Strong stability of 2D viscoelastic Poiseuille-type flows

Y. Giga, J. Sauer and K. Schade

Abstract. We investigate L_p stability issues of small viscoelastic Poiseuille-type flows in two dimensions stemming from a model considered in Fang-Hua Lin, Chun Liu, and Ping Zhang (2005). We show local existence of perturbed flows of locally-in-time existing Poiseuille-type flows and global existence of the peturbed flows whenever the initial perturbation is small enough. In this case the perturbed flow decays exponentially. In all cases, the perturbations immediately regularize.

Mathematics Subject Classification (2010). 35A01, 35A02, 35Q35, 76A05, 76A10, 76D03, 35K59 .

1. Introduction

Viscoelasticity describes a property of materials exhibiting both viscous and elastic characteristics under deformation. Such a material may exhibit elastic behavior like memory effects as well as fluid properties. In two space dimensions in a layer, a Poiseuille-type flow has a horizontal flow-profile that is completely determined by the vertical component.

We are interested in stability of the viscoelastic Poiseuille flow. There is an earlier work by Dario Götz, Chun Liu and the first auther [11], where they proved L_2-type stability results for small Poiseuille flow by an energy argument. This paper considers a similar problem in an L_p-setting.

Given a fixed viscosity $\nu > 0$ and a small original viscoelastic Poiseuille-type flow we consider L_p-dynamics of perturbations of this original flow. By a change of variables introduced in [16], we can transform the equation into a parabolic quasilinear evolution equation. Its linear part is a diagonal operator matrix with the Stokes operator and the Dirichlet-Laplacian on the

The first author is partly supported by Grants-in-Aid for Scientific Research No. 21224001 (Kiban S), No. 23244015 (Kiban A) and No. 25610025 (Houga) of Japan Society for the Promotion of Science.
The second and the third author are supported by the DFG International Research Training Group 1529 on Mathematical Fluid Dynamics at TU Darmstadt.

diagonal with lower order perturbations. We can then show unique local-in-time existence of the perturbed flow for small initial perturbations. We furthermore establish unique existence on $\mathbb{R}_+$ of the perturbed flow given small initial perturbations as well. This is possible due to the invertibility and maximal regularity of the Dirichlet-Laplacian and the Stokes operator on the layer. This global-in-time perturbation decays exponentially to the initial flow. All obtained solutions immediately regularize due to Angenent's trick.

The considered viscoelastic model is due to considerations in [16]. The authors use weak theory to obtain local-in-time smooth solutions in bounded domains in $\mathbb{R}^2$ and $\mathbb{R}^3$ with smooth boundary, the whole space $\mathbb{R}^2$ and $\mathbb{R}^3$ or a periodic box. They show global-in-time existence of solutions with small initial data in the case of $\mathbb{R}^2$ and the periodic box.

Strong theory of quasilinear evolution equation is based on a local-in-time existence result of Clément and Li [5] in 1992. Prüss et al. [20, 15, 23] have subsequently developed a broad quasilinear theory including methods to analyze asymptotic behavior using the theory of dynamical systems and exploiting spectral properties of a linearization around equilibria with the so called *generalized principle of linearized stability.*

The main ingredient is maximal L_p-regularity of the linear part of the quasilinear problem. This property is known for a large class of linear equations and the associated linear operators on various domains, including elliptic operators ([6]) and the Stokes operator on layer domains. These techniques have been applied to several Navier-Stokes related models, such as nematic liquid crystals to obtain strong local dynamics and asymptotic behavior close to equilibria [13]. In our case of a layer domain, both the Dirichlet-Laplacian and the Stokes operator do not only admit maximal L_p-regularity, but are also invertible. This fact makes it possible to control also the long-time asymptotic behavior for small perturbations of lower order.

Stability of a flow parallel to the boundary like the Poiseuille flow or the Couette flow is a very important topic in fluid mechanics. In fact it is known that the Couette flow for the incompressible Navier-Stokes equations in a layer domain is stable under a small perturbation, irrespective of how large its velocity is [12]; see [24] for a pioneering work.

In the remaining part of the introduction we first introduce the model for viscoelastic fluids. Then we establish a model for a perturbation of a Poiseuille-type flow in 2D. In the third part we apply a transformation to the stream-function equation which reveals the hidden diffusive characteristic of the equation. In Section 2 we give a short overview over the tools of quasilinear evolution equations. In Section 3, we (equivalently) reformulate our model for our Poiseuille-type flow in the language of quasilinear evolution equations. Section 4 is concerned with issues of maximal L_p-L_q regularity of the linearization of the model. In Section 5 we conclude with our main results.

1.1. Viscoelastic Fluids

We consider a general system describing the flow of viscoelastic fluids, which has been considered in [16].

$$\begin{cases} \partial_t F + u \cdot \nabla F &= F \nabla u, \\ \operatorname{div} u &= 0, \\ \partial_t u - \nu \Delta u + u \cdot \nabla u + \nabla \pi &= \operatorname{div} F^T F, \end{cases} \tag{1}$$

where F denotes the deformation tensor, u the velocity, π the pressure and ν the viscosity.

In two space dimensions, one can obtain an $\mathbb{R}^2$-valued stream function ζ_0 such that

$$F_0 = \nabla^\perp \zeta_0 = \begin{pmatrix} -\partial_2 \zeta_{0_1} & \partial_1 \zeta_{0_1} \\ -\partial_2 \zeta_{0_2} & \partial_1 \zeta_{0_2} \end{pmatrix}.$$

Moreover, if, for a divergence-free function u, this quantity is propagated in time subject to the transport equation

$$\begin{aligned} \partial_t \zeta + u \cdot \nabla \zeta &= 0, \\ \zeta(0) &= \zeta_0, \end{aligned} \tag{2}$$

then one can easily show that for $F = \nabla^\perp \zeta$, the first equation of (1), i.e. $F_t + u \cdot \nabla F = F \nabla u$, is fulfilled. This system is much more friendly to analyse and hence we will in the following consider the function ζ instead of F. With this new variable, one calculates

$$\operatorname{div} F^T F = \frac{1}{2} \nabla |\nabla \zeta|^2 - \Delta \zeta_1 \nabla \zeta_1 - \Delta \zeta_2 \nabla \zeta_2.$$

Note here, that the first term is a gradient that can be absorbed into the pressure function in the momentum balance equation in (1). So let us introduce a new pressure function $\tilde{\pi} = \pi - \frac{1}{2}|\nabla \zeta|^2$, which is again denoted by π in the following. With this, we end up with an equivalent system that is valid in two space-dimensions for $(u, F, \pi) = (u, \nabla^\perp \zeta, \pi)$, when we apply Einstein's sum convention adding terms with the same indices $k = 1, 2$:

$$\begin{cases} \partial_t \zeta + u \cdot \nabla \zeta = 0, \\ \operatorname{div} u = 0, \\ \partial_t u - \nu \Delta u + u \cdot \nabla u + \nabla \pi = -\Delta \zeta_k \nabla \zeta_k. \end{cases} \tag{3}$$

In this paper, we want to consider a flow through a two-dimensional layer $\Omega = \mathbb{R} \times (0, 1)$. In particular, we study the stability of a one-dimensional flow of Poiseuille-type $(\bar{u}, \nabla^\perp \eta, \bar{\pi})$ subject to Dirichlet boundary conditions.

