Experimental identification of the dynamic characteristics of a flexible rocking structure

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This paper presents the results of free vibration and earthquake excitation tests to investigate the dynamic behaviour of freely rocking flexible structures with different geometric and vibration characteristics. The primary objective of these tests was to identify the complex interaction of elasticity and rocking and discuss its salient effects on the rocking and vibration mode frequencies, shapes and excitation mechanisms. The variability of response is discussed, including critical investigation of the repeatability of the tests. It was found that the variability in energy dissipation and energy transfer to vibrations at impact may lead to significantly different responses to almost identical excitations.

Keyword Rocking; Uplift; Impact; Vibration modes; Shake Table

1. INTRODUCTION

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There is a growing interest in utilizing rocking as a means of seismic isolation, which has inspired many experimental studies (see [Hajjar et al., 2013] for a review). Earlier investigations on freely rocking rigid blocks highlighted the notorious sensitivity of rocking motion to small changes in boundary conditions and ground motions [Aslam et al., 1980; ElGawady et al., 2010; Zhang, and Makris, 2001]. Identifying the negative stiffness and the lack of inherent energy dissipation mechanisms upon uplift as the primary reason for these sensitivities, many researchers proposed the use of rocking configurations with self-centring restoring elements and dampers to regulate the response, while maintaining the beneficial effect of rocking isolation [Kelly, and Tsztoo, 1977; Toranzo-Dianderas, 2002]. Since then, significant research effort has concentrated on proof-of-concept studies investigating the efficiency of various rocking configurations [Huckelbridge and Clough, 1978; Priestley et al., 1999; Midorikawa et al., 2006; Buchanan et al., 2008; Anastasopoulos et al., 2009; Palermo et al., 2007; Tremblay et al., 2008; Ma, 2010a; Wiebe et al., 2012].

However, recent experimental studies indicate that response sensitivity due to variable radiation damping is observed even for structures with controlled rocking [Cheng, 2007; Ma, 2010b]. Additionally, several studies have raised concern regarding the base isolation provided by rocking. Researchers demonstrated that vibrations may be excited by the ground motion during rocking action [Roke et al., 2010; Ma, 2010a; Wiebe et al., 2012] and impacts [Toranzo-Dianderas, 2002; Pollino, and Bruneau, 2010]. Further observations of excitations of vibrations at impact were noted in a limited number of experimental studies [Ma, 2010b; Evison, 1977; McManus, 1980; Acikgoz, and DeJong, 2013a; Truniger et al., 2015], where significant changes in vibration characteristics were detected during rocking action.

The findings from previous studies, as well as limited fundamental research in the area, highlight the need for a better general understanding of the dynamic characteristics of flexible rocking structures. To achieve this, discerning changes in vibration characteristics and excitation mechanisms during rocking and identifying the reasons of response variability are essential. In this paper, results of experimental tests which aim to provide further understanding in the aforementioned areas are discussed. Free vibration and earthquake excitation tests are carried out on a freely rocking experimental model. By focusing on a freely rocking specimen, the effect of rocking alone on vibration response could be quantified and energy dissipation mechanisms at impact could be examined. Within this context, questions posed by earlier researchers concerning the coupling of rocking and vibrations [Psycharis, 1983; Yim, and Chopra, 1985] can be posed. How do the vibrations of the superstructure influence the rocking motion? In return, how does the rocking motion change the vibration characteristics and excitation mechanisms of the superstructure?

In order to generalise the problem and consider the effects of different superstructure characteristics, a modular experimental specimen was used. This experimental model is introduced in Sections 2 and 3, and the free vibration test results are presented in Section 4, where a number of dynamic characteristics of flexible rocking motion are identified. Section 5 presents the rocking and acceleration response to a scaled earthquake record. In this section, earthquake response variability and the role of impact forces and ground motion accelerations in exciting the superstructure vibrations are evaluated. Section 6 presents the conclusions.

2. EXPERIMENTAL MODEL

The model used in this study was fabricated and tested at the University of Auckland (see Figure 1, left). The intention was to produce a specimen identical to that used by McManus [McManus, 1980]. The model is comprised of an approximately 2 m tall rectangular hollow section welded to a 16mm thick base plate with four small rocking feet (threaded M16 bolts welded on to 6mm thin and 20mm wide square plates) which provide contact with the shake table. Before assembling the structure, a thin aluminium plate was bolted on the table to provide protection for the table surface. The structure was placed on this plate and rocked on it during tests. In addition, a 30 mm base plate was welded under the original 16mm base plate which had distorted noticeably during the fabrication of the specimen. This resulted in a model with a heavy base (weighing approximately 70kg) and light column (weighing approximately 30kg). A side view of the as-built column is shown in Figure 1 (left). This drawing also shows elements that were welded to the base plate in order to stiffen the column and plate connection, to simulate a rigid structure-foundation interface. Additionally, key dimensions of the column highlight the locations of accelerometers A2 and A4 (shown later in Figure 1) which were strategically placed at heights H_{A2} and H_{A4} where additional masses were attached.

