On decay rate of quenching profile at space infinity for axisymmetric mean curvature flow*†

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Abstract

We study the motion of noncompact hypersurfaces moved by their mean curvature obtained by a rotation around x-axis of the graph a function $y = u(x,t)$ (defined for all $x \in \mathbf{R}$). We are interested to estimate its profile when the hypersurface closes open ends at the quenching (pinching) time T. We estimate its profile at the quenching time from above and below. We in particular prove that $u(x,T) \sim |x|^{-a}$ as $|x| \to \infty$ if $u(x,0)$ tends to its infimum with algebraic rate $|x|^{-2a}$ (as $|x| \to \infty$ with $a > 0$).

1 Introduction and main theorem

This is a continuation of our study [4] on motion of noncompact axisymmetric n-dimensional hypersurface Γ_t moved by its mean curvature. Let Γ_t be given by a rotation of the graph of a function $y = u(x,t)$ (defined on $x \in \mathbf{R}$) around the x-axis (cf [1, 2]). In our previous paper [4], among other results, we have proved that if $u(x,0) \to m := \inf_{x\in\mathbf{R}} u(x,0) > 0$ as $|x| \to \infty$, then Γ_t closes open ends at the time $T(m)$, where $T(m)$ is the quenching (pinching) time of the regular cylinder with radius m. (Moreover, there is no neck-pinch in $\mathbf{R}$ at $t = T(m)$.) These results imply that

$$\lim_{x\to\infty} u(x,T(m)) = 0 \quad \text{or} \quad \lim_{x\to-\infty} u(x,T(m)) = 0,$$

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but it does not provide the convergence rate.

We are interested in studying the profile of $u(x, T(m))$, especially the behavior as $|x| \to \infty$ which is affected by initial data.

The equation for u is of the form

$$u_t = \frac{u_{xx}}{1+u_x^2} - \frac{n-1}{u}, \quad x \in \mathbf{R},\ t > 0 \tag{1}$$

supplemented by initial data

$$u(x,0) = u_0(x) > 0, \quad x \in \mathbf{R}. \tag{2}$$

The function u_0 is assumed to satisfy

$$u_0 \text{ is bounded and uniformly continuous in } \mathbf{R}, \tag{3}$$

$$m := \inf_{x \in \mathbf{R}} u_0(x) > 0. \tag{4}$$

The Cauchy problem (1)-(2) has a unique positive classical solution with the conditions (3)-(4) to the initial data (cf [4]). However, the solution quenches in finite time. For a given initial datum u_0, we see

$$T(u_0) = \sup\{t > 0; \inf_{x \in \mathbf{R}} u(x,t) > 0\} < \infty$$

and call it the *quenching time* of u. It is clear that

$$\lim_{t \to T(u_0)} \inf_{x \in \mathbf{R}} u(x,t) = 0.$$

Let v be a solution of (1) with initial datum $m = \inf_{x \in \mathbf{R}} u_0(x)$. It is easily seen that

$$v' = -\frac{n-1}{v},\ t > 0, \quad v(0) = m, \tag{5}$$

and

$$v(t) = \sqrt{2(n-1)(T(m)-t)} \quad \text{with} \quad T(m) = \frac{m^2}{2(n-1)}. \tag{6}$$

It is immediate that $T(u_0) \geq T(m)$ by a comparison argument. We treat the case $T(u_0) = T(m)$. The notion of "minimal quenching time" was defined in [4], which is recalled below.

Definition 1.1. *A solution of the Cauchy problem* (1)-(2) *is said to have a minimal quenching time, if*

$$T(u_0) = T(m).$$

In [4] we characterized solutions of (1)-(2) quenching only at space infinity. The following conditions on initial data u_0 play essential roles in [4].

A. There exists a sequence $\{x_k\} \in \mathbf{R}$ such that $x_k \to \infty$ and $u_0(x+x_k) \to m$ a.e. in $\mathbf{R}$ as $k \to \infty$.

B. There exists a sequence $\{x_k\} \in \mathbf{R}$ such that $x_k \to -\infty$ and $u_0(x+x_k) \to m$ a.e. in $\mathbf{R}$ as $k \to \infty$.

