Energy and Spectrum Efficiency Trade-off for Green Small Cell Networks

Haris Pervaiz, Leila Musavian and Qiang Ni
School of Computing and Communications, InfoLab 21,
Lancaster University, Lancaster, UK
Email: {h.pervaiz, l.musavian, q.ni}@lancaster.ac.uk

***Abstract*—Green Small Cell Networks aim at achieving high rates and low powers by offloading users with low signal-to-noise-ratios from macrocell to the pico base station. In this work, we propose to jointly optimise energy efficiency (EE) and spectrum efficiency (SE) such that the network providers can dynamically tune the trade-off parameter for different design requirements. This paper formulates the EE-SE trade-off as a multi-objective optimisation problem (MOP) in the uplink of multi-user two-tier Orthogonal Frequency Division Multiplexing Heterogeneous Networks. Using the weighted sum method, the MOP can be transformed into a single-objective optimisation problem (SOP). The proposed EE and SE trade-off optimisation problem is strictly quasi-concave. Hence, using Dual Decomposition approach, we derive the unique optimal solution. Numerical results demonstrate the effectiveness of the proposed approach and illustrate the fundamental tradeoff between EE and SE for different tradeoff parameters such as maximum transmission power and circuit power.**

***Index Terms*—HetNets, Green Communications, Energy and Spectrum Efficiency, Resource Allocation, Small Cells.**

I. Introduction

ONE of the emerging technologies towards enabling Fifth Generation (5G) is heterogeneous networks (HetNets) which include Green Small Cell Networks consisting of low-power base station (BS), (e.g., microcells, picocells, and femtocells), overlaid within the macrocell geographical area, deployed by either users or network operators who share the same spectrum with the macrocells [1]. The purpose of HetNets is to allow user equipments (UEs) to access small cells even though the UEs are within the coverage of macrocell. The deployment of small cells has a great potential to improve the spatial reuse of radio resources and also to enhance the energy efficiency (EE) of the network [2] [3].

In traditional networks, the spectrum efficiency (SE) metric is considered the main performance indicator which measures how efficiently the frequency resources are utilized regardless of the efficient power consumption. On the other hand, EE is emerging as one of the key performance indicators for the next generation wireless communications systems. The motivation behind EE arises due to the current energy cost payable by operators for running their access networks as a significant factor of their operational expenditures (OPEX). Hence, green networking paradigm, which focuses on reducing energy consumption by bringing the BS closer to the UEs [4], motivates using HetNets for higher EE.

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Currently, most of EE gains in HetNets are achieved with sacrificing SE [4]. We note that the user lying within the coverage area of heterogeneous network can efficiently utilise its resources in order to either improve its achievable EE or SE. In this trend, the EE-maximisation problem in an uplink of HetNets is analytically solved for a single user case under minimum target rate and maximum transmission power constraints in [5]. A joint bandwidth and power allocation scheme to optimise EE for a set of users within the heterogeneous networks is proposed in [6]. This scheme is implemented for the multi-user system in a distributed manner in [6]. A joint BS association and power control scheme which intent to satisfy the user's target signal-to-interference-plus-noise ratio (SINR) for the uplink of a large-scale HetNets is proposed in [7]. An efficient power allocation scheme to investigate the power consumption and EE without degrading the network throughput in Long Term Evolution (LTE) HetNets is proposed in [8]. In [9], a distributed non-cooperative game is proposed to improve the system EE in the downlink transmission of HetNets. In this work, the BSs autonomously choose their optimal transmission strategies while balancing the load among themselves and satisfying the users' quality-of-service requirements.

Considering that simply maximising either EE or SE does not utilise the resources efficiently, there is an increasing attention for 5G networks to jointly optimise the two conflicting objectives, i.e., EE and SE. However, most of the current literature mainly focus on the EE-SE tradeoff in the downlink transmission scheme of traditional Orthogonal Frequency Division Modulation Access (OFDMA) based macrocell only systems (for example, [10] and [11]).

According to the best of our knowledge, there is no work on joint EE-SE tradeoff in the HetNets considering multi-user multi-carrier systems. In this work, we address an EE-SE tradeoff resource allocation technique in an uplink HetNet as a multi-objective optimization problem (MOP) to simultaneously maximise both EE and SE considering maximum input power constraint. We transform the formulated MOP into a single-objective optimization problem (SOP) using weighted sum method. Proving that the EE-SE tradeoff SOP is strictly quasi-concave with respect to the transmit power, we derive the optimal solution. By exploiting the fractional programming concept, the formulated SOP can be transformed into an equivalent subtractive form which is tractable. Numerical results demonstrate the impact of maximum transmit power, the channel-to-noise ratio, the circuit power and the tradeoff parameter on EE-SE tradeoff.