We now want to construct a suitable Poiseuille-type flow solution $\bar{u}$ to (1) or equivalently (3), i.e. a solution with horizontal flow-profile that is completely determined by the vertical component. Hence, we assume that $\bar{u}$ takes the form

$$\bar{u}(t, x) = \begin{pmatrix} \psi(t, x_2) \\ 0 \end{pmatrix},$$

with homogeneous Dirichlet boundary conditions. Then the divergence condition in (1) is trivially fulfilled.

In order to adequately determine the corresponding deformation tensor $\bar{F}$ or equivalently the corresponding stream function η, we introduce the flow map $x_i(t, X)$, $0 \le t < T$, corresponding to Lagrangian coordinates X. These flow maps are given by the system of ordinary differential equations

$$\begin{aligned} \frac{d}{dt}x_1(t,X) &= \bar{u}^1(t, x_1(t,X), x_2(t,X)) = \psi(t, x_2(t,X)), \qquad & x_1(0) &= X_1, \\ \frac{d}{dt}x_2(t,X) &= \bar{u}^2(t, x_1(t,X), x_2(t,X)) = 0, & x_2(0) &= X_2, \end{aligned}$$

which can easily be solved by

$$\begin{aligned} x_1(t,X) &= X_1 + \int_0^t \psi(s, x_2(s,X))\,\mathrm{d}s = X_1 + \int_0^t \psi(s, X_2)\,\mathrm{d}s, \\ x_2(t,X) &= X_2, \end{aligned}$$

as long as ψ admits sufficient regularity. Let us abbreviate

$$\phi(t, x_2) = \int_0^t \psi(s, x_2)\,\mathrm{d}s. \tag{4}$$

Then, we can calculate the deformation tensor and the resulting elastic force

$$\bar{F} = \begin{pmatrix} 1 & 0 \\ \partial_2\phi & 1 \end{pmatrix}, \quad \bar{F}^T\bar{F} = \begin{pmatrix} 1+(\partial_2\phi)^2 & \partial_2\phi \\ \partial_2\phi & 1 \end{pmatrix} \quad \text{and} \quad \operatorname{div}\bar{F}^T\bar{F} = \begin{pmatrix} \partial_2^2\phi \\ 0 \end{pmatrix}.$$

Note here, that with $x_2(t,X) = X_2$ it is also $\frac{\partial}{\partial X_2} = \frac{\partial}{\partial x_2} = \partial_2$. Let us also remark at this point, that $\operatorname{div}\bar{F} = 0$.

The stream function η corresponding to $\bar{F}$ may be chosen as

$$\eta(t,x) = \begin{pmatrix} -x_2 \\ x_1 - \phi(t, x_2) \end{pmatrix} \tag{5}$$

solving the system

$$\begin{cases} \partial_t\eta + \bar{u}\cdot\nabla\eta &= 0, & \text{in} \quad (0,T)\times\Omega, \\ \eta(0,x) &= (-x_2, x_1)^T, & \text{for} \quad x\in\Omega. \end{cases}$$

1.2. Perturbation of the flow through the layer

It is our aim to examine the stability of system (3) (or equivalently (1)) with respect to the Poiseuille-type flow $(\bar{u}, \eta, \bar{\pi})$ constructed in the previous section. For this, we introduce the perturbation

$$(v, \alpha, p) = (u, \zeta, \pi) - (\bar{u}, \eta, \bar{\pi})$$

of the solution (u, ζ, π) (with corresponding deformation tensor G) of (3) around the Poiseuille-type flow $(\bar{u}, \eta, \bar{\pi})$ with deformation tensor F.

We are interested in solutions (u, ζ, π) that satisfy homogeneous Dirichlet boundary conditions $u|_{\partial\Omega} = 0$ and have initial values ζ_0 and u_0. Let u_0 satisfy the compatibility condition

$$\operatorname{div} u_0 = 0.$$

Let us moreover assume that the initial stream function satisfies

$$\zeta_0|_{\partial\Omega} = \begin{pmatrix} -x_2 \\ x_1 \end{pmatrix} \quad \text{and} \quad \partial_1\zeta_{0_1}\partial_2\zeta_{0_2} - \partial_1\zeta_{0_2}\partial_2\zeta_{0_1} = 1.$$

The first assumption together with the homogeneous Dirichlet boundary conditions for u guarantees $\zeta|_{\partial\Omega} = (-x_2, x_1)^T$ for all times. The second assumption is a reformulation of the incompressibility condition $\det F_0 = 1$, which ensures $\det F = 1$ for all times and hence $\partial_1\zeta_1\partial_2\zeta_2 - \partial_1\zeta_2\partial_2\zeta_1 = 1$.
Then (v, α, p) solves

$$\begin{cases} \partial_t\alpha + v\cdot\nabla\alpha + \bar{u}\cdot\nabla\alpha = -v\cdot\nabla\eta & \text{in } (0,T)\times\Omega, \\ \operatorname{div} v = 0 & \text{in } (0,T)\times\Omega, \\ \partial_t v - \nu\Delta v + v\cdot\nabla v + v\cdot\nabla\bar{u} + \bar{u}\cdot\nabla v + \nabla p & \\ \qquad = -\Delta\alpha_k\nabla\alpha_k - \Delta\eta_k\nabla\alpha_k - \Delta\alpha_k\nabla\eta_k & \text{in } (0,T)\times\Omega, \\ v|_{\partial\Omega} = 0, & \text{in } (0,T), \\ \alpha|_{\partial\Omega} = 0, & \text{in } (0,T), \\ \alpha(0,x) = \zeta_0(x) - (-x_2, x_1)^T & \text{for } x\in\Omega, \\ v(0) = u_0 - (\psi_0, 0)^T & \text{in } \Omega. \end{cases}$$

