for such locally reacting materials this approach is seen to be appropriate. In 1946 Scott (Ref. 41) produced a detailed theory for ducts with a non-locally reacting boundary, and showed that in many practical cases propagation within the lining material had to be considered. In his own experiments
Scott demonstrated that for an unpartitioned and lightly packed rock wool lining the Morse approach breaks down and produces generally much too high results. In 1969 independent results were produced by A. Bokor (Ref. 42) which experimentally supported this work.

Chapter II Theory
2.1. The Duct Boundary Condition
The detail specification of the lining materials used (by the aircraft industry) was determined mainly by a series of rig and engine trials. Because of the severe environmental conditions and strict safety regulations the more usual forms of industrial duct silencers (see Fig. 3(a)) were not practicable; for instance large areas of minimally supported facing sheet are not sufficiently rigid to withstand the stresses of aero-engine life.

Also a volume filled with foam or fibreglass is peculiarly susceptible to 'wicking' i.e. the soaking up of fuel or water which in turn produces weight and stress problems. The soaking up of fuel would in addition to be a considerable fire hazard. It is clear too that for the aero industry the designer is likely to have severe limitations on the weight, length and depth of lining material which he can use so that it is important to obtain as efficient a design as possible.

As a consequence of these various factors the basic structure took on the shape of Fig. 3(b). Here the absorbent foam packing has been replaced by a thin resistive sheet which is mounted over a network of air cavities, formed by the supporting honeycomb which provides the necessary stiffness to the structure. Acoustically there is a fundamental difference in the behaviour of these two types of absorber. The type of Fig. 3(a) is potentially less efficient because it allows sound propagation or leakage through the material, whereas the honeycomb cells of Fig. 3(b) prevent this
lateral transmission of sound. Furthermore the cells are normally of such a size (of order of 1\textquoteleft diameter) so that at normal audio frequencies only the lowest order mode will be above its cut-off frequency and so any acoustic motion will tend to be confined to the normal direction. A material which behaves in this way is called 'locally reacting' and the term may be defined as follows (from Zwicker and Kosten Ref. 3).

"A layer of material is called locally reacting if the velocity component perpendicular to the surface only depends upon the pressure at the point and not upon the angle of incidence." As is mentioned by Morse (Ref. 4, page 360) the amount of dependence of the impedance on the angle of incidence in turn depends on how well wave motion can travel in the surface material, both as to its attenuation rate and velocity. The former property effectively prevents propagation if the attenuation rate is high enough whilst a lower acoustic velocity than that in air means that, from Snell's law of refraction, the direction of propagation in the material will always tend to the normal to the surface. A honeycombed or partitioned structure can be seem to combine both these actions and so can normally be described as locally reacting. A bulk absorber will also be locally reacting if it provides a sufficiently high attenuation rate and low propagation velocity compared with that within the duct proper.
2.1. (a) **LOCALLY REACTING MATERIALS**

For locally reacting materials the boundary condition is dictated solely by the local impedance.

\[ \mathbf{Z}' = \frac{P}{V} \]  

where

\[ \mathbf{Z}' = \text{Impedance (R + i x) Rayls (Mks)} \]
\[ V = \text{Normal Acoustic Velocity M/S} \]
\[ P = \text{Acoustic Pressure N/M}^2 \]
\[ \mathbf{Z} = \text{Normalised impedance} = \frac{\mathbf{Z}'}{pc} \]

The acoustic pressure and velocity are related to the velocity potential by the equations

\[ V_y = \frac{\partial \phi}{\partial y} \quad \text{and} \quad P = \rho \frac{\partial \phi}{\partial t} \]

where the first relation is by definition and the second follows from the equation of motion for a fluid with no external forces and no mean flow.

The solution for the velocity potential is (see Appendix 1) for a two dimensional system.

\[ \phi = \frac{\cos}{\sin} (Ky, y) \exp j (wt - Kx x) \]  

The boundary condition for a normal impedance \( Z \) at \( y = y \) yields the conditions

\[ jK \frac{1}{Z} = Ky \tan ( Ky \ y / 2 ) \]  
\[ jK \frac{1}{Z} = - Ky \cot ( Ky \ y / 2 ) \]

for the cosine and sine solutions respectively.

For a rigid wall \( (Z \rightarrow \infty) \) these conditions simplify to

\[ \tan ( Ky \ y / 2 ) = \cot ( Ky \ y / 2 ) = 0 \]

so that

\[ ( Ky \ y / 2 ) = \frac{n \pi}{2} \]

\[ n = 0, 1, 2, \ldots \]

A duct with only one wall lined and the other rigid is almost equivalent to a duct with twice the width and both sides lined. We may realise this by interposing a rigid wall at \( y = 0 \) into the two lined system: so that the condition \( \frac{\partial \phi}{\partial y} = 0 \) applies at \( y = 0 \).

This requirement means that only the solutions of

\[ \phi = \frac{\cos (Ky y) \exp j (wt - Kx x)}{\sin} \]
are tenable and thus the modes given by Eq. (3) as before. The
modes given by Eq. (4) no longer exist.
Equation (3) may be expressed in the notation due to Morse which
is perhaps more familiar, by utilising the hyperbolic notation and
writing
\[ \Xi = \frac{1}{\beta} \quad \text{and} \quad Ky = 2 \pi g_1 / \lambda \]
so that we obtain
\[ \Pi g \tanh (\Pi g) = i \Pi \beta \left( \frac{g}{\lambda} \right) \]  \( \text{(5)} \)
where \( g = (\xi + i \mu) \) is a cross-sectional wave number called a
distribution parameter by Morse. Equation (5) has an infinity
of solutions which correspond to the normal modes of the tube and
these may be designated \( g_1, g_2, \ldots, g_n \). Fig (4) represents
Eq. (4) in chart form. It was prepared by Morse (Ref. 14) and used
to obtain numerical solutions, by for example Beranek (Ref. 27),
before the general availability of computer techniques. The modal
attenuation coefficients are not simply related to their respective
wave numbers but may be calculated directly from them by the
relation
\[ K_x^2 = K^2 - K_y^2 \]  \( \text{(Ky = 2 \pi g_1 / \lambda)} \)
Morse has prepared charts which enable the solutions \( g_1 \) and \( g_2 \) to
be obtained for a range of the parameters \( \Xi \) and \( \phi \). From this
study Morse drew the conclusions "the behaviour of sound
waves, even of the principal wave, is quite complicated unless the
admittance ratio if the walls is small compared with unity" (i.e. the
walls are nearly hard) "very few sweeping generalities can be
made concerning the behaviour of sound in ducts with highly absorbent
walls" "slight changes in impedance and in size of duct can produce
very considerable changes in the shape of the curves of \( \xi \) and \( \mu \) and of
all the quantities derived from them". Such conclusions help to explain
why the simpler propagation models of other investigators did not
produce satisfactory results.

2.1. (b) NON-LOCALLY REACTING MATERIALS

As had already been mentioned not all materials satisfy the relatively
simple local boundary condition. Morse (Ref. 4 Page 364) describes hair-felt as being such a non-locally reacting material. It is interesting in this respect that some later measurements by Shaw (Ref. 6, 1953) confirm this judgement and clearly show a strong variation of impedance with angle of incidence for this material. When acoustic velocities are not confined to a direction normal to the wall surface the boundary condition becomes, as written by Scott (Ref. 41)

\[
\rho_0 \frac{\coth(-i\sqrt{1^2 + k^2} \cdot \frac{L}{2})}{\sqrt{1^2 + k^2}} = \rho' \frac{\coth j\sqrt{1^2 + h^2} \cdot \frac{L}{2}}{\sqrt{1^2 + h^2}} 
\]

where \( L \) = Lining depth

\( \Gamma \) = Complex propagation constant through airspace

\( h \) = Complex propagation constant through absorber

\( \rho'c' \) = Characteristic Impedance of the absorber

The acoustic impedance of the lining may be written as

\[
\mathcal{Z} = \rho' \frac{\coth j\sqrt{1^2 + h^2} \cdot L}{h}
\]

so that, as Scott showed, if \( \Gamma \) is small compared with \( h \) the conditions above tend to the local impedance boundary condition. This will occur if the lining material has either a very low propagation velocity or a very high damping factor.

2.2. ATTENUATION COEFFICIENT AND THE OPTIMUM SOLUTION

For a wave motion such as is described by Equation (2) the attenuation in decibels per unit length in the 'x' direction is given by

\[
\text{dB/ft.length} = 8.68 \text{ Im (Kx)}
\]

where

\[
Kx^2 = K^2 - Ky^2
\]

\[
Kx^2 = K^2 - \frac{4\pi^2}{l^2} \frac{(\xi + i\mu)^2}{(\xi + i\mu)^2}^{\frac{1}{2}}
\]

\[
Kx = 2\pi \sqrt{\left\{ \mathcal{Q}^2 - (\xi + i\mu)^2 \right\}}
\]

where \( \mathcal{Q} \) is the duct to wavelength ratio

It is apparent from Equation (8) that the attenuation/duct width is a function only of \( \mathcal{Q} \) and the distribution parameters \( \xi \) and \( \mu \). For a duct with nearly hard walls solutions will be obtained, from the chart of Fig. 4, which give \( \mu = 0, 1.0, 2.0... \) and small value to \( \xi \).
The different modes which are thus defined will have pressure profiles across the duct almost identical to perfect sinusoidal distributions and attenuations which increase steadily for increasing mode orders. Other values of the wall impedance and duct to wavelength ratio will produce modes of a profile given by Equation (9) below and an attenuation rate not necessarily the smallest for the lowest order mode:

\[ P(y) = \cosh \left( \frac{2\pi y h}{\lambda} (\xi + j\mu) \right) \]

so that

\[ \left[ P(y) \right]^2 = \cos^2 \left( \frac{2\pi y h}{\lambda} \xi \right) + \sinh^2 \left( \frac{2\pi y h}{\lambda} \xi \right) \] (9)

Examination of Fig. (4) reveals that most values of impedance and duct width will result in distinct first and second order mode solutions. Generally the second mode will be more rapidly attenuated than the first but for low impedance values (or large \( \xi/\lambda \)) and a negative reactance this order may be reversed. In this regime the first mode solutions will have large values of \( \xi \) which indicate from Equation (9) that the pressure is increasing exponentially towards the duct wall. This solution is called a 'surface wave' by Cremer (6) from the analogy with surface water waves which also exhibit an exponential variation of disturbance with depth. At one particular point on the chart the first and second mode solutions become indistinguishable. This point (\( \Phi = 38.7^0 \) and \( h = 0.84 \)) occurs at a definite wall impedance for a fixed value of duct to wavelength ratio. The relation is given by Equation (10) below

\[ Z_{\text{opt}} = (0.928758 - j0.744190) \frac{\xi}{\lambda} \] (10)

This identity was obtained mathematically to the above degree of accuracy by Tester (3). The nomenclature \( Z_{\text{opt}} \) is used because as realised by Cremer (6), this impedance is indeed the optimum value under normal circumstances. Although individual solutions can be found which have a higher attenuation rate than is consistent with \( Z_{\text{opt}} \), all other values of impedance will generally produce at least one mode with a lower decay rate. Thus if all modes are present the best attenuation in a long duct will be produced by the wall impedance value \( Z_{\text{opt}} \). Abnormal circumstances can be imagined in which a
source excites only a high order or surface wave but generally a mixture of modes will be produced and the optimum wall impedance will then be given by \( Z_{opt} \). This value of impedance will produce a mode with constant values of \( \phi \) and \( \mu \) so that, from Equation (8), the attenuation rate/duct width is a function of \( \eta \) only. A graph of this function was drawn by Cremer (6) and is reproduced here as Fig. 5. It shows that for values of \( \phi \lambda < 2 \) there is a maximum possible decay rate of approximately 19 dB/duct width and that this rate then decreases rapidly to \( \sim 9 \) dB/duct width at \( \phi \lambda = 1 \). and \( \sim 1 \) dB at \( \phi \lambda = 10 \).

### 2.3 Calculation of an Insertion Loss

It is assumed that the fundamental quantity, with regard to the far field sound radiation, is the reduction of sound energy flow in the duct. The energy at a particular cross-sectional location of the duct is given by the integral of the acoustic intensity over that area, or over a strip of unit width in the two-dimensional case.

\[
E = \int_{y_1}^{y_2} I \, dy
\]

(11)

where

\[
I = \frac{1}{2} \text{Re} \left\{ \rho \cdot \nabla x \right\}^* \quad \text{N/m}^2\text{s} 
\]

(12)

and the asterisk indicates the complex conjugate.