II. System Model

We consider an uplink two-tier HetNet composed of one macrocell overlaid with one pico BS with total number of users N and total number of subacrriers K. We denote the index set of all subcarriers as $k = \{1, \cdots, K\}$, the set of all users as $n = \{1, \cdots, N\}$ and the set of networks as $m = \{1, \cdots, M\}$. We further assume that the channel state information (CSI) corresponding to each subcarrier is perfectly known to the UEs transmitters. Further, we consider an orthogonal subcarrier selection scheme which assigns each subcarrier exclusively to either pico BS (PB) or macrocell (Mc) at any time such that $K_{\text{PB}} \bigcap K_{\text{Mc}} = \emptyset$ where K_{Mc} and K_{PB} indicate the set of subcarriers assigned to the macrocell and pico BS, respectively. Assume $\sigma_{k,n}^{(\text{PB})}$ and $\sigma_{k,n}^{(\text{Mc})}$ denote the subcarrier allocation indices for pico BS and macrocell, and the assignment of subcarriers to the users as well. Particularly, when subcarrier $k \in K_{\text{PB}}$, for $k = \{1, \cdots, K\}$, is allocated to user n, for $n = \{1, \cdots, N\}$, then $\sigma_{k,n}^{(\text{PB})} = 1$, and otherwise, $\sigma_{k,n}^{(\text{PB})} = 0$. Similarly, if the subcarrier $k \in K_{\text{Mc}}$ is allocated to user n, $\sigma_{k,n}^{(\text{Mc})} = 1$, and otherwise, $\sigma_{k,n}^{(\text{Mc})} = 0$.

The instantaneous rate achieved on each subcarrier k by user n for macrocell and pico BS can be hence written as:

$$\text{r}_{k,n}^{(\text{Mc})} = \sigma_{k,n}^{(\text{Mc})} B_k^{(\text{Mc})} \log_2 \left(1 + \gamma_{k,n}^{(\text{Mc})} \times p_{k,n}^{(\text{Mc})}\right), \forall k \in K_{\text{Mc}}, \quad (1a)$$

$$\text{r}_{k,n}^{(\text{PB})} = \sigma_{k,n}^{(\text{PB})} B_k^{(\text{PB})} \log_2 \left(1 + \gamma_{k,n}^{(\text{PB})} \times p_{k,n}^{(\text{PB})}\right), \forall k \in K_{\text{PB}}, \quad (1b)$$

where $p_{k,n}^{(\text{PB})}$ and $p_{k,n}^{(\text{Mc})}$ indicate the power allocated to the subcarrier k for user n in the pico BS and macrocell, respectively. Similarly, the rate of user n using subcarrier k choosing macrocell or pico BS is represented by $r_{k,n}^{(\text{Mc})}$ and $r_{k,n}^{(\text{PB})}$, respectively. Each network $m \in \{\text{Mc}, \text{PB}\}$ has its own bandwidth equally divided among its subcarriers. $B_k^{(\text{Mc})}$ and $B_k^{(\text{PB})}$ represent the subcarrier spacing in macrocell and pico BS, respectively. $\gamma_{k,n}^{(\text{Mc})}$ and $\gamma_{k,n}^{(\text{PB})}$ represent the channel-to-noise-ratio (CNR) of user n on subcarrier k in the macrocell and pico BS, respectively, and can be defined as:

$$\gamma_{k,n}^{(\text{Mc})} = \frac{|h_{k,n}^{(\text{Mc})}|^2}{\sigma_{k,n}^{(\text{Mc})} \rho_{\text{Mc}}^2 \text{PL}^{(\text{Mc})}}, \quad (2a)$$

$$\gamma_{k,n}^{(\text{PB})} = \frac{|h_{k,n}^{(\text{PB})}|^2}{\sigma_{k,n}^{(\text{PB})} \rho_{\text{PB}}^2 \text{PL}^{(\text{PB})}}, \quad (2b)$$

where $h_{k,n}^{(\text{Mc})}$ and $h_{k,n}^{(\text{PB})}$ represent the channel amplitude gain for user n on subcarrier $k \in K_{\text{Mc}}$ and $k \in K_{\text{PB}}$, respectively. The distance-based path loss in macrocell and pico BS are denoted by $\text{PL}^{(\text{Mc})}$ and $\text{PL}^{(\text{PB})}$. The noise power in subcarrier k for macrocell and pico BS are respectively given by $\rho_{\text{Mc}}^2 = B_k^{(\text{Mc})} N_0$ and $\rho_{\text{PB}}^2 = B_k^{(\text{PB})} N_0$, where N_0 is the noise spectrum density.

For simplicity, we assume that a set of available networks are known in two-tier HetNets. In practice, the transmission power available at user n, P_n, is limited to a maximum threshold, i.e., $P_n^{\max}$ which can be formulated as:

$$\text{P}_n \leq P_n^{\max}, \forall n = \{1, \cdots, N\}, \quad (3a)$$

$$\text{P}_n = \sum_{k \in K} p_{k,n}^{(m)}, \forall m. \quad (3b)$$

In an uplink transmission scenario, multiple users transmit data towards a BS so each communication link between user and BS introduces an individual circuit power P_{C}. Since the circuit power is related to the UE handsets, we assume $P_{\text{C}}^{\text{Mc}} = P_{\text{C}}^{\text{PB}} = P_{\text{C}}$. Hence, the overall power consumption model and the transmission power in an uplink of HetNets are modelled as below:

$$P = \epsilon_0 \sum_{m \in M} \sum_{k \in K} \sum_{n \in N} p_{k,n}^{(m)} + N \times P_{\text{C}}, \quad (4)$$

where N represents the total number of active users and ϵ_0 is an inverse of power amplifier efficiency.