1.3. Change of variables and dissipation

Using the definition of η in (5), we obtain

$$\nabla\eta = \begin{pmatrix} 0 & 1 \\ -1 & -\partial_2\phi(t,x_2) \end{pmatrix}, \quad \Delta\eta = \begin{pmatrix} 0 \\ -\partial_2^2\phi(t,x_2) \end{pmatrix}, \quad \partial_1\partial_2^2\phi(t,x_2) = 0.$$

Hence

$$\begin{aligned} -\Delta\alpha_k\nabla\eta^k - \Delta\eta^k\nabla\alpha_k &= -\Delta\alpha_1\begin{pmatrix} 0 \\ -1 \end{pmatrix} - \Delta\alpha_2\begin{pmatrix} 1 \\ -\partial_2\phi \end{pmatrix} - \Delta(-x_2)\nabla\alpha_1 \\ &\quad - \Delta(x_1-\phi)\nabla\alpha_2 \\ &= \Delta\begin{pmatrix} -\alpha_2 \\ \alpha_1 \end{pmatrix} + \nabla\phi\Delta\alpha_2 + \partial_2^2\phi\nabla\alpha_2. \end{aligned}$$

Inserting this into the momentum equation, we see

$$\begin{aligned} \partial_t v - \nu\Delta\Big(v + \frac{1}{\nu}\begin{pmatrix} -\alpha_2 \\ \alpha_1 \end{pmatrix}\Big) + v\cdot\nabla v + v\cdot\nabla\bar{u} + \bar{u}\cdot\nabla v + \nabla p & \\ = -\Delta\alpha_k\nabla\alpha_k + \nabla\phi\Delta\alpha_2 + \partial_2^2\phi\nabla\alpha_2. & \end{aligned} \tag{6}$$

To obtain additional dissipative structure to use for α we introduce the new variable

$$w = v + \frac{1}{\nu}\begin{pmatrix} -\alpha_2 \\ \alpha_1 \end{pmatrix} \quad \text{such that} \quad \alpha = \nu \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} (w - v).$$

1.4. Equation for the new variable

The next step is to determine the right system that defines w. It is easy to see with

$$-v \cdot \nabla \eta = -(\nabla \eta)^T v = -\begin{pmatrix} 0 & -1 \\ 1 & -\partial_2 \phi \end{pmatrix} v = \begin{pmatrix} v^2 \\ -v^1 + \partial_2 \phi v^2 \end{pmatrix},$$

that $\frac{1}{\nu}\alpha_1$ satisfies

$$\partial_t\Big(\frac{1}{\nu}\alpha_1\Big) + v \cdot \nabla\Big(\frac{1}{\nu}\alpha_1\Big) + \bar{u} \cdot \nabla\Big(\frac{1}{\nu}\alpha_1\Big) = \frac{1}{\nu} v_2.$$

and for $-\frac{1}{\nu}\alpha_2$ we have

$$\partial_t\Big(-\frac{1}{\nu}\alpha_2\Big) + v \cdot \nabla\Big(-\frac{1}{\nu}\alpha_2\Big) + \bar{u} \cdot \nabla\Big(-\frac{1}{\nu}\alpha_2\Big) = \frac{1}{\nu}(v_1 - \partial_2 \phi v_2)$$

Adding these equations to the system for v, we receive

$$\begin{aligned} &\partial_t w - \nu \Delta w + v \cdot \nabla w + v \cdot \nabla \bar{u} + \bar{u} \cdot \nabla w + \nabla p \\ &\qquad = -\Delta \alpha_k \nabla \alpha_k + \frac{1}{\nu} v - \frac{1}{\nu}\begin{pmatrix} \partial_2 \phi v_2 \\ 0 \end{pmatrix} + \partial_2^2 \phi \nabla \alpha_2 + \nabla \phi \Delta \alpha_2. \end{aligned} \tag{7}$$

Now our system takes the form

$$\begin{cases} \partial_t v - \nu \Delta v + v \cdot \nabla v + v \cdot \nabla \bar{u} + \bar{u} \cdot \nabla v + \nabla p \\ \qquad = -\Delta \alpha_k \nabla \alpha_k - \Delta \eta^k \nabla \alpha_k - \Delta \alpha_k \nabla \eta^k \\ \partial_t w - \nu \Delta w + v \cdot \nabla w + v \cdot \nabla \bar{u} + \bar{u} \cdot \nabla w + \nabla p \\ \qquad = -\Delta \alpha_k \nabla \alpha_k + \frac{1}{\nu}\begin{pmatrix} v_1 - \partial_2 \phi v_2 \\ v_2 \end{pmatrix} + \partial_2^2 \phi \nabla \alpha_2 + \nabla \phi \Delta \alpha_2 \\ \qquad \operatorname{div} v = 0 \end{cases} \tag{P}$$

in $(0, T) \times \Omega$, with boundary and initial conditions

$$\begin{cases} v|_{\partial\Omega} = 0, & \text{on } (0,T) \times \partial\Omega, \\ \alpha|_{\partial\Omega} = 0, & \text{on } (0,T) \times \partial\Omega, \\ \alpha(0) = \alpha_0 := \zeta_0 - (-x_2, x_1)^T, & \text{in } \Omega, \\ v(0) = v_0 := u_0 - (\psi_0, 0)^T, & \text{in } \Omega. \end{cases}$$

2. Quasilinear Evolution Equations

Let X_0 and X_1 be Banach spaces such that $X_1 \overset{d}{\hookrightarrow} X_0$, i.e. X_1 is continuously and densely embedded in X_0. Assume $T > 0$ or $T = \infty$. By a *quasilinear*

autonomous parabolic evolution equation we understand an equation of the form

$$\dot{z}(t) + A(z(t))z(t) = F(z(t)), \quad t \in (0,T), \quad z(0) = z_0, \tag{QL}$$

where A is a mapping from a real interpolation space $X_{\gamma,\mu}$ with suitable weights between X_0 and X_1 into $\mathcal{L}(X_0, X_1)$. An *equilibrium* of (QL) is a stationary solution $z \in X_1$, *i.e.*, $A(z)z = F(z)$.

Our approach relies on the maximal L_p-regularity of $A(v)$ for $v \in X_{\gamma,\mu}$. For details we refer e.g. to [6].