For an initial datum satisfying (3)-(4), we proved in [4] the following results for the Cauchy problem (1)-(2):

1. A solution of (1)-(2) has a minimal quenching time, if and only if the conditions A or B holds.

Moreover, if u_0 is not constant as well as the conditions A or B holds, then:

2. For an initial datum satisfying $u_0 \not\equiv m$, the solution (1)-(2) quenches only at space infinity.

3. There exists a function $u(\cdot, T(m)) \in C^\infty(\mathbf{R})$ such that $u(\cdot, t) \to u(\cdot, T(m))$ in the Frechét space $C^\infty(\mathbf{R})$ as $t \to T(m)$, $u(x, T(m)) > 0$ in the whole $\mathbf{R}$ and

$$\liminf_{x \to -\infty} u(x, T(m)) = 0 \quad \text{or} \quad \liminf_{x \to -\infty} u(x, T(m)) = 0.$$

For a solution u of (1)-(2) with minimal quenching time $T(m)$, we call $u(\cdot, T(m))$ the profile of u (at the quenching time T(m)). The hypersurface corresponding to $u(\cdot, T(m))$ is called limit surface.

These are related studies on blow-up at infinity for the reaction-diffusion equations [8, 5, 6, 3, 10, 9, 11] (see also [7]). We shall explain these papers at the end of this introduction. In particular, blow-up profile was discussed, for example, in [8] and [11] for a semilinear heat equation.

In this paper we consider the relation between the profile of a quenching solution at quenching time $T(m)$ and the form of initial data. Our goal, which is investigating the shape of limit surface, is similar to studying blow-up profile. Inspired by the method used in [8, §2b] and [11, Theorems 1.3 and 1.5], we construct a subsolution and a supersolution of the form $\phi(T(m)-t+g(x,t))$ with some function $g(x,t)$ decaying to zero at space infinity, where

$$\phi(s) = v(T(m) - s) = \sqrt{2(n-1)s}, \tag{7}$$

in order to estimate the profile at the quenching time. Let $\psi(x)$ be a positive function satisfying the following conditions:

$$\sqrt{\psi(x)} \text{ is bounded and uniformly continuous in } \mathbf{R}; \tag{8}$$

$$\psi(x) > 0 \text{ for } x \in \mathbf{R}; \tag{9}$$

$$\lim_{x\to\infty} \psi(x) = 0 \text{ or } \lim_{x\to-\infty} \psi(x) = 0; \tag{10}$$

there exist constants $C_1 > 0$ and $C_2 > 0$ such that

$$\psi(x) \le C_1 \max\{\inf_{z\in[x-1,x]} \psi(z), \inf_{z\in[x,x+1]} \psi(z)\} \quad \text{for } x \in \mathbf{R}; \tag{11}$$

$$\psi(x-y) \le C_2 \exp\left(a|y|^2\right) \psi(x) \quad \text{for } x, y \in \mathbf{R},\ a \in \left(0, \frac{1}{4T(m)}\right). \tag{12}$$

Example 1.2. *The functions* $\psi(x) = (|x|^2+1)^{-b/2}$, $e^{-b|x|}$ *and* $(\log(|x|+e))^{-b}$ *with* $b > 0$ *satisfy* (8)-(12).

Theorem 1.3. *Let* ψ *be a function satisfying* (8)-(12). *Assume that* (3)-(4) *hold and that there exist constants* $C_I > 0$ *and* $C_{II} > 0$ *such that*

$$u_0^2(x) - m^2 \ge C_I \psi(x) \quad (\text{or } \le C_{II}\psi(x)). \tag{13}$$

Then there exists $C = C(C_1, C_2, a, T(m), C_I) > 0$ *(or* $C' = C'(C_1, C_2, a, T(m), C_{II}) > 0$*) such that the solution of the Cauchy problem* (1)-(2) *satisfies*

$$u(x, T(m)) \ge C\sqrt{\psi(x)} \quad \left(\text{or } \le C'\sqrt{\psi(x)}\right).$$

By setting $\psi(x) = \langle x\rangle^{-2a_1}$ (or $\langle x\rangle^{-2a_2}$) with $\langle x\rangle = (1+|x|^2)^{1/2}$, we obtain algebraic decay at the space infinity.