Energy Efficiency (η_{EE}) is defined as the amount of data transferred per unit energy consumed by the system (usually measured in (b/J) and is defined as:

$$\eta_{\text{EE}} = \frac{R}{P} = \frac{\sum\limits_{m \in M} \sum\limits_{k \in K} \sum\limits_{n \in N} r_{k,n}^{(m)}}{\epsilon_0 \left(\sum\limits_{m \in M} \sum\limits_{k \in K} \sum\limits_{n \in N} p_{k,n}^{(m)} \right) + N \times P_{\text{C}}}, \quad (5)$$

where R denotes the total achievable data rate. η_{EE} is strictly quasi-concave with respect to transmission power P_{T} [11]. Hence, there exists one and only one optimal solution that maximises η_{EE} which strictly increases with $P_{\text{T}} \in \left[0, P_{\eta_{\text{EE}}}^*\right]$ while strictly decreases with $P_{\text{T}} \in \left[P_{\eta_{\text{EE}}}^*, \infty\right]$. SE ($\eta_{\text{SE}}$), on the other hand, is a measure that reflects the efficient utilization of the available spectrum in terms of throughput and it is commonly expressed in units of b/s/Hz. η_{SE} is strictly increasing with transmission power P_{T}, is concave in P_{T}, and can be defined as:

$$\eta_{\text{SE}} = \frac{R}{B} = \frac{\sum\limits_{m \in M} \sum\limits_{k \in K} \sum\limits_{n \in N} r_{k,n}^{(m)}}{\frac{K_{\text{a}}}{K} \times \sum\limits_{m \in M} \sum\limits_{k \in K_m} B_k^{(m)}}, \quad (6)$$

where B denotes the total occupied bandwidth and K_a is the number of active subcarriers. It is usually not always possible to maximise both EE and SE simultaneously. It is also worthwhile to mention that in most of the power regions, the power allocation strategies to increase these metrics are conflicting approaches. In detail, EE and SE both increase with transmission power P_{T} until it reaches the energy-efficient transmission power $P_{\text{T}} = P_{\eta_{\text{EE}}}^*$. After this point, EE decreases with an increase in SE. These fact motivate us to dynamically tune the EE and SE trade-off dependent on the available resources, in terms of bandwidth and the transmission power.

In the following sections, we propose an energy-efficient user association scheme in which the user associates to the BS with the maximum achievable EE. Unique association of users with the macrocell or pico BS is assumed [1]. Specifically, each user can only be associated with one BS. More detail can be found in Section IV.

III. Problem Formulation of EE-SE Tradeoff

Our goal is to optimise EE and SE simultaneously. In this section, we formulate EE and SE trade-off with the maximum input power constraint in an uplink transmission scheme of Two-Tier HetNets. We formulate the EE-SE trade-off as an MOP according to

$$\max_{\sigma_{k,n}^{(m)}, p_{k,n}^{(m)}} \eta_{\text{EE}} \quad \text{and} \quad \max_{\sigma_{k,n}^{(m)}, p_{k,n}^{(m)}} \eta_{\text{SE}}. \quad (7)$$

In order to maintain the balance between EE and SE in the considered MOP, we transform the optimisation problem using normalised factors θ_{EE} and θ_{SE} such that EE and SE are in the similar scale. Using the weighted sum method [12], we convert the MOP in (7) into an SOP defined by $\overline{\eta}$, yielding

$$\max_{\sigma_{k,n}^{(m)}, p_{k,n}^{(m)}} \quad \alpha\theta_{\text{EE}}\eta_{\text{EE}} + (1-\alpha)\theta_{\text{SE}}\eta_{\text{SE}} \tag{8a}$$

s.t.

$$0 \leq \alpha \leq 1 \tag{8b}$$

$$\sum_{m=1}^{M}\sum_{k=1}^{K} p_{k,n}^{(m)} \leq P_n^{\max}, \forall n. \tag{8c}$$

$$\sum_{m=1}^{M} \sigma_{k,n}^{(m)} \leq 1, \forall k, \forall n. \tag{8d}$$

$$p_{k,n}^{(m)} \geq 0, \sigma_{k,n}^{(m)} \in \{0,1\}, \ \forall n, \forall k, \forall m. \tag{8e}$$

Here, (8a) represents the EE-SE tradeoff optimisation problem and α is the tradeoff parameter such that $0 \leq \alpha \leq 1$. (8a) can further be simplified to

$$\overline{\eta} = \max_{\sigma_{k,n}^{(m)}, p_{k,n}^{(m)}} \quad \theta_{\text{EE}}\eta_{\text{EE}} + \left(\frac{1-\alpha}{\alpha}\right)\theta_{\text{SE}}\eta_{\text{SE}}. \tag{9}$$

In (9), we replace $\left(\frac{1-\alpha}{\alpha}\right)$ with β which can be from 0 to ∞. We further simplify (9) as

$$\eta = \frac{\overline{\eta}}{\theta_{\text{EE}}} = \max_{\sigma_{k,n}^{(m)}, p_{k,n}^{(m)}} \quad \eta_{\text{EE}} + \beta\left(\frac{\theta_{\text{SE}}\eta_{\text{SE}}}{\theta_{\text{EE}}}\right). \tag{10}$$