The equation (QL) is investigated in spaces of the form $L_p(0,T;X_0)$ with temporal weights. More precisely, for $p \in (1,\infty)$ and $\mu \in (1/p, 1]$, the spaces $L_{p,\mu}$ and $H^1_{p,\mu}$ are defined by

$$L_{p,\mu}(0,T;X_1) := \{z\colon [0,T) \to X_1 \colon t^{1-\mu} z \in L_p(0,T;X_1)\},$$
$$H^1_{p,\mu}(0,T;X_0) := \{z \in L_{p,\mu}(0,T;X_0) \cap W^1_1(0,T;X_0)\colon \dot{z} \in L_{p,\mu}(0,T;X_0)\}.$$

It is clear, that

$$L_p(0,T;X) \hookrightarrow L_{p,\mu}(0,T;X) \quad \text{and} \quad L_{p,\mu}([0,a];X) \hookrightarrow L_p([\tau,a];X),$$

for all Banach spaces X and $\tau \in (0,a)$ for all $a > 0$. It has been shown in [21, Theorem 2.4] that L_p-maximal regularity implies also $L_{p,\mu}$-maximal regularity, provided $p \in (1,\infty)$ and $\mu \in (1/p, 1]$. The trace space of the maximal regularity class containing temporal weights,

$$z \in H^1_{p,\mu}(0,T;X_0) \cap L_{p,\mu}(0,T;X_1)$$

has been characterized in [21, Theorem 2.4] as

$$X_{\gamma,\mu} = (X_0, X_1)_{\mu - 1/p, p},$$

provided $p \in (1,\infty)$ and $\mu \in (1/p, 1]$; see also [19, Theorem 4.2].

We now impose precise regularity assumptions on A and F.

(A_{-1}) $A : X_{\gamma,\mu} \to \mathcal{L}(X_1, X_0))$ locally Lipschitz,
(F_{-1}) $F : X_{\gamma,\mu} \to X_0$ locally Lipschitz.

Local in time existence of (QL) for a more general non-autonomous case was shown by Clément-Li [5] in the case $\mu = 1$ and by Köhne-Prüss-Wilke [15, Theorem 2.1, Corollary 2.2] for the case $\mu \in (1/p, 1]$.

Proposition 1. *Let $1 < p < \infty$, $\mu \in (1/p, 1]$, $z_0 \in X_{\gamma,\mu}$, and suppose that the assumptions (A_{-1}) and (F_{-1}) are satisfied. Furthermore assume that $A(z_0)$ has the property of maximal L_p-regularity. Then, there exists $a > 0$, such that* (QL) *admits a unique solution z on $J = [0,a]$ in the regularity class*

$$z \in H^1_{p,\mu}(J;X_0) \cap L_{p,\mu}(J;X_1) \hookrightarrow C(J;X_{\gamma,\mu}) \cap C((0,a];X_\gamma).$$

The solution depends continuously on z_0, and can be extended to a maximal interval of existence $J(z_0) = [0, t^+(z_0))$.

The following result on global existence and stability of was proved in [20, Theorem 6.1] in the case $\mu = 1$ and see [15, Theorem 4.1] for $\mu \in (1/p, 1]$.

For this we need slightly stronger conditions on A and F. We now impose regularity assumptions regarding the Fréchet differentiability of A and F.

(A) $A \in C^1(X_{\gamma,\mu}; \mathcal{L}(X_1, X_0))$.
(F) $F \in C^1(X_{\gamma,\mu}; X_0)$.

Proposition 2. *Let $1 < p < \infty$, $\mu \in (1/p, 1]$ and $z^* \in X_\gamma$ be an equilibrium of* (QL)*, such that $A(z^*)$ has maximal L_p-regularity on $\mathbb{R}_+$ and the assumptions (A) and (F) are satisfied. Let A_0 be the linearization of* (QL)*, i.e., let*

$$A_0 w = A(z^*)w + (A'(z^*)w)z^* - F'(z^*)w, \quad w \in X_1.$$

Suppose that $\sigma(A_0)$ is contained in the open right half plane $\mathbb{C}_+ = \Sigma_{\pi/2}$. Then, there is $\varepsilon > 0$ such that for each $z_0 \in B_\varepsilon(z^*) \subset X_{\gamma,\mu}$ there exists a unique global solution $z \in H^1_{p,\mu,\mathrm{loc}}(\mathbb{R}_+; X_0) \cap L_{p,\mu,\mathrm{loc}}(\mathbb{R}_+; X_1)$ of (QL)*. Furthermore, there is a $\beta > 0$ such that*

$$e^{\beta t} z \in H^1_{p,\mu}(\mathbb{R}_+; X_0) \cap L_{p,\mu}(\mathbb{R}_+; X_1) \cap C_0(\mathbb{R}_+; X_{\gamma,\mu}).$$

In particular, the equilibrium z^* is exponentially stable in $X_{\gamma,\mu}$.

We remark that the constant $\varepsilon > 0$ depends only on the maximal regularity constant of A_0 and the local Lipschitz constants of A and F.

Parabolic problems allow for additional smoothing effects. In this respect, a method due to Angenent [3] is well known. We will state a variant of it which is adapted to (QL); see [20, Theorem 5.1] for the case $\mu = 1$. We remark, that in the context of spatial regularity of Navier-Stokes equations, a similar technique has already been used before by Kyûya Masuda [17, 18]. We give a slight adjustment to the situation of temporal weights together with the adaption to space regularity in domains as discussed in [8], which can easily be transfered to the case of layer domains. To this end, we need to strengthen our assumptions (A) and (F) with an order of differentiability $k \in \mathbb{N} \cup \{\infty, \omega\}$, where the index ω refers to real analyticity.

(A_k) $A \in C^k(X_{\gamma,\mu}; \mathcal{L}(X_1, X_0))$.
(F_k) $F \in C^k(X_{\gamma,\mu}; X_0)$.

Proposition 3. *Let $1 < p < \infty$, $k \in \mathbb{N} \cup \{\infty, \omega\}$ and $\mu \in (1/p, 1]$, $J = [0, a)$ for some $a > 0$ and assume that (A_k) and (F_k) hold. Let $z \in H^1_{p,\mu}(J; X_0) \cap L_{p,\mu}(J; X_1)$ be a solution of* (QL) *on J and assume $A(z(t))$ has maximal L_p-regularity for all $t \in J$. Then*

$$t^j [\frac{d}{dt}]^j z \in H^{j+1}_{p,\mu}(J; X_0) \cap H^j_{p,\mu}(J; X_1), \quad j \le k.$$

Furthermore, if $k = \infty$, then $z \in C^\infty(J; X_1)$ and if $k = \omega$, then z is real analytic with values in X_1 on J.