Rectangular steel blocks were utilized as additional masses. These masses were hung on threaded rods and were firmly tightened to the column. The bolts were regularly checked during testing to ensure that they were tight. The lowest vibration frequency of the attached masses was higher than 50Hz. To observe the dynamic response of structures with varying geometry and vibration characteristics, different mass arrangements were specified were specified as Specimen 1, 2 and 3 (referred from here onwards as S1, S2 and S3). All specimens had a total mass (m_t) of 227 kg, although other notable characteristics were different and are presented in Figure 2 (right). These characteristics include the distance from the edge of steel feet to the column centreline (B), first and second mode vibration frequencies in the full contact stage (ω_{n1} and ω_{n2}), the height of the centre of gravity (H_{cg}), the mass moment of inertia about the pivot point P (J_{P}).

Figure 1 (middle) depicts the rocking column (with mass arrangement of S1) and the instrumentation. Four piezoelectric accelerometers were mounted on steel brackets that were welded onto the column. Accelerometers A1-A4 (see Figure 1, left) measured accelerations in the direction of the rotating coordinate u while A1 also measured accelerations in the perpendicular direction v (see Figure 2, middle, for a description of the coordinates). An additional accelerometer was used to measure table accelerations. In order to measure the rocking angle (denoted by θ in Figure 2, middle), four vertical LVDTs were fixed on wooden brackets firmly attached to the base side restraints (see Figure 1, right, and Figure 2, left). Due to this arrangement, the horizontal distance between LVDTs on opposite corners of the base remained the same during planar motion and this allowed accurate calculation of the rocking angle. Differences in the estimated rocking angle for each plane of LVDTs was used to detect out of plane rocking motion. Sliding in the rocking plane was measured with the use of a 5^{th} LVDT fixed horizontally on the shake table.Steel guides were provided around the perimeter of the rocking model to ensure planar rocking motion would take place in a controlled manner. Figure 1 (right) illustrates these restraints. Base restraints were fixed on the shake table approximately 5 mm away from the rocking feet on all sides to prevent excessive sliding. The steel guides in the rocking plane had an inclined face and did not provide resistance against rocking in case of contact, which was occasionally observed for the specimen S1. Contact with the other side restraints due to out of plane motion was rare. Regardless, the clearances between the feet and all steel guides were checked before each test. The structure was recentred when the structure was within 1mm of the steel guides at the end of the tests. Finally, a laser displacement sensor fixed to the safety rig was used to measure the expected large lateral displacements (around 200mm) at the top of the column.

Tests were performed on a 3.6m × 2.4m uniaxial shake table with maximum velocity of 0.276 m/s [O'Hagan, and Ma, 2012]. During the tests, data was logged using a National Instruments Compact DAQ device. Thirteen channels of data were recorded at a sampling rate of 1000 Hz. After identifying the vibration frequencies of interest, all data was low-pass filtered with a 4^{th} order zero phase Butterworth filter (50 Hz cut-off frequency). This limited the ability of the measurements to record the transient shock waves that arise at the moment of impact but the structural vibrations due to impact were captured.

3. SPECIMEN CHARACTERISTICS

Several important dynamic characteristics of the test specimens are listed in Figure 2 (right) and were defined in Section 2. In addition, the slenderness $\alpha_{cg} = \tan^{-1}(B/H_{cg})$ describes the aspect ratio of the structure. Note that S1 is effectively the stockiest structure while S2 is the most slender. The frequency parameter p, originally proposed [Housner, 1963] for a rigid body rocking formulation, is a strong indicator of the resistance of the structure against rocking. In this paper, a modified frequency parameter p_l, which approximates the rigid body solution for the structure under consideration, was derived by assuming small rocking angles and a non-slender rigid structure. Thus, as opposed to p, p_l describes the rocking resistance of less slender structures for small rocking motion:

$$p_l = \sqrt{\frac{m_t g H_{cg}}{J_{\mathrm{P}}}} \qquad (1)$$

where g denotes the gravitational acceleration.

Figure 2 (right) also lists the full contact vibration frequencies for the first two modes of the structure, ω_{n1} and ω_{n2}. These values were determined by exciting the structure with a soft-tipped hammer. Using the Structural Identification Toolbox, modal parameters were retrieved from output-only data using five different modal estimation techniques [Beskhyroun, 2011]. The frequencies listed in Figure 2 (right) were obtained via the simple peak-picking method and agree well with other methods. Furthermore, by using the magnitude and phase of the Fourier spectrum at the identified modal frequencies, operational deflection shapes can be identified [Ewins, 2000]. These provide an approximation to the mode shape of the structure and are herein called operational acceleration profiles or approximate mode shapes. They are presented later in Figure 8 for the identified modes. Finally, to find modal damping factors ζ_1 and ζ_2, the acceleration time histories were band-pass filtered in a narrow pass-band around the identified modal frequencies. Then, by using the logarithmic decay approach, an exponential function was fitted to the peaks and damping factors were identified. These factors were observed to vary with the amplitude of accelerations. The constant values specified in Figure 2 (right) are approximate average values.