The maximisation problem (8a) – (8e) is an integer combinatorial fractional programming problem and is generally NP-hard. For better tractability, we first relax the integer variables, $\sigma_{k,n}^{(m)} \in \{0,1\}$ into continuous variables, $\tilde{\sigma}_{k,n}^{(m)} \in [0,1]$. After some mathematical manipulations, the modified optimisation problem for (8a) – (8e) can be written as

$$\eta = \max_{\tilde{\sigma}_{k,n}^{(m)}, p_{k,n}^{(m)}} \quad \eta_{\text{EE}}\left(1 + \beta\frac{\theta_{\text{SE}}P}{\theta_{\text{EE}}B}\right), \tag{11a}$$

s.t.

$$\sum_{m=1}^{M}\sum_{k=1}^{K} p_{k,n}^{(m)} \leq P_n^{\max}, \forall n. \tag{11b}$$

$$\sum_{m=1}^{M}\sum_{n=1}^{N} \tilde{\sigma}_{k,n}^{(m)} \leq 1, \forall k. \tag{11c}$$

$$\beta \geq 0, p_{k,n}^{(m)} \geq 0, \tilde{\sigma}_{k,n}^{(m)} \in [0,1], \ \forall n, \forall k, \forall m. \tag{11d}$$

Note, the unit of η is (b/J). η_{EE} is quasi-concave in P_T and R is strictly concave in P_T. Hence, η is continuously differentiable and quasi-concave with respect to the transmission power P_{T}.[1]

As mentioned in [13], any optimisation problem in fractional form can be transformed into an equivalent optimisation problem in subtractive form. Hence, the non-linear fractional optimization problem in (11a) can be transformed into the parameterized function as shown in (12). The constraints in (11c)$-$(11d) are later considered by dual decomposition method such that each subcarrier can be exclusively assigned to a single user and the non-negative optimal powers are computed. The optimal solution can be determined by finding the root to the $U(\eta)$ as shown in (12) using various root finding methods [14].

$$U(\eta) = \max_{\tilde{\sigma}_{k,n}^{(m)}, p_{k,n}^{(m)}} \left(\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{k=1}^{K} r_{k,n}^{(m)}\left(1 + \beta\frac{\theta_{\text{SE}}P}{\theta_{\text{EE}}B}\right) - \eta\left(N \times P_{\text{C}} + \epsilon_0 \sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{k=1}^{K} p_{k,n}^{(m)}\right)\right) \tag{12}$$

From (12), it implies that $U(\eta)$ strictly decreases with respect to η. It also imply that $\eta \to -\infty, U(\eta) > 0$ and $\eta \to \infty, U(\eta) < 0$. From (12), it is quite obvious that $U(\eta) > 0$, when $\eta \leq 0$. In this work, we will solve (12) for $\eta > 0$.

IV. EE and SE Trade-off Resource Allocation Scheme

The solution to EE-SE tradeoff optimisation problem is formulated as an iterative two-layer solution combining Dinkelbach type method (outer layer) and Lagrangian dual decomposition approach (inner layer). This process is repeated until both procedures converge to an optimum value. We have proposed an iterative Dinkelbach type method as an outer layer solution to find an optimal solution to (12) by determining a root to $U(\eta) = 0$. At an iteration $i-1$, the value of η is initialised and the $U(\eta)$ is solved for a given value of η, i.e., η_{i-1}, and the optimal power p_{i-1}^* is computed using dual decomposition approach (i.e., inner layer solution). The optimal power computed in iteration $i-1$ can be used to update the value of η for iteration i. This process is repeated until convergence. The pseudo code for the Dikenlbach method is shown in Algorithm-I.

We utilise the dual decomposition approach [15] to solve $U(\eta) = 0$ in each iteration of Dinkelbach type method. It is shown that the dual-composition approach has lower computational complexity and the duality gap for non-convex optimisation approaches to zero for sufficiently large number of subcarriers [14]. In order to apply dual decomposition method, we first need to find the Lagrangian function of (12). Using standard optimisation methods proposed in [14], the Lagrangian function of (12) can be written as:

$$L(p_{k,n}^{(m)}, \underline{\mu}) = \sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{k=1}^{K} r_{k,n}^{(m)}\left(1 + \beta\frac{\tau_{\text{EE}}}{\tau_{\text{SE}}}\right) - \eta\left(\varepsilon_0 \sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{k=1}^{K} p_{k,n}^{(m)} + N \times P_{\text{C}}\right) + \sum_{n=1}^{N}\mu_n\left(P_n^{\max} - \sum_{m=1}^{M}\sum_{k=1}^{K} p_{k,n}^{(m)}\right), \tag{13}$$

where $\tau_{\text{EE}} = \frac{P}{\theta_{\text{EE}}}$ and $\tau_{\text{SE}} = \frac{B}{\theta_{\text{SE}}}$. Following (13), the Lagrangian dual function corresponding to problem (12) is

$$g(\underline{\mu}) = \max_{\tilde{\sigma}_{k,n}^{(m)}, p_{k,n}^{(m)}} L(p_{k,n}^{(m)}, \underline{\mu}). \tag{14}$$

The corresponding dual problem to (12) is

$$\min_{\underline{\mu}} g(\underline{\mu}) \quad \text{s.t.} \quad \underline{\mu} \geq 0, \tag{15}$$