3. Quasilinear Formulation of the Perturbation

For layer domains Ω, the Helmholtz decomposition exists on $L^q(\Omega)$, see [9]. We denote by $P_q : L_q(\Omega) \to L_{q,\sigma}(\Omega)$ the Helmholtz projection, where

$$L_{q,\sigma}(\Omega) := \{u \in L_q(\Omega) : \operatorname{div} u = 0, u \cdot N = 0 \text{ on } \partial\Omega\};$$

here, N denotes the unit exterior vector field of $\partial\Omega$. The Poiseuille problem (P) is equivalent to the problem

$$\begin{cases} \partial_t z + \nu \tilde{P}_q(L + S(z))z = F(z) \\ z(0) = z_0, \end{cases} \tag{PQL}$$

where $z = (v, w)$, $\tilde{P}_q(v, w) := (P_q v, w)$, and $z_0 = (v_0, w_0)$ with

$$\begin{aligned} v_0 &= u_0 - (\psi_0, 0)^T, \\ w_0 &= v_0 + \frac{1}{\nu}(-\zeta_{0_2} + x_1, \zeta_{0_1} + x_2)^T. \end{aligned}$$

Moreover,

$$\begin{aligned} L &= \begin{pmatrix} -\Delta & 0 \\ -\frac{1}{\nu^2} & -\Delta \end{pmatrix}, \qquad F(v, w) = -\begin{pmatrix} P_q(v \cdot \nabla v) \\ v \cdot \nabla w + (I - P_q)(v \cdot \nabla v) \end{pmatrix}, \\ S(z) &= S_1 + S_2 + S_3(z), \end{aligned}$$

and with the notation $z = (v_1, v_2, w_1, w_2)$,

$$\begin{aligned} S_1 &= S_1(\nu, \bar{u}, \nabla\bar{u}, \nabla\phi, \nabla^2\phi) \\ &= \begin{pmatrix} S_1^{11} & S_1^{12} \\ S_1^{21} & S_1^{22} \end{pmatrix}, \end{aligned}$$

where

$$\begin{aligned} S_1^{11} &= \begin{pmatrix} \frac{1}{\nu}\partial_1\bar{u}_1 + \Delta\eta_2\partial_1 & \frac{1}{\nu}\partial_2\bar{u}_1 - \Delta\eta_1\partial_1 \\ \frac{1}{\nu}\partial_1\bar{u}_2 + \Delta\eta_2\partial_2 & \frac{1}{\nu}\partial_2\bar{u}_2 - \Delta\eta_1\partial_2 \end{pmatrix}, \\ S_1^{12} &= \begin{pmatrix} \frac{1}{\nu}\bar{u}_1\partial_1 + \frac{1}{\nu}\bar{u}_2\partial_2 - \Delta\eta_2\partial_1 & \Delta\eta_1\partial_1 \\ -\Delta\eta_2\partial_2 & \frac{1}{\nu}\bar{u}_1\partial_1 + \frac{1}{\nu}\bar{u}_2\partial_2 + \Delta\eta_1\partial_2 \end{pmatrix}, \\ S_1^{21} &= \begin{pmatrix} \frac{1}{\nu}\partial_1\bar{u}_1 - \partial_2^2\phi\partial_1 & \frac{1}{\nu^2}\partial_2\phi + \frac{1}{\nu}\partial_2\bar{u}_1 \\ \frac{1}{\nu}\partial_1\bar{u}_2 - \partial_2^2\phi\partial_2 & \frac{1}{\nu}\partial_2\bar{u}_2 \end{pmatrix}, \\ S_1^{22} &= \begin{pmatrix} \frac{1}{\nu}\bar{u}_1\partial_1 + \frac{1}{\nu}\bar{u}_2\partial_2 + \partial_2^2\phi\partial_1 & 0 \\ \partial_2^2\phi\partial_2 & \frac{1}{\nu}\bar{u}_1\partial_1 + \frac{1}{\nu}\bar{u}_2\partial_2 \end{pmatrix}, \\ S_2 &= S_2(\nabla\eta, \nabla\phi) = \begin{pmatrix} \partial_1\eta_2\Delta & -\partial_1\eta_1\Delta & -\partial_1\eta_2\Delta & \partial_1\eta_1\Delta \\ \partial_2\eta_2\Delta & -\partial_2\eta_1\Delta & -\partial_2\eta_2\Delta & \partial_2\eta_1\Delta \\ -\partial_1\phi\Delta & 0 & \partial_1\phi\Delta & 0 \\ -\partial_2\phi\Delta & 0 & \partial_2\phi\Delta & 0 \end{pmatrix}, \\ S_3(z) &= S_3(\nu, \nabla v, \nabla w) \\ &= \nu \begin{pmatrix} \partial_1(v_1 - w_1)\Delta & \partial_1(v_2 - w_2)\Delta & \partial_1(w_1 - v_1)\Delta & \partial_1(w_2 - v_2)\Delta \\ \partial_2(v_1 - w_1)\Delta & \partial_2(v_2 - w_2)\Delta & \partial_2(w_1 - v_1)\Delta & \partial_2(w_2 - v_2)\Delta \\ \partial_1(v_1 - w_1)\Delta & \partial_1(v_2 - w_2)\Delta & \partial_1(w_1 - v_1)\Delta & \partial_1(w_2 - v_2)\Delta \\ \partial_2(v_1 - w_1)\Delta & \partial_2(v_2 - w_2)\Delta & \partial_2(w_1 - v_1)\Delta & \partial_2(w_2 - v_2)\Delta \end{pmatrix}. \end{aligned}$$

Note that $S_3(0) = 0$ and that $S_3(z)$ acts on z as

$$\nu S_3(z)z = \Delta\alpha_k(z)\nabla\alpha_k(z).$$

4. The Linear Problem

Fix $1 < p, q < \infty$ and $\mu \in (1/p, 1]$. We denote by

$$\begin{aligned} D(A_q) &:= H^2_q(\Omega) \cap H^1_{q,0}(\Omega) \cap L_{q,\sigma}(\Omega), \\ A_q u &:= P_q \Delta u \end{aligned}$$

the Stokes operator on Ω and by

$$\begin{aligned} D(\Delta_q) &:= H^2_q(\Omega) \cap H^1_{q,0}(\Omega), \\ \Delta_q u &:= \Delta u \end{aligned}$$

the Dirichlet Laplacian on Ω. Then we write

$$X_0 := L_{q,\sigma}(\Omega) \times L_q(\Omega), \qquad X_1 := D(A_q) \times D(\Delta_q),$$

and

$$X_\gamma := (X_0, X_1)_{1-1/p,p}, \qquad X_{\gamma,\mu} := (X_0, X_1)_{\mu-1/p,p}.$$