Corollary 1.4. *Assume that there exist constants* $a_1 > 0$, $a_2 > 0$, $C_I > 0$ *and* $C_{II} > 0$ *such that*

$$u_0^2(x) - m^2 \ge C_I\langle x\rangle^{-2a_1} \quad \left(\text{or } \le C_{II}\langle x\rangle^{-2a_2}\right). \tag{14}$$

Then there exists $C = C(a_1, T(m), C_I) > 0$ *(or* $C' = C'(a_2, T(m), C_{II}) > 0$*) such that*

$$u(x, T(m)) \ge C\langle x\rangle^{-a_1} \quad \left(\text{or } \le C'\langle x\rangle^{-a_2}\right).$$

We conclude this introduction by giving a short review on blow-up (or quenching) at the space infinity. Lacey [8] considered problems in a half line of $u_t = u_{xx} + f(u)$ in $\mathbf{R}^+ = \{x : x > 0\}$ and constructed solutions blowing up only at space infinity. Gladkov [7] studied problems of the equation

$u_t = u_{xx} + f(x,t,u)$ in $\mathbf{R}^+$ and showed that solutions of the problem uniformly converge as $x \to \infty$ to the solution of the ODE obtained by dropping u_{xx} in the equation.

Giga-Umeda [5] proved that blow-up only at space infinity occurs under the condition $\lim_{|x|\to\infty} u_0(x) = \sup_{x\in\mathbf{R}} u_0(x) =: M$ and $u_0 \not\equiv M$ for nonnegative solutions of $u_t = \Delta u + u^p$ in $\mathbf{R}^n$ (cf. also [12] for a related study). For generalization, see [6] and a review article by Giga-Seki-Umeda [3]. More recently, Shimojō [11] discussed blow-up profile $u(x,T) := \lim_{t\to T} u(x,t)$ for $x \in \mathbf{R}^n$. See also Seki-Suzuki-Umeda [10] and Seki [9] for quasilinear parabolic equations, which generalized the result of [6]. They also gave necessary and sufficient conditions for a solution to have "minimal blow-up time (or the least blow-up time)". See [9, 10, 3] for the precise definition of the last notion.

2 Profile at quenching

In order to prove Theorem 1.3, we construct a subsolution and supersolution of the form $\phi(T(m) - t + g(x,t))$, as we have explained before. This is a modification of the method employed in [8] and [11] to study blow-up profile for a semilinear heat equation. The function

$$g(x,t) = \int_{-\infty}^{\infty} G(x-y,t)\psi(y)dy$$

with the Gauss kernel of heat equation

$$G(x,t) = (4\pi t)^{-1/2} \exp\left(-\frac{x^2}{4t}\right)$$

is used there. However, because the problem which we treat here is a quasilinear equation, the Gauss kernel is not appropriate in our problem. We use the following function instead of $G(x,t)$:

$$g^{\gamma}_{\alpha,\beta}(x,t) = g^{\gamma,\psi}_{\alpha,\beta}(x,t) = \int_{-\infty}^{\infty} G^{\gamma}_{\alpha,\beta}(x-y,t)\psi(y)dy, \tag{15}$$

where

$$G^{\gamma}_{\alpha,\beta}(x,t) = \frac{|x|^{\beta}}{(t+\gamma)^{\alpha}} \exp\left(-\frac{x^2}{4(t+\gamma)}\right)$$

with $\alpha \geq 0$, $\beta \geq 0$ and $\gamma > 0$ being constants. Note that this $g^{\gamma}_{\alpha,\beta}$ may be expressed by

$$g^{\gamma}_{\alpha,\beta}(x,t) = \int_{-\infty}^{\infty} G^{\gamma}_{\alpha,\beta}(y,t)\psi(x-y)dy.$$