[1] Due to the space limitation, the proof is omitted.

where $g(\underline{\mu})$ is the dual function given as

$$g(\underline{\mu}) = \sum_{k=1}^{K} g_k(\underline{\mu}) - \eta N P_{\mathrm{C}} + \sum_{n=1}^{N} \mu_n P_n^{\max}, \tag{16}$$

and $g_k(\underline{\mu})$ is defined by

$$g_k(\underline{\mu}) = \max_{\tilde{\sigma},p} \left(\sum_{m=1}^{M}\sum_{n=1}^{N} r_{k,n}^{(m)} \left(1+\beta\frac{\tau_{\mathrm{EE}}}{\tau_{\mathrm{SE}}}\right) - \eta\varepsilon_0 \sum_{m=1}^{M}\sum_{n=1}^{N} p_{k,n}^{(m)} - \sum_{m=1}^{M}\sum_{n=1}^{N} \mu_n p_{k,n}^{(m)} \right). \tag{17}$$

The dual problem can be given by:

$$\min_{\underline{\mu}\geq 0} \max_{\tilde{\sigma}_{k,n}^{(m)}, p_{k,n}^{(m)}} L(p_{k,n}^{(m)}, \underline{\mu}).$$

The dual problem can be decomposed into two layers namely as lower layer and master layer. In the lower layer, *K* subproblems are solved in parallel to compute the power and subcarrier allocation on each subcarrier $k \in K$ for the given values of μ and η. In the master layer, the Lagrangian multipliers are updated using subgradient method. For fixed set of Lagrange multipliers and a given parameter η, the power for user n on subcarrier k can be computed by taking the derivative of (17) with respect to $p_{k,n}^{(m)}$ as follows:

$$\frac{\partial g_k(\underline{\mu})}{\partial p_{k,n}^{(m)}} = \frac{B_k^{(m)}\left(1+\beta\frac{\tau_{\mathrm{EE}}}{\tau_{\mathrm{SE}}}\right)\times \gamma_{k,n}^{(m)}}{\ln(2)\left(1+\gamma_{k,n}^{(m)}p_{k,n}^{(m)}\right)} - (\mu_n + \eta\varepsilon_0) \tag{18}$$

Algorithm-I: Iterative EE and SE Tradeoff Algorithm:-
Initialize
iter = max number of iterations = 10,
$\triangle$= maximum acceptable tolerance = 10^{-3},
Set i=1 and $\eta(i) = 0$,
While $(|U(\eta)| < \triangle) \,||\, (i < \text{iter})$ **do**

Solve (12) for a given value of $\eta(i)$ using Algorithm-II.
Update $\eta(i+1) = \frac{\left(\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{k=1}^{K} r_{k,n}^{(m)}\left(1+\beta\frac{\tau_{\mathrm{EE}}}{\tau_{\mathrm{SE}}}\right)\right)}{\left(\mathrm{N}\times P_{\mathrm{C}}+\epsilon_0\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{k=1}^{K} p_{k,n}^{(m)}\right)}$
Update $i = i+1$
end While
Output: $[\eta]$

By applying the KKT conditions, we get

$$\frac{\partial L(p_{k,n}^{(m)},\underline{\mu})}{\partial p_{k,n}^{(m)}} = \begin{cases} >0, & p_{k,n}^{(m)} = p_n^{\max} \\ =0, & 0 < p_{k,n}^{(m)} < p_n^{\max} \\ <0, & p_{k,n}^{(m)} = 0 \end{cases}$$

Hence,

$$p_{k,n}^{(m)} = \begin{cases} \left(\frac{B_k^{(m)}\left(1+\beta\frac{\tau_{\mathrm{EE}}}{\tau_{\mathrm{SE}}}\right)}{\ln 2(\mu_n+\eta\varepsilon_0)} - \frac{1}{\gamma_{k,n}^{(m)}}\right)^{+}, & \text{if } \tilde{\sigma}_{k,n}^{(m)} = 1. \\ 0, & \text{otherwise.} \end{cases} \tag{19}$$

where $(x)^{+} = \max(0,x)$. The optimal solution of (11a) can then be expressed as

$$p_{k,n}^{(m)*} = \min\left(p_{k,n}^{(m)}, P_n^{\max}\right).$$

The dual variable μ must satisfy the KKT conditions in order to be optimal. Each subcarrier k is allocated to the corresponding user n which maximises (20). Therefore, a feasible subcarrier assignment matrix is given as:

$$\tilde{\sigma}_{k,n}^{(m)} = \begin{cases} 1, & \text{if } (k,m^*,n^*) = \arg\max_{m,n} \eta_{k,n}^{(m)}, \\ 0, & \text{otherwise.} \end{cases} \tag{20}$$

where $\tilde{\sigma}_{k,n}^{(m)} = 1$ indicates that the subcarrier k is assigned to user n associated with network m and $\eta_{k,n}^{(m)} = \frac{B_k^{(m)}\log_2\left(1+\gamma_{k,n}^{(m)}p_{k,n}^{(m)}\right)}{\epsilon_0 p_{k,n}^{(m)}+P_{\mathrm{C}}}$. To minimise the dual function $g(\underline{\mu})$, the subgradient method [14] can be used to update the dual variable μ. Then, we can update the Lagrange multiplier μ according to

$$\mu_n(i+1) = \left(\mu_n(i) - \frac{s^i}{\sqrt{i}}\left(P_n^{\max} - \sum_{m=1}^{M}\sum_{k=1}^{K} p_{k,n}^{(m)}\right)\right)^{+}. \tag{21}$$