As before the acoustic pressure and velocity are related to the velocity potential by equations (13) and (14) below

\[
\nabla x = \frac{\partial \phi}{\partial x} \quad \text{m/s} 
\]

(13)

and

\[
\rho = \frac{\partial \phi}{\partial t} \quad \text{N/kg}^2 
\]

(14)

Now \( \rho \) and \( V_x \) in Equation (12) are each expressed as a series of terms which represent the sum of individual modal solutions. The true value of the acoustic energy or intensity includes a whole series...
of cross terms.

Thus in a two-dimensional rectangular duct we have

\[
I = \frac{1}{2} Re \left\{ \sum_{m=0}^{+\infty} \frac{B_m \sin (k_y^m y)}{\alpha_m \omega} \sin (k_y^m y) \frac{j(\omega t - k_x^m x)}{2} \right\}
\]

\[
\times \left[ \sum_{n=0}^{+\infty} \frac{B_n \cos (k_y^n y)}{\alpha_n \omega} \cos (k_y^n y) \frac{j(\omega t - k_x^n x)}{2} \right]^* \right\} \frac{1}{\sqrt{m/s}} (15)
\]

and this relation must now be substituted in Equation (11) to find the energy flow at a particular duct station. This is then repeated at some further duct station, \( x' \) and the dB transmission loss is given by

\[
\text{TRANSMISSION LOSS (dB)} = 10 \log_{10} \left\{ \frac{E(x)}{E(x')} \right\}
\]

(16)

Clearly values for the amplitude coefficients \( A_m \) and \( B_m \) must be assumed or the transmission loss is incalculable. One possibility is to put \( A_m = B_m = A_n = B_n \) so that all modes then have equal amplitude coefficients. However the following assumption is perhaps more justifiable and in addition results in a simpler calculation.

We put:

\[
\frac{1}{2} \int_{-l/2}^{+l/2} Re \left\{ P_n \cdot U_m^* \right\} dy = 0 \quad n \neq m \quad N/M
\]

(17) \( N/N \)

and

\[
\frac{1}{2} \int_{-l/2}^{+l/2} Re \left\{ P_n \cdot U_m^* \right\} dy = \frac{1}{2} \quad n = m
\]

We are saying here that equal energy is carried by all modes when considered on their own, and that no energy is transmitted by cross-modal terms. The assumption of equi-partition of energy is consistent with the theoretical work of I. Dyer (Ref. I) and is, in the absence of other information, the most reasonable input. Using these assumptions
and summing over $N$ possible modes the expression for transmission loss ($T.L.$) can be simplified to

$$T.L.(dB) = 10 \log_{10} \left\{ \sum_{n=0}^{\infty} \exp\left(-2 \cdot \frac{\text{Im}(k_n^x)(x'-x)}{N}\right) \right\}$$  \hspace{1cm} (18)

It is necessary to limit the number of modes $N$ to a reasonable number. One solution is to use only that number of modes which are above cut-off in the equivalent sized hard walled duct. Rice (Ref 7) alternatively selects the first ten modes for his calculations and finds modal amplitudes such that a 'plane wave' is synthesised at the duct entry, in addition using the cross modal terms also. The 'best' assumption to make regarding the modal amplitudes and number of modes will depend on the type of acoustic source. For a given source these amplitudes can be measured, by analysing the acoustic field in detail at a given $x$ plane, and this information then used to feed into a more sophisticated attenuation calculation. However, unless a highly attenuated mode dominates the acoustic field, or the duct is very short, the difference between the fairly reasonable assumptions outlined above is not likely to be large. (see section 4.2).

* The cross modal terms vanish only for certain restrictive values of wall impedance e.g. an infinitely hard wall. However, computer calculations by the author and by Ref. (34) have shown that these terms are generally of second order.
CHAPTER 3 EFFECTS OF FLOW AND HIGH SOUND PRESSURE

A uniform mean axial flow can be considered theoretically when the wave equation is used in the convective form

\[ \frac{\partial^2 \phi}{\partial t^2} = c^2 \nabla^2 \phi \quad \frac{\partial}{\partial t} \equiv \frac{\partial}{\partial t} + U_x \frac{\partial}{\partial x} \]

so that

\[ \frac{\partial^2 \phi}{\partial t^2} + 2 U_x \frac{\partial^2 \phi}{\partial x \partial t} + U_x^2 \frac{\partial^2 \phi}{\partial x^2} = c^2 \nabla^2 \phi \]

The solution to this equation is as before

\[ \phi = \cos \left( k_y y \right) e^{j \omega t + k_x x} \]

where the wave number relation is now given by

\[ (k - k_x M_x)^2 = k_x^2 + k_y^2 \quad \text{and} \quad M_x = \frac{U_x}{c} \]

The new boundary condition becomes

\[ j k \left( 1 - \frac{k_x M_x}{k} \right) \frac{n}{2} = k_y \tan(k_y y) \]

where \( n = 1 \) if particle velocity is assumed continuous at the duct wall

\( n = 2 \) if particle displacement is assumed continuous

Some discussion has taken place over the correct choice of boundary conditions (Refs. 19, 33) but for porous materials the assumption of continuity of particle velocity is generally preferred. Solutions to Equation (19) can be obtained by computer techniques (Refs. 1-3) and reveal the dependence of attenuation on flow mach number. A mean flow against the direction of sound propagation generally reduces the frequency of peak attenuation, and vice versa. The optimum impedance varies with flow velocity but the attenuation rate corresponding to this optimum value is almost constant. (Ref. 3). For flow velocities less than approx 0.1M the effects computed in this way are not large so that although flow effects are important to the aero-engine designer, they need hardly be considered by the ventilation engineer.

Unfortunately the practical effects of a high duct mean flow cannot be so easily accounted for as may be implied above. Other effects need to be considered. The duct boundary layer refracts sound waves either to or away from the duct boundary, depending on the direction of flow. These
effects have been studied theoretically by Mungur and others (Refs. 15, 19, 22). Also the sound field will be scattered by the turbulent velocity fields inevitably present in even the slowest flow regimes. The relative importance of refraction and scattering effects depends on the acoustic wavelength to 'flow disturbance' size. If the wavelength is smaller than the distance over which there is any noticeable change in the flow properties then the important phenomena is refraction as described by geometric acoustics. The calculation of acoustic intensity in a duct with flow is more complex than given by the zero flow equations described in section 2.3. and must take account of interaction terms describing the coupling of the acoustic and aerodynamic fields. (See Refs. 15-19).

However, the primary effect of duct flow is not normally its reaction on the form of propagation but on the behaviour of the lining material. It has been found both by inference from observed attenuation results and by direct measurement that the impedance, primarily the resistive component, of porous materials is a function of flow velocity. This increase in resistance can be explained qualitatively on the basis that large air velocities are induced in the lining material by the moving air stream. The resistance of a porous material normally behaves like:

$$ R = A + BV^n $$

so that for sufficiently low velocities the resistance will be constant. The velocity $V$ is the sum of the aerodynamic and acoustic velocities. Thus for very high sound pressures alone and no air flow the same
effect can result. The non-linear form of Equation (20) arises from the departure from uniform flow within the porous material and the resultant increase in resistance above the viscous 'Poiseuille' background level. For holes of larger size a Helmholtz rather than a Poiseuille law applies and here energy is lost by the formation of jets and eddies at the exit of the material holes. The constants $A$, $B$ and $n$ of Equation (20) will depend on the properties of the particular material such as pore size, shape etc.

An indication of the non-linearity of a material can be obtained by measuring the resistance to steady or D.C. flow. This is a much simpler measurement than that of acoustic resistance and is often the first step in examining a new material. At high flow velocities and sound pressure levels open perforated materials can be used which have almost zero resistance under normal conditions but become quite resistive in their operating environment. Such materials have the advantage of cheapness and ease of manufacture over other porous metals.

In the experimental work conducted here the complications incurred by a mean air-flow and a high sound pressure level have been avoided. Although the effects are important they would tend to obscure the data under investigation.
CHAPTER 4 THE SOLUTION IN CYLINDRICAL CO-ORDINATES

4.1 Formulation

We proceed, as in section 2.1., to apply the local impedance boundary condition \( P = \frac{Z}{U_r} \) to the wave equation solution in cylindrical co-ordinates. This solution may be written in pressure form

\[ P_{mn} = A_{mn} J_m(K_r^n r) \cos(m \theta) \exp(j(Kc - Kxx)) \]  

(21)

where \( K^2 = K_r^2 + Kx^2 \).

The Neumann functions \( Y_m \) go to infinity for zero argument and so can only provide a valid solution for an annular duct where this difficulty does not arise. The wavenumbers \( K_r \) (and therefore \( Kx \)) are particular to a single \( n \) solution for a given \( m \) value. A whole series of values exist for \( K_r \) and this quantity defines the properties of a succession of radial modes. For every circumferential 'm' mode a similar series of radial terms exist and the overall pressure field may be represented by a summation of \((m \times n)\) terms like that of Equation (21).

The wavenumber \( K_r \) may be determined from the outer wall boundary condition \( P = \frac{Z}{U_r} \) at \( r = b \)

(22)

Also

\[ Ur = \frac{1}{\rho \omega} \frac{\partial P}{\partial r} \]  

(23)

The boundary condition contained within equations (22) and (23) is obtained in a usable form by differentiating equation (21) to find \( \frac{\partial P}{\partial r} \) and hence \( Ur \) from Equation (23). Now substitution for \( Ur \) and \( P \) may be made in Equation (22) giving the result

\[ \frac{\omega \rho b}{i \pi} = m - (k_r b) \frac{J_{m+1}(k_r)}{J_m(k_r)} \]  

(24)

It was found useful at this stage to change the notation to that used by Morse and Ingard (1). The reason for the change is that this reference provides a range of numerical solutions to Equation (24), in a chart form, and is thus a convenient check on the developed computer programme. The chart was not considered as an alternative to the computer programme because of the inconvenience of using it and because it only provided solutions over a limited range of parameter values.
The substitutions required are

(i) \( h = \frac{1}{10} \cdot \left( \frac{2L}{\lambda} \right) \)

(ii) \( \psi = \tan^{-1}\left( \frac{X}{R} \right) \)

(iii) \( K_r = \pi \left( \mu - i\nu \right) = -i\pi w \)

so that Equation (24) becomes

\[
i h e^{i\psi} = -\frac{m}{\Pi} - iw \frac{J_{m-1}(-i\pi w)}{J_m(-i\pi w)}
\]  
(25)

The chart representing this equation is shown in Fig. (6)

Equation (25) was solved for \( K_r \) by the iterative method due to Newton, as was used successfully by Tester (1–3) for the rectangular duct equations. The details of this method have been relegated to the appendices, which also describe the subroutine used to evaluate the Bessel functions, \( J_m \), with complex argument. The iteration subroutine was checked by using it to find solutions on the chart of Morse and Ingard (Fig. 6).

The attenuation of a particular mode is then determined by the imaginary part of the wavenumber \( K_x \), which is related to \( K_r \) by

\[
\frac{K_x^2}{K_r^2} = K^2 - K_r^2
\]  
(26)

More precisely

Attenuation/unit length = 8.68 Im \((K_x)\) dB

By using measured values of the wall impedance (\( Z = R + iX \)) and this programme it is then possible to compare directly experimental and theoretical decay values for particular modal solutions.
4.2 Some Results of the Theoretical Model

The analysis of the previous section has not been confined to a particular circumferential mode order. In many circumstances only the plane wave (m = 0) mode is of interest. However, for some noise sources, particularly those of rotating machinery, higher order 'm' modes are liable to be generated. Figure (7) illustrates the different kinds of attenuation curve which might be expected from an m = 0 and m = 1 mode. The higher order mode is always more highly attenuated but some convergence at high frequencies is apparent. The attenuation curve for the hard walled duct and the m = 1 mode is extremely steep and below approx 1 kHz it exceeds that of the equivalent lined duct. Any mode other than the plane wave will have a 'cut-off' frequency below which the mode will rapidly decay in level as it travels along the duct. This frequency is clearly defined and occurs when K = Kr, so that the axial wave-number Kx becomes purely imaginary at this point (see Equation 26). If the duct wall is other than rigid the wavenumber Kr will be complex and so the behaviour below cut-off will be modified and the cut-off point less distinct, although the general trend of a rapidly increasing attenuation rate with reducing frequency will still dominate.