4. FREE VIBRATION TESTS

After identifying characteristics of the experimental specimens in their full contact stage, free vibration tests were performed by releasing the specimens from a specified rocking angle. The ensuing 'free' rocking motion allowed the investigation of the relationship between rocking amplitude and duration of a rocking cycle (defined as twice the time in between consecutive zero crossings in the rocking angle trace). Additionally, these tests allow the extraction of vibration mode shapes and frequencies during rockingand serve as a benchmark in investigating the role of impacts in exciting vibrations.

4.1. Time domain response from a single test

For each specimen, 24 tests with varying initial rocking angles were repeated. In Figure 3, a large amplitude free vibration test on S1 is presented. The top row shows the full rocking angle time history where the structure was displaced to a negative rocking amplitude and released with zero initial velocity. At each impact energy is lost, and the duration of each rocking half-cycle reduces. The rocking angle trace is not entirely smooth and contains small high frequency oscillations. These oscillations can be observed from the phase plot where the rocking angular velocity is plotted against the rocking amplitude (Figure 3, bottom row). The second rocking half-cycle has been highlighted with a thick black line in all the sub-figures and is considered in detail in Section 4.2.

The second row of Figure 3 shows the top horizontal displacement (δ_t) time history. The horizontal displacement trace is similar to the rocking trace, demonstrating that rocking motion dominates the lateral response. The third row of the same figure shows the sliding time history. It can be observed that the predominant sliding direction coincides with the direction of rocking motion. However, changes in the direction of base shear forces due to vibrations lead to sliding action in both directions. The magnitude of the sliding excursions is limited by the base restraints to approximately 5mm in either direction. In Figure 3, large sliding excursions, approximately 4mm, are observed during the highlighted half-cycle. However, it is noteworthy that most of this sliding occurs during impact. For instance, the sliding trace in Figure 3 indicates that for the first six rocking half-cycles, sliding during the rocking phase away from impact (defined as the period 0.1s after rocking cycle starts and 0.1 s before it ends) is less than 1mm for all half-cycles.

Figure 4 presents the lateral accelerations a_{A2} and a_{A4} recorded by accelerometers A2 and A4 during the same test. These accelerometers register the accelerations due to rocking of the structure as well as the superstructure vibrations. This explains the underlying step function that is observed in both measurements, and why the 'steps' are significantly larger for a_{A4}. The characteristic step function arises due to the relatively constant rotational accelerations induced by gravity forces during rocking and was observed previously [Evison, 1977; McManus, 1980; Acikgoz, and DeJong, 2013a]. The frequency characteristics of this rigid body mode will be discussed further in Section 4.3. Note that in the first rocking half-cycle, where the structure is released from an initial displacement, only the 'step accelerations' due to rocking are observed. Subsequently, large acceleration spikes arise at the moment of impact and are followed by smaller magnitude high frequency structural vibrations that decay freely. The phenomena at impact which leads to the large acceleration spikes is complex and may feature rocking, sliding, full contact elastic response and free flight of the structure. A comprehensive exploration of these spikes and their structural significance is outside the scope of this study, which instead focuses on the induced structural vibrations which have a more obvious structural significance. The primary source of vibration excitation at impact is due to the change in direction of the horizontal component of the rotational acceleration as the pivot point migrates from one corner of the base to the other. As the horizontal rotational accelerations and elastic translational accelerations are coupled, the vibrations of the superstructure are excited at impact. The ensuing vibration frequencies are notably higher than the first vibration mode frequency ω_{n1}, confirming that rocking causes an increase in the first vibration mode frequency [Acikgoz, and DeJong, 2012; Acikgoz, and DeJong, 2013b].

The third row of Figure 4 shows the axial acceleration a_{A1v} recorded by the triaxial accelerometer A1. These accelerations have an analogous form to lateral accelerations a_{A2} and a_{A4} with similar frequency content, but less prominent step functions. A comparison of induced rotational accelerations $B\ddot{\theta}$ and a_{A4} (not shown) confirm that the oscillations observed are due to the rigid body rotations and not by vertical vibration modes of the structure, which can be assumed axially rigid. Finally, the fourth row of Figure 4 displays the sliding acceleration $\ddot{\delta}_s$ computed by numerical differentiation. Due to the observed forward and back sliding motion sliding accelerations manifest themselves in high frequency large amplitude (with amplitudes close to 2*g*) pulses. These pulses diminish shortly after impact where sliding effects are less. Likewise, sliding decreases as the rocking motion is damped. For small rocking half-cycles (defined in this paper as half-cycles where $\theta < 0.01$rad), sliding effects and consequently sliding accelerations are minimal. Therefore, for the following discussions on the vibration response during rocking between impacts, sliding is neglected.

4.2. Frequency domain response from a single test

To explore the characteristics of oscillations in the rocking and acceleration traces, the frequency characteristics of a specific rocking half-cycle, highlighted in Figures 3 and 4, is investigated in this section. In the top left corner of Figure 5, the highlighted rocking half-cycle is plotted, where the previously discussed small oscillations on the rocking trace can be observed. Using empirical mode decomposition [Huang et al., 1998], the signal was decomposed into an oscillatory intrinsic mode function (IMF) plus a residual function describing smooth rocking action (see Figure 5, top right). The periodicity of the dominant smooth rocking action will be discussed further in Section 4.3 and the remainder of the section focuses on the IMF component of response.