Here, i is the iteration number and s^i is the constant size of the step. The Lagrangian multipliers are updated accordingly until the convergence is achieved indicating that the dual optimal point is achieved. The subgradient update is guaranteed to converge to optimal μ as long as s^i is chosen to be sufficiently small [14]. A common practice is to choose square summable step sizes in contrast to absolute step sizes [15]. In this paper, we have used $s^i = \frac{0.1}{\sqrt{i}}$ as a step size.

V. Simulation Results

We consider a two-tier HetNets environment with a single macrocell with 500 m radius overlaid with a pico BS with a radius of 50 m. The bandwidth of each subcarrier is 30 kHz. The maximum transmission power of users considered in the simulation vary from 200 mW to 500 mW, respectively, whereas the value of circuit power of users is set fixed to P_{C} =100 mW. We assume that the users are uniformly distributed within the simulated scenario. The path-loss model for macrocell and pico BS are given as $\mathrm{PL(dB)} = 34 + 40\log_{10}(d_n)$ and $\mathrm{PL(dB)} = 37 + 30\log_{10}(d_n)$ [1], where d_n is the distance of user n from the BS in km, and therefore, $\mathrm{PL}_n^{(Mc)} = 10^{(\mathrm{PL}_n^{(\mathrm{Mc})}(\mathrm{dB})/10)}$ and $\mathrm{PL}_n^{(\mathrm{PB})} = 10^{(PL_n^{(\mathrm{PB})}(\mathrm{dB})/10)}$. The noise spectrum density is assumed to be $N_0 = -174$dBm/Hz. In this work, the power amplifier efficiency is assumed as 38%, i.e., $\epsilon_0 = \frac{1}{0.38}$. The maximum transmission power for all users are same, hence, $P_n^{\max}$ will be referred to as $P^{\max}$. The normalization factors used in our work are assumed to be $\theta_{\mathrm{EE}} = \epsilon_0 P^{\max} + P_{\mathrm{C}}$ and $\theta_{\mathrm{SE}} = \sum_{m\in M}\sum_{k\in K_m} B_k^{(m)}$. All the simulation results presented are averaged over 10,000 channel realizations.

The convergence of Algorithm I and II for a given maximum uplink transmission power of $P^{\max} = 0.2$ W is given in Fig. 1a and Fig. 1b which show that Algorithms I and II converge to optimal values within 4 and 83 iterations, respectively.

Fig. 2 analyses the maximum achievable $\overline{\eta}$ versus varying $P^{\max}$ for different values of β. Fig. 2 reveals that $\overline{\eta}$ increases with an increase in β, whereas $\overline{\eta}$ first increases with $P^{\max}$. Then after particular value of $P^{\max}$, it starts decreasing due to the considered ratio, i.e., $\left(\tau_{\mathrm{EE}} = \frac{P}{\theta_{\mathrm{EE}}}\right)$, in the optimisation problem. For smaller values of $P^{\max}$, the achievable $\overline{\eta}$ increases when $P^{\max}$ increases. Furthermore, for higher values of $P^{\max}$, the achievable $\overline{\eta}$ decreases with $P^{\max}$.

Algorithm-II: Joint User association, Subcarrier and Power Allocation

Input: $[\eta, \beta, \epsilon_0, \gamma_{k,n}^{(m)}]$

Step 1: Initialize

$i = 0,\ p_{k,n}^{(m)} = 0, \mu_n^{(i)} = 0.01,$ **for** $n = 1, \cdots, N,$
$k = 1, \cdots, K, m = 1, \cdots, M.$

Step 2:

For $k = 1 : K$

For $n = 1 : N$

Calculate $p_{k,n}^{(m)}$ according to (19).

end **For**

Obtain the user association and sub-carrier assignment according to (20) respectively.

end **For**

Step 3:

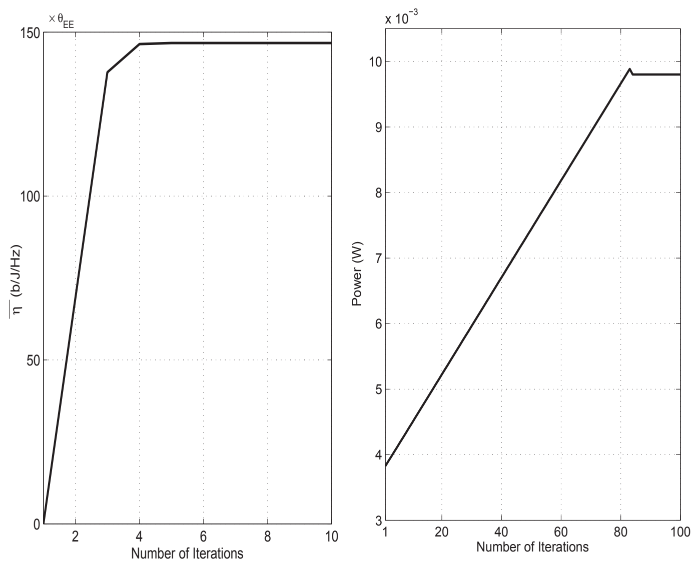
i=i+1

Update $\mu_n^{(i+1)}$ according to (21).