Thus it is technically possible that the inclusion of acoustic linings could, in the wrong circumstances, result in increased radiation to the far field. A more detailed study of this effect has been carried out by D. Brown (Ref. 53) and his theoretical results also form a useful means of checking the computer programme. The two sets of results show precise agreement and those for an m = 3 mode are reproduced in Fig. 8. These curves show the change in attenuation rate below the hard-wall value and vice versa. Also, as might be expected, the change in decay rate tends to zero as the impedance becomes large.

In Fig. 9 a comparison is made with some results by Rice (7) and again satisfactory agreement is obtained. It is interesting here that the two different models used result in only small differences in the final answer.
As was outlined in Section 2.3 the differing assumptions are (i) Equal energy in all modes above cut-off and neglect of cross-modal terms and (ii) Modal amplitudes such as to create a 'plane-wave' at the duct start and the inclusion of cross terms. In spite of these differences the agreement is good although a divergence begins to appear when more than two modes are present. The attenuation rate is primarily controlled by the least damped mode for ducts of this length to diameter ratio \( L/D = 5 \). It is to be expected that the original assumptions of the correct modal amplitudes would become increasingly important for ducts of a reduced length to diameter ratio.

However, a detail discussion of the nature of the source modal amplitude distribution is beyond the intended scope of this thesis. The comparisons with the work of other people reasonably establishes the correctness of the computer programme and its validity as a check on any suitable experimental data. In addition some individual modal attenuation results were also checked by Mungur's computer programme (ref. 15 and 16) to whom thanks are due for this cooperation. Again precise agreement was obtained.
4.3 The Annular Duct

The annular duct solution is in principle no different to either the cylindrical or rectangular duct. However the detail work involved in obtaining the exact modal solutions was not attempted here. For a very narrow annulus the annular solution will tend to the equivalent width rectangular duct solution, see Ref. 3(b), so that these calculations later provide an interesting basis of comparison with the experimental annular results. The available rectangular duct computer programme was easily adopted to handle three dimensional modes by including the third, i.e. circumferential wavenumber. Thus spiral modes in a narrow annular can be modelled and some of these calculations are produced in Fig. 10 for a simulated annulus of hub-tip ratio 0.5, diameter 6\" and a wall impedance similar to that of Fig. 11.

For the infinitely narrow annulus the circumferential wavenumbers are given by $K_3 = \frac{m}{r}$, however as we wish to imitate a real annulus it is more reasonable to use the true wavenumbers which for HTR= 0.5, $m = 1, 2, 4$ are given by $K_3 = (\frac{m}{r}) \times 1.36, (\frac{m}{r}) \times 1.34$ and $\frac{m}{r} \times 1.29$ respectively. These values are taken from available tables such as those of Ref. 8 (c). The 'radial' wavenumbers are then found as before from the solutions to equations (3) and (4) of section 2.1 (a).

The solutions are then combined to give the axial wavenumber, $k_x$, which of course determines the axial decay rate.

$$k_x = \sqrt{k^2 - k_y^2 - k_3^2}$$ (27)

The results shown in Fig. 10 again show the interesting feature of a reduced attenuation rate below the hard wall cut-off frequency. Another interesting feature is that the frequency of peak attenuation is found to be independent of mode number.

* hub-tip ratio (HTR) 0.5, diameter 6\"
4.4. Computer Programme Description (See Appendix V)

The program used for the cylindrical duct was constructed within the framework of a rectangular duct program written by B.J. Tester and described in Refs. (1) to (3). Various forms of input are available with the original program but these are confined to that shown in Fig. (12) for the purposes of this discussion. The blank cards are a consequence of these omitted options. The input card format is largely self explanatory. Card 5 allows an unlimited nos of frequencies to be examined in equal frequency steps. Mode order means the circumferential mode order and may take any integer value. The nos of radial modes can be selected to suit or the space can be left black, in the latter case the programme will use the number of modes above cut-off in the equivalent hard wall duct. The final two cards list the wall resistance and reactance values, corresponding to each selected frequency, which must of course be known.

The working of the program can be followed from the simplified chart of Fig. (13) and the listing which appears as Appendix V. The satisfactory development of the programme was primarily dependent on the evaluation of the complex Bessel function and the solution of the boundary condition equation. The mathematics of these steps appear as Appendix II & III and as separate subroutines within the programme. The starting points for the iteration require the solutions of the equations \( J_m = 0 \) and \( J'_m = 0 \).

These solutions are found outside the programme and inserted as arrays within the appropriate sub-routines. Because the iteration may occasionally diverge and because two starting points may produce the same solution it is necessary to use more starting points than there are solutions.

The main programme then sorts and lists these solutions, discards any duplicate solutions, and calculates an overall attenuation for all the modes found below their cut-off frequency. For each individual solution the following output is given; resistance, reactance, Morse parameters \( h \) and \( \mathcal{C} \) (see Section 4.1), real and imaginary parts of the cross-sectional wavelength \( k_m \), axial phase velocity.
and finally the axial attenuation is decibels over the specified duct length. The specimen output of Appendix V also shows three columns of print before the normal output described above. These three figures reveal how the iteration progresses to the final wavenumber \( k_m n \); the third figure is a measure of the rate of change of the iteration and will tend to zero on convergence. This additional output is only used to examine the detail iterative behaviour from different starting points, and is not normally used. Thanks are due to Mr. B.J. Tester of Southampton University for helping in developing this programme and for providing the framework within which it was written.
CHAPTER 5. CONSTRUCTION OF THE EXPERIMENTAL RIG

5.1. General Description

The overall layout of the rig is shown in Figs. 14 and 15. The mode of operation is, in principle, extremely simple. At one end of the acoustically lined duct a noise source is provided. For these experiments the form of this noise source is important but such details are considered later. The modes produced by the source travel along the duct, being gradually attenuated, until they either reach negligibly low levels or are absorbed by the duct termination. The acoustic pressure in the duct is measured by a probe microphone and it is this rate of decay per unit length which is the main factor of interest. The attenuating portion of the duct is made sufficiently long so that over a considerable length only the least damped mode will be present. Clearly complications will arise if more than one mode must be considered simultaneously.

Two different noise sources were used and either of these could be connected to an adjustable length, up to six feet, of six inch diameter metal ducting. Inside this duct a series of inner tubes could be inserted so that an annular channel of hub-tip ratio equal to 0.5 or 0.75 was created. To measure pressure distribution within the duct a probe microphone or 1/8" diameter microphone cartridge was used. This microphone could be traversed along the duct axis at any angular location and it was also possible to traverse circumferentially at any axial position. The latter manoeuvre was accomplished by either rotating the microphone or rotating the inner duct and microphone together about the duct centre line. Radial traverses could be made at a number of fixed positions.

Various measurements using the above system were made in the hard walled duct and are reported in Chapter 6. These experiments were useful in obtaining a familiarity with the rig under known conditions where the results could be fairly easily interpreted.
A three foot length of attenuating duct was designed to form part of the outside wall of the above rig. It was constructed of a 'rigimesh' material (PMS 1526) chosen primarily because regional variations in flow resistance, common in this type of material, were minimised in this particular specification. The original flow resistance of this material was approximately 40 c.g.s. Rayls due to the viscous air flow through the miniature and regular perforate structure. This material was mounted over a honeycomb backing of half inch cell size which divides and compartments the combined structure so as to form a locally reacting boundary i.e. there is no communication between adjacent cells. The honeycomb was bonded to the rigimesh which formed the inner duct wall and was also bonded to the rigid metal outer backing (Fig. 15). The bonding of the rigimesh material resulted in a blockage of approximately 50%, rather more than was allowed for, so that the true flow resistance of the material increased to about 60 Rayls or 1.5 pc.

A more precise impedance measurement of the material was made using a Brüel & Kjaer standing wave tube. These results are presented in Fig. 11.

The forming of this double sided duct and the bonding of the honeycomb into a tight six inch diameter circle presented some quite difficult manufacturing problems. These were overcome by making the outer duct skin in two halves and constructing a wooden jig in which the structure could be held whilst bonding took place. This work was done by CIBA Ltd., Duxford.
5.2 Noise Sources

5.2.(a) Siren Rig

This rig has been used extensively as a means of studying the propagation and radiation characteristics of various duct modes. Different modes are excited by changing the hole numbers in the rotor and stator discs, thus creating an array of point monopole sources around the duct and towards its outer wall. The number, spacing, and relative phasing, of the sources are adjusted via the rotor and stator hole numbers so as to create a required mode. The rules giving the conditions for excitation of a particular mode are as given by Tyler and Sofrin, viz

\[ m = cB - kV \]

where

- \( m \) = order of circumferential mode
- \( c \) = harmonic number = 1, 2, 3 etc.,
- \( B \) = nos. of holes in rotor disc (or nos. of rotor blades)
- \( V \) = nos. of holes in stator disc (or nos. of stator vanes)
- \( k \) = 0, ±1, ±2 etc.

The above formula however only considers the conditions for excitation. Before a particular \( m \) mode can propagate its frequency must exceed a critical value known as its 'cut-off' frequency, which is a function of the duct size and \( m \) number only (for no duct mean flow). Below this frequency the mode will decay exponentially as it travels along the duct axis.

For each \( m \) number there exists a series of radial(\( n \)) order modes which are also liable to be generated. Their propagation, or otherwise, will again depend on the generating frequency, duct size and individual cut-off frequency. The cut-off frequency increases for increasing \( m \) and \( n \) values so that for a given discrete frequency there are only a finite number of possible propagating modes.

A photograph of the rig, and a sample spectrum produced by it are shown in Fig. 16.

At first sight the siren rig would appear an adequate instrument for
obtaining a suitable sound spectrum. In practice however it
does pose a series of problems. In order to maintain a constant
acoustic output it is necessary to monitor the speed and air
pressure both of which are subject to oscillation and creep.
Variation of the speed means that the frequency and acoustic
power are changing simultaneously.
The noise spectrum produced by the siren shows a series of discrete
tones protruding above what one assumes to be a continuous broad­
band noise floor. By using the closest rotor-stator spacing the
ratio between the discrete tone and broad band levels can be
maximised. Even so the spectrum of Fig. (3) shows this signal
to noise ratio to be only if the order 10 to 20 dB. Larger values
than this are preferable for the type of measurements proposed.
Also the high levels of vibration and rig noise do not aid accurate
measurements and are also subjectively undesirable.

5.2 (b) **Loudspeakers.**
Because of the kind of problems discussed briefly above the loud­
speaker rig was adopted. This rig has the advantages of stability
and ease of control.
It consists of two similar electro-mechanical loudspeakers which
are linked to the ducting, at the source plane, in order to represent
the particular mode it is desired to create. The two drivers used
here will, when driven in anti-phase, tend to excite the first order
circumferential mode. In principle a far larger number of loud­
speakers could be used so that any desired mode could be created.

Prior to any measurements the two loudspeaker positions and
relative levels must be carefully adjusted until precisely the
correct circumferential pressure distribution is obtained. In
these experiments this distribution was either plane or first order.
In the latter case the pattern varies as \( \cos \theta \), thus showing two
nodes and antinodes around the duct circumference. (See Fig. 19)
It was found that the setting up of a symmetrical pressure distribution obeying the above law necessitated also an accurate alignment of the ducting so that it was normal to the source plane. The care needed in setting up increased as higher frequencies were approached. Close to the cut-off frequency, it was difficult not to excite almost a pure 1,0 mode.

For the plane wave mode zero pressure variation is required around the circumference. This could be obtained by using the two loudspeaker system but this time connecting them in phase. Alternatively a single loudspeaker could be used. Again care is necessary to obtain precise symmetry.

The procedure for the cylindrical and annular ducts is very similar. However the setting up and traversing in the annular case does become a little more involved, as previously discussed.

Apparatus of this kind was used as long ago as 1938 by Hartig and Swanson (7) to measure "values of wavelength, threshold frequency, and pressure distribution for some of the possible harmonic wave types", in a rigid walled cylindrical duct. These interesting experiments were repeated here as a preliminary to any attenuation measurements and are described in Chapter 6.

