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Local Varying-Alpha Theories

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In a recent paper we demonstrated how the simplest model for varying alpha may be interpreted as the effect of a dielectric material, generalized to be consistent with Lorentz invariance. Unlike normal dielectrics, such a medium cannot change the speed of light, and its dynamics obey a Klein-Gordon equation. This work immediately suggests an extension of the standard theory, even if we require compliance with Lorentz invariance. Instead of a wave equation, the dynamics may satisfy a local algebraic relation involving the permittivity and the properties of the electromagnetic field, in analogy with more conventional dielectric (but still preserving Lorentz invariance). We develop the formalism for such theories and investigate some phenomenological implications. The problem of the divergence of the classical self-energy can be solved, or at least softened, in this framework. Some interesting new cosmological solutions for the very early universe are found, including the possibility of a bounce, inflation and expansion with a loitering phase, all of which are induced by early variations in alpha.

1. Introduction

Since 1999 there have been a series of fascinating high-precision observational investigations into the possible space and time variation of some of the traditional constants of physics, notably the fine structure constant, $\alpha \equiv e^2/\hbar c$ and the proton-electron mass ratio $\mu \equiv m_p/m_e$, (and combinations of the two) which have been extensively reviewed.[1] Indications of possible variations of α in time[2] and space[3] and time variations in μ[4] have been reported. These observations have inspired the creation of self-consistent theories in which 'constants' like α and μ are promoted to become scalar fields that gravitate with their own dynamics that conserve energy and momentum.[5] Such theories extend the philosophy of Jordan[6] and Brans-Dicke,[7] who first created extensions of general relativity to accommodate variations in the Newtonian gravitation constant G, to variations of other non-gravitational "constants".

Variations in the traditional (low-energy) constants of physics offer a new observational window on fundamental physics at very high-energies. Theories with a non-unique vacuum state, possessing "extra" dimensions, or permitting new light scalar fields, can all lead to space-time variations of the fundamental low-energy "constants" of nature.[1,8] Simultaneous variations of different gauge couplings may

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be significantly constrained by nay form of grand unification at sub-Planckian energies.[9]

Small variations of non-gravitational constants have negligible effects on the expansion dynamics of the universe but have potentially observable influences on astronomical spectra from atomic and molecular transitions.[10] Self-consistent scalar-tensor theories are needed to evaluate their full cosmological consequences. Theoretical studies have focused on a varying fine-structure constant α, which is simplest to develop because of its gauge symmetry,[5] and a varying proton-electron mass ratio $\mu = m_p/m_e$,[11,12] Scaling arguments have also been used to relate changes in α to changes in μ using the internal structure of the standard model, including supersymmetry.[13] Typically (in the absence of unusual cancellations involving the rates of change of α, and the supersymmetry-breaking and grand unification energy scales), they predict that changes in μ at low energies should be about an order of magnitude greater than those in α. However, high-redshift cosmological bounds on μ variation are expected to be weaker than those from laboratory tests of the equivalence principle.[11] Systematic investigations of the spectra of cold H_2 towards quasar sources have now produced a constraint on μ-variation over cosmological time scales yielding $\Delta\mu/\mu < 1 \times 10^{-5}$ at redshifts $z = 2 - 3.5$, corresponding to look-back times of 10-12 Gyr.[14,15]   Radio studies have surpassed optical ones in limiting changes in μ at lower redshifts, with 2σ limits of $\Delta\mu/\mu < $ few $\times 10^{-7}$ from comparisons between NH_3 and rotational lines,[16,17] and $\Delta\mu/\mu < 1.5 \pm 1.5 \times 10^{-7}$ from multiple methanol lines in a lensing galaxy at $z = 0.89$.[18] Also, at low redshifts, the conjugate satellite OH method is sensitive to changes in alpha[19] at the level of $\Delta\alpha/\alpha < (-3.1 \pm 1.2) \times 10^{-6}$ at $z = 0.247$.

Besides producing effects on cosmological scales, the couplings between light scalar fields and other fields can generate dependencies of coupling strengths on the local matter density,[5,20] or on local gravitational fields.[21,22] Such couplings violate the Einstein equivalence principle.[7,23] The gravitational potential at distance R from an object of mass M is commonly expressed in dimensionless units of $\phi = GM/Rc^2$. A number of studies have been performed using ultrastable lasers and atomic clocks exploiting the eccentricity of the Earth's orbit[24–27] causing sinusoidal changes of $\Delta\phi = 3 \times 10^{-10}$. Recently, a spectroscopic study of Fe V and Ni V ions in the local environment of the photosphere of a white dwarf was employed to assess the dependence of α in a strong gravitational field ($\phi = 4.9 \times 10^{-5}$).[28] Most recently, spectra of molecular hydrogen (H_2) were employed to search for any dependence of μ on gravity. The Lyman transitions of H_2, observed with the COS on the Hubble Space Telescope by Xu et al[29] towards white dwarf stars GD133 (WD 1116+026) and G29−38 (WD 2326+049) are compared to accurate laboratory spectra taking into account the high-temperature conditions ($T \sim 13\,000\ K$) of their photospheres to probe possible dependence of μ on a gravitational potential that is $\sim 10^4$ times stronger than its value at the Earth's surface. The spectrum of white dwarf star GD133 yields a $\Delta\mu/\mu$ constraint of $(-2.7 \pm 4.7_{\text{stat}} \pm 0.2_{\text{sys}}) \times 10^{-5}$ for a local environment with gravitational potential $\phi \sim 10^4\ \phi_{\text{Earth}}$, while that of G29−38

yields $\Delta\mu/\mu = (-5.8 \pm 3.8_{\mathrm{stat}} \pm 0.3_{\mathrm{sys}}) \times 10^{-5}$ for $\phi \sim 2 \times 10^4 \; \phi_{\mathrm{Earth}}$,.[30]