Note that for sufficiently large p and q,

$$\begin{aligned} X_{\gamma,\mu} =& \{v \in B^{2\mu-2/p}_{qp}(\Omega) : v = 0 \text{ on } \partial\Omega\} \cap L_{q,\sigma}(\Omega) \\ & \times \{w \in B^{2\mu-2/p}_{qp}(\Omega) : w = 0 \text{ on } \partial\Omega\} \end{aligned} \tag{8}$$

and an analogue definition for X_γ. Fix $z_0 \in X_{\gamma,\mu}$, $z^* \in X_\gamma$ and $f \in L_{p,\mu}(\mathbb{R}_+; X_0)$. Then we call the system

$$\begin{cases} \partial_t z + \nu \tilde{P}_q (L + S(z^*))z = f, \\ z(0) = z_0, \end{cases} \tag{PL}$$

the *linear Poiseuille perturbation problem.*

Lemma 4. *$\tilde{P}_q L$ has maximal $L_{p,\mu}$-regularity on X_0. More precisely, the problem*

$$\begin{cases} \partial_t z + \nu \tilde{P}_q L z = f, \\ z(0) = z_0, \end{cases}$$

has a unique solution $z = (v, w)$ in the maximal regularity class

$$(v, w) \in H^1_{p,\mu}(\mathbb{R}_+; X_0) \cap L_{p,\mu}(\mathbb{R}_+; X_1)$$

if and only if $z_0 \in X_{\gamma,\mu}$ and $f \in L_{p,\mu}(\mathbb{R}_+; X_0)$ and the solution depends continuously on the data. Moreover, $0 \in \rho(L)$.

Proof. The Dirichlet-Laplacian has maximal L_p-regularity on the layer. This follows by an argument from 1997 by Hieber and Prüss [14] in $\mathbb{R}^n$ which can be adapted to the layer case as described in [4, Remark 3.7].

The Stokes operator enjoys the property of maximal regularity on the layer, since the Helmholtz decomposition exists, and maximal regularity follows by [10]. The off-diagonal entries can be dealt with by first solving for v and then substituting this solution in the equation for w.

The invertibility of the Stokes operator has been proven in [1], while the case of the Dirichlet Laplacian can be verified using standard arguments using Fourier transformation and a reflection principle. □

Proposition 5. *There are $\varepsilon = \varepsilon(1/\nu) > 0$ and $\delta = \delta(\nu) > 0$ such that if $z^* \in BUC^1(\Omega)$ with $\|z^*\|_{BUC^1(\Omega)} < \delta$ and $\|(\bar{u}, \eta, \bar{\pi})\|_{BUC(\mathbb{R}_+;BUC^1(\bar{\Omega}))} + \|\nabla^2\eta, \nabla^2\phi\|_\infty < \varepsilon$, then problem* (PL) *has maximal $L_{p,\mu}$-regularity on $\mathbb{R}_+$.*

Proof. In virtue of Lemma 4, $\tilde{P}_q L$ is invertible and possesses maximal $L_{p,\mu}$-regularity. Thus, also $\tilde{P}_q L - \omega$ possesses maximal $L_{p,\mu}$-regularity for some small $\omega > 0$ depending on L. We use a standard argument of maximal regularity using relative perturbation, see e.g. [20, Proposition 1.5]. We need to establish the estimate

$$\|\tilde{P}_q S(z^*)z\|_{X_0} \le a\|z\|_{X_0} + b\|(\tilde{P}_q L - \omega)z\|_{X_0} \tag{9}$$

such that

$$bC_0(L) < 1 \text{ and } \omega \ge \frac{aM_0(L)}{1 - bC_0(L)}, \tag{10}$$

where $M_0(L)$ and $C_0(L)$ are positive constants depending on the L. The entries of the lower order perturbation S_1 are controlled by

$$c_\nu \left(\|(\bar{u}, \eta, \bar{\pi})\|_{BUC(\mathbb{R}_+;BUC^1(\bar{\Omega}))} + \|\nabla^2\eta, \nabla^2\phi\|_\infty \right) < c_\nu \varepsilon,$$

where $c_\nu = \max\{1/\nu, 1/\nu^2\}$. Thus, with the interpolation estimate, cf. [7, Example III.2.2],

$$\|\nabla z\|_{X_0} \le C\|z\|_{X_0} + \|\tilde{P}_q L z\|_{X_0},$$

we can estimate the norm of $S_1 z$ by

$$\begin{aligned}\|\tilde{P}_q S_1 z\|_{X_0} \le c_\nu\varepsilon\|z, \nabla z\|_{X_0} &\le c_\nu\varepsilon \left((1 + C)\|z\|_{X_0} + \|\tilde{P}_q L z\|_{X_0} \right) \\ &\le c_\nu\varepsilon \left((1 + C + \omega)\|z\|_{X_0} + \|(\tilde{P}_q L - \omega)z\|_{X_0} \right).\end{aligned}$$

For the highest order perturbations S_2 and $S_3(z^*)$ we calculate

$$\begin{aligned}\|\tilde{P}_q S_2 z\|_{X_0} &\le \varepsilon\|\tilde{P}_q L z\|_{X_0} \le \varepsilon \left(\omega\|z\|_{X_0} + \|(\tilde{P}_q L - \omega)z\|_{X_0} \right), \\ \|\tilde{P}_q S_3(z^*)z\|_{X_0} &\le \nu\|\nabla z^*\|_\infty\|\tilde{P}_q L z\|_{X_0} \le \nu\delta \left(\omega\|z\|_{X_0} + \|(\tilde{P}_q L - \omega)z\|_{X_0} \right).\end{aligned}$$

In total, we choose

$$\begin{aligned} a &:= \varepsilon(c_\nu(1 + C + \omega) + \omega) + \nu\delta\omega, \\ b &:= \varepsilon(c_\nu + 1) + \nu\delta. \end{aligned} \tag{11}$$

Hence, for $\varepsilon, \delta > 0$ sufficiently small we obtain (10) and the maximal $L_{p,\mu}$-regularity follows for $(\tilde{P}_q L - \omega) + \tilde{P}_q S(z^*) + \omega = \tilde{P}_q(L + S(z^*))$. □

Remark 6. The smallness assumption on z_* is indeed essential for maximal regularity even in the case of the shifted operator $\tilde{P}_q(L + S(z_*)) + \omega$: For fixed viscosity $\nu > 0$, a bounded perturbation argument requires b as defined in (11) to be suffciently small to preserve sectoriality. Hence, it is necessary for both ε and δ to be sufficiently small.

Lemma 7. *There is an $\varepsilon > 0$ such that if*

$$\|(\bar{u}, \eta, \bar{\pi})\|_{BUC(\mathbb{R}_+; BUC^1(\bar{\Omega}))} + \|\nabla^2 \eta, \nabla^2 \phi\|_\infty < \varepsilon,$$

then there holds $\sigma(\tilde{P}_q(L + S_1 + S_2)) \subset \mathbb{C}_+ := \Sigma_{\pi/2}$.