Step 4:

Repeat steps **2** and **3** until $\mu_n^{(i+1)}$ are converged.

Output: $[p_{k,n}^{(m)}, \tilde{\sigma}_{k,n}^{(m)}]$



(a) Algorithm I (b) Algorithm II

Fig. 1: Convergence of Proposed Algorithms I & II.

Fig. 3 shows the plots for ratio of optimal average transmit power and $P^{\max}$ versus weighted coefficient β. In Fig. 3, $P^*_{\eta_{\rm EE}}$ denotes the optimal transmit power that maximises EE (or η) at $\beta = 0$, whereas P^*_{η} denotes the proposed optimal transmit power level that maximises η at any given value of $\beta > 0$.[2] It can be seen that the optimal transmit power P^*_{η} monotonically increases with β. Fig. 3 shows that at β close to 8.8, P^*_{η} converges to the maximum transmission power $P^{\max} = 0.2$ W, whereas specifically when $P^{\max} = 0.5$ W, P^*_{η} converges to the maximum transmission power at β close to 12. This happens due to the fact that when $P^{\max}$ increases, the normalising factor $\left(\frac{\theta_{\rm SE}}{\theta_{\rm EE}}\right)$ decreases, which in turn results in reducing the impact of the tradeoff parameter β. This is an important observation indicating that to achieve maximum SE for higher values of $P^{\max}$, a higher value of β needs to be chosen when compared to a smaller value of $P^{\max}$.

[2]For clarity purpose, it is mentioned that $\eta = \frac{\overline{\eta}}{\theta_{\rm EE}}$.

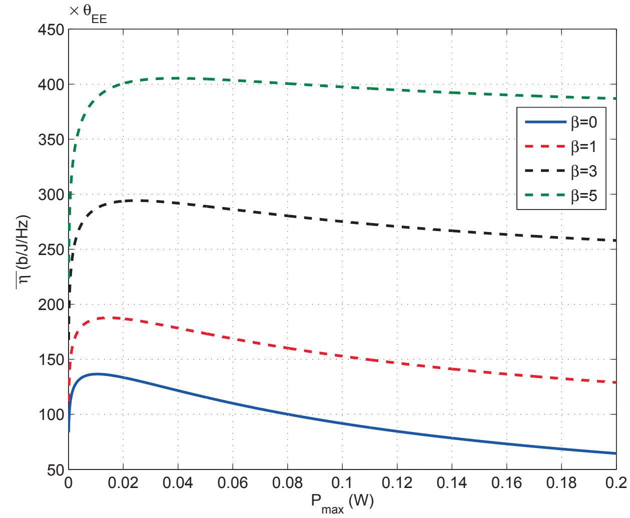


Fig. 2: $\overline{\eta}$ versus $P^{\max}$ for different values of β.

Fig. 4 shows the plots for maximum achievable EE and SE at the optimal tradeoff transmit power values (as previously shown in Fig. 3) versus β. It shows that SE is non-decreasing with respect to β, whereas EE is non-increasing with β. When β is small, i.e., $\beta = 0$, the tradeoff solution maximise EE, whereas SE is maximised when β is large, i.e., $\beta \to \infty$. Furthermore, both EE and SE become constant as the transmission power approaches to $P^{\max}$ when β is close to 8.8 (in case of $P^{\max} = 0.2$ W) and $\beta \approx 12.6$ (in case of $P^{\max} = 0.5$ W). This phenomena justifies the fact that increasing β gives more weightage to SE, and therefore, more transmit power is consumed, and in turn, higher SE can be achieved. For example, for the case of $P^{\max} = 0.2$ W and required EE level of 120 b/J/Hz, the optimal $\beta = 3$, which results in achievable SE of 15 b/s/Hz. Similalry, for the requirement to achieve average SE of 18 b/s/Hz, then the optimal $\beta = 10$, which results in achievable EE of 61 b/J/Hz. Intuitionally, we can say that EE is always maximised at $\beta = 0$ whereas SE is maximised at different values of β, which depend on the maximum transmission power. We also study the impact of transmission power budget ratio to the maximum available transmission power on the EE and SE tradeoff. For example, the minimum achievable EE is 61 b/J/Hz for $P^{\max} = 0.2$ W and drops to 34 b/J/Hz for $P^{\max} = 0.5$ W. Similarly, the maximum SE is 18.1 b/s/Hz for $P^{\max} = 0.2$ W and increases to 19.5 b/s/Hz for $P^{\max} = 0.5$ W. This indicates that more power can be saved by lowering the maximum transmission power which provides a good metric for green communications.