These observational advances lead us to refine our theoretical models and in what follows we review a new way of viewing the theory of varying α introduced by Bekenstein, Sandvik, Barrow and Magueijo (BSBM) in[5,8] and extended in References[31] and.[32] Following recent work[34] in Section 2 we show how BSBM may be seen as the actions of a dielectric medium permittivity encoded in scalar field, ψ. Specifically, we will see that ψ acts like a relativistic generalisation of a common dielectric or insulator. Unlike in standard media, ϵ and μ^{-1} obey a relativistic Klein-Gordon equation sourced by the EM lagrangian, $E^2 - B^2$. The dielectric analogy suggests a number of extensions of the original BSBM theory, but if we seek to preserve Lorentz invariance the options are limited. Regarding the effects of ψ upon electromagnetism, the theory is fully fixed by Lorentz, parity and gauge invariances, but we may still change the dynamics of ψ. If we wish to extend further the analogy with a standard dielectric, then it would make more sense to make ψ a local function of the electromagnetic field. If this function is to be Lorentz invariant, it can only depend on the scalar $E^2 - B^2$ and the pseudo-scalar $\mathbf{E} \cdot \mathbf{B}$, but the latter induces parity violations (and also permits modifications to Maxwell's equations, which are beyond the scope of this discussion). The formalism for such *local* varying alpha theories is developed in Section 3.

The rest of this paper is devoted to exploring some of the phenomenology of these theories. As with torsion theories, it is extremely difficult to constrain the model by local particle physics experiments. In Section 4 we show how the problem of the divergence of the classical self-energy of a point particle might be removed or ameliorated within these theories. In Section 5 we start exploring the cosmological solutions, particularly as models for the very early universe. We close the paper with a discussion of the wider implications.

Throughout this paper we shall use Planck units, and employ metrics with signature is $-, +, +, +$.

2. Varying-alpha as a relativistic dielectric effect

In this Section we describe how the scalar field, ψ, which self-consistently drives variations in the electron charge $e \equiv e_0 \exp(2\psi)$, and hence in α, in BSBM theory, may be interpreted as a dielectric medium.[33] The dielectric is linear (in the sense that $\mathbf{D}$ and $\mathbf{H}$ are proportional to $\mathbf{E}$ and $\mathbf{B}$) and the proportionality constants ϵ and μ^{-1} are isotropic and frequency independent. Unlike in standard media, ϵ and μ^{-1} obey a relativistic Klein-Gordon equation sourced by the EM lagrangian $E^2 - B^2$. Since $\epsilon = 1/\mu$, the medium is non-dispersive, and so it induces no frequency shift or photon production. The argument was presented in,[34] and in fact applies to any varying-alpha theory which preserves relativistic Lorentz invariance and the gauge principle.

In setting up electromagnetism under a varying alpha there is an "ambiguity" in the definition of electric and magnetic fields similar to that found for insulators,

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where one can use $\mathbf{E}$ or $\mathbf{D}$ for the electric field, and $\mathbf{B}$ or $\mathbf{H}$ for the magnetic field. In reality both concepts play a role, with $\mathbf{E}$ and $\mathbf{B}$ convenient for writing the homogeneous Maxwell equations, and $\mathbf{D}$ and $\mathbf{H}$ better suited for writing the inhomogeneous equations, even when there are no sources. Varying-alpha theories may be phrased either in terms of A_μ (as in[8]), or a_μ (as in[35]), with the two quantities related by:

$$a_\mu = e^\psi A_\mu \,, \tag{1}$$

where

$$\psi = \ln \tilde{\epsilon} = \ln \frac{e}{e_0} = \frac{1}{2} \ln \alpha. \tag{2}$$

The last expression links ψ to the fine structure "constant", α. Here $\tilde{\epsilon}$ corresponds to the "ϵ" used in,[8] which we stress is *not* the relative permittivity of the "medium"(ϵ, in our notation here), as we shall see. Gauge transformations can be performed as

$$a_\mu \to a_\mu + \partial_\mu \Lambda \tag{3}$$

or as

$$A_\mu \to A_\mu + \frac{\partial_\mu \Lambda}{\tilde{\epsilon}}. \tag{4}$$

This fork in the development propagates into the definition of gauge-invariant field tensors, with[8] led to make the natural definition:

$$F_{\mu\nu} = e^{-\psi} \left[\partial_\mu(e^\psi A_\nu) - \partial_\nu(e^\psi A_\mu) \right], \tag{5}$$

and[35] to choose

$$f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu. \tag{6}$$

The two are related by

$$F_{\mu\nu} = e^{-\psi} f_{\mu\nu}. \tag{7}$$

The electromagnetic action, from which the non-homogeneous Maxwell's equations are derived, can be written in the two forms:

$$S_{EM} = -\frac{1}{4} \int d^4x \, F^2 = -\frac{1}{4} \int d^4x \, e^{-2\psi} f^2. \tag{8}$$