The power spectral density (PSD) of the IMF was evaluated with a Hamming window and zero-padding in order to explore its frequency content. This is illustrated in Figure 5 (bottom left). A frequency peak is identified at about 20 Hz, which is distinct from the frequencies ω_{n1} and ω_{n2}. The time history of the IMF suggests that there are various other high frequency components in this signal. These components might not have been clearly captured in the Fourier analysis due to their short duration or the averaging of uplift data in the calculation of the signal θ (see Section 2). Additionally, it is of interest to observe how the vibration frequency of the oscillatory component changes in the time domain. To do this, the instantaneous frequency ω_{ins}, derived via the use of the Hilbert spectrum, is evaluated [Huang et al., 1998]. First, the IMF was band-pass filtered to a range of 12.5-27.5 Hz to obtain a mono-component signal. The Hilbert spectrum was then computed to reveal that the instantaneous frequency is approximately 20 Hz (Figure 5, bottom right). At the beginning and end of the rocking half-cycle, this frequency appears to change. These changes may be due to gradual phase transition of the structure due to base plate flexibility, excitation of the structure during impact, or end effects which arise during the application of empirical mode decomposition. Therefore, as mentioned earlier in Section 4.1, investigation of vibration frequencies during rocking will be carried out away from impact; analyses consider data from 0.1 s after the rocking half-cycle begins at $\theta = 0$ and 0.1 s before a new rocking half-cycle starts again at $\theta = 0$.

Figure 6 (top row, left) shows the acceleration time history of the highlighted rocking half-cycle. The time histories indicate that A2 and A4 accelerometers record opposing acceleration directions at almost all times and the structure appears to be vibrating predominantly at two distinct frequencies. A periodogram of the highlighted acceleration trace, trimmed to include data between the vertical dashed lines, is presented in Figure 6 (top row, right). The PSD estimate reveals the presence of three peaks, the first around zero frequency which may be assumed to describe rigid body motion, the second at 20.5 Hz describing the 1st uplifted vibration mode and the third at 39.1 Hz describing the 2nd uplifted vibration mode. These vibration frequencies during rocking (also referred to as 'uplifted vibration frequencies') are distinct from the vibration frequencies of the structure during full contact (see Figure 2, right). A significant increase in frequency is observed for the 1st vibration mode, and a modest increase is observed for the 2nd vibration mode once rocking initiates. It is also important to note that the first uplifted vibration mode frequency shows excellent agreement with the predominant frequency observed for the IMF in Figure 5. These findings suggest that the superstructure vibration frequencies increase during rocking and cause small oscillatory rocking motion. Then, this oscillatory rocking motion leads to axial accelerations on the structure (Figure 4, 3rd row)

In the 2nd row of Figure 6, the acceleration signals from A2 and A4 have been (i) low-pass filtered with a 15 Hz cut-off frequency, (ii) band-pass filtered to a 15-27.5 Hz range and

(iii) band-pass filtered to a 27.5-50 Hz range, with 4th order zero-phase Butterworth filters. These respective traces are intended to estimate accelerations due to the (i) rocking mode, (ii) 1st uplifted vibration mode and (iii) 2nd uplifted vibration mode. While these filtered modal responses neglect response nonlinearity, they are useful guides in exploring components of response. For instance, the acceleration trace due to the rocking mode is illustrated in Figure 6 (2nd row, left). A step-like acceleration profile is observed similar to previous studies [Ma, 2010b; Evison, 1977; McManus, 1980; Acikgoz, and DeJong, 2013a; Truniger et al., 2015] . The unexpected decay in this step profile is a result of the inability of the piezoelectric accelerometers to register low-frequency dynamic effects. The observed decay conforms with the indicated rate of charge leakage given in the accelerometer specifications. This measurement error might have contributed to the imprecise estimation of the rocking mode operational acceleration profile (Figure 6, 3rd row, left). A perfect triangular mode shape was expected, although the resulting mode shape (scaled to a Euclidean norm) is reasonably similar.

The filtered 1st uplifted vibration mode signal features an initial large spike due to impact. The ensuing acceleration trace in between the dashed lines shows consistent and lightly damped structural vibration. The approximate mode shape for the 1st uplifted vibration mode, illustrated under the filtered signal, indicates a radical departure from the first mode operational acceleration profile during full contact stage (see Figure 8). As mentioned earlier, the accelerometers A2 and A4 record opposing accelerations for this mode. It is noteworthy that vibrations in the superstructure induce a certain amount of rocking rotation (see Figure 5). Therefore the operational acceleration profiles arise as a combination of two components: the modal vibration of the structure with reference to the rotating frame (in the direction u) and a modal rigid body rotation [Acikgoz, and DeJong, 2013c]. Hence, the presented result does not necessarily indicate that the masses at A2 and A4 are vibrating out of phase. Instead, as demonstrated in Acikgoz and DeJong [2015], the modal vibration of the structure with reference to the rotating frame (in the direction u) is very similar during full contact and rocking; both masses vibrate in phase with respect to the rotating frame. When the effects of the rocking rotations which are induced by the superstructure are also considered, then the out of phase approximate mode shape that is presented is obtained. For the 2nd vibration mode during rocking, the filtered time history suggests that the vibrations are excited gradually during impact. Free decay starts only after 0.8s, with significant damping until 1.0s. Despite a modest increase in 2nd vibration mode frequency from full contact phase to rocking phase, the corresponding operational acceleration profile is similar during full contact and rocking (see Figure 8)