The modes thus generated will be of the form

\[ P_{mn} = A_{mn} \cos (m\Theta) J_m (K_{mn} r) \exp(-iK_x x) \]  

where for this rigid \( m = 0 \) or 1. That a given mode is in fact the predominant one is established by examining the pressure distribution over the duct face, which when the system is correctly adjusted will behave as

\[ f(r, \Theta) = \cos (m\Theta) J_m (K_{mn} r) \]

The angular distribution of pressure may then be used to distinguish the \( m = 0 \) and \( m = 1 \) modes. At some angular position the \( m = 1 \) contribution to the pressure field will vanish whereas a constant value is expected for the \( m = 0 \) mode. This means of distinguishing the two mode types is possible in this case because a standing rather
than a spinning mode has been created. A fan or compressor, or indeed the siren rig also used in this study, will because of the particular generating mechanism, produce modes of the form

\[ P_{mn} = A_{mn} \cos (m\Theta - wt) J_m (K_{mn}r) \exp (-ikx) \] (2)

The different, \( m \), modes are now not separable by simply measuring the form of the pressure distribution in \( \Theta \), since it will be invariant in its mean amplitude, as dictated by the term \( \cos (m\Theta - wt) \).

It is possible to regard this spinning mode as the sum of two equal components in time and space quadrature, one of which may be represented by Eq (1), above. This is made clearer by writing out the familiar expansion.

\[ \cos (m\Theta - wt) = \cos m\Theta \cos wt + \sin m\Theta \sin wt \]

It is convenient that the properties of cut-off and attenuation which we wish to investigate are identical for both the spinning mode and either of its constituent parts. The advantage which can be taken of this fact is that the standing wave modes can, if the sound field is not too complex, be identified without resorting to correlation techniques.

Thus the use of loudspeakers not only enables a stable well regulated sound field to be produced but also, in simple cases, eases the problems of modal identification.
5.3 Instrumentation

All measurements and analysis was performed 'on-line', using a microphone power supply, microphone, filter and meter or pen recorder.

The simplest system involved used a Bruel and Kjaer 1/12 Octave filter and combined microphone power supply. The filter was tuned to the appropriate frequency, and pressure dB readings were recorder versus microphone position. The probe microphone was as supplied by B & K for their impedance tube, and utilised an electromagnetic detector, but a special 54 ins probe of 2 or 4 mm diameter was used.

As only comparative measurements were required at a series of line frequencies a strict calibration of the system was not necessary. An 'S' shaped removable end to the probe enabled a circumferential traverse to be made of the open cylinder by rotating the whole probe (Fig. 12). To allow this manoeuvre an adapter of very fine thread was manufactured and used to connect the probe to the microphone carriage. Because of the fine thread several full turns of the probe could be made before noticeably altering the response of the probe due to its consequent change in length.

For other traverses a 1/8 inch microphone cartridge and cathode follower unit via either a 12 inch or 22 inch long 1/8 inch diameter sting was used with a flexible cable to the power supply and analyser. This equipment was used to obtain circumferential pressure profiles in the annular duct. In this arrangement the microphone was mounted on the inner core of the duct which was able to rotate. Axial traverses were carried out by simply sliding the microphone support along the locating groove in the rotatable tube, which was also calibrated along its length.

To measure the frequency response of the system a General Radio narrow band (2.5 or 10 Hz Band width) analyser was used. This instrument incorporates a sweep device so that the analysis frequency
moved slowly over a given range. Simultaneously a tracking oscillator gave a small output at the same frequency so that this could be connected through an amplifier to the loudspeaker(s). Thus a closed loop is obtained which enables the frequency response to be obtained as a continuous progress. The 1/8 inch microphone cartridge was used for this response measurement because of its own flat response over the relevant frequency range. A diagrammatic layout of the instrumentation used is shown in Fig. 17.
CHAPTER 6 MODES IN A RIGID WALLED DUCT

The frequency response of the open cylinder was measured at a point in the duct wall. Strong excitations occurred at various particular frequencies. The largest of these resonances occur at the normal mode cut-off frequencies for the duct which are tabulated below.

<table>
<thead>
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<th>Mode (m, n)</th>
<th>fm, n Hz</th>
<th>Mode (m, n)</th>
<th>fm, n Hz</th>
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<td>6922.2</td>
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</tr>
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<td>3, 0</td>
<td>3014.4</td>
<td>9, 0</td>
<td>7685.6</td>
</tr>
<tr>
<td>4, 0</td>
<td>3815.4</td>
<td>3, 2</td>
<td>8140.9</td>
</tr>
<tr>
<td>1, 1</td>
<td>3825.4</td>
<td>1, 3</td>
<td>8399.2</td>
</tr>
<tr>
<td>5, 0</td>
<td>4603.3</td>
<td>6, 1</td>
<td>8420.8</td>
</tr>
<tr>
<td>2, 1</td>
<td>4811.7</td>
<td>10, 0</td>
<td>8445.8</td>
</tr>
<tr>
<td>0, 2</td>
<td>5033.8</td>
<td>4, 2</td>
<td>9099.4</td>
</tr>
<tr>
<td>6, 0</td>
<td>5382.2</td>
<td>11, 0</td>
<td>9203.2</td>
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<tr>
<td>3, 1</td>
<td>5751.0</td>
<td>7, 1</td>
<td>9279.2</td>
</tr>
<tr>
<td>1, 2</td>
<td>6124.9</td>
<td>2, 3</td>
<td>9449.9</td>
</tr>
<tr>
<td>7, 0</td>
<td>6154.7</td>
<td>0, 4</td>
<td>9559.9</td>
</tr>
<tr>
<td>4, 1</td>
<td>6660.2</td>
<td>12, 0</td>
<td>9958.2</td>
</tr>
</tbody>
</table>

**TABLE 1** The theoretical cut-off frequencies for all modes occurring below 10,000 Hz in a 6.0 inch diameter pipe at 20°C.

(c = 34,353 cms/sec)

A typical response curve is shown in Fig. 18 for the two loudspeaker system with the speakers out of phase. The rapid changes in pressure occurring at the resonant frequencies are clearly visible. Not surprisingly the odd modes \( m = 1 \) and \( m = 3 \) are strongly excited for this source.
The response which has been measured is the total response of the system, i.e. it includes the effects of changes in oscillator output, loudspeaker resonances etc. However, it does serve to show that the normal modes occur where we expect to see them and that certain modes are likely to be strongly excited.

To illustrate some of the properties of the 1, 0 mode the radial and circumferential pressure distributions were measured (Fig. 19) at a frequency of 1500 c/s, in a similar way to that of Hartig and Swanson ref. (7). At 1500 c/s it seems fairly certain from the response diagram that the 1, 0 mode will be dominant. The pressure traverses confirm this in that the measured patterns agree very well with the theoretical distribution...

\[ f(r, \Theta) = J_1 (k_1 r), \cos(\Theta) \]

It is interesting also to measure the wavelength of this (1, 0) mode and to show that it tends to infinity as the cut-off frequency is approached (Fig. 19). This was done by blocking the end of the tube with a solid metal reflector and measuring the distance between successive minima (i.e. a half-wave length), again as was done by Hartig and Swanson (7).

This rig was also used to establish a suitable reflectionless termination for a given frequency and mode and also to check the apparatus for unwanted resonances and sound transmission paths. As a consequence of this work various joints were improved and it was found beneficial to immerse the whole tube in a box of sand in order to subdue pipe vibrations.

Having established the feasibility of the proposed methods of mode excitation the next step was to insert the soft-walled duct and to examine the properties of the modes created under the new boundary conditions.
CHAPTER 7 MEASUREMENT OF MODAL DECAY RATES

The primary object of these experiments was to investigate the difference in attenuation, if any, between modes of varying circumferential order. The means of doing this, using the loudspeaker rig, was to create at the source plane a well defined mode shape, and to then measure its rate of decay as it was transmitted along the duct. Using the single loudspeaker a plane wave could be created and by careful alignment of the rig the deviations in pressure around the circumference could be kept minimal (< 1 dB). To excite the first order (m = 1) mode the phased loudspeakers were used. Accurate adjustment of their relative outputs and rig alignment were necessary to obtain a sufficient signal to noise ratio. The signal to noise ratio is used in this context to mean the excess of the required modal signal over the remaining unwanted modes. By examining the circumferential pressure distribution this quantity could be estimated. For instance when the plane wave mode is dominantly excited any deviation from a perfectly uniform distribution indicates the presence of other modes, it is easily calculated that if the variation is always less than ± 1 dB then the signal to noise ratio is approximately 18 dB and so other modes may be safely neglected. Modes of a higher order will generally decay at a higher rate and so the modal purity will increase in this case along the propagational axis. When the (1, 0) mode is being excited the modal purity is indicated by the general uniformity of the pattern, as in Fig. 19, and more precisely by the decrease in level at the two anti-nodal points. The minimum level at these points represents the contribution of all other modes as the (1, 0) mode contribution is, by definition zero. If this total pressure distribution contains a mode of a lower attenuation rate, generally
the plane wave, then the signal to noise ratio will decrease further from the source, and so care must be taken in interpreting measurements in these circumstances.

A signal to noise ratio of at least 20 dB was generally obtainable although this quantity is somewhat frequency dependent, so that just above the cut-off frequency a ratio of 40 dB was possible. Nevertheless, the simplicity of this method of excitation should not be taken too much for granted. For instance, when trying to excite the (1,0) mode it was found difficult to suppress components of the (3,0) mode also.

Although an almost perfectly defined circumferential pressure distribution can be constructed the resultant radial structure has so far been ignored. This distribution may be analysed into a series of radial components, which in the hard walled duct, will be of familiar Bessel function shape. In the soft walled duct these shapes will be distorted and in addition all the modes, including those above cut-off will suffer an amplitude decay as they travel along the duct. In general these rates will be different for different modes and will increase with increasing modal order. Thus, if the duct is sufficiently long, the higher order modes will preferentially decay so that eventually there will be only the lowest damped mode present. Subsequent to this point the pressure decay will be linear with distance as the remaining mode is gradually reduced in amplitude. Prior to this point the pressure field will be formed as an interference pattern between modes of different decay rate and phase velocity. The curves of Fig. 20 have been drawn to illustrate the kind of pattern which can result from this phenomena. Because of differences in phase velocity it is possible that the pressure may increase along the duct in some regions, and this was in fact frequently observed to be the case.

To prevent distortion of the axial pressure pattern by reflected waves a 'pc' termination of a porous foam construction was fitted to the end of the duct. Such reflected waves will also decay as they
return along the lined duct so that for reasonable decay rates they will soon become negligible. For this reason it was found that the presence of a $\rho c$ termination was rarely of significant benefit. For the majority of test runs the attenuation rates were high enough for the reflected wave to be of no importance, the signal having been lost in the background noise level before the duct termination was reached. In general the measurement approach for the relatively undamped modes was dependent on frequency. Above the peak of the attenuation curve, at the higher frequencies around 6 KHz, it was found to be fairly simple to provide a good reflectionless termination. However, at low frequencies this proved to be a more difficult exercise, not helped in the annular case by the need for supports for the inner ducting. In these circumstances an alternative approach is to substitute a termination of known impedance and to calculate the attenuation rate from the form of the standing wave pattern. This calculation becomes simplest if a rigid rather than an absorbing termination is used. Furthermore the low frequency and attenuation rate enable the equations to be simplified so that, following Ref. and the derivation in Appendix IV we may write: \[
\text{Attenuation Rate (dB/metre)} = 868.6 \left( \frac{1}{\text{dn, S}} \right)
\]

where \( d_N \) = distance from the rigid termination to the first minima,
\( S = \frac{P_{\text{max}}}{P_{\text{min}}} \) the standing wave ratio between the first maxima (at the termination) and the first minima (at \( d_N \)).

This approximation gave good agreement for decay rates particularly in the range 10 to 30 dB/metre where the normal method of simply measuring a linear decay rate could not be used. The above equation was also checked by re-computing the axial decay rate from a calculated pressure traverse. These computed pressure traverses are illustrated in Fig.21. Figures 21, 22, 23 and 24 show similar traces for various decay rates and terminations of absorption coefficients 90%, 95% and 99%. These results were useful in interpreting the experimental measurements as it is clearly important to recognise and distinguish the effects of a
non absorbing termination and those of interference between different modes.

Figures 25 to 31 show a selection of experimental results. In particular Fig. 25 is an example of the type of measurement just discussed. These pressure traces clearly show the kind of distortions discussed above, but it is generally possible though to measure a decay rate in the central portion of the duct where the decay is linear. A very large number of these results were recorded. The pressure was mapped every 5 or 10 mm along the axis for frequencies between 1000 and 6000 c/s and for hub tip ratios of 0, 0.5 and 0.75. Also the modes \( m = 0 \) and \( m = 1 \) were both examined. At several points along the linear portion of the pressure trace the radial and circumferential pressure profiles were measured. Clearly if only one mode is present and if there is no change in the properties of the tube, i.e., no misalignment or change of impedance this profile will remain identical except for a uniform decrease in amplitude. This supposition was frequently checked and found to be correct.