In order to study which quantities play the role of $\mathbf{E}$ and $\mathbf{B}$ (and so give the equivalent of the Faraday tensor) in,[34] we examined the non-homogeneous Maxwell equations. These are best written in terms of $f_{\mu\nu}$, in the form of the integrability condition:

$$\epsilon^{\alpha\beta\mu\nu} \partial_\beta f_{\mu\nu} = 0. \tag{9}$$

This is obviously a necessary condition for (6), but note that the same argument cannot be made directly for $F_{\mu\nu}$ (derivatives of ψ would appear in the corresponding condition in terms of $F_{\mu\nu}$; cf. (5) and (6)). Thus, in order to parallel the usual

theory of electrodynamics in media we should associate **E** and **B** (appearing in the inhomogeneous Maxwell equations) with $f_{\mu\nu}$, with entries in the usual places. With this identification we obtain the standard inhomogeneous Maxwell equations:

$$\nabla \cdot \mathbf{B} = 0, \tag{10}$$

$$\nabla \wedge \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \,. \tag{11}$$

This was already noted in[8] (however, the wrong identification was made in ref.,[37] cf. their Eq.(25)).

In order to find the equivalent of **D** and **H**, in[34] we examined instead the inhomogeneous Maxwell equations. In the absence of currents these can be written in the two forms:

$$\partial_\mu(e^{-2\psi} f^{\mu\nu}) = \partial_\mu(e^{-\psi} F^{\mu\nu}) = 0, \tag{12}$$

and we see that neither of them leads to the equivalent standard expression for dielectric media (in both cases extra terms in the derivatives of ψ appear). Therefore, we should define the alternative "Maxwell" tensor:

$$\mathcal{F}_{\mu\nu} = e^{-\psi} F_{\mu\nu} = e^{-2\psi} f_{\mu\nu}, \tag{13}$$

in terms of which we have

$$\partial_\mu \mathcal{F}^{\mu\nu} = 0 \,. \tag{14}$$

We should then define **D** and **H** from the appropriate entries in $\mathcal{F}_{\mu\nu}$, so as to get:

$$\nabla \cdot \mathbf{D} = 0, \tag{15}$$

$$\nabla \wedge \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = 0 \,. \tag{16}$$

With these identifications BSBM becomes equivalent to electromagnetism in dielectric media with only small adaptions. We have:

$$\mathbf{D} = \epsilon \mathbf{E} = e^{-2\psi} \mathbf{E}, \tag{17}$$

$$\mathbf{H} = \mu^{-1} \mathbf{B} = e^{-2\psi} \mathbf{B}, \tag{18}$$

and so

$$\epsilon = \frac{1}{\mu} = e^{-2\psi}. \tag{19}$$

D and **H** are proportional to **E** and **B**, and the proportionality constants ϵ and μ^{-1} are isotropic and frequency-independent.

3. The permittivity as an algebraic function of the EM field

The identifications found above suggest an obvious extension of BSBM. In BSBM, ψ satisfies a driven Klein-Gordon equation, but in standard electromagnetism the

permittivity would be a local function of the fields $\mathbf{E}$ and $\mathbf{B}$. The only relativistically invariant such functions take the form:

$$\psi = \psi(E^2 - B^2, \mathbf{E} \cdot \mathbf{B}). \tag{20}$$

In this paper we will focus on the first argument of this function, and explicitly develop the formalism and applications for this case. Setting up the formalism for the dependence of ψ on the pseudo-scalar $\mathbf{E} \cdot \mathbf{B}$ is a straightforward extension (and we will briefly present it at the end of this Section). However the phenomenology is entirely different, involving parity violation effects, and a whole new set of phenomena. We will defer the study of these theories to a future publication.

Imposing $\psi = \psi(E^2 - B^2)$ can be easily implemented in a Lagrangian formulation by setting:

$$S_{EM} = \int d^4x \, (-\frac{1}{4} e^{-2\psi} f^2 - V(\psi)) \tag{21}$$

with no kinetic term for ψ. Then the Euler-Lagrange equation implies the algebraic relation:

$$V'^{2\psi} = \frac{f^2}{2} = -(E^2 - B^2) \tag{22}$$

from which we can deduce $\psi = \psi(E^2 - B^2)$. This theory implements non-linear EM in dielectrics without breaking Lorentz invariance. It differs from BSBM in many ways, most importantly in that it does not induce a fifth force or violations of the weak equivalence principle. In fact the theory has something in common with the basic torsion theories (of the form of the Einstein-Cartan-Sciama-Kibble type), in that its field is non-propagating. It is algebraically related to electromagnetic field in this case, rather than to the spin density, as is the case with torsion.

The Lagrangian of BSBM contains only a kinetic term; instead, the theories we would like to explore, when recast in the Lagrangian formulation, derive from Lagrangians which contain only a potential term (and no kinetic term). Obviously we could add a potential to BSBM, but that would not change its qualitative nature, since it would still contain propagating degrees of freedom. We will not consider that possibility here, because we want to explore the qualitatively different possibility that the permittivity is not a propagating degree of freedom, but like in normal dielectrics, it is an algebraic function of the EM field itself.

