

# Stochastic Homogenization: An introduction to some recent variants and to numerical approaches

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The series of lectures will overview some recent contributions on several theoretical aspects and numerical approaches in stochastic homogenization. After an introduction to the elementary aspects of stochastic homogenization, a variant of the classical theory will be presented. It has been introduced in [6], and further studied in [11, 16]. The relation between stochastic homogenization problems and other multiscale problems in materials science [7] will be emphasized. Several numerical approaches will be presented: some for genuinely stochastic problems (where variance issues are a practical concern and need to be addressed for efficiency purposes, [9, 10, 13]), and some for approximations of stochastic problems when the random character is only a perturbation of a deterministic model [1, 2, 3, 12, 14]. Most of these contributions are summarized in [15, 4]. Further details will be available in [5, 11, 17].

The series of talks will be centered around a simple, *linear elliptic* situation, since the focus is

- (a) deliberately elementary,
- (b) more on the stochastic setting, its relation with situations relevant for applications and its requirements in terms of numerical approaches, than
- (c) on the complexity of the equation itself.

However, given the topic of the conference, one purpose of the series of talks will be to draw connections with more elaborate cases involving *nonlinear equations*. Several suggestions for application to the nonlinear setting of the ideas and techniques currently developed in the simple situation will be given. The idea is to hopefully spark interest in, and foster strong interaction with the audience, expert in *nonlinear equations*.

## References

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