Most of the remarks applied to the above rig also apply to the siren rig. The same method of measurement was used in both cases. Unfortunately however, the same degree of control cannot be exercised over the circumferential mode distribution, although by choosing particular rotor and stator numbers a given mode may be encouraged. If these numbers are chosen correctly it can be arranged that, over a limited frequency range, any other modes which might be excited are below their cut-off frequency and so will decay extremely rapidly. For example to try and excite the \( m = 2 \) mode, 32 rotor holes were used in conjunction with 30 stator holes. Thus from the relation \( m = 32 + k \) (30) of Tyler and Sofrin the likely circumferential mode numbers at the fundamental rotor frequency form the series:

\[-52, -28, 2, 32, 62 \quad k = 0, \pm 1, \pm 2 \text{ etc.}\]

of which only the \( m = +2 \) mode will propagate at any frequencies up to about 25 KHz.
Despite this method of eliminating unwanted modes it is still difficult to obtain a sufficiently high modal signal to noise ratio. Also, because the higher order modes decay at ever increasing rates, the available duct length in which this decay can be measured rapidly decreases. Another factor which limits the usefulness of the siren rig is the degree of signal variation occurring over both long and short time scales. Because of these factors the decay rates measured on the siren rig for mode numbers $m = 0, 1, 2$ and $4$ are not so rigorously defined as for the loudspeaker rig.

Table II shows the rotor and stator hole numbers and the possible modes used in this series of experiments. Only for the $m = 1$ mode is the frequency range at all curtailed by the possible theoretical excitation of other modes. In this case the $m = 6$ mode may appear at frequencies above 5380 c/s.

A more detailed discussion of all the measured results and a comparison with theoretical values follows in the next section.

<table>
<thead>
<tr>
<th>Mode to be Excited</th>
<th>Rotor Hole Numbers</th>
<th>Stator Hole Numbers</th>
<th>Possible Modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>32</td>
<td>32</td>
<td>-64 -32 0 32 64</td>
</tr>
<tr>
<td>1</td>
<td>8</td>
<td>7</td>
<td>-13 -6 1 8 15</td>
</tr>
<tr>
<td>2</td>
<td>32</td>
<td>30</td>
<td>-58 -28 2 32 62</td>
</tr>
<tr>
<td>4</td>
<td>32</td>
<td>28</td>
<td>-52 -24 4 32 60</td>
</tr>
</tbody>
</table>

**TABLE II** A list of the possible Modes generated by given Rotor and Stator numbers at the Rotor Fundamental Frequencies.
8.1 Decay Rates

The figures 25 to 31 are specimen pressure trace measurements. They have been selected from the available results in order to illustrate a range of cases from very small to very large attenuation rate. Figure 25 represents a low decay rate and also a low frequency. As was explained in Chapter 7 such situations are often best handled by using a known, preferably rigid, duct termination so that the decay rate can be obtained from details of the standing wave pattern as follows.

\[
\text{Attenuation rate (dB/metre)} = 868.6 \frac{1}{(d_n \cdot S)}
\]

where \(d_n\) = distance to first minima = 8.5 cms

\[
S = \frac{P_{\text{max}}}{P_{\text{min}}} = \frac{\text{Pamplitude}}{\text{Pamplitude}} \left(\frac{d_0}{2d_0}\right) = 6.31
\]

\(d_0 = 16\)

Thus in this case \(\text{Attenuation Rate} = 16.2 \text{ dB/metre}\)

Subsequent traces show attenuation rates measured in the normal way from a linear pressure decay plot. The early part of such traces often show pressure maxima and minima caused by interference effects between several modes. The higher order modes then decay more rapidly so that soon only a single mode remains. This mode decays linearly until it either disappears into the background noise level or it forms a standing wave pattern with its own reflected wave from the duct termination.
8.2. Comparison with Rectangular and Cylindrical Duct Theories

The theoretical and experimental results are presented in a summarised form in Figures 32 et seq, as charts of attenuation rate versus frequency. The first figure shows the measured effect of the presence of an inner annulus on the attenuation rate of the plane wave mode. For a hub-tip ratio of 0.5 there is almost no change of decay rate for frequencies below the peak attenuation frequency, whereas at higher frequencies there is often a doubling or trebling of the decay rate compared with the completely open duct. Decreasing the annulus width further to a hub-tip ratio of 0.75 produces a curve of similar shape but in addition to small increases at low frequencies there are now very large increases of six or seven times at the higher frequencies. At these frequencies the improvement in attenuation rate is far greater than the proportionate decrease in duct cross sectional area.

It is encouraging to note that the theoretical model, when directly applied to the open cylinder configuration, shows excellent agreement. In Section 4.3 the use of an approximate annular model was outlined. The approximation was then used to show how high order modes might behave in the limit of a thin annulus. This model was used to produce the two other theoretical curves of Fig. 32, and shows the measured attenuation rates to be noticeably larger than the predicted values.

The excess attenuation over the rectangular duct model is a maximum at the peak of the attenuation curve but falls off away from resonance. It is important to note however that this comparison has been made with an annular duct which was lined on the outside wall only. More often the duct would be lined on both sides and in this case the apparent advantage of the annular
configuration may disappear. This is because the inner wall having a smaller area, will in general absorb less energy than the outer one.

It is interesting to translate these results for the plane wave mode onto the, perhaps more equitable scale, of dB/duct width as opposed to dB/unit length. The narrower annulus is now no longer always the most efficient configuration.

In fact the preferred configuration is now, as might be expected from equation (8) of section 2.2, a function of the duct width to wavelength ratio \( \frac{w}{\lambda} \), i.e. for a fixed impedance the narrower duct 'prefers' higher frequencies, and vice versa.

The following graph shows a similar set of curves for the \( m = 1 \) mode although there are one or two noticeable differences.

Strangely the intermediate hub-tip ratio of 0.5 failed to exhibit any significant increase in decay rate, particular at the high frequencies, where such a change has just been noted for the plane wave mode. This trend is not followed by the approximate annular duct theory. Also at the low frequency end the theoretical curve begins to rise again because the frequency has dropped below the cut-off point. Experimental evidence of this feature was only possible when using the siren rig. It proved impossible to excite, the \( 1,0 \) mode below its cut-off frequency with the loudspeaker rig.

In the table below are tabulated the relevant cut-off frequencies for the modes examined.

<table>
<thead>
<tr>
<th>MODE</th>
<th>HUB-TIP RATIO</th>
<th>0</th>
<th>0.5</th>
<th>0.75</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,0</td>
<td></td>
<td>1321</td>
<td>972</td>
<td>822</td>
</tr>
<tr>
<td>2,0</td>
<td></td>
<td>2191</td>
<td>1924</td>
<td>1645</td>
</tr>
<tr>
<td>4,0</td>
<td></td>
<td>3816</td>
<td>3713</td>
<td>3285</td>
</tr>
</tbody>
</table>

**TABLE II**

Showing the cut-off frequencies for the modes examined in detail. A speed of sound = 343.53 M/Sec at 20°C has been assumed.
In the next figure (35) results are shown for all four modes of the open cylindrical duct including those previously examined, of the $m = 0$ and $m = 1$ modes for comparison. Very close agreement is obtained with the theoretical curves for the higher modes $m = 2$ and $m = 4$. As these modes were produced by the siren rig no positive mode identification was carried out in this case. However, the wide differences in modal attenuation rates and the good agreement between theory and experiment give, indirectly strong evidence in favour of the modal generation processes outlined by Tyler and Sofrin. The rapid increase of attenuation rate with modal order, even above the respective cut-off frequencies, adds another factor to these which must be considered when choosing vane and blade numbers for noise sensitive machinery.

Figure 36 compares these attenuation rates with those which would have prevailed in an unlined duct. The slight loss in efficiency below cut-off is in agreement with the theoretical prediction for a wall lining of negative reactance. The impedance curve for the wall lining material is illustrated in Fig. 11. This change from a very large decay rate to a very large, but slightly smaller rate, would not be expected to be significant in practice.

The remaining graphs, Figs 37 and 38 compare the experimental and theoretical decay values obtained for the different circumferential modes at the two hub-tip ratios of 0.5 and 0.75 respectively. Well above the modal cut-off frequencies the individual decay rates will tend towards each other. Evidence for this can be seen in the results already discussed and is also apparent in the form of the equations of section 4.3. Further evidence is provided by Figs. 37 and 38. When the hub-tip ratio is changed from 0.5 to 0.75 all the modal cut-off frequencies are reduced (see Table II) and this is reflected in the greater conformity of the four sets of results at high frequencies. The approximate annular theory is again seen to underestimate the measured results except where the cut-off effect is dominant. Agreement in this area is to be expected from the construction of the theoretical model (see 4.3), as the attenuation below cut-off is controlled by the circumferential wave number which was taken directly from the exact boundary condition.
CONCLUSIONS
A series of experiments have been described where results are clearly predicted by the theoretical model used to simulate the physical situation. The central conclusion is therefore that the duct attenuation mechanism is truthfully portrayed by classic modal theory with the use of the appropriate boundary condition, which in this instance was that of a locally reacting boundary. It is necessary to add the limitation that the work was confined to a regime of low sound pressure levels and zero mean airflow. The particular conclusions regarding the detailed behaviour of the modes studied, was revealed by both theory and experiment, are as follows:

1. The attenuation rate is dependent on mode number and increases with increasing 'm' number.
2. As in the rigid walled duct there is a large increase in rate immediately below the cut-off frequency.
3. The influence of a soft duct wall on the attenuation of a mode below cut-off is to marginally change the attenuation rate, either positively or negatively. The direction of the change depends on the wall reactance, the decay rate being reduced for negative values. Also the magnitude of the change is largest for very small resistances and is progressively reduced as the resistance increases.
4. Above cut-off the decay rates of higher order modes tend to the plane wave rate as the frequency is raised well above the cut-off value. In this experiment this effect was most noticeable for the narrower annuli.
5. The peak attenuation frequency does not vary with mode number.
6. The approximate annular theory compared only qualitatively with the measured results. It was found that this theory underestimated the measured results by up to 35% (at a hub-tip ratio of 0.5) and 20% (at a hub-tip ratio of 0.75). Too much significance should not be attached to these figures as the annular duct was lined only on its outer wall.
whereas the normal practical situation involves lining both the inner and outer wall.

7. The exact cylindrical duct theory gave results which were clearly followed by experimental values and were within the experimental tolerance.

With regard to aero-engine noise work it is worthwhile rephrasing some of the above points as follows:-

The cut-off mechanism is well understood in a hard walled duct as are the consequent design requirements (choice of blade and vane numbers) needed to take advantage of the physical situation. If a soft walled duct is assumed these clear cut rules need to be modified. There no longer exists a sharp dividing line between decaying and propagating modes. Instead there is a more gradual change of attenuation rate as a given mode passes through its cut-off frequency, and considerable attenuation may still be attained over the ensuing frequency range.

For most engine configurations it is inevitable that at some harmonic a possible interaction tone will fall within the range of propagating mode numbers for all practical choices of blade and vane numbers. To select one of these choices then rests upon the consideration of a number of secondary factors. It is now clear that a previously unconsidered factor exists, namely that there is some advantage to be gained by selecting those blade numbers which will produce the interaction mode closest to, although above, its cut-off frequency i.e. with the highest 'm' number.

Well above their cut-off frequency the higher order circumferential modes are attenuated more rapidly in the soft walled duct than they would be in an unlined one of identical size, when the attenuation rate is zero. However, this situation is not maintained and at lower frequencies i.e. below cut-off it appears
that higher attenuation can be obtained from the rigid walled duct, although both rates of decay are, in absolute terms, still very large. Thus in a short length duct, in which modes below their cut-off frequency may contribute significantly to the far field radiation, it appears that a lined duct could be noisier than the untreated counterpart. However, this possibility takes no account of other benefits of the lined duct such as their efficient action on buzz-saw noise.
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ACKNOWLEDGEMENTS

The work was sponsored by Rolls-Royce Limited., who gave
direct financial aid and by Loughborough University who provided the
working facilities. Thanks are due to the staff of both institutes
for their help and support, particularly the project supervisor
Dr. M.V. Lowson.