Note that a *possible* generalization of the formalism to include the second argument in (20) would follow from

$$S_{EM} = \int d^4x \, \{ -\frac{1}{4} e^{-2\psi} f^2 - \frac{\beta}{4} e^{-2\phi} f_{\mu\nu} \tilde{f}^{\mu\nu} - V(\psi, \phi) \} \tag{23}$$

resulting in conditions:

$$\frac{\partial V}{\partial \psi} = -e^{2\psi}(E^2 - B^2) \tag{24}$$

$$\frac{\partial V}{\partial \phi} = -\beta e^{2\phi}(\mathbf{E} \cdot \mathbf{B}). \tag{25}$$

In these theories, the Maxwell equations would receive corrections due to the parity violating term. Other ways to accommodate chirality are possible, and will be further explored elsewhere.

4. Removal of the classical self-energy divergence

A suitable choice of ϵ (or $V(\psi)$) can lead to the removal or softening of classical divergences, specifically in the particle electromagnetic self-energy. This was once perceived as a major problem (citations) and may still interact in non-trivial ways with the more general issue of quantum divergences. The potential V may be seen as a classical effective way to describe quantum effects in the self-interaction. Conversely if we were to postulate it at tree-level, it would affect loop corrections in non-trivial ways. Regardless of this we now exhibit one theory where the Coulomb self-energy is finite.

Let us assume that:

$$V(\psi) = \frac{1}{\ell^4}\left(\frac{1}{2}e^{2\psi} - \psi\right).$$ (26)

Then, solving Eq. 22 leads to two solutions. The branch with the correct limit when the field is small is:

$$\epsilon = \frac{1 - \sqrt{1 - 4\ell^4(E^2 - B^2)}}{2\ell^4(E^2 - B^2)},$$ (27)

and we see that this is equivalent to:

$$\mathbf{E} = \frac{\mathbf{D}}{\epsilon} = \frac{\mathbf{D}}{1 + \ell^4 D^2}$$ (28)

when $\mathbf{B} = 0$. Since $\mathbf{D}$ satisfies the Poisson equation (cf. Eq. (15)) we have that for a point charge:

$$\mathbf{D} = \frac{e_0}{r^2}\mathbf{e_r}\,.$$ (29)

However now the electric field reaches a maximum and then drops to zero:

$$\mathbf{E} = \frac{e_0}{r^2 + \frac{\ell^4 e_0^2}{r^2}}\,.$$ (30)

Clearly there is no singularity in the energy density around a point charge. As $r \to 0$ we find that

$$E \sim \frac{r^2}{l^4 e_0}\,.$$ (31)

As explained in,[34] the electrostatic energy density in a dielectric (generalized or otherwise) is given by:

$$\tilde{\rho}_{EM} = \frac{1}{2}\mathbf{E} \cdot \mathbf{D}$$ (32)

and so, as $r \to 0$, the energy density tends to a constant:

$$\tilde{\rho}_{EM} \to \frac{1}{2\ell^4} \ . \tag{33}$$

We note that there is still a divergence in the energy associated with the dielectric itself, but this is logarithmic. Indeed the energy associated with the dielectric can be written as:

$$\rho_\psi = V(\psi) = \frac{1}{\ell^4} \left(\frac{1}{2\epsilon} + \frac{1}{2} \log \epsilon \right) \tag{34}$$

and since $\epsilon \to 1/r^4$, the second term diverges only logarithmically.

5. Cosmological equations

A closed set of cosmological equations can be found by appealing to energy conservation. We first review how this is the case in BSBM, before adapting the argument to the theory proposed in this paper. In BSBM, the Lagrangian for ψ is:

$$\mathcal{L}_\psi = -\frac{\omega_B}{2} \partial_\mu \psi \partial^\mu \psi \tag{35}$$

leading to a forced Klein-Gordon equation for ψ:

$$\nabla^2 \psi = \frac{2}{\omega_B} \widetilde{\mathcal{L}}_{EM} \ . \tag{36}$$

where $\widetilde{\mathcal{L}}_{EM} = e^{-2\psi}(E^2 - B^2)/2$ (in what follows we denote by tilded variables those which have factors which are a function of ψ absorbed into their definitions). Under the assumption of homogeneity and isotropy, this equation becomes the ODE:

$$\ddot{\psi} + 3\frac{\dot{a}}{a}\dot{\psi} = -\frac{2}{\omega_B} \widetilde{\mathcal{L}}_{EM} \ . \tag{37}$$

and this equation can be interpreted as an energy balance equation, with the driving terms representing energy exchange between ψ and other forms of matter. Homogeneity and isotropy imply that ψ behaves like a perfect fluid, and computing the stress-energy tensor reveals:

$$p_\psi = \rho_\psi = \omega_B \frac{\dot{\psi}^2}{2}. \tag{38}$$

Equation (37) is then equivalent to:

$$\dot{\rho}_\psi + 3\frac{\dot{a}}{a}(p_\psi + \rho_\psi) = -2\dot{\psi}\widetilde{\mathcal{L}}_{EM} \ . \tag{39}$$

Each component i contributes a term proportional to $\widetilde{\mathcal{L}}_{EM_i}$ to the right-hand side of (39). This should be balanced by a counter-term with opposite sign in the right hand side of the the conservation equation for i:

$$\dot{\widetilde{\rho}}_i + 3\frac{\dot{a}}{a}(\widetilde{\rho}_i + \widetilde{p}_i) = 2\dot{\psi}\widetilde{\mathcal{L}}_{EM_i} \ . \tag{40}$$

For all i components (including the dark matter) we need equations of state relating their energy density with their EM Lagrangian content. One possibility is to define parameters:

$$\zeta_i = \frac{\widetilde{\mathcal{L}}_i^{EM}}{\widetilde{\rho}_i}\,.\tag{41}$$

For radiation $\zeta_r = 0$, but $\zeta_m \neq 0$ for baryonic as well as for some types of dark matter. We stress that the statement that ζ_i is a constant is part of the model (and such a model is *not* the model employed for matter in[35]).