Finally, the Hilbert Spectrum of the filtered 1st and 2nd uplifted vibration mode acceleration traces is evaluated to determine the quasi-instantaneous vibration frequencies in the time domain. The bottom row of Figure 6 shows these frequencies. The figure generally suggests a mean frequency of oscillation that is independent of rocking amplitude and consistent with PSD estimations. The observed variations in the instantaneous frequency of the 2nd uplifted vibration mode are more significant and may be attributed to the nonlinear components of response (e.g. mode mixing due to Coriolis forces) and the high signal to noise ratio. These results indicate that an exact modal description would not be precise but these estimates are useful in investigating the approximately periodic nature of the motion.

4.3. General evaluation of free vibration tests

Sections 4.1 and 4.2 described the particular effects of superstructure vibrations on rocking and vice versa. The results were specific to a single test of a single specimen. This section investigates the general implications of the interaction of elasticity and rocking in terms of the displacement and acceleration response by collating the free vibration test results of all three specimens.

Figure 7 describes the relationship between rocking amplitude and duration of rocking cycles for S1 and S2. Since rocking typically governs the total displacement demands, exploring this relationship is useful in determining the frequency characteristics which govern the lateral motion of the structure after the initiation of rocking. The duration of a rocking cycle T_r is defined as twice the time in between consecutive zero crossings in the rocking angle trace while rocking amplitude is assumed as the maximum rocking angle recorded during the half-cycle, $\theta_{c\max}$. Alongside experimental data from various tests, theoretical predictions are shown in Figure 7. These predictions are made by re-deriving a well-known formula that describes the relationship between rocking amplitude and period for rigid blocks [Housner, 1963]. The new formula was derived for small rocking angles of a non-slender rigid structure:

$$T_r = \frac{4}{p_l}\cosh^{-1}\left(\frac{1}{1-\frac{\theta_{c\max}}{\tan\alpha_{cg}}}\right) \tag{2}$$

The predictions provided by Equation (2) are plotted with an error band of $\pm 5\%$ in Figure 7. Although the formula was derived assuming a rigid body, it provides excellent predictions over a wide range of rocking amplitudes for both specimens. This is possible due to the comparatively small and oscillatory nature of the rocking motion induced by the vibrations (see Figure 5). However, for small rocking half-cycles where the duration of rocking cycles approach the first mode vibration period during full contact, the influence of oscillations on the maximum rocking response becomes more significant, although the majority of the experimental data still remain in the $\pm 5\%$ error band. Figure 7 suggests that the rocking oscillations induced by the uplifted first and second vibration modes have a minimal influence on the duration of rocking cycles of the structure. As a result, the rocking mode of the structure appears to be weakly coupled to the elastic response and it is primarily influenced by geometry and scale. It should be noted that the relationship in Equation (2) might not be accurate for very flexible structures where elastic displacements are significant in comparison to the base half-width. In these cases, elastic displacements can influence the available restoring moments and rotational inertia. However, in general elastic displacements are expected to be small and Equation (2) can be reliably used to predict duration of rocking cycles for design displacements of flexible rocking structures.

In Section 4.2, operational acceleration profiles were defined during full contact and rocking phases for S1. Figure 8 presents all of the operational acceleration profiles obtained from all free vibration tests for all three specimens. The approximate frequencies associated to these profiles are also compared. The numbers in brackets under each figure (e.g. [7.5, 20.5] Hz), respectively specify the frequency of a particular vibration mode during full contact and rocking phases. While determining the operational acceleration profiles, only rocking half-cycles allowing a frequency resolution higher than 2.5 Hz were considered. These results are plotted with a grey line in the background and are remarkably similar to one another. Amongst these results, an arbitrary reference mode shape was chosen for each vibration mode as specified in Figure 8. The reference mode shapes were then compared to corresponding operational acceleration profiles (grey lines) by evaluating associated frequencies and modal assurance criterion (MAC) values [Ewins, 2000]. An excellent agreement was observed for the identified modal frequencies where more than 95% of the estimations were within 2.5Hz of the estimates specified in Figure 8. Correspondingly, MAC values higher than 0.98 were obtained when comparing individual test results to the reference mode shape. The high values of MAC are partially due to the small number degrees of freedom considered, but nonetheless these findings suggest a coherent estimation of the operational acceleration profiles. The identified modal characteristics were further validated in Acikgoz and DeJong [2015], where a procedure to estimate the approximate vibration mode characteristics during rocking is presented. For instance, in that study the first vibration mode frequency increased from 7.6 Hz to 20.7 Hz during rocking for the analytical model of S1. Similarly, the second vibration mode frequency of S1 increased from 41.4 Hz to 46.9 Hz.

