Bursting Dynamics of the 3D Euler Equations in Cylindrical Domains

Alex Mahalov
Department of Mathematics and Statistics
Arizona State University
Tempe, AZ 85287-1804, USA

Abstract

A class of three-dimensional initial data characterized by uniformly large vorticity is considered for the 3D incompressible Euler equations in bounded cylindrical domains. The fast singular oscillating limits of the 3D Euler equations are investigated for parametrically resonant cylinders. Resonances of fast oscillating swirling Beltrami waves deplete the Euler nonlinearity. These waves are exact solutions of the 3D Euler equations. We construct the 3D resonant Euler systems; the latter are countable uncoupled and coupled $SO(3;\mathbf{C})$ and $SO(3;\mathbf{R})$ rigid body systems. They conserve both energy and helicity. The 3D resonant Euler systems are vested with bursting dynamics, where the ratio of the enstrophy at time $t = t^*$ to the enstrophy at $t = 0$ of some remarkable orbits becomes very large for very small times t^*; similarly for higher norms $\mathbf{H}^s,\ s \geq 2$. These orbits are topologically close to homoclinic cycles. For the time intervals where $\mathbf{H}^s$ norms, $s \geq 7/2$ of the limit resonant orbits do not blow up, we prove that the full 3D Euler equations possess smooth solutions close to the resonant orbits uniformly in strong norms.

References

[1] Arnold, V.I. and Khesin, B.A., Topological Methods in Hydrodynamics, Applied Mathematical Sciences, **125**, Springer, 1997.

[2] Babin, A., Mahalov, A. and Nicolaenko, B., Global regularity of 3D rotating Navier-Stokes equations for resonant domains, *Indiana Univ. Math. J.*, **48** (1999), No. 3, p. 1133-1176.

[3] Babin, A., Mahalov, A. and Nicolaenko, B., 3D Navier-Stokes and Euler equations with initial data characterized by uniformly large vorticity, *Indiana Univ. Math. J.*, **50** (2001), p. 1-35.

[4] F. Golse, A. Mahalov and B. Nicolaenko (2008), Bursting Dynamics of the 3D Euler Equations in Cylindrical Domains, *Instability in Models Connected with Fluid Flows.* International Mathematical Series, Volumes 6 and 7, p. 300-338. Springer.

[5] Kato, T., Nonstationary flows of viscous and ideal fluids in $\mathbf{R^3}$, *J. Func. Anal.*, **9** (1972), p. 296-305.

[6] Mahalov, A., Nicolaenko, B., Bardos, C. and Golse, F., Non blow-up of the 3D Euler equations for a class of three-dimensional initial data in cylindrical domains, *Methods and Applications of Analysis*, **11** (2004), No. 4, p. 605-634.

[7] Moffatt, H.K., The degree of knottedness of tangled vortex lines, *J. Fluid Mech.*, **106** (1969), p. 117-129.

[8] Poincaré, H., Sur la précession des corps déformables, *Bull. Astronomique*, **27** (1910), p. 321-356.

[9] Sobolev, S.L., Ob odnoi novoi zadache matematicheskoi fiziki, *Izvestiia Akademii Nauk SSSR, Ser. Matematicheskaia*, **18** (1954), No. 1, p. 3–50.