Given these considerations we concluded in[34] that in BSBM theory, a full closed set of cosmological equations for a matter and radiation universe is:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}\left(\widetilde{\rho}_m + \widetilde{\rho}_r + \rho_\psi\right),\tag{42}$$

$$\dot{\widetilde{\rho}}_r + 4\frac{\dot{a}}{a}\widetilde{\rho}_r = 0,\tag{43}$$

$$\dot{\widetilde{\rho}}_m + 3\frac{\dot{a}}{a}\widetilde{\rho}_m = 2\dot{\psi}\zeta_m\widetilde{\rho}_m,\tag{44}$$

$$\dot{\rho}_\psi + 6\frac{\dot{a}}{a}\rho_\psi = -2\dot{\psi}\zeta_m\widetilde{\rho}_m.\tag{45}$$

Here, $\dot{\psi}$ on the right-hand side of the last two equations is to be written as

$$\dot{\psi} = \sqrt{\frac{2\rho_\psi}{\omega_B}}\tag{46}$$

(where we used (38)) to form a closed system.

A similar construction can be set up for the theory proposed in this paper, with (37) replaced by (22), or, using the notation in this Section:

$$V' = -2\widetilde{\mathcal{L}}_{EM}.\tag{47}$$

In view of the considerations leading to (41), we can rewrite this as:

$$V' = -2\zeta_m\widetilde{\rho}_m\,.\tag{48}$$

Assuming homogeneity and isotropy, the ψ fluid is now made up of pure potential energy, instead of kinetic energy (as was the case for BSBM), so, instead of (38), we have

$$-p_\psi = \rho_\psi = V(\psi).\tag{49}$$

The conservation equation for the ψ-fluid therefore reads:

$$\dot{\rho}_\psi + 3\frac{\dot{a}}{a}(\rho_\psi + p_\psi) = \dot{\rho}_\psi = \dot{\psi}V' = -2\dot{\psi}\widetilde{\mathcal{L}}_{EM},\tag{50}$$

where in the last identity we have used (47), and in the first two we have used (49). We therefore obtain an equation identical to (39) but with a different equation of state for the ψ fluid. It can be further expressed as:

$$\dot{\rho}_\psi = -2\dot{\psi}\zeta_m\widetilde{\rho}_m\tag{51}$$

As a result, we know that the right-hand side (RHS) of this equation should appear with a reversed sign as a source term to the the matter conservation equation. The cosmological equations for a radiation-matter universe are therefore:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}\left(\widetilde{\rho}_m + \widetilde{\rho}_r + \rho_\psi\right) \tag{52}$$

$$\dot{\widetilde{\rho}}_r + 4\frac{\dot{a}}{a}\widetilde{\rho}_r = 0 \tag{53}$$

$$\dot{\widetilde{\rho}}_m + 3\frac{\dot{a}}{a}\widetilde{\rho}_m = 2\dot{\psi}\zeta_m\widetilde{\rho}_m \tag{54}$$

$$\dot{\rho}_\psi = -2\dot{\psi}\zeta_m\widetilde{\rho}_m. \tag{55}$$

This system looks formally identical to the equations obtained for BSBM (once one accounts for the different equation of state for ψ), but in fact the system is entirely different and, in fact in this form, it does not constitute a closed set of ODEs. Given that we have (49) instead of (38) we cannot write $\dot{\psi}$ on the RHS of the last two equations as a function of ρ_ψ. Instead, we should use (48) to write ψ as a function of $\widetilde{\rho}_m$. We can then write $\dot{\psi}$ as:

$$\dot{\psi} = \frac{d\psi}{d\widetilde{\rho}_m}\dot{\rho}_m. \tag{56}$$

Given that $\psi = \psi(\widetilde{\rho}_m)$, we can also eliminate ρ_ψ from the equations, since ρ_ψ in the Friedman equation can be written as $\rho_\psi = V(\psi) = V(\psi(\widetilde{\rho}_m))$. These two steps allow for a rearrangement of the equations into a closed system:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}\left(\widetilde{\rho}_m + \widetilde{\rho}_r + V(\widetilde{\rho}_m)\right), \tag{57}$$

$$\dot{\widetilde{\rho}}_r + 4\frac{\dot{a}}{a}\widetilde{\rho}_r = 0, \tag{58}$$

$$\dot{\widetilde{\rho}}_m\left(1 - 2\zeta_m\widetilde{\rho}_m\frac{d\psi}{d\widetilde{\rho}_m}\right) + 3\frac{\dot{a}}{a}\widetilde{\rho}_m = 0. \tag{59}$$

As our worked examples will now show, in practice these steps are always folded into finding a solution to the system.