It is easily seen that the derivatives are calculated and estimated as follows:

$$|\partial_x g^{\gamma}_{\alpha,0}| \le \frac{g^{\gamma}_{\alpha+1,1}}{2}, \tag{16}$$

$$\partial_{xx} g^{\gamma}_{\alpha,0} = \frac{g^{\gamma}_{\alpha+2,2}}{4} - \frac{g^{\gamma}_{\alpha+1,0}}{2}, \tag{17}$$

$$\partial_t g^{\gamma}_{\alpha,0} = \frac{g^{\gamma}_{\alpha+2,2}}{4} - \alpha g^{\gamma}_{\alpha+1,0}, \tag{18}$$

and

$$g^{\gamma}_{\alpha,\beta}(x,t) = \frac{g^{\gamma}_{0,\beta}}{(t+\gamma)^{\alpha}}. \tag{19}$$

Before proving the Theorem 1.3 we prepare two propositions.

Proposition 2.1. *Let ψ be a positive bounded uniformly continuous function. For any $C > 0$ and $\gamma > 0$ the function*

$$W(x,t) = \phi(T(m) - t + C g^{\gamma}_{0,0}(x,t)) \tag{20}$$

is a supersolution of (1) *in* $\mathbf{R} \times (0, T(m))$, *where* ϕ *is defined in* (7).

Proof. By a direct calculation we have

$$\begin{aligned} W_t - \frac{W_{xx}}{1+W_x^2} + \frac{n-1}{W} \\ = -\phi' + C\phi'\partial_t g^{\gamma}_{0,0} - \frac{C\phi'\partial_{xx} g^{\gamma}_{0,0} + (C\partial_x g^{\gamma}_{0,0})^2\phi''}{1+(C\phi'\partial_x g^{\gamma}_{0,0})^2} + \frac{n-1}{\phi}. \end{aligned}$$

Noting that $\phi'\partial_t g^{\gamma}_{0,0} \ge 0$ from (18) and $\phi' = (n-1)/\phi$, we obtain

$$W_t - \frac{W_{xx}}{1+W_x^2} + \frac{n-1}{W} \ge \frac{C\phi'\partial_t g^{\gamma}_{0,0} - C\phi'\partial_{xx} g^{\gamma}_{0,0} - (C\partial_x g^{\gamma}_{0,0})^2\phi''}{1+(C\phi'\partial_x g^{\gamma}_{0,0})^2}.$$

Since $(\partial_t - \partial_{xx}) g^{\gamma}_{0,0} = g^{\gamma}_{1,0}/2$ by (17)-(18), we have

$$W_t - \frac{W_{xx}}{1+W_x^2} + \frac{n-1}{W} \ge \frac{1}{1+(C\phi'\partial_x g^{\gamma}_{0,0})^2}\left(\frac{C\phi' g^{\gamma}_{1,0}}{2} - (C\partial_x g^{\gamma}_{0,0})^2\phi''\right).$$

Due to the fact that $\phi'' \le 0$, we see that W is a supersolution of (1). □

Proposition 2.2. *Assume that ψ is a function satisfying* (8)-(12) *and*

$$\gamma \in \left(0, \frac{1}{a} - 4T(m)\right) \tag{21}$$

with the constant a in (12). *Then, for each constant $C > 0$, the function*

$$w(x,t) = \phi(T(m) - t + C g^{\gamma}_{\alpha,0}(x,t)) \tag{22}$$

is a subsolution of (1) *in $\mathbf{R} \times (0, T(m))$ provided that α satisfies $\alpha \geq \alpha_0$ with some constant $\alpha_0 = \alpha_0(C_1, C_2, a, T(m), \gamma) > 0$, where ϕ is the function defined in* (7).

Before proving Proposition 2.2, we prepare a lemma on estimates for $g^{\gamma}_{0,\beta}$.