Upon observing that the frequency and mode shape estimations are consistent, Figure 8 provides significant insight on how vibration modes change during rocking. The changing ‘rocking mode frequency’ was discussed earlier in Figure 7 and its approximately triangular mode shape is identical for each specimen and shown on the left of Figure 8. This mode shape estimation indicates a rigid body rotation about the base for the rocking mode with negligible elastic motion. Next, the first vibration mode shape approximations for the full contact and rocking phase are shown. The ratio of first mode vibration frequencies during rocking phase to full contact phase is 2.75 for S1 and 3.70 for S2. While all specimens demonstrate very similar 1st vibration mode shapes during full contact, the 1st uplifted vibration mode shapes are different for each specimen. However, all of these mode shapes have a distinct profile where lateral accelerations at top and mid-height levels have opposite polarity. The last two columns of the figure depict the approximate second vibration mode shape in full contact and rocking stages for S1 and S3. A modest increase in structural vibration frequency is observed for both specimens, as evidenced by the ratio of second mode vibration frequencies during rocking phase to full contact phase (1.1 for S1 and 1.05 for S3). This result is significant as previous studies assumed that second and higher vibration mode characteristics did not change once rocking is initiated [Psycharis, 1983; Yim, and Chopra, 1985]. Due to modal orthogonality, an increase in vibration frequency would not be possible without a change in mode shape. Although in this case the change in mode shape is minimal; the approximate second vibration mode shapes during full contact and rocking (see Figure 8) are similar. Nonetheless, it is important to emphasize that the changes in vibration characteristics are due to the introduction of oscillatory angular accelerations and changes in effective mass participating in vibrations. These changes may lead to significant differences in force demands along the height of the structure and their effects need to be considered for analysis and design.

Impacts are the primary energy dissipation mechanism for the stiff, lightly damped and freely rocking experimental structures and free vibration tests allow the approximate determination of energy dissipated at impacts. For large rocking half-cycles where elastic displacements are negligible, the total system energy is dominated by gravitational potential energy or kinetic rotational energy. Consequently, an approximate coefficient of restitution may be defined empirically as:

$$r_e = \sqrt{\frac{\cos\left(\alpha_{cg} - \left|\theta^{+}_{c\max}\right|\right) - \cos\left(\alpha_{cg}\right)}{\cos\left(\alpha_{cg} - \left|\theta^{-}_{c\max}\right|\right) - \cos\left(\alpha_{cg}\right)}} \tag{3}$$

where r_e is empirical coefficient of restitution for a specific impact and $\theta^{-}_{c\max}$ and $\theta^{+}_{c\max}$ indicate the maximum rocking amplitude of the half-cycles before and after the impact. It is noteworthy that this equation describes the coefficient of restitution for rigid blocks rocking freely on a rigid surface (e.g. [ElGawady et al., 2010]).

Utilizing Equation (3) and by evaluating all free vibration tests, the coefficient of restitution for each impact was determined. In Figure 9, the coefficients of restitution are plotted against rocking amplitude for S1 and S2, where left and right corner impacts were separated. Primarily, the results indicate the variability of energy dissipation, particularly for small rocking motion. Also, a clear difference in coefficients of restitution for right and left corners is observed. A similar result was noted during previous tests on the same experimental model, and is attributed to the imperfections in the geometry of the constructed specimen, particularly at the contact locations.

Figure 9 depicts the mean coefficient of restitution about each corner, respectively denoted by $\overline{r_{el}}$ and $\overline{r_{er}}$ for left and right corners, calculated for large rocking angles (described by $\theta^{+}_{c\max} > 0.01$ rad). The mean coefficient of restitution estimates are compared with a conservative theoretical estimate for the coefficient of restitution, denoted by r_h. This estimate was derived by using Housner's [Housner, 1963] assumptions of a rigid superstructure and conserving angular momentum about the impacting corner. As indicated by the theoretical estimate, the energy dissipation at impact is smaller for the slender specimen S2. However, on average, r_h overestimates energy dissipation for both specimens. Finally, it is important to note that a consistent correlation was not observed between the amplitude of rocking and coefficient of restitution for both specimens. For the large rocking angles, the standard deviation of the coefficient of restitution was approximately determined as $\sigma_r = 0.025$ for S1 and $\sigma_r = 0.0175$ for S2. For smaller rocking angles, further variability can be observed, as was noted in previous work [ElGawady et al., 2010].

5. EARTHQUAKE TESTS

In Section 4, the free vibration tests were utilized to identify important dynamic characteristics. Following the free vibration tests, a number of pulse tests were conducted. These tests revealed an orderly and repeatable pulse response where the observed variability of coefficient of restitution did not significantly affect the amplification mechanisms and maximum rocking motion. To illustrate, rocking angle traces from three repeated tests are presented in Figure 10, alongside sliding and acceleration traces from one of the tests. In all these tests, the shake table produced the input motion with fidelity. Furthermore, the sliding and acceleration traces display several salient aspects of response that were observed also during free vibration tests (e.g. opposing accelerations at A2 and A4, the excitation of vibrations at impact). Due to the predictable response, the complete set of pulse test results are not presented here. Instead, this section investigates the earthquake response of the flexible rocking structures and describes how the highlighted dynamic characteristics influence the response trends.