6. Some examples

We now consider some cosmological application of this theory. We construct models for the early universe, assuming for simplicity a single form of matter ($i = 1$) with constant $\zeta \neq 0$ and a general equation of state $p/\rho = w$, with w constant. It is curious that the simplest choices of potential lead to interesting solutions, namely bouncing, loitering and inflationary dynamics.

6.1. *Quadratic potential: bouncing models*

Let us assume a quadratic potential, leaving the sign undefined for the time being:

$$V(\psi) = \pm\frac{1}{2}M^4\psi^2. \tag{60}$$

The Euler-Lagrange equation leads to:

$$V'^4 \psi = -2\zeta\widetilde{\rho}, \tag{61}$$

and so we learn that this a model in which the early universe is filled with a dielectric rendering the electric charge exponentially dependent on the density:

$$e = e_0 \exp\psi = e_0 \exp\frac{\mp 2\zeta\widetilde{\rho}}{M^4}. \tag{62}$$

Furthermore, we have:

$$\psi = \mp 2\frac{\zeta}{M^4}\widetilde{\rho} \tag{63}$$

$$V(\widetilde{\rho}) = \pm 2\frac{\zeta^2}{M^4}\widetilde{\rho}^2. \tag{64}$$

Therefore, following the procedure outlined at the end of last Section, we find that the Friedmann equation (see Eq. (57)) resembles that obtained in brane-world cosmology when the minus sign is picked:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}\left(\widetilde{\rho} \pm 2\frac{\zeta^2}{M^4}\widetilde{\rho}^2\right). \tag{65}$$

However, this theory is very different because the Friedmann equation is supplemented by a modified conservation equation (cf. Eq. (59)):

$$\dot{\widetilde{\rho}}\left(1 \pm 4\frac{\zeta^2}{M^4}\widetilde{\rho}\right) + 3(1+w)\frac{\dot{a}}{a}\widetilde{\rho} = 0, \tag{66}$$

and therefore the dynamics is potentially very different.

It is easy to prove that the theory still leads to bouncing behaviour. The conservation equation integrates to:

$$\ln\widetilde{\rho} \pm \frac{4\zeta^2}{M^4}\widetilde{\rho} = -3\ln a \tag{67}$$

or:

$$\widetilde{\rho}\exp\left(\pm 4\frac{\zeta^2}{M^4}\widetilde{\rho}\right) \propto \frac{1}{a^{3(1+w)}} \tag{68}$$

If we choose the minus sign for the potential this model leads to a bounce when the density reaches the maximum:

$$\widetilde{\rho}_{max} = \frac{M^4}{2\zeta^2}. \tag{69}$$

We would also expect the ρ^2 term to dominate the effects of simple shear anisotropies on approach to the expansion minimum and create a quasi-isotropic bounce whenever $w > 0$.

6.2. *Exponential potential: Inflation and loitering*

Another interesting case is that with an exponential potential:

$$V(\psi) = V_0 e^{-\lambda\psi}. \tag{70}$$

Then:

$$V' = -\lambda V_0 e^{-\lambda\psi} = -2\zeta\widetilde{\rho}, \tag{71}$$

so that

$$\psi = -\frac{1}{\lambda}\ln\widetilde{\rho} + C \tag{72}$$

$$V = \frac{2\zeta}{\lambda}\widetilde{\rho}. \tag{73}$$

We therefore now have a model in which the early universe behaves like a dielectric for which the electric charge is a power-law of the energy density

$$e = e_0 \exp\psi \propto \widetilde{\rho}^{-\frac{1}{\lambda}}. \tag{74}$$

Inserting the solution into Eq. (59) gives us the modified conservation equation

$$\dot{\widetilde{\rho}}(1+B) + 3(1+w)\frac{\dot{a}}{a}\widetilde{\rho} = 0 \tag{75}$$

with $B = 2\zeta/\lambda$. Thus, regarding matter evolution, there is an effective shift in the equation of state:

$$w \to w_{eff} = \frac{w - B}{1 + B}. \tag{76}$$

We see that as $B \to \infty$ we get $w_{eff} \to -1$, i.e. inflation. If $w = 0$, for $B > 1/2$ we have $w_{eff} < -1/3$. This shift in the equation of state transfers into the expected change in the expansion rate, because $V \propto \widetilde{\rho}$, and so the Friedmann equation reads:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}\left(1 + \frac{2\zeta}{\lambda}\right)\widetilde{\rho}. \tag{77}$$

The only effect in this equation is an effective shift in the gravitational constant

$$G \to G_{eff} = G(1 + B). \tag{78}$$

We can therefore generate acceleration (certainly an early-time, inflationary one) with this model.

The extreme case $B = -1$, corresponding to

$$\lambda = -2\zeta, \tag{79}$$

is interesting in that gravity seemingly switches off (since $G_{eff} = 0$). A static universe is then possible. Close to this value we may induce loitering.

7. Conclusions

In this paper we have examined an alternative to BSBM varying-alpha theory inspired by conventional dielectrics. The properties of BSBM theory may be understood from the standard electrodynamics of dielectrics with suitable definitions for the fields $\mathbf{E}$ and $\mathbf{B}$ and the displacements $\mathbf{D}$ and $\mathbf{H}$ (associated with the Faraday and Maxwell tensors, respectively). We must, however be prepared to regard the "vacuum" as a dielectric medium with unusual properties: Lorentz invariant and with $\epsilon = 1/\mu$ satisfying a driven Klein-Gordon equation. Conventional dielectrics, however, are not dynamical but, rather, have properties which are local functions of the EM field. We explored this possibility in this paper without letting go of Lorentz invariance. The result is a theory with *formal* analogies to torsion in the Einstein-Cartan-Sciama-Kibble theory of gravity. If the connection and metric are to be seen as independent degrees of freedom, then torsion is inevitable, but it appears as a non-dynamical degree of freedom, algebraically related to the spin-density. Here we find an algebraic relation between $\epsilon = 1/\mu$ and the EM Lagrangian. In both cases it is extremely difficult to constrain the ensuing theory.