The help of Southampton University is appreciated in the person
of Mr. B.J. Tester who gave much encouragement and
Dr. J. Mungur who provided numerical checks with the computer
programme. Finally thanks are also due to the typist Mrs. J. Hodges.
APPENDIX 1

Derivation of the Wave Equation

Although simpler concepts are perhaps possible, it is useful to introduce the method of the velocity potential. We may define the velocity potential for an irrotational fluid by the identity

\[ \mathbf{U} = -\nabla \phi = -\nabla \phi \]

so that the coordinate velocities \( U_x, U_y \) and \( U_z \) are given by

\[ U_x = -\frac{\partial \phi}{\partial x}, \quad U_y = -\frac{\partial \phi}{\partial y}, \quad U_z = -\frac{\partial \phi}{\partial z} \]

For an incompressible fluid the equation of continuity formalises the idea of mass conservation in the form

\[ \nabla \cdot \mathbf{U} = \frac{\partial U_x}{\partial x} + \frac{\partial U_y}{\partial y} + \frac{\partial U_z}{\partial z} = 0 \]

which on substituting from Equation (1) results in Laplace's equation

\[ \nabla^2 \phi \equiv \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \]

The equation of continuity for a compressible fluid is

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{U} = 0 \]

On substituting again from Equation (1) and introducing the variable \( s \), the condensation \( \frac{\rho - \rho_0}{\rho_0} \) we have,

\[ \nabla^2 \phi = -\frac{\partial s}{\partial t} \]

instead of Laplace's Equation (2).

For a system containing no external forces Bernoulli's integral equation of motion (e.g. Ref. 6) may be written

\[ \int \frac{d\rho}{\rho} + \frac{1}{2} \mathbf{U}^2 - \frac{\partial \phi}{\partial t} = \text{const} \]

which after some manipulation, assuming \( \mathbf{U}^2 \) to be a small quantity, and that \( \frac{\partial \rho}{\partial t} = c^2 \) results in the expression

\[ c^2 S - \frac{\partial \phi}{\partial t} = 0 \]

By elimination of \( s \) between Equations (5) and (7) we then have the
classical wave equation

\[ \nabla^2 \phi = \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} \]  \hspace{0.5cm} (8)

which shows that the constant c may indeed be interpreted as the velocity of sound.

In the text we are only concerned with either two dimensional rectangular or cylindrical coordinates, in these two cases, Equation (8) takes the form

\[ \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} \]  \hspace{0.5cm} (9)

or

\[ \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta^2} + \frac{\partial^2 \phi}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} \]  \hspace{0.5cm} (10)

Solutions to Equations (9) and (10) may be obtained by the method of separation of variables and we can obtain from Equation (9) the possible solutions

\[ \phi = \frac{\cos (k_y y)}{\sin (k_x x)} \cos (k \gamma t) \cos (k \gamma t) \]  \hspace{0.5cm} (11)

where the sine and cosine functions are interchangeable and

\[ k^2 = k_y^2 + k_x^2 \]  \hspace{0.5cm} (12)

Alternatively, and more usually, exponential notation is used so that

\[ \phi = \frac{\cosh (k_y y)}{\sinh (k_x x)} \exp \pm (k_x x - k \gamma t) \]  \hspace{0.5cm} (13)

and in cylindrical coordinates

\[ \phi = \frac{F_m (k_r r)}{G_m (m \gamma)} \exp \pm (k_x x - k \gamma t) \]  \hspace{0.5cm} (14)

where

\[ k^2 = k_x^2 + k_r^2 \]

To particularise the solution further requires knowledge of the local boundary conditions as is discussed in sections 2.1 and 2.2.
The equation to be solved is of the form
\[ y = x \frac{J_{m-1}(x)}{J_m(x)} = f(x) \]
It was solved here by using Newton's iteration method, where successive solutions are given by
\[ x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \]
and
\[ f(x_n) = y - f(x_n) \]
This process is repeated until sufficient accuracy is obtained.
The starting points (S, P, ′s) are varied to obtain the required series of solutions. The S, P, ′s used are as follows:

(i) **Positive Reactances**
   (a) Put Re (x) equal to successive solutions of \( J_m = 0 \) less .05, so that the series of starting points for say \( m = 4 \) reads
   7.588 -.05 ; 11.064 -.05 ; 14.372 -.05 etc.
   Im (x) = .05
   or what was found to be somewhat more satisfactory put
   Im (x) = .05 for the first mode only and for succeeding modes put the starting point equal to the answer found for the previous mode.

(ii) **Negative Reactances**
   (a) put Re (x) equal to successive solutions of \( J_m = 0 \) plus .05. Therefore for \( m = 4 \) we have 7.588 + .05 ; 11.064 + .05 ; 14.372 + .05 Im (x) = .05 always.
   (b) put Re (x) equal to successive solutions of less a small quantity so that for \( m = 4 \) we have
   5.317 -.05 ; 9.282 -.05 ; 12.682 -.05
   Im (x) = .05 always.
   (c) Re (x) = Im (y)
   Im (x) = -Re (y)
The solutions to the equations \( J_m = 0 \) and \( J'_m = 0 \) are generally found by a separate iteration program and this data then used as input to the main program. However, for low \( m \) numbers, say less than eight, suitable arithmetic expressions may be used.

(See ref. 2c)
APPENDIX III

Evaluation of $J_m(R e^{i\phi})$

The first method to be tried utilised the series expansion

$$J_m(R e^{i\phi}) = \sum_{k=0}^{\infty} \frac{(-1)^k (R e^{i\phi})^{2k+n}}{k! (k+n)!}$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k (R/2)^{2k+n}}{k! (k+n)!} \left[ \cos((2k+n)\phi) + i \sin((2k+n)\phi) \right]$$

For small values of $R$ this can be satisfactory but care must be taken not to 'overflow' in the evaluation of the factorial quantities. The series will normally converge after a small number of terms.

For large values of $R$ the integral expression below may be used

$$J_m(z) = \frac{1}{i} \int_{\phi} \cos(z \sin \theta - m \theta) \, d\theta$$

After some trials these methods were rejected in favour of a recurrence technique developed by Goldstein and Thaler (Ref. 3). This method was generally more rapid and also has the advantage that in the evaluation of $J_m$, the functions $J_{n-1}$ to $J_c$ are also calculated. The method depends upon the recurrence relation

$$J_{n-1}(z) = J_{n+1}(z) - \frac{2n}{z} J_n(z)$$

where $n$ is chosen to be significantly larger than $m$ so that the starting point of the iteration may be taken as $J_{n+1}(z) = 0$

and $J_n(z) = \text{a small quantity, say } 10^{-6}$
Derivation of Approximate Equation for an Attenuating Standing Wave Pattern

For waves of only small decay rate it is normally difficult to measure this decay rate by simply measuring the slope of axial pressure trace, because of interference caused by the reflected wave. At high frequencies it is relatively easy to construct a very efficient duct termination so that the incident wave is almost completely absorbed. The kind of pressure traces produced in this way are illustrated in Figs. 21-24. Unfortunately at low frequencies it was often found to be difficult to build an absorbent termination of the required efficiency. However, it was found that by using a completely rigid termination the attenuation rate could be readily determined from the properties of the standing wave pattern, at least for small attenuation rates. The necessary algebra is outlined below.

Following Ref. 51, we may write the following relation for an attenuated standing wave pattern

$$\frac{P_{\text{max}}}{P_{\text{min}}} = \left[ \frac{2 \cos^2 (\alpha DM + \gamma) - \frac{\alpha^2}{2} \beta^2 \sinh^2 2 (\alpha DN + \gamma)}{2 \sinh^2 (\alpha DN + \gamma) + \frac{\alpha^2}{2} \beta^2 \sinh^2 2 (\alpha DN + \gamma)} \right]$$

where

DM = distance to $M^{th}$ pressure maxima \( \text{cm} \)

$dN = " \ " N^{th} " \ " \text{minima cm}$

$\alpha$ = attenuation coefficient of acoustic wave

$\frac{\beta}{2\gamma}$ = phase velocity. " " "

$\alpha^2$ = ratio of amplitudes on reflection

For the typical values under consideration we may put $\frac{\alpha^2}{\beta^2} \ll 1$

An attenuation rate of 20 dB/metre and a frequency of 1200 c/s produce the approximate values

$$\alpha = 20 \div 864 \times 10^{-3} \approx 24 \times 10^{-3} \quad \alpha/\beta \approx 10^{-1}$$

So that the factor $\frac{\alpha^2}{\beta^2} = 0.005$ and thus the secondary terms of Eq (1) may be neglected.
Under the particular circumstances of a rigid termination 
\( \gamma = 0 \) and \( \beta_1 = 0 \) so that Equation (1) becomes:
\[
\frac{P_{\text{max}}}{P_{\text{min}}} = \frac{1}{\sinh(\lambda d_N)}
\]

Now \( dN \) is of the order of magnitude \( \frac{1}{4} \) so \( \lambda d_N = 0.2 \)
and for arguments of this magnitude \( \sinh(x) \approx x \) to within 1\% accuracy.

Thus we have
\[
\frac{P_{\text{max}}}{P_{\text{min}}} = S = \frac{1}{\lambda d_N}
\]

\( \text{dB (Attenuation Rate)/metre} = 868.6 \lambda = 868.6 \left[ \frac{1}{d_N S} \right] \)
APPENDIX V

COMPUTER PROGRAMME

LISTING
SUBROUTINE DO457(MM, KK, STP2)
DIMENSION STP2(10)
N=MM
C=4.0*N*N
DO 20 K=1,KK
B=2.0*N+4.0*K-1.0
A=B*0.785398
D=(C-1.0)/(8.0*A)
E=(4.0*(C-1.0)*(7.0*C-31.0))/(3.0*((8.0*A)**3))
F=(32.0*(C-1.0)*(83.0*(C**2)-982.0*C+3779.0))/(15.0*(8.0*A)**5)
G=(64.0*(C-1.0)*(6949.0*(C**3)-153855.0*(C**2)+1585743.0*C-62772371.0))/(105.0*(8.0*A)**7)
20 STP2(K)=A-(D+E+F+G)
RETURN
END

SUBROUTINE UU425(MM, DTWR, NRM, KK, STP1)
DIMENSION STP1(10)
DIMENSION ARR(10)
ARR(1)=3.8317
ARR(2)=7.0156
ARR(3)=10.1735
ARR(4)=13.3237
ARR(5)=16.4706
ARR(6)=19.6159
-ARR(7)=22.7601
ARR(8)=25.9037
ARR(9)=29.0468
ARR(10)=32.1897
IF(NRM)10,25,10
10 DO 20 N=1,NRM
20 STP1(N)=ARR(N)
KK=NRM
RETURN
25 DO 40 N=1,10
STP1(N)=ARR(N)
KK=N
DT=DTWR*3.141593
IF(DT-STP1(N))50,40,40
40 CONTINUE
KK=10
RETURN
50 IF(MM)51,51,52
51 KK=N
RETURN
52 KK=N-1
60 RETURN
END
SUBROUTINE DQ477(A,B,M,REANS,IMANS,IND,RANS2,IANS2)
REAL JAY(200),IJAY(200),IMANS,IMBES,IANS2
C**** CALCULATION OF COMPLEX BESSEL FUNCTIONS - MARK 2 *****
MOUT = 3
DENOM = A**2. + B**2.
X = SQRT(DENOM)
C
C ***** FORMS THE STARTING VALUE OF KA *****
C
50 IF (X - 5.) 50, 70, 70
50 KA = X + 6.
GO TO 80
70 KA = 1.4*X + 60./X
80 KB = M + IFIX(X)/4 + 2
IF(KA-KB)71,72,72
71 KK = KB
GO TO 1222
72 KK = KA
1222 CONTINUE
C
C***** FORM COMPLEX 1/(A+B) USING COMPLEX CONJUGATE *****
P = A/(DENOM)
Q = -B/(DENOM)
C
C***** LOOP GENERATES J(N) AND PICKS OUT J(M), THE REQUIRED FN *****
C
N = 0
60 N = N + 1
KK = KK + 4
K = KK
CALL TSTRT
991 JAY(K+2) = 0
IJAY(K+2) = 0
JAY(K+1) = 1.0E-35
IJAY(K+1) = 0
C
110 RP = P*JAY(K+1) - Q*IJAY(K+1)