The El Centro 1940 (EC) record obtained from the PEER database was chosen to investigate the rocking response. This record has a peak ground acceleration of 0.32g. Although the specific record has limited significance, the EC record was chosen because it has a rich broadband frequency content. To investigate how the response changes with ground motion amplitude, the original earthquake amplitude was scaled by a factor A_{sc} which ranged from 0.5-1.1. Each earthquake test was repeated a minimum of three times to gauge the repeatability of response.

Figure 11 (bottom row) shows an example table acceleration trace for a single test. The table appears to reproduce the record with fidelity although various high frequency components are observed in the table motion. These result in a peak table acceleration of 0.46g, which is higher than the peak ground acceleration of the scaled input motion. Figure 11 also shows the response of S1 to consecutive runs of the scaled EC record with $A_{sc} = 1.1$. In the first row, the rocking traces for three repeated tests demonstrate a very similar rocking response until $t = 3$ s. However, the following rocking response is significantly different. These differences do not stem from table accelerations, as they were near identical. Instead, it is likely that the observed variability of the coefficient of restitution (see Figure 9), coupled with differences in energy transfer to vibrations at impact, led to significant variability. Furthermore, the lateral acceleration trace in Figure 11 for A2 (3rd row, right) is quite similar to the corresponding free vibration lateral acceleration traces (see Figure 4); previously highlighted free vibration acceleration trace characteristics such as step functions, spikes and the excitation of first and second uplifted mode vibrations at impact and the following decay of these vibrations are observed. Sliding trends observed for the earthquake excitation (Figure 11, 2nd row) is also similar to those observed in free vibration tests.

To investigate earthquake response trends and repeatability systematically, Figure 12 (top left) compares the undamped response spectra of the input and table motions (normalized by the earthquake amplitude scale). For spectral periods smaller than 0.1s, the recorded table motion contains random high frequency content which is not present in the input motion. This resulted in a mean table peak ground acceleration of 0.42g. On the other hand, for spectral periods larger than 0.1s (which is lower than the full contact first vibration mode frequencies), the table has replicated the input motion with precision. Then, it is clear that the variance in the rocking response is not due to differences in ground motion as high frequency noise would not significantly affect the rocking trace. However, the significant spectral amplitudes around 0.05s (which approximately corresponds to the frequency of the first vibration modes of all specimens during rocking) indicates that the table motion may excite vibrations during rocking. Figure 12 additionally shows the maximum rocking amplitude versus the amplitude scale of the input EC motion for all three specimens. In order to trace how maximum rocking response varies in average with A_{sc}, rocking spectra shows the individual test results as well as mean spectra.

The rocking spectrum in Figure 12 for S1 (top right) suggests a repeatable low-amplitude response for all earthquake amplitude scales except for $A_{sc} = 1.1$, for which the maximum motion is large for three of the tests. However, an almost identical maximum response of $\theta_{\max} / \alpha_{cg} \approx 0.03$is also observed for three other tests. In this case, there appears to be a clear response threshold at around $t = 4.2$ s (see Figure 11). If the rocking response of the structure at this time is above a certain value, or has a certain phase with respect to the ground motion, the acceleration pulses quickly amplify the motion. For S2 and S3 (Figure 12, bottom row), a large variability is observed again, but clear trends in motion exist. For the amplitude scaling range $A_{sc} = 0.6 - 0.8$, the slender specimen S2 experiences variable and large rocking motion, while the least slender structure S1 experiences small and consistent rocking. On the other hand, for S2 and S3, the increase of rocking with increasing earthquake amplitude seems to become more gradual for larger earthquake amplitudes. This suggests the following. For certain amplitude scale factors ($A_{sc} < 1.1$ for S1, $A_{sc} < 0.6$ for S2 and $A_{sc} < 0.7$ for S3), the structure can uplift as a result of resonant vibrations, even when the peak ground acceleration of the table is smaller than the slenderness of the structure. However, due to changes in the lateral response frequency and damping characteristics with the initiation of rocking (see Figures 7 and 9), often the rocking motion cannot be sustained and is dissipated. Hence, the rocking motion starts and stops repeatedly, and it remains minimal and consistent. However, when the aforementioned amplitude scale factor threshold is exceeded, larger rocking motion continues throughout the earthquake, and allows the response to be continually amplified or de-amplified, yielding a larger and more variable response [DeJong, 2012]. Then the energy dissipation and energy transfer to vibration mechanisms at impact play a critical role of in determining the rocking amplification mechanisms. It is important to note that the mean peak ground acceleration to slenderness ratio at the specified threshold is approximately 1.3 for S1, 0.97 for S2 and 0.98 for S3. Noting how the differences in energy dissipation and specimen geometry appear to affect the onset of large rocking motion, an approximate peak ground acceleration to slenderness ratio of 1.0 may be specified for the threshold on the basis of the experimental results. However, in unique circumstances, this value could theoretically be lower .