Nonetheless the theory leads to interesting results. At a very fundamental level it could change the outlook on the problem of the divergence of the self-energy of particles, as we have shown in Section 4. It also leads to interesting early universe cosmologies, with simple choices for the potential inducing bounces, loitering and acceleration. The dynamics is fundamentally different from that found in BSBM, because the conservation equation must be modified so as to obtain a closed system of governing equations. The late-time implications are likely to be less dramatic. With a quadratic potential the effects are suppressed by factors $O(\rho_m/M^4)$ and so are naturally small (one can easily adapt the calculations in Section 6.1 to a late-time matter universe). With an exponential potential we obtain tracking solutions, just as with quintessence, but these induce a shift in the equation of state. We should therefore either ensure that for dark matter the constant B is small, or else induce a feature in the potential (i.e. a local minimum) taking the field away from rolling behaviour at late-times.

Further extensions of this theory are possible. We may regard BSBM as a "purely kinetic" varying-alpha theory (in which the Lagrangian for ψ only has a kinetic term). By contrast, the theory proposed here is endowed with a Lagrangian with only a potential term, leading to an algebraic relation between α and EM field. We could of course have both a kinetic and a potential term, but such a theory would ontologically be more like a BSBM theory, where ψ is a truly dynamical field. Quantitative differences would arise, and one suspects that scaling solutions similar to those found in the quintessence scenario would exist. A more dramatic possible generalisation of these theories would arise from giving up Lorentz symmetry. Then a much larger class of theories can be written down, with a richer phenomenology. We are currently exploring this possibility.

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Acknowledgments

References

1. J-P. Uzan, *Living Rev. Relativity* **14**, 2 (2011), J. Magueijo, *Rept. Prog. Phys.* **66,** 2025 (2003)
2. J.K. Webb, V. V. Flambaum, C. W. Churchill, M. J. Drinkwater and J. D. Barrow, *Phys. Rev. Lett.*, **82**, 884 (1999), J. K. Webb, M. T. Murphy, V. V. Flambaum , V. A. Dzuba, J. D. Barrow, C. W. Churchill, J. X. Prochaska and A. M. Wolfe, *Phys. Rev. Lett.* **87**, 0913012001, H. Chand et al, *Astron. Astrophys.* **417**, 853 (2004), R. Srianand et al, *Phys. Rev. Lett.* **92**, 121302 (2004), M. T. Murphy et al, *Lect. Notes Phys.* **648**, 131 (2004), M. Murphy, J. K. Webb and V. V. Flambaum, *Phys. Rev. Lett.* **99**, 239001 (2007), S. A. Levshakov, P. Molaro, S. Lopez, et al., *Astron. Astrophys.* **466**, 1077 (2007), M. Murphy, J. K. Webb and V. V. Flambaum, *Mon. Not. Roy. Astron. Soc.* **384**, 1053 (2008)
3. J. K. Webb et al, *Phys. Rev. Lett.* **107**, 191101 (2011), J. A. King et al, *Mon. Not. Roy. Astron. Soc.* **422**, 3370 (2012), P. Molaro et al, *Astron. Astrophys.* in press, (2013) arXiv:1305.1884
4. E. Reinhold, R. Buning, U. Hollenstein, A. Ivanchik, P. Jean, and W. Ubachs, *Phys. Rev. Lett.* **96** 151101 (2006).
5. H. B. Sandvik, J. D. Barrow, and J. Magueijo, *Phys. Rev. Lett.* **88**, 031302 (2002); J. D. Barrow, H. B. Sandvik, and J. Magueijo, *Phys. Rev. D* **65**, 063504 (2002).
6. P. Jordan, *Schwerkraft und Weltall*, (Vieweg, Braunschweig, 1955)
7. C. Brans and R. H. Dicke, *Phys. Rev.* **124**, 925 (1961), R. H. Dicke, *Phys. Rev.* **125**, 2163 (1962)
8. J. D. Bekenstein, *Phys. Rev. D* **25**, 1527 (1982)
9. M.J. Drinkwater, J.K.Webb, J.D. Barrow and V.V. Flambaum, *Mon. Not. Roy. Astron. Soc.* 295, 457 (1998), X. Calmet and M. Keller, arXiv:1410.2765
10. M.P. Savedoff, *Nature* **178**, 689 (1956), J.N. Bahcall and E.E. Salpeter, *Astroph. J.* 642, 1677 (1965)
11. J. D. Barrow and J. Magueijo, *Phys. Rev. D* **72**, 043521 (2005), J. D. Barrow, *Phys. Rev. D* **71,** 083520 (2005), S. Lee, *Mod. Phys. Lett. A* **22**, 2003 (2007), C. G. Scóccola, M. E. Mosquera, S. J. Landau and H. Vucetich, *Ap. J.* **681**,737 (2008)
12. C. G. Scoccola, M. E. Mosquera, S. J. Landau and H. Vucetich, *Ap. J.* **681**, 737 (2008).
13. X. Calmet and H. Fritsch, Eur. J. Phys. C **24**, 639 (2002); X. Calmet and H. Fritzsch, *Europhys. Lett.* **76**, 1064 (2006).
14. A. L. Malec, R. Buning, M. T. Murphy, N. Milutinovic, S. L. Ellison, J. X. Prochaska, L. Kaper, J. Tumlinson, R. F. Carswell, and W. Ubachs, MNRAS **403**, 1541 (2010); F. van Weerdenburg, M. T. Murphy, A. L. Malec, L. Kaper, and W. Ubachs, *Phys. Rev. Lett.* **106**, 180802 (2011).
15. J. Bagdonaite, W. Ubachs, M. T. Murphy, and J. B. Whitmore, *Ap. J.* **782**, 10 (2014).
16. N. Kanekar, G. I. Langston, J. T. Stocke, C. L. Carilli and K. L. Menten, *Ap. J. Lett.,* **728**, L12 (2011).
17. N. Kanekar, W. Ubachs, K. M. Menten, J. Bagdonaite, A. Brunthaler, C. Henkel, S. Muller, H. L. Bethlem, and M. Dapra, *Mon. Not. Roy. Astron. Soc.,* in press,