YP = Q*JAY(K+1) + P*IJAY(K+1)
112 JAY(K) = 2*K *RP - JAY(K+2)
113 IJAY(K) = 2*K *YP - IJAY(K+2)
K = K - 1
IF(K) 114,114,110
114 CONTINUE
C
C***** NEXT ROUTINE CALCULATES THE COMPLEX CONSTANT OF THE FORM (C+ID)
L = 3
SUM1 = JAY(L)
SUM2 = IJAY(L)
121 L = L + 2
SUM1 = SUM1 + JAY(L)
SUM2 = SUM2 + IJAY(L)
IF(L-KK)121,122,122
122 CONTINUE
SUM3 = JAY(1) + 2*SUM1
SUM4 = IJAY(1) + 2*SUM2
CALL TSTOP
IF(SUM3)550,551,550
551 C = 0.0
GO TO 555
550 C = 1./((SUM4/SUM3)*SUM4 + SUM3)
555 IF(SUM4)552,553,552
553 D = 0.0
GO TO 192
552 D = -1./((SUM3/SUM4)*SUM3 + SUM4)
***** FORM THE COMPLEX BESSEL FUNCTION *****
192 REBES = C*JAY(M+1) - D*IJAY(M+1)
IMBES = D*JAY(M+1) + C*IJAY(M+1)
134 IF(REBES)625,621,625
621 IF(IMBES)625,992,625
625 IF(N-1)130,130,132
132 DIFF =(|GESS - REBES|/GESS
IF(DIFF - 0.000001)133,133,620
133 IF(DIFF + 0.000001)620,135,135
620 IF( 8 - N)137,130,130
130 GESS = REBES
GO TO 60
137 REANS = REBES
IMANS = IMBES
IANS2 = D*JAY(M) + C*IJAY(M)
RANS2 = C*JAY(M) - D*IJAY(M)
IND = 10
RETURN
135 REANS = REBES
IMANS = IMBES
RANS2 = C*JAY(M) - D*IJAY(M)
IANS2 = D*JAY(M) + C*IJAY(M)
RETURN
992 IND = 1
CALL TSTOP
RETURN
END
SUBROUTINE DO458(H,PHY,M,DL,DD,VK,SPR,SPI,ZI,BET,PHV,IIR)
MOUT=3
ADJ=0.001
ICN=20
IIR=0
HR=-H*SIN(PHY)*3.14159
HI=H*COS(PHY)*3.14159
ARGR=SPR*3.141593
ARGI=-3.141593*SPI
KKK=0
40 STR=ARGR/3.14159
STI=-ARGI/3.14159
KKK=KKK+1
IF(KKK-ICN)215,215,4
215 SPR=STR
SPI=STI
ASPR=ABS(SPR)
IF(ASPR-100.)235,4,4
235 ASPI=ABS(SPI)
IF(ASPI-100.0)240,4,4
240 CONTINUE
IF(SPR)241,242,242
241 IF(SPI)4,242,242
242 CONTINUE
LM=M+1
IF(4)39,39,45
39 CALL DO477(ARGR,ARGI,LM,A2,B2,IND,A1,Bl)
A2=-A2
B2=-B2
45 CALL DO477(ARGR,ARGI,LM,A1,B1,IND,A2,B2)
IF(IND)11,10,11
11 GO TO 100
10 CONTINUE
16 CONTINUE
GR=-(ARGR=A2-ARGI*B2)
GI=-(ARGI=A2+ARGR*B2)
DEN=A1*A1+B1*B1
FR=(GR=A1+GI*B1)/DEN
FI=(GI=A1-GR*B1)/DEN
TSTR=FR+HR+M
TSTY=HI+FI
TSR=ABS(TSTR)
TSY=ABS(TSTY)
IF(TSR-ADJ)20,20,35
20 IF(TSY-ADJ)25,25,35
25 ZR=SPR
ZI=SPI
B=DD/2.
BB=B*B
PIB=9.869604/BB
DIF=SPR*SPR-PI*SPI
FAC=PIB*DIF
A=VK*VK-FAC
B=Z.*PIB*SPR*SPI
RT=SQR(A*A+B*B)
VXIM=SQRT(0.5*(RT-A))
VXRE=SQR(0.5*(RT+A))
BET=DL*VXIM=8.68589
PHV=VXRE/VK
RETURN
CONTINUE
DON=AI*A1+B1*B1
CR=(A2*A1+B2*B1)/DON
CI=(A1*B2-A2*B1)/DON
DONN=ARGR*ARGR+ARGI*ARGI
DR=(2.0*M*ARGR/DONN)-CR
DI=(-2.0*M*ARGI/DONN)-CI
ER=CR*DR-CI*DI
EI=CR*DI+CI*DR
ER=1.0-ER
EI=-EI
FDPR=ER*ARGR-EI*ARGI
FDPI=EI*ARGR+ER*ARGI
DAN=FDPR*FDPR+FDPI*FDPI
RAR=(TSTR*FDPR-TSTY*FDPI)/DAN
RAI=(TSTY*FDPR-TSTR*FDPI)/DAN
CONV=SQR(RAR*RAR+RAI*RAI)
WRITE(MOUT,230)STRI,ST1,CONV
230 FORMAT(30X,3(10X,F11.7))
TRAR=ABS(RAR)
IF(TRAR-0.1E-05)210,210,220
210 TRAI=ABS(RAI)
IF(TRAI-0.1E-05)4,4,220
220 ARGR=ARGR-RAR
ARGI=ARGI-RAI
GO TO 40
100 GO TO(140,150),IND
140 WRITE(MOUT,141)
141 FORMAT(12X,'OVERFLOW CONDITION')
GO TO 4
150 WRITE(MOUT,151)
151 FORMAT(2X,'FAILED TO CONVERGE AFTER 8 CONFIGURATIONS')
GO TO 4
4 IIR=1
RETURN
END
C
MASTER MAIN
DIMENSION FT(100),XR(3),XX(3),XL(3),STP1(10),GRS(100),GRE(100)
DIMENSION STP2(10),STPN(20),VR(21),VL(21),ATN(21)
DIMENSION PHVE(20)
DIMENSION AVN(21),WR(21),WI(21)
MIN=2
MOUT=3
18 READ(MIN,15) (FT(I),I=1,20)
15 FORMAT(20A4)
READ(MIN,150) NLL
150 FORMAT(15)
IF(NLL) 13,13,12
12 CONTINUE
READ(MIN,152) (XR(I),I=1,NLL)
152 FORMAT(8F10.6)
READ(MIN,152) (XX(I),I=1,NLL)
READ(MIN,152) (XL(I),I=1,NLL)
13 CONTINUE
READ(MIN,20) CS,DL,DD,XM,NZZ,NUT,FX,F
20 FORMAT(F10.3,2F10.6/F10.6/2I5,2F10.3)
READ(MIN,21) MM,NRM
21 FORMAT(2I5)
WRITE(MOUT,16) (FT(I),I=1,20)
16 FORMAT(//////1X,20A4)
IF(NLL) 2000,2000,2001
2001 CONTINUE
WRITE(MOUT,109) NLL
109 FORMAT(5X,'NO. OF LAYERS',I5)
WRITE(MOUT,106) (XR(I),I=1,NLL)
106 FORMAT(5X10HRESISTANCE8E13.6)
WRITE(MOUT,107) (XX(I),I=1,NLL)
107 FORMAT(5X4HMSS6X8E13.6)
WRITE(MOUT,108) (XL(I),I=1,NLL)
108 FORMAT(5X7HLENGTHS3X8E13.6)
2000 CONTINUE
WRITE(MOUT,1530) CS,DD,DL,XM,MM
1530 FORMAT(4X,'SPEED OF SOUND',3X,F10.3,/,4X,'DUCT DIAMETER',4X,F10.3,
1/,4X,'DUCT LENGTH',6X,F10.3,/,4X,'MACH NO',10X,F10.3,/,4X,'MODE OR
1DER',10X,13//)
IF(NZZ)4,4,2
2 FT (1)=40.
FT (2)=50.
FT (3)=63.
FT (4)=80.
FT ( 5)=100.
FT ( 6)=125.
FT ( 7)=160.
FT ( 8)=200.
FT ( 9)=250.
FT (10)=315.
FT (11)=400.
FT (12)=500.
FT (13)=630.
FT (14)=800.
FT (15)=1000.
FT (16)=1250.
FT (17)=1600.
FT(18)=2000.
FT(19)=2500.
FT(20)=3150.
FT(21)=4000.
FT(22)=5000.
FT(23)=6300.
FT(24)=8000.
FT(25)=10000.
FT(26)=12500.
FT(27)=16000.
FT(28)=20000.
GO TO 7
4 NZZ=NZZ+1
DO 6 IV=1,NUT
FT(IV)=FX
6 FX=FX+F
2008 READ(MIN,152) (GRS(LT),LT=NZZ,NUT)
READ(MIN,152) (GRE(LT),LT=NZZ,NUT)
2007 DO 80 JJJ=NZZ,NUT
FX=FT(JJJ)
WVL=CS*12./FX
DTWR=DD/WVL
CALL DQ455(MM,DTWR,NRM,KK,STP1)
WRITE(MOUT,1000)FX,DTWR
1000 FORMAT(1X,'AT THIS FREQUENCY OF',F10.3,'HZ AND DUCT TO WAVELENGTH
1RATIO OF',F10.4)
IF (KK)1001, 1001, 1005
1001 WRITE(MOUT,1002)
1002 FORMAT(1X,'THERE ARE NO MODES ABOVE CUT-OFF AND SO THE LOWEST ORDE
1R MODE ONLY HAS BEEN USED')
KK=1
GO TO 1019
1005 IF(KK-10)1006,2005,2006
2005 CONTINUE
1006 WRITE(MOUT,1007)KK
1007 FORMAT(1X,'THE FIRST',I3,1X,'MODES ARE ABOVE CUT-OFF AND THESE HAV
1E ALL BEEN USED')
GO TO 1019
1008 WRITE(MOUT,1009)
1009 FORMAT(1X,'THERE ARE TEN OR MORE MODES ABOVE CUT-OFF AND SO ONLY T
1HE FIRST TEN HAVE BEEN USED')
1019 CONTINUE
WRITE(MOUT,1020)
1020 FORMAT(1X,'RESISTANCE REACTANCE H AUTEN 1
1G M(WVN) PH. VEL ATTENUATION')
VK=2.*3.141593/WVL
IF(NLL)2005,2005,2006
2006 CONTINUE
CALL DQ456(VK,NLL,XR,XX,XL,R,X)
GO TO 2009
2005 R=GRS(JJJ)
X=GRE(JJJ)
2009 CONTINUE
ZMOD=SQRT(R*R+X*X)
H=DTWR/ZMOD
PHY=ATAN(X/R)
CALL DQ457(MM,KK,STP2)
IF(PHY)40,25,25
DO 30 N=1,KK
SPR=STP2(N)-.01
SPR=SPR/3.1415926536
SPI=.025
CALL DO458(H,PHY,MM,DL,DD,VK,SPR,SPI,ZR,ZI,BET,PHV,IIR)
IF(IIR).GT.1030,31,1030
31 CONTINUE
VR(N)=ZR
VI(N)=ZI
PHV(N)=PHV
ATN(N)=BET
IF(KK-1).GT.32,32,60
32 KJ=1
AVN(1)=ATN(1)
WR(1)=VR(1)
WI(1)=VI(1)
GO TO 1550
40 N=1
SPR=H*COS(PHY)
SPI=MM/3.14159
SPI=SPI-H*SIN(PHY)
CALL DO458(H,PHY,MM,DL,DD,VK,SPR,SPI,ZR,ZI,BET,PHV,IIR)
IF(IIR).GT.1040,41,1040
41 CONTINUE
VR(1)=ZR
VI(1)=ZI
PHV(1)=PHV
ATN(1)=BET
III=0
DO 42 NNN=1,KK
III=III+1
STPN(III)=STP2(NNN)
III=III+1
42 STPN(III)=STP1(III)
DO 50 N=1,III
SPR=STPN(N)-((-1)**N)*.01
SPR=SPR/3.1415926536
SPI=.025
CALL DO458(H,PHY,MM,DL,DD,VK,SPR,SPI,ZR,ZI,BET,PHV,IIR)
44 IF(IIR).GT.45,46,45
45 IJK=N+1
VR(IJK)=VR(1)
VI(IJK)=VI(1)
ATN(IJK)=ATN(1)
PHVE(IJK)=PHV(1)
GO TO 50
46 IJK=N+1
VR(IJK)=ZR
VI(IJK)=ZI
ATN(IJK)=BET
PHVE(IJK)=PHV
50 CONTINUE
60 IF(PHY).GT.61,62,62
61 J=2*KK+1
GO TO 1495
62 J=KK
1495 NUN=J-1
1500 ISW=0
DO 1502 I=1,NUN
BB=ABS(VR(I)-VR(I+1))
1497 IF(BB-.001) 1503,1503,1497
1501 S=VR(I+1)
    T=VI(I+1)
    V=ATN(I+1)
    W=PHVE(I+1)
    VR(I+1)=VR(I)
    VI(I+1)=VI(I)
    ATN(I+1)=ATN(I)
    PHVE(I+1)=PHVE(I)
    VR(I)=S
    VI(I)=T
    ATN(I)=V
    PHVE(I)=W
ISW=1
GO TO 1502
1503 CC=ABS(VI(I)-VI(I+1))
    IF(CC-.001) 1506,1506,1505
1505 IF(VI(I)-VI(I+1)) 1501,1506,1502
1506 VR(I)=0.
1507 CONTINUE
IF(ISW)1509,1509,1500
1509 CONTINUE
    KJ=0
    DO 1514 I=1,J
        KJ=KJ+1
        AVN(KJ)=ATN(I)
        WR(KJ)=VR(I)
        WI(KJ)=VI(I)
        PHVE(KJ)=PHVE(I)
    1514 CONTINUE
1515 CONTINUE
    DO 1516 I=1,KJ
        WRITE(MOUT,1517)R,X,H,PHY,WR(I),WI(I),PHVE(I),AVN(I),I
    1517 FORMAT(IX,F10.6,3X,4(F10.6),3(6X,F10.6),5X,13)
G=0.
    DO 1520 I=1,KK
1520 G=G+10.**(AVN(I)/10.0)
G=G/KK
G=4.3424*ALOG(G)
WRITE(MOUT,1525)G
1525 FORMAT(/IX,'THE OVERALL ATTENUATION FOR THESE MODES IS',F9.3,IX,'D 16',//)
    IF(KJ=KK)1531,80,80
1531 WRITE(MOUT,1535)
1535 FORMAT(IX,'N.B....ALL THE REQUIRED SOLUTIONS HAVE NOT BEEN FOUND')
80 CONTINUE
GO TO 18
1030 WRITE(MOUT,1035)R,X,H,PHY,N
1035 FORMAT(IX,F10.6,3X,3(F10.6),5X,'THE ITERATION HAS FAILED FOR',/12,'THE ,12,TH ORDER RADIAL MODE'/)
GO TO 80
1040 WRITE(MOUT,1045)R,X,H,PHY
1045 FORMAT(IX,F10.6,3X,3(F10.6),5X,'THE ITERATION HAS FAILED FOR',/12,'THE FIRST ORDER RADIAL MODE'/)
GO TO 80
END
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<tr>
<th>Resistance</th>
<th>Reactance</th>
<th>H</th>
<th>Phy</th>
<th>Ref (V/m)</th>
<th>Im (V/m)</th>
<th>Ph. Vel</th>
<th>Attenuation</th>
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The overall attenuation for these modes is -3.790 dB.