The tradeoff between EE and SE for various normalised circuit power consumption values, i.e., $w = \frac{P_{\rm C}}{P^{\max}}$ at $\beta = 10$, is shown in Fig. 5. We observe that EE and SE contradicts each other when the transmit power is higher than $P^*_{\eta_{\rm EE}}$. A small loss in EE can result in a significant gain in SE. On the other hand, both EE and SE increase when the transmit power is lower than $P^*_{\eta_{\rm EE}}$. For $w = 0$, the EE-SE tradeoff curve is linear and an increase in w causes reduction in the EE. From Fig. 5, it is evident that for $P_{\eta} > P^*_{\eta_{\rm EE}}$, there is always a tradeoff between EE and SE no matter how the parameter w changes. The lower the value of w, the flatter is the EE-SE tradeoff curve.

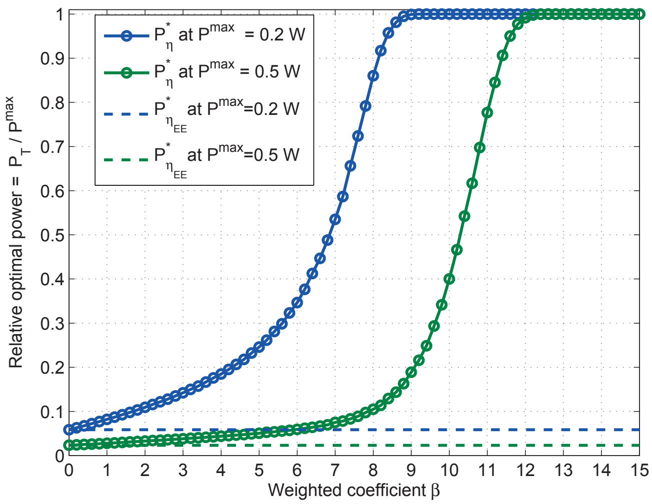


Fig. 3: Optimal transmit power versus weighted coefficient β with $P^{\max} = 0.5$ W, $P_C = 0.1$ W, and $B_k^{(m)} = 30$ kHz.

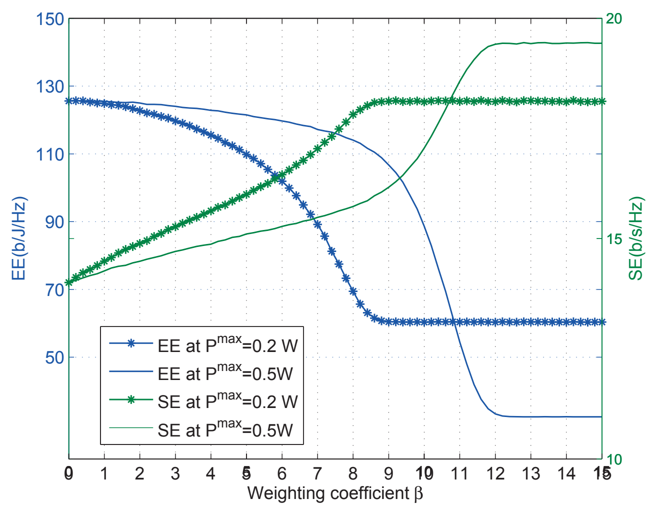


Fig. 4: EE and SE versus weighted coefficient β for various values of $P^{\max}$ with $P_C = 0.1$ W and $B_k^{(m)} = 30$ kHz.

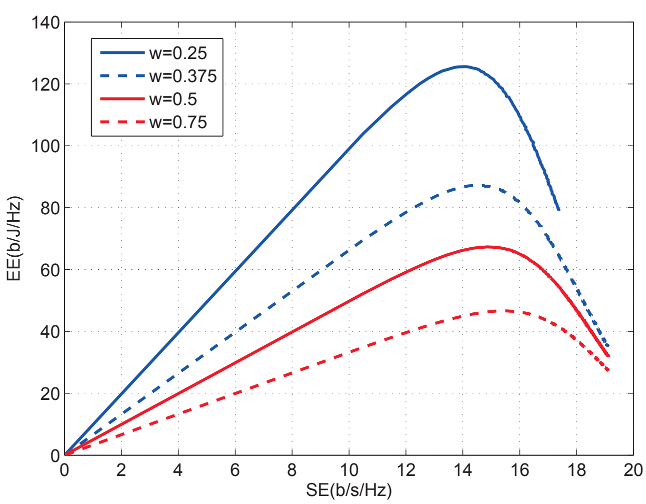


Fig. 5: EE versus SE at $\beta = 10$ for various ratios of $\frac{P_C}{P^{\max}}$.

VI. Conclusions

In this paper, the multi objective problem of simultaneously maximizing EE and SE of an uplink of a two-tier OFDMA-based HetNets with maximum input power constraint is solved. At first, the problem is converted into an SOP and then is solved using a two layer optimisation approach in which the outer layer is solved by Dikelbach method (as shown in Algorithm-I) whereas the inner layer is solved using LDD approach (as shown in Algorithm-II). Due to the quasi-concavity nature of the proposed approach, the global optimal solution is derived using LDD. From the simulation results, we can obtain two main observations. Firstly, SE is maximised at different values of tradeoff factor β depending on the maximum transmission power. Secondly, the proposed tradeoff factor β can help saving power by lowering the operational power. The tradeoff performance, $\overline{\eta}$, is an increasing function of transmission power for smaller values of $P^{\max}$, whereas $\overline{\eta}$ is a decreasing function of transmission power for higher values of $P^{\max}$.

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