As demonstrated by Figure 11, the energy dissipation and transfer to vibrations at impacts is critical in determining earthquake response. Furthermore, it is important to examine if the ground accelerations directly excite the vibrations of the superstructure while it is rocking. Figure 13 investigates these aspects for the specimen S1 by comparing various acceleration related parameters. Specifically, a new parameter a_{A4cmid} is examined for this purpose. This parameter intends to capture the elastic first vibration mode oscillations induced at impacts. Maximum acceleration recorded during a half-cycle (denoted by a_{A4cmax}) is influenced by the complex phenomena at impact and is not indicative of the ensuing vibration amplitudes. Therefore accelerations away from the effects of impact were considered. The acceleration traces for a specific half-cycle were first filtered in the uplifted first vibration mode band pass filter range (see Section 4.2). Then, the acceleration peak nearest to the time of the maximum rocking amplitude for the half-cycle is identified and its amplitude was recorded as a_{A4cmid} at height H_{A4}. Due to light first vibration mode damping, a_{A4cmid} approximates the accelerations due to first vibration mode elastic oscillations excited at impact during free vibration tests. These are examined to observe the variability of energy transmitted to vibrations at impacts. For earthquake excitations, the parameter a_{A4cmid} represents a combined effect due to impact and potential direct earthquake excitation of vibrations. Therefore, a comparison of a_{A4cmid} parameters from free vibration and earthquake tests demonstrates if the first vibration mode during rocking was excited further by the ground motion.

Figure 13 (left) plots a_{A4cmax} and the associated maximum rocking angles for free vibration and earthquake tests on S1. a_{A4cmax} increases with increasing rocking half-cycle amplitude and is significantly higher than corresponding a_{A4cmid} values, which are plotted in Figure 13 (right). This confirms that the maximum accelerations during a rocking half-cycle are transient artefacts of impact and do not describe the induced vibrations. For this, a_{A4cmid} values need to be examined. First, the free vibration results indicate that larger rocking cycles have the potential to induce larger first vibration mode response, although this is not always the case. A range of $a_{\mathrm{A}4cmid}$ values are also observed for the small rocking range, suggesting that the induced elastic vibrations depend on the specific impact. Hence, the figure demonstrates that that the energy transfer to vibrations at impact is highly variable. As mentioned earlier, this might have contributed to the differences in rocking amplification (see Figure 11 and [Acikgoz, and DeJong, 2015]). Furthermore, $a_{\mathrm{A}4cmid}$ data from the earthquake excitations display similar results to free vibration tests. While further investigations would be required to demonstrate conclusively that the rocking action isolates the superstructure, the results suggest that there was little interaction between high frequency components of ground motion and the first vibration mode response during rocking. This claim is supported by the fact that the accelerations observed in Figure 13 (right) are smaller than the values that would be expected for the same structure if uplift was not allowed. More generally, the lack of correlation between increased rocking response and increased vibrations is noteworthy as it suggests that the energy from the ground motion is input largely into the rocking mode and the vibrations are predominantly excited at impacts. However, there remains a need to further investigate if the rocking motion isolates all vibration modes against direct excitation of ground motion and how various structural and ground motion characteristics affect this isolation. To further quantify the effects of pure rocking isolation experimentally, more flexible superstructures and a mechanism to eliminate sliding altogether is recommended.

6. CONCLUSIONS

This paper summarised an experimental investigation of the free vibration and earthquake response of a modular flexible rocking column. The tests were conducted to identify the dynamic characteristics of the structure, and to determine how these characteristics influence displacement and acceleration demands. The primary conclusions are:

- The superstructure vibrations were experimentally confirmed to be coupled with rocking. The superstructure vibrations during rocking motion caused oscillatory rocking action which resulted in axial and lateral accelerations on the system, which would not have been observed if the vibration and rocking response was uncoupled.
- The vibrations of the superstructure had a limited effect on the duration of rocking half-cycles of the structure, indicating a weakly coupled rocking mode response. An established formula which assumes the superstructure as rigid effectively predicted the duration of rocking cycles, which increases with rocking amplitude.
- The superstructure vibration mode frequencies and shapes changed upon uplift. These changes were influenced by the geometric characteristics of the structure and suggest drastically different vibration response during full contact and during rocking.
- The superstructure vibration modes were strongly excited at impact. Larger rocking cycles may inspire larger accelerations, although the magnitude of induced vibrations appears to be random and impact-specific.
- The amount of energy dissipated and energy transferred to vibrations at impact was significantly variable, although important trends in energy dissipation were observed statistically. This variability played a critical role during earthquake response, resulting in inconsistent rocking motion which is often unrepeatable.
- The rocking response to earthquake excitations varied nonlinearly with the amplitude of the excitation. Typically, when the peak ground accelerations are smaller than slenderness, the structure uplifted but the rocking motion could not be sustained and was dissipated quickly. This is due to changes in lateral response characteristics with the initiation of rocking and resulted in small and consistent rocking motion.

However, above the aforementioned threshold, rocking motion was continually amplified or de-amplified by the acceleration pulses, yielding a larger and more variable response.

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