Lemma 2.3. *Assume the same hypotheses as in Proposition 2.2. Then for $\beta = 0,\ 1,\ 2$, there exist constants $C_3 = C_3(C_1, \gamma) > 0$ and $C_4 = C_4(C_2, a, T(m), \gamma) > 0$ such that*

$$C_3\psi(x) \leq g^{\gamma}_{0,\beta}(x,t) \leq C_4\psi(x) \quad \text{in } \mathbf{R} \times [0, T(m)],$$

where C_1 and C_2 are the constants in (11) *and* (12), *respectively.*

Proof. First we show $g^{\gamma}_{0,\beta} \geq C_3\psi(x)$ with some $C_3 > 0$. From (11)

$$\psi(x) \leq C_1 \inf_{z \in [x-1,x]} \psi(z) \tag{23}$$

or

$$\psi(x) \leq C_1 \inf_{z \in [x,x+1]} \psi(z) \tag{24}$$

for each $x \in \mathbf{R}$. If (23) holds, then there exists a constant $C_3 > 0$ such that

$$\begin{aligned} g^{\gamma}_{0,\beta}(x,t) &\geq \inf_{z \in (x-1,x)} \psi(z) \times \int_0^1 |y|^{\beta} \exp\left(-\frac{|y|^2}{4\gamma}\right) dy \\ &\geq \psi(x) \frac{1}{C_1} \int_0^1 |y|^{\beta} \exp\left(-\frac{|y|^2}{4\gamma}\right) dy. \end{aligned}$$

Set

$$C_3 = \min_{\beta=0,1,2} \frac{1}{C_1} \int_0^1 |y|^{\beta} \exp\left(-\frac{|y|^2}{4\gamma}\right) dy = \frac{1}{C_1} \int_0^1 |y|^2 \exp\left(-\frac{|y|^2}{4\gamma}\right) dy.$$

We then see that

$$g^{\gamma}_{0,\beta}(x,t) \geq C_3\psi(x).$$

A similar argument shows that if (24) holds, then

$$g^{\gamma}_{0,\beta}(x,t) \geq C_3\psi(x).$$

Thus we see that

$$g^{\gamma}_{0,\beta}(x,t) \geq C_3\psi(x)$$

for any $x \in \mathbf{R}$.

We next prove $g^{\gamma}_{0,\beta}(x,t) \leq C_4\psi(x)$ with some $C_4 > 0$. For (21) it is possible to take a constant $\gamma > 0$ depending only a and m that satisfies

$$\frac{1}{4(T(m)+\gamma)} - a > 0.$$

Thus we see that from (12)

$$g^{\gamma}_{0,\beta}(x,t) \leq C_2\psi(x)\int_{-\infty}^{\infty}|y|^{\beta}\exp\left\{-\left(\frac{1}{4(T(m)+\gamma)}-a\right)|y|^2\right\}dy$$

for $t \in [0, T(m)]$. Let

$$C_4 = \max_{\beta=0,1,2} C_2\int_{-\infty}^{\infty}|y|^{\beta}\exp\left\{-\left(\frac{1}{4(T(m)+\gamma)}-a\right)|y|^2\right\}dy.$$

Then we see

$$g^{\gamma}_{0,\beta}(x,t) \leq C_4\psi(x)$$

for $t \in [0, T(m)]$. □

Proof of Proposition 2.2. As before, for $\phi = \phi(T(m) - t + Cg^{\gamma}_{\alpha,0}(x,t))$ we have

$$\begin{aligned} w_t - &\frac{w_{xx}}{1+w_x^2} + \frac{n-1}{w} \\ &= -\phi' + C\phi'\partial_t g_{\alpha,0} - \frac{C\phi'\partial_{xx}g^{\gamma}_{\alpha,0} + (C\partial_x g^{\gamma}_{\alpha,0})^2\phi''}{1+(C\phi'\partial_x g^{\gamma}_{\alpha,0})^2} + \frac{n-1}{\phi} \\ &\leq \frac{C(n-1)\partial_t g^{\gamma}_{\alpha,0}}{\phi} + \frac{C(n-1)|\partial_{xx}g^{\gamma}_{\alpha,0}|}{\phi} + \frac{\{C(n-1)\partial_x g^{\gamma}_{\alpha,0}\}^2}{\phi^3} \end{aligned} \tag{25}$$

by using the fact that $\phi' = (n-1)/\phi$ and $\phi'' = -(n-1)^2/\phi^3$. It is easily seen that

$$\phi^2 = 2(n-1)(T(m) - t + Cg^{\gamma}_{\alpha,0}) \geq 2(n-1)(Cg^{\gamma}_{\alpha,0}). \tag{26}$$