At this frequency of 1500.000 Hz and duct to wavelength ratio of 0.6696, the first 1 modes are above cut-off and these have all been used.
MEAN GEOMETRIC FREQUENCIES HZ

APPLICATION OF VARIOUS ATTENUATION FORMULAE (After Kerka Ref. 24)

- Parkinson
- Sabine
- Roberts
- Brueel

Morse (R,£)
Morse (Flow Resistance)
Kerka (Measured)

Fig 1
Some Results From Ref. 50 for a duct of width 4 ins and length 24 ins (L/H = 6)

Experimental from Ref. 50

Theoretical from Morse by method of Tester et al.

FIGURE 2
Figure 3: Illustrating the difference in structure between (a) non-locally and (b) locally reacting lining materials.

(a) The absorbent materials allows limited sound propagation through itself and its behaviour is dependent on the amount of this propagation so that the material is non-locally reacting. The perforated top surface is used here simply to contain the absorber and is not itself a significant acoustic element if sufficiently porous.

(b) Propagation through the air backing is prevented by the honeycomb walls and through the thin resistive layer by joining it to the stiff honeycomb walls. The material is therefore locally reacting.
\[ z_1 = (pc \phi / \lambda h) e^{-i\phi} \]

\[ \varphi = \tan h(\pi \varphi) = ihe^{-i\phi} \]

\[ h = \left| \beta \right| \ell / \lambda \]

\[ \beta = \left| \beta \right| e^{i \phi} \]

FIG 4
Attenuation Duct to Wavelength Ratio, from Cremer (Ref 6)

\[ \frac{\eta}{\lambda} \] for 1 side lined

\[ \eta = 2\epsilon/\lambda \] for 2 side lined

FIGURE 5
SOLUTION OF THE CIRCULAR DUCT BOUNDARY CONDITION FOR AN M = 0 MODE. FROM MORSE AND INGARD REF(1)
Attenuation of (1,0) and (0,0) modes in a 6" diameter cylinder (least damped mode only)

Fig 7
DECAY CURVES for M=3 MODE. SEE BROWN D. (REF 53)
DUCT DIAMETER = 6" ) \( L/D = 5 \)
DUCT LENGTH = 30"

---

RESULTS FROM REF (5) FOR \( L/D = 5 \)
O RESULTS FROM THIS ANALYSIS FOR AN IDENTICAL CONFIGURATION

THEORETICAL SOUND PRESSURE LEVEL ATTENUATIONS FOR RADIAL MODE DISTRIBUTIONS
(a) "PLANE WAVE" SYNTHESIS FROM RICE (5), (b) O O EQUAL ENERGY

Fig 9.
Theoretical Attenuation curves for different ‘m’ modes using the approximate annular model.

Fig 10.
INPEDELANCE OF WALL LINING MATERIAL

Resistance ($R/p_o$)

Normalized Impedance Components ($R/p_o$ and $X/p_o$)
### Input Cards for Use of Cylindrical Duct Attenuation Programme

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**Card 1**
- 20.64

**Card 2**
- Leave blank

**Card 3**
- Speed of Sound
- Inner Diameter
- Liner Length

**Card 4**
- F10.6 (ft/sec)
- F10.6 (ins)
- Leave blank

**Card 5**
- Nos. & Frequencies
- Frq. Increment

**Card 6**
- 8 X 12
- F10.6 (Hz)
- F10.6 (Hz)

**Card 7**
- Mode
- NOS. ORDER
- Radial Modes
- 15

**Card 8**
- Resistance (1)
- Resistance (2)
- Resistance (3)

**Card 9**
- Reactance (1)
- Reactance (2)
- Reactance (3)

**Card 10**
- ETC
START

READ INPUT

CALL DQ455

WRITE HEADINGS

CALL DQ456

CALL DQ457

Form Array of starting points

TEST REACTANCE

Form array of relevant starting points (see Appendix III)

CALL DQ458

Sort and list soln's found; eliminating duplicate soln's

Calculate Overall Attenuation

PRINT OUTPUT

END

Calculates nos. of radial modes above cut-off. Holds array of soln's Jm = 0, to be used as starting points.

Calculates material impedance from dimensional properties and surface resistance; not used with form of input shown.

Holds array of soln's Jm = 0 to be used as starting points.

Solves boundary condition equation for the complete array of starting points.
The Absorbent Duct and Loudspeaker
Rig Showing the Traverse Mechanisms

Fig 14
Fig. 15 (a) Showing an Annular Configuration of the Loudspeaker Rig and (b) A cut-away View of the Lining Material
SIREN RIG. (Showing facing into 'anechoic chamber' not used here)

Typical siren spectrum showing first eight harmonics.
The General Circuit Arrangement for (TOP) Normal two-speaker Operation and (BOTTOM) frequency - response measurements

**General Radio Unit**
(a) analyser
(b) microphone power supply
(c) tracking filter

**B & K Analyzer**

Probe microphone & carriage

OR

1" microphone cartridge & stem

**Pen Recorder**
The frequency response of the hard walled duct at a position close to the source plane with the two loudspeakers in anti-phase.
The (1,0) mode at 1500 c/s - showing circumferential and radial pressure profiles. The lower diagram shows the variation of wavelength with frequency for the (1,0) mode ΔΔΔΔΔ and the plane wave ・・・. As measured in the rigid-walled duct.
Illustrating the Interference Patterns Produced by Two Nodes of Different Decay Rate as they Travel Along the Duct Axis

Theoretical Results

S.P.L. dB

DISTANCE FROM SOURCE (CMS)

Fig 20
Standing wave patterns for varying decay rates and a rigid termination

Theoretical Results.
Standing wave patterns for varying decay rates (see nomenclature of Fig. 8) and a termination of absorption coefficient 90%
Standing wave patterns for varying decay rates (see nomenclature of FIG. 8) and a termination of absorption coefficient 95%
Standing wave patterns for varying decay rates (see nomenclature of FIG. 8) and a termination of absorption coefficient 95%.
Fig 25

Single Loudspeaker
Kjeld Terminated
Full Duct Damping

Mode M = 0

Pres 1000 cl

TEMP 23°C

Probe 2.0 mm diam
at 75 mm from outer wall.

Annulus He = 0.0

Attenuation 16 dB/metre

Other Traverses

Circumferential Checks
For uniformity - max deviation ≤ 1.5 dB
Radial: None.
RIG
Single Loudspeaker
Semi-absorbent termination

MODE M = 0
FREQ 1500 c/s
TEMP 22°C
PROBE 2 mm. diam.

ANNULUS Hc = 0.0
ATTENUATION 22.5 dB/metre.

OTHER TRAVERSES
Circumferential profile checked.
Radial at 5, 10, 15, 25 cms.

FIG 26
FIG 27

RIG
Single Loudspeaker
Semi-absorbent Permeation

MODE M-0

FREQ 2000 Hz

TEMP 23.5°C

PROBE 2 mm

ANNULUS He = 0.5

ATTEN RATE 44 dB/metre

OTHER TRAVERSES
Circumferential checked.
Radial at 15, 25, 35, 45 cm

DISTANCE FROM ENTRANCE PLANE OF SOFT WALL DUCT (CMS)
SPL

DISTANCE FROM ENTRANCE PLANE OF SOFT WALL DUCT

FIG. 28

RIG
Anti-phased loudspeakers
Open termination

MODE
M = 1

FREQ
2600 c/s

TEMP
22.5 °C

PROBE
2 mm.

ANNULUS
He = 0.

ATTENUATION
90 dB/metre

OTHER
TRAVERSES
Circumferential checked.
Radial at 5, 15, 25 cm.
RIG
Anti-phased loudspeakers.
Open Termination.

MODE M = 1

FREQ 3500 c/s

TEMP 23°C

PROBE 2 mm.

ANNULUS He = 0.75

ATTEN. RATE 168 dB/metre

OTHER TRAVERSALS
Circumferential checked.
Radial at 5 and 15 cm.
RIG
SIREN (B = 32, V = 28)

MODE M = 4
FREQ 4600 Hz
TEMP 18°C
PROBE 4 mm

ANNULUS H_b = 0.0
ATTENUATE 310 dB/metre

OTHER TRAVERSE:
NONE

DISTANCE FROM ENTRANCE PLANE OF SOFT WALLED DUCT (CMS)
Figure 32

Fluctuation of Duct Diameter

- Exact Cylindrical Duct Theory
- Approximate Annular Duct Theory

Experimental Measurements

- M = 0, 0
- M = 0, 0.5
- M = 0, 0.75
The Results of Fig. 32 transferred to a dB/duct with scale (M = 0 mode)
Attenuation Rate

\[
\text{dB/duct diameter}
\]

Frequency KHZ

\[
\text{M = 1 HTR = 0}
\]

\[
\text{Experimental Results}
\]

\[
\text{Approximate Annular Theory}
\]

\[
\text{Exact Cylindrical Duct Theory}
\]

\[
\text{FIG 34}
\]
HTR = 0  M = 0, 1, 2, 4. Numbers denote experimental results.

Exact Cylindrical Duct Theory

Fig. 35
Experimental results for the Open Cylindrical Duct and modes \( M = 0, 1, 2 \) & 4. For comparison the equivalent theoretical, hard walled duct, attenuation rates are shown (continuous lines).
Experimental and theoretical results for the annular duct, HTR 0.5 and modes M = 0, 1, 2 & 4

Approximate Annular Theory

Fig 37
Experimental and theoretical results for the Annular duct, HTR = 0.75 and modes M = 0, 1, 2, & 4.