Proof. Observe that $S_3(0) = 0$. Thus, by Proposition 5 applied to $z^* = 0$, $\tilde{P}_q(L + S_1 + S_2)$ is sectorial. Therefore the invertibility of $\tilde{P}_q L$ implies the assertion. □

5. Main result

We need to clarify what we mean by local and global stability of a given Poiseuille flow.

Definition 8. We call a Poiseuille flow $(\bar{u}, \eta, \bar{\pi})$ *stable on* $[0, T_0)$ *of level* $\delta > 0$ if

$$\|(v_0, \alpha_0)\|_{X_{\gamma,\mu}} < \delta$$

implies that the problem (3) has a unique strong solution (v, α, p) on $[0, T_0)$ with

$$\begin{aligned}(v, \alpha) &\in H^1_p(0, T_0; X_0) \cap L_p(0, T_0; X_1) \cap C_0(0, T_0; X_{\gamma,\mu})\\ \nabla p &\in L_p(0, T_0; L_q(\Omega)),\end{aligned}$$

depending continuously on (v_0, α_0).

Secondly, we need a notion for global-in-time stability which quantifies its asymptotic behavior for $t \to \infty$.

Definition 9. A Poiseuille flow $(\bar{u}, \eta, \bar{\pi})$ is called *exponentially stable of level* $\delta > 0$ *with rate* $\beta > 0$, if

$$\|(v_0, \alpha_0)\|_{X_{\gamma,\mu}} < \delta$$

implies that Problem (3) has a unique strong solution (v, α, p) such that

$$\begin{aligned}e^{t\beta}(v, \alpha) &\in H^1_p(\mathbb{R}_+; X_0) \cap L_p(\mathbb{R}_+; X_1) \cap C_0(\mathbb{R}_+; X_{\gamma,\mu}),\\ e^{t\beta}\nabla p &\in L_p(\mathbb{R}_+; L_q(\Omega)),\end{aligned}$$

depending continuously on the initial data (v_0, α_0).

Permissible initial data

We impose the following condition on the exponents $1 < p, q < \infty$ and the temporal weight $\mu \in (1/p, 1]$.

$$\frac{1}{p} + \frac{1}{q} < \mu - \frac{1}{2} \tag{I}$$

Then the identity (8) holds by [19, Theorem 4.2] for the half-space case which can then be transfered to the layer via reflection.

Then $X_{\gamma,\mu} \hookrightarrow BUC^1(\bar{\Omega})$, see [2, Theorem 4.12], and

$$H^1_p(\mathbb{R}_+; L_{q,\sigma}(\Omega)) \cap L_p(\mathbb{R}_+; D(A_q)) \hookrightarrow BUC(\mathbb{R}_+; BUC^1(\bar{\Omega})).$$

Hence we may state our main theorems. We begin with local stability for finite-time Poiseuille flows.

Main Theorem 1 (Local Stability). *Assume the condition* (I)*, let* $\nu > 0$ *and* $T > 0$ *be given. There are* $\varepsilon, \delta > 0$ *and a* $T_0 \in (0, T)$ *such that whenever* $(\bar{u}, \eta, \bar{\pi})$ *solves the Poiseuille problem* (3) *on* $[0, T)$ *with* $\|(\bar{u}, \eta, \bar{\pi})\|_{BUC(\mathbb{R}_+; BUC^1(\bar{\Omega}))} + \|\nabla^2 \eta, \nabla^2 \phi\|_\infty < \varepsilon$*, then* $(\bar{u}, \eta, \bar{\pi})$ *is stable of level* δ *on* $[0, T_0)$*.*

Proof. Maximal regularity on finite times of the linearization of (PQL) is a direct consequence of Proposition 5. Let us now verify the regularity conditions (A) and (F). For $z \in X_1$ and $z^* \in X_{\gamma,\mu}$, we estimate similarly as in (9),

$$\|\tilde{P}_q(L + S(z^*))z\|_{X_0} \le c\|z\|_{X_1} < \infty,$$

where $c = c(z^*) > 0$. For the right-hand side F we note that $L_p(\mathbb{R}_+; L_q(\Omega)) \cap BUC(\mathbb{R}_+; BUC^1(\bar{\Omega}))$ is an algebra if equipped with the pointwise multiplication. Therefore $F(z) \in X_0$. The local Lipschitz assertions in (A) and (F) are trivially fulfilled, since F and $L + S(z)$ are polynomial in z. Then Proposition 1 yields the unique existence of a solution to the perturbed Poiseuille problem (PQL) on an interval $[0, T_0)$ for some $T_0 \in (0, T)$ and hence stability of $(\bar{u}, \eta, \bar{\pi})$ on $[0, T_0)$ follows by definition. □

We remark that T_0 depends only on the chosen perturbation and is independent of the choice of the original flow.

Main Theorem 2 (Global Stability). *Assume the condition* (I) *and* $\nu > 0$ *be given. Then, there are constants* $\varepsilon, \delta, \beta > 0$ *such that every global strong solution* $(\bar{u}, \eta, \bar{\pi})$ *to the Poiseuille problem* (3) *with* $\|(\bar{u}, \eta, \bar{\pi})\|_{BUC(\mathbb{R}_+; BUC^1(\bar{\Omega}))} + \|\nabla^2 \eta, \nabla^2 \phi\|_\infty < \varepsilon$ *is exponentially stable of level* δ *and rate* β*. Here,* ϕ *is defined as in* (4)*.*

Proof. We want to apply Proposition 2. Maximal regularity of the shifted linear problem with $z^* = 0$ has already been proven in Proposition 5. Let us now verify the regularity conditions (A) and (F). Observe that (A) and (F) are fulfilled by the same argumentation as in Main Theorem 1. Since F and $L + S(z)$ are polynomial in z, Fréchet differentiability follows as well.

By Lemma 7, the spectrum of the linearization

$$A_0 = \nu \tilde{P}_q(L + S_1 + S_2)$$

is contained in the right half plane. Hence, Proposition 2 yields the existence of a level $\delta > 0$ such that the perturbed flow in (PQL) has a unique global solution for initial data $\|(v_0, \alpha_0)\|_{X_{\gamma,\mu}} < \delta$ which decays exponentially with rate $\beta > 0$. □

By the smoothing effects of parabolic equations, further regularity follows directly from Angenent's Trick, Proposition 3.