arXiv:1412.7757

18. J. Bagdonaite, M.Daprà, P. Jansen, H. L. Bethlem, W. Ubachs, S. Muller, C. Henkel, and K. M. Menten, *Phys. Rev. Lett.* **111**, 231101 (2013).
19. N. Kanekar, J. N. Chengalur and T. Ghosh, *Ap. J. Lett.* **716**, L23 (2010).
20. J. Khoury and A. Weltman, *Phys. Rev. Lett.* **93**, 171104 (2004).
21. J. Magueijo, J. D. Barrow and H. Sandvik, *Phys. Lett. B* **549**, 284 (2002).
22. V. V. Flambaum, and E. V. Shuryak, Nuclei and Mesoscopic Physics - WNMP 2007, AIPC Proceedings **995**, 1 (2008).
23. C. M. Will, *Living Rev. Relativity,* **17**, 4 (2014).
24. S. J. Ferrell, A. Cingöz, A. Lapierre, A.-T. Nguyen, N. Leefer, D. Budker, V. V. Flambaum, S. K. Lamoreaux, and J. R. Torgerson, *Phys. Rev. A* **76**, 062104 (2007).
25. T. M. Fortier, N. Ashby, J. C. Bergquist, M. J. Delaney, S. A. Diddams, T. P. Heavner, L. Hollberg, W. M. Itano, S. R. Jefferts, K. Kim, F. Levi, L. Lorini, W. H. Oksay, T. E. Parker, J. Shirley, and J. E. Stalnaker, *Phys. Rev. Lett.* **98**, 070801 (2007).
26. S. Blatt, A. D. Ludlow, G. K. Campbell, J. W. Thomsen, T. Zelevinsky, M. M. Boyd, J. Ye, X. Baillard, M. Fouché, R. Le Targat, A. Brusch, P. Lemonde, M. Takamoto, F.-L. Hong, H. Katori, and V. V. Flambaum, *Phys. Rev. Lett.* **100**, 140801 (2008).
27. J. D. Barrow and D. J. Shaw, *Phys. Rev. D* **78**, 067304 (2008).
28. J. C. Berengut, V. V. Flambaum, A. Ong, J. K. Webb, J. D. Barrow, M. A. Barstow, S. P. Preval, and J. B. Holberg, *Phys. Rev. Lett.* **111**, 010801 (2013).
29. S. Xu, M. Jura, D. Koester, B. Klein, and B. Zuckerman, *Astroph. J. Lett.* **766**, L18 (2013), S. Xu, M. Jura, D. Koester, B. Klein, and B. Zuckerman, *Phys. Rev. A* **783**, 79 (2014).
30. J. Bagdonaite, E.J. Salumbides, S.P. Preval, M.A. Barstow, J.D. Barrow, M.T. Murphy and W. Ubachs, P*hys. Rev. Lett.* **113**, 123002
31. J. D. Barrow and S. Z. W. Lip, *Phys. Rev. D* **85**, 023514 (2012), J. D. Barrow and B. Li, *Phys. Rev. D* **78**, 083536 (2008)
32. J.D. Barrow and A. A. H. Graham, *Phys. Rev. D* **88**, 103513 (2013)
33. L. D. Landau, L. P. Pitaevskii and E. M. Lifshitz, *Electrodynamics of Continuous Media,* 2nd ed. (Pergamon Press, Oxford, 1979).
34. J. D. Barrow and J. Magueijo, *Phys. Rev. D* **90**, *123506* (2014).
35. H. B. Sandvik, J. D. Barrow and J. Magueijo, *Phys. Rev. Lett.* **88**, 031302 (2002)
36. J. D. Barrow, H. B. Sandvik and J. Magueijo, *Phys. Rev. D* **65**, 063504 (2002), J. D. Barrow, J. Magueijo and H. B. Sandvik, *Phys. Rev. D* **66**, 043515 (2002)
37. L. Kraiselburd and H. Vucetich, *Phys. Lett. B* **718**, 21 (2012)