From Lemma 2.3, (16), (19) and (26), it follows that

$$\left|\frac{\partial_x g^\gamma_{\alpha,0}}{\phi^2}\right| \le \frac{g^\gamma_{0,1}}{4(n-1)(t+\gamma)g^\gamma_{0,0}} \le \frac{C_4}{4\gamma(n-1)CC_3}. \tag{27}$$

Substituting (27) for (25), and using (17)-(19), we have

$$\begin{aligned} w_t - & \frac{w_{xx}}{1+w_x^2} + \frac{n-1}{w} \\ & \le \frac{C(n-1)}{2(t+\gamma)^{\alpha+2}\phi}\left[g^\gamma_{0,2} + (t+\gamma)\left\{-2\alpha g^\gamma_{0,0} + g^\gamma_{0,0} + \frac{C_4 g^\gamma_{0,1}}{4C_3}\right\}\right] \\ & \le \frac{C(n-1)\psi}{2(t+\gamma)^{\alpha+2}\phi}\left[-2\alpha\gamma C_3 + C_4\left\{1 + (T(m)+\gamma)\left(1+\frac{C_4}{4C_3}\right)\right\}\right] \end{aligned}$$

in $\mathbf{R} \times [0, T(m)]$. If α satisfies

$$\alpha \ge \alpha_0 \equiv \frac{C_4}{2\gamma C_3}\left\{1 + (T(m)+\gamma)\left(1+\frac{C_4}{4C_3}\right)\right\}, \tag{28}$$

then w is a subsolution of (1) in $\mathbf{R} \times (0, T(m))$. □

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Proof of Theorem 1.3. There exist positive constants $c_1 = c_1(C_2, a, \gamma, \alpha)$ and $c_2 = c_2(C_1, \gamma)$ such that

$$g^\gamma_{\alpha,0}(x,0) \le c_1\psi(x), \qquad g^\gamma_{0,0}(x,0) \ge c_2\psi(x)$$

by Lemma 2.3 and (19), and thus

$$u_0^2(x) \ge m^2 + C_l g^\gamma_{\alpha,0}(x,0) \ \left(\text{or } \le m^2 + C_h g^\gamma_{0,0}(x,0)\right)$$

with $C_l = C_I/c_1$ (or $C_h = C_{II}/c_2$).

Since $m^2 = 2(n-1)T(m)$ by (6), we have

$$\begin{aligned} u_0(x) & \ge \sqrt{2(n-1)T(m) + C_l g^\gamma_{\alpha,0}(x,0)} \ge w(x,0) \\ & \left(\text{or } \le \sqrt{2(n-1)T(m) + C_h g^\gamma_{0,0}(x,0)} \le W(x,0)\right). \end{aligned}$$

Propositions 2.1, 2.2 and the comparison principle yield

$$u(x,t) \ge w(x,t) \ \ (\text{or } \le W(x,t)) \qquad \text{in } \mathbf{R} \times [0, T(m)).$$

We thereby get

$$u(x,T(m)) \ge \sqrt{C_l g^\gamma_{\alpha,0}(x,T(m))} \ \left(\text{or } \le \sqrt{C_h g^\gamma_{0,0}(x,T(m))}\right).$$

By using Lemma 2.3 and letting $C = \sqrt{C_l C_3}$ and $C' = \sqrt{C_h C_4}$, we obtain

$$u(x, T(m)) \geq C\psi^{1/2}(x) \ \big(\text{or } \leq C'\psi^{1/2}(x)\big) .$$

We may choose

$$\gamma = \frac{1}{2a} - 2T(m), \quad \alpha = \alpha_0$$

with α_0 in (28), and then the constant C (or C') depends only on C_1, C_2, a, $T(m)$, C_I (or C_1, C_2, a, $T(m)$, C_{II}). □

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