Main Theorem 3 (Regularity). *Let* $T > 0$ *or* $T = \infty$ *and let* $(v, \alpha, \nabla p)$ *be a solution to the perturbed Poseuille problem as obtained by either Main*

Theorem 1 or 2. Then $(v, \alpha, \nabla p)$ *is real analytic with values in* X_1 *on* $[0, T)$ *and for* $k \in \mathbb{N}$ *it holds*

$$t^k [\frac{d}{dt}]^k (v, \alpha, \nabla p) \in H^{k+1}_{p,\mu}(0, T; X_0) \cap H^k_{p,\mu}(0, T; X_1) \times H^k_{p,\mu}(0, T; L_q(\Omega)).$$

Employing scaling techniques jointly in time and space, it is possible to show via maximal regularity and the implicit function theorem that $(v, \alpha, \nabla p)$ are real analytic in $(0, T) \times \Omega$; see [20, Section 5] for parabolic problems, and specifically for a Navier-Stokes problem [22].

References

1. T. Abe and Y. Shibata, *On a resolvent estimate of the Stokes equation on an infinite layer. II. $\lambda = 0$ case*, J. Math. Fluid Mech. **5** (2003), no. 3, 245–274. MR **1994781 (2004e:**35173)
2. R. A. Adams and J. J. F. Fournier, *Sobolev spaces*, second ed., Pure and Applied Mathematics (Amsterdam), vol. 140, Elsevier/Academic Press, Amsterdam, 2003.
3. S. B. Angenent, *Nonlinear analytic semiflows*, Proc. Roy. Soc. Edinburgh Sect. A **115** (1990), no. 1-2, 91–107.
4. W. Arendt, *Semigroup properties by Gaussian estimates*, Sūrikaisekikenkyūsho Kōkyūroku (1997), no. 1009, 162–180, Nonlinear evolution equations and their applications (Japanese) (Kyoto, 1996). MR **1630389 (99g:**47088)
5. P. Clément and S. Li, *Abstract parabolic quasilinear equations and application to a groundwater flow problem*, Adv. Math. Sci. Appl. **3** (1993/94), no. Special Issue, 17–32.
6. R. Denk, M. Hieber, and J. Prüss, *$\mathcal{R}$-boundedness, Fourier multipliers and problems of elliptic and parabolic type*, Mem. Amer. Math. Soc. **166** (2003), no. 788, viii+114.
7. K.-J. Engel and R. Nagel, *One-parameter semigroups for linear evolution equations*, Graduate Texts in Mathematics, vol. 194, Springer-Verlag, New York, 2000, With contributions by S. Brendle, M. Campiti, T. Hahn, G. Metafune, G. Nickel, D. Pallara, C. Perazzoli, A. Rhandi, S. Romanelli and R. Schnaubelt. MR **1721989 (2000i:**47075)
8. J. Escher, J. Prüss, and G. Simonett, *Analytic solutions for a Stefan problem with Gibbs-Thomson correction*, J. Reine Angew. Math. **563** (2003), 1–52.
9. R. Farwig, *Weighted L^q-Helmholtz decompositions in infinite cylinders and in infinite layers*, Adv. Differential Equations **8** (2003), no. 3, 357–384. MR **1948530 (2003j:**35248)
10. M. Geissert, H. Heck, M. Hieber, and O. Sawada, *Weak Neumann implies Stokes*, J. Reine Angew. Math. **669** (2012), 75–100. MR 2980452
11. Y. Giga, D. Götz, and C. Liu, *Stability of a two-dimensional poiseuille-type flow for a viscoelastic fluid*, preliminary notes (2013).
12. Horst Heck, Hyunseok Kim, and Hideo Kozono, *Stability of plane Couette flows with respect to small periodic perturbations*, Nonlinear Anal. **71** (2009), no. 9, 3739–3758. MR **2536284 (2010g:**35223)

13. M. Hieber, M. Nesensohn, J. Prüss, and K. Schade, *Dynamics of nematic liquid crystal flows*, (submitted).
14. M. Hieber and J. Prüss, *Heat kernels and maximal L^p-L^q estimates for parabolic evolution equations*, Comm. Partial Differential Equations **22** (1997), no. 9-10, 1647–1669. MR **1469585 (98k:**34096)
15. M. Köhne, J. Prüss, and M. Wilke, *On quasilinear parabolic evolution equations in weighted L_p-spaces*, J. Evol. Equ. **10** (2010), no. 2, 443–463.
16. F.-H. Lin, C. Liu, and P. Zhang, *On hydrodynamics of viscoelastic fluids*, Comm. Pure Appl. Math. **58** (2005), no. 11, 1437–1471. MR **2165379 (2006d:**76005)
17. K. Masuda, *On the analyticity and the unique continuation theorem for solutions of the Navier-Stokes equation*, Proc. Japan Acad. **43** (1967), 827–832. MR 0247304 (40 #572)
18. ———, *On the regularity of solutions of the nonstationary Navier-Stokes equations*, Approximation methods for Navier-Stokes problems (Proc. Sympos., Univ. Paderborn, Paderborn, 1979), Lecture Notes in Math., vol. 771, Springer, Berlin, 1980, pp. 360–370. MR **566007 (81f:**35095)
19. M. Meyries and R. Schnaubelt, *Interpolation, embeddings and traces of anisotropic fractional Sobolev spaces with temporal weights*, J. Funct. Anal. **262** (2012), no. 3, 1200–1229.
20. J. Prüss, *Maximal regularity for evolution equations in L_p-spaces*, Conf. Semin. Mat. Univ. Bari (2002), no. 285, 1–39 (2003).
21. J. Prüss and G. Simonett, *Maximal regularity for evolution equations in weighted L_p-spaces*, Arch. Math.(Basel) **82** (2004), no. 5, 415–431.
22. ———, *Analytic solutions for the two-phase Navier-Stokes equations with surface tension and gravity*, Parabolic Problems, Progress in Nonlinear Differential Equations and Their Applications, vol. 80, Springer Basel, 2011, pp. 507–540.
23. J. Prüss, G. Simonett, and R. Zacher, *On convergence of solutions to equilibria for quasilinear parabolic problems*, J. Differential Equations **246** (2009), no. 10, 3902–3931.
24. V. A. Romanov, *Stability of plane-parallel Couette flow*, Dokl. Akad. Nauk SSSR **196** (1971), 1049–1051. MR 0311201 (46 #10294)

Y. Giga
The University of Tokyo, Department of Mathematics, Graduate School of Mathematical Sciences, 3-8-1 Komaba Meguro-ku Tokyo 153-8914, Japan
e-mail: labgiga@ms.u-tokyo.ac.jp

J. Sauer
Technische Universität Darmstadt, Fachbereich Mathematik, Schlossgartenstr. 7, D-64298 Darmstadt, Germany
e-mail: jsauer@mathematik.tu-darmstadt.de

K. Schade
Technische Universität Darmstadt, Fachbereich Mathematik, Schlossgartenstr. 7, D-64298 Darmstadt, Germany
e-mail: schade@mathematik.tu-